

---

## References

- [ABB02] M. J. Alejandro and A. Ballester-Bolinches. On a theorem of Berkovich. *Israel J. Math.*, 131:149–156, 2002.
- [AJBBPR00] P. Arroyo-Jordá, A. Ballester-Bolinches, and M. D. Pérez-Ramos. Fitting sets pairs. *J. Algebra*, 231:574–588, 2000.
- [AJPR01] M. Arroyo-Jordá and M. D. Pérez-Ramos. On the lattice of  $\mathfrak{F}$ -Dnormal subgroups in finite soluble groups. *J. Algebra*, 242:198–212, 2001.
- [AJPR04a] M. Arroyo-Jordá and M. D. Pérez-Ramos. Fitting classes and lattice formations. I. *J. Aust. Math. Soc.*, 76(1):93–108, 2004.
- [AJPR04b] M. Arroyo-Jordá and M. D. Pérez-Ramos. Fitting classes and lattice formations. II. *J. Aust. Math. Soc.*, 76(2):175–188, 2004.
- [And75] W. Anderson. Injectors in finite solvable groups. *J. Algebra*, 36:333–338, 1975.
- [AS85] M. Aschbacher and L. Scott. Maximal subgroups of finite groups. *J. Algebra*, 92:44–80, 1985.
- [AS89] M. Asaad and A. Shaalan. On the supersolvability of finite groups. *Arch. Math. (Basel)*, 53(4):318–326, 1989.
- [Bae57] R. Baer. Classes of finite groups and their properties. *Illinois J. Math.*, 1:115–187, 1957.
- [Bar72] D. W. Barnes. On complemented chief factors of finite soluble groups. *Bull. Austral. Math. Soc.*, 7:101–104, 1972.
- [Bar77] D. Bartels. Subnormality and invariant relations on conjugacy classes in finite groups. *Math. Z.*, 157:13–17, 1977.
- [BB74] J. Beidleman and B. Brewster.  $\mathfrak{F}$ -normalizers in finite  $\pi$ -solvable groups. *Boll. Un. Mat. Ital. (4)*, 10:14–27, 1974.
- [BB76] D. Blessenohl and B. Brewster. Über Formationen und komplementierbare Hauptfaktoren. *Arch. Math.*, 27:347–351, 1976.
- [BB89a] A. Ballester-Bolinches.  $\mathfrak{H}$ -normalizers and local definitions of saturated formations of finite groups. *Israel J. Math.*, 67:312–326, 1989.
- [BB89b] A. Ballester-Bolinches. *Normalizadores y subgrupos de prefrittini en grupos finitos*. PhD thesis, Facultat de Matemàtiques, Universitat de València, 1989.
- [BB91] A. Ballester-Bolinches. Remarks on formations. *Israel J. Math.*, 73(1):97–106, 1991.

- [BB92] A. Ballester-Bolinches. A note on saturated formations. *Arch. Math. (Basel)*, 58(2):110–113, 1992.
- [BB05] A. Ballester-Bolinches.  $\mathfrak{F}$ -critical groups,  $\mathfrak{F}$ -subnormal subgroups and the generalised Wielandt property for residuals. Preprint, 2005.
- [BBCE01] A. Ballester-Bolinches, J. Cossey, and L. M. Ezquerro. On formations of finite groups with the Wielandt property for residuals. *J. Algebra*, 243(2):717–737, 2001.
- [BBCER03] A. Ballester-Bolinches, C. Calvo, and R. Esteban-Romero. A question from the Kourovka Notebook on formation products. *Bull. Austral. Math. Soc.*, 68(3):461–470, 2003.
- [BBCER05] A. Ballester-Bolinches, C. Calvo, and R. Esteban-Romero.  $\mathfrak{X}$ -saturated formations of finite groups. *Comm. Algebra*, 33(4):1053–1064, 2005.
- [BBCER06] A. Ballester-Bolinches, C. Calvo, and R. Esteban-Romero. Products of formations of finite groups. To appear in *J. Algebra*, 2006.
- [BBCS05] A. Ballester-Bolinches, C. Calvo, and L. A. Shemetkov. On partially saturated formations. Preprint, 2005.
- [BBDPR92] A. Ballester-Bolinches, K. Doerk, and M. D. Pérez-Ramos. On the lattice of  $\mathfrak{F}$ -subnormal subgroups. *J. Algebra*, 148(1):42–52, 1992.
- [BBDPR95] A. Ballester-Bolinches, K. Doerk, and M. D. Pérez-Ramos. On  $\mathfrak{F}$ -normal subgroups of finite soluble groups. *J. Algebra*, 171(1):189–203, 1995.
- [BBE91] A. Ballester-Bolinches and L. M. Ezquerro. On maximal subgroups of finite groups. *Comm. Algebra*, 19(8):2373–2394, 1991.
- [BBE95] A. Ballester-Bolinches and L. M. Ezquerro. The Jordan-Hölder theorem and pre-Frattini subgroups of finite groups. *Glasgow Math. J.*, 37:265–277, 1995.
- [BBE98] A. Ballester-Bolinches and L. M. Ezquerro. On a theorem of Bryce and Cossey. *J. Austral. Math. Soc. Ser. A*, 57:455–460, 1998.
- [BBE05] A. Ballester-Bolinches and L. M. Ezquerro. On formations with the Kegel property. *J. Group Theory*, 8(5):605–611, 2005.
- [BBEPA02] A. Ballester-Bolinches, L. M. Ezquerro, and M. C. Pedraza-Aguilera. A characterization of the class of finite groups with nilpotent derived subgroup. *Math. Nachr.*, 239–240:5–10, 2002.
- [BBERR05] A. Ballester-Bolinches, R. Esteban-Romero, and D. J. S. Robinson. On finite minimal non-nilpotent groups. *Proc. Amer. Math. Soc.*, 133(12):3455–3462, 2005.
- [BBERss] A. Ballester-Bolinches and R. Esteban-Romero. On minimal non-supersoluble groups. *Rev. Mat. Iberoamericana*, in press.
- [BBH70] R. M. Bryant, R. A. Bryce, and B. Hartley. The formation generated by a finite group. *Bull. Austral. Math. Soc.*, 2:347–357, 1970.
- [BBMPPR00] A. Ballester-Bolinches, A. Martínez-Pastor, and M. D. Pérez-Ramos. Nilpotent-like Fitting formations of finite soluble groups. *Bull. Austral. Math. Soc.*, 62(3):427–433, 2000.
- [BBPA96] A. Ballester-Bolinches and M. C. Pedraza-Aguilera. On minimal subgroups of finite groups. *Acta Math. Hungar.*, 73(4):335–342, 1996.
- [BBPAMP00] A. Ballester-Bolinches, M. C. Pedraza-Aguilera, and A. Martínez-Pastor. Finite trifactorized groups and formations. *J. Algebra*, 226:990–1000, 2000.

- [BBPAPR96] A. Ballester-Bolinches, M. C. Pedraza-Aguilera, and M. D. Pérez-Ramos. On  $\mathfrak{F}$ -subnormal subgroups and  $\mathfrak{F}$ -residuals of finite groups. *J. Algebra*, 186(1):314–322, 1996.
- [BBPR90] A. Ballester-Bolinches and M. D. Pérez-Ramos. On  $\mathfrak{F}$ -subnormal subgroups. *Suppl. Rend. Circ. Mat. Palermo (2)*, 23:25–28, 1990.
- [BBPR91] A. Ballester-Bolinches and M. D. Pérez-Ramos.  $\mathfrak{F}$ -subnormal closure. *J. Algebra*, 138(1):91–98, 1991.
- [BBPR94a] A. Ballester-Bolinches and M. D. Pérez-Ramos. A note on the  $\mathfrak{F}$ -length of maximal subgroups in finite soluble groups. *Math. Nachr.*, 166:67–70, 1994.
- [BBPR94b] A. Ballester-Bolinches and M. D. Pérez-Ramos. On  $\mathfrak{F}$ -subnormal subgroups and Frattini-like subgroups of a finite group. *Glasgow Math. J.*, 36(2):241–247, 1994.
- [BBPR95] A. Ballester-Bolinches and M. D. Pérez-Ramos. On  $\mathfrak{F}$ -critical groups. *J. Algebra*, 174(3):948–958, 1995.
- [BBPR96a] A. Ballester-Bolinches and M. D. Pérez-Ramos. A question of R. Maier concerning formations. *J. Algebra*, 182(3):738–747, 1996.
- [BBPR96b] A. Ballester-Bolinches and M. D. Pérez-Ramos. Two questions of L. A. Shemetkov on critical groups. *J. Algebra*, 179(3):905–917, 1996.
- [BBPR98] A. Ballester-Bolinches and M. D. Pérez-Ramos. Some questions of the Kourovka notebook concerning formation products. *Comm. Algebra*, 26(5):1581–1587, 1998.
- [BBS97] A. Ballester-Bolinches and L. A. Shemetkov. On lattices of  $p$ -local formations of finite groups. *Math. Nachr.*, 186:57–65, 1997.
- [BC72] R. A. Bryce and J. Cossey. Fitting formations of finite soluble groups. *Math. Z.*, 127:217–223, 1972.
- [BC75] R. A. Bryce and John Cossey. A problem in the theory of normal Fitting classes. *Math. Z.*, 141:99–110, 1975.
- [BC78] T. R. Berger and J. Cossey. More Fitting formations. *J. Algebra*, 51:573–578, 1978.
- [BC82] R. A. Bryce and J. Cossey. Subgroup-closed Fitting classes are formations. *Math. Proc. Cambridge Philos. Soc.*, 91:225–258, 1982.
- [BCMV84] A. Bolado-Caballero and J. R. Martínez-Verdúch. The Fitting class  $\mathfrak{F}\mathfrak{S}$ . *Arch. Math. (Basel)*, 42:307–310, 1984.
- [Bec64] H. Bechtell. Pseudo-Frattini subgroups. *Pacific J. Math.*, 14:1129–1136, 1964.
- [Ben70] H. Bender. On groups with abelian sylow 2-subgroups. *Math. Z.*, 117:164–176, 1970.
- [Ber99] Y. Berkovich. Some corollaries to Frobenius’ normal  $p$ -complement theorem. *Proc. Amer. Math. Soc.*, 127:2505–2509, 1999.
- [BG70] D. Blessenohl and W. Gaschütz. Über normale Schunck- und Fittingklassen. *Math. Z.*, 118:1–8, 1970.
- [BH03] J. C. Beidleman and H. Heineken. On the Fitting core of a formation. *Bull. Austral. Math. Soc.*, 68(1):107–112, 2003.
- [Bir69] G. Birkhoff. *Lattice Theory*, volume 25 of *Amer. Math. Soc. Colloquium Pub.* Amer. Math. Soc. Providence, RI, USA, 1969.
- [BK66] D. W. Barnes and O. H. Kegel. Gaschütz functors on finite soluble groups. *Math. Z.*, 94:134–142, 1966.

- [BL79]     D. Blessenohl and H. Laue. Fittingklassen endlicher Gruppen, in denen gewisse Hauptfaktoren einfach sind. *J. Algebra*, 56:516–532, 1979.
- [Bra88]     A. Brandis. Moduln und verschränkte Homomorphismen endlicher Gruppen. *J. Reine Angew. Math.*, 385:102–116, 1988.
- [Cam81]     P. J. Cameron. Finite permutation groups and finite simple groups. *Bull. London Math. Soc.*, 13:1–22, 1981.
- [Car61]     R. Carter. Nilpotent self-normalizing subgroups of soluble groups. *Math. Z.*, 75:136–139, 1960/1961.
- [CCN<sup>+</sup>85]     J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker, and R. A. Wilson. *Atlas of Finite Groups*. Oxford Univ. Press, London, 1985.
- [CFH68]     R. W. Carter, B. Fischer, and T. O. Hawkes. Extreme classes of finite soluble groups. *J. Algebra*, 9:285–313, 1968.
- [CH67]     R. Carter and T. Hawkes. The  $\mathcal{F}$ -normalizers of a finite soluble group. *J. Algebra*, 5:175–202, 1967.
- [Cha72]     G. A. Chambers. On  $f$ -prefrattini subgroups. *Canad. Math. Bull.*, 15(3):345–348, 1972.
- [CK87]     J. Cossey and C. Kanes. A construction for Fitting formations. *J. Algebra*, 107:117–133, 1987.
- [CM98]     A. Carocca and M. Maier. Hypercentral embedding and pronormality. *Arch. Math. (Basel)*, 71:433–436, 1998.
- [CO87]     J. Cossey and E. A. Ormerod. A construction for Fitting-Schunck classes. *J. Austral. Math. Soc. Ser. A*, 43:91–94, 1987.
- [COM71]     J. Cossey and S. Oates-MacDonald. On the definition of saturated formations of groups. *Bull. Austral. Math. Soc.*, 4:9–15, 1971.
- [Cos89]     J. Cossey. A construction for Fitting formations II. *J. Austral. Math. Soc. Ser. A*, 47:95–102, 1989.
- [Dar72]     R. S. Dark. Some examples in the theory of injectors of finite soluble groups. *Math. Z.*, 127:145–156, 1972.
- [Des59]     W. E. Deskins. On maximal subgroups. *Proc. Symp. in Pure Math. Amer. Math. Soc.*, 1:100–104, 1959.
- [DF]     R. Dark and A. Feldman. A characterization of injectors in finite groups. Private communication.
- [DH78]     K. Doerk and T. O. Hawkes. On the residual of a direct product. *Arch. Math.*, 30:458–468, 1978.
- [DH92]     K. Doerk and T. Hawkes. *Finite Soluble Groups*. Number 4 in De Gruyter Expositions in Mathematics. Walter de Gruyter, Berlin, New York, 1992.
- [Doe66]     K. Doerk. Minimal nicht überauflösbare, endliche Gruppen. *Math. Z.*, 91:198–205, 1966.
- [Doe71]     K. Doerk. *Über Homomorphe und Formationen endlicher auflösbarer Gruppen*. Habilitationsschrift, Johannes Gutenberg-Universität Mainz, Mainz, 1971.
- [Doe73]     K. Doerk. Die maximale lokale Erklärung einer gesättigten Formation. *Math. Z.*, 133:133–135, 1973.
- [Doe74]     K. Doerk. Über Homomorphe endlicher auflösbarer Gruppen. *J. Algebra*, 30:12–30, 1974.
- [Eri82]     R. P. Erickson. Projectors of finite groups. *Comm. Alg.*, 10:1919–1938, 1982.

- [ESE] L. M. Ezquerro and X. Soler-Escrivà. On certain distributive lattices of subgroups of finite soluble groups. To appear in *Acta Math. Sinica*.
- [ESE05] L. M. Ezquerro and X. Soler-Escrivà. Some new permutability properties of hypercentrally embedded groups. *J. Austral. Math. Soc. Ser. A*, 79(2):243–255, 2005.
- [Ezq86] L. M. Ezquerro. On generalized covering subgroups and normalizers of finite soluble groups. *Arch. Math.*, 47:385–394, 1986.
- [FGH67] B. Fischer, W. Gaschütz, and B. Hartley. Injektoren endlichen auflösbarer Gruppen. *Math. Z.*, 102:337–339, 1967.
- [Fis66] B. Fischer. *Klassen konjugierter und Untergruppen in endlichen auflösbarer Gruppen*. Habilitationsschrift, Universität Frankfurt am Mainz, Frankfurt, 1966.
- [För78] P. Förster. Charakterisierungen einiger Schunckklassen endlicher auflösbarer Gruppen. *J. Algebra*, 55:155–187, 1978.
- [För79] P. Förster. Closure operations for Schunck classes and formations of finite solvable groups. *Math. Proc. Cambridge Philos. Soc.*, 85(2):253–259, 1979.
- [För82] P. Förster. Homomorphs and wreath product extensions. *Math. Proc. Cambridge Philos. Soc.*, 92(1):93–99, 1982.
- [För83] P. Förster. Praefrattini subgroups. *J. Austral. Math. Soc. (Ser. A)*, 34:234–247, 1983.
- [För84a] P. Förster. A note on primitive groups with small maximal subgroups. *Publ. Sec. Mat. Univ. Autònoma Barcelona*, 28(2-3):19–27, 1984.
- [För84b] P. Förster. Projektive Klassen endlicher Gruppen. I: Schunck- und Gaschützklassen. *Math. Z.*, 186:149–178, 1984.
- [För85a] P. Förster. Nilpotent injectors in finite groups. *Bull. Austral. Math. Soc.*, 32:293–298, 1985.
- [För85b] P. Förster. Projektive Klassen endlicher Gruppen. IIa. Gesättigte Formationen: ein allgemeiner Satz von Gaschütz-Lubeseder-Baer-Typ. *Publ. Sec. Mat. Univ. Autònoma Barcelona*, 29(2-3):39–76, 1985.
- [För85c] P. Förster. Projektive Klassen endlicher Gruppen. IIb. Gesättigte Formationen: Projektoren. *Arch. Math. (Basel)*, 44(3):193–209, 1985.
- [För87] P. Förster. Maximal quasinilpotent subgroups and injectors for Fitting classes in finite groups. *Southeast Asian Bull. Math.*, 11:1–11, 1987.
- [För88] P. Förster. Chief factors, crowns, and the generalised Jordan-Hölder theorem. *Comm. Algebra*, 16(8):1627–1638, 1988.
- [För89] P. Förster. An elementary proof of Lubeseder’s theorem. *Arch. Math. (Basel)*, 52(5):417–419, 1989.
- [Föra] P. Förster. Projectors of Soluble type in Finite Groups. Preprint.
- [Förb] P. Förster. Salomon Subgroups in Finite Groups. Preprint.
- [FS85] P. Förster and E. Salomon. Local definitions of local homomorphs and formations of finite groups. *Bull. Austral. Math. Soc.*, 31(1):5–34, 1985.
- [FT63] W. Feit and J. G. Thompson. Solvability of groups of odd order. *Pacific J. Math.*, 13:775–1029, 1963.
- [Gaj79] D. Gajendragadkar. A characteristic class of characters of finite  $\pi$ -separable groups. *J. Algebra*, 59:237–259, 1979.
- [Gas62] W. Gaschütz. Praefrattinigruppen. *Arch. Math.*, 13:418–426, 1962.
- [Gas63] W. Gaschütz. Zur Theorie der endlichen auflösbaren Gruppen. *Math. Z.*, 80:300–305, 1963.

- [Gas69] W. Gaschütz. Selected topics in the theory of soluble groups. Canberra, 1969. Lectures given at the 9th Summer Research Institute of the Australian Math. Soc. Notes by J. Looker.
- [GK84] F. Gross and L. G. Kovács. On normal subgroups which are direct products. *J. Algebra*, 90:133–168, 1984.
- [GL63] W. Gaschütz and U. Lubeseder. Kennzeichnung gesättigter Formationen. *Math. Z.*, 82:198–199, 1963.
- [Gla66] G. Glauberman. On the automorphism groups of a finite group having no non-identity normal subgroups of odd order. *Math. Z.*, 93:154–160, 1966.
- [Gor80] D. Gorenstein. *Finite Groups*. Chelsea Pub. Co., New York, 1980.
- [GS78] R. L. Griess and P. Schmid. The Frattini module. *Arch. Math.*, 30:256–266, 1978.
- [Hal28] P. Hall. A note of soluble groups. *J. London Math. Soc.*, 3:98–105, 1928.
- [Hal37] P. Hall. On the system normalizers of a soluble group. *Proc. London Math. Soc.*, 43:507–525, 1937.
- [Hal59] P. Hall. On the finiteness of certain soluble groups. *Proc. London Math. Soc. (3)*, 9:595–622, 1959.
- [Hal63] P. Hall. On non-strictly simple groups. *Proc. Cambridge Philos. Soc.*, 59:531–553, 1963.
- [Har72] M. E. Harris. On normal subgroups of  $p$ -solvable groups. *Math. Z.*, 129:55, 1972.
- [Haw67] T. Hawkes. Analogues of Prefrattini subgroups. In *Proc. Internat. Conf. Theory of Groups (Canberra, 1965)*, pages 145–150. Gordon and Breach, New York, 1967.
- [Haw69] T. Hawkes. On formation subgroups of a finite soluble group. *J. London Math. Soc.*, 44:243–250, 1969.
- [Haw70] T. O. Hawkes. On Fitting formations. *Math. Z.*, 117:177–182, 1970.
- [Haw73] T. Hawkes. Closure operations for Schunck classes. *J. Austral. Math. Soc. Ser. A*, 16:316–318, 1973.
- [Haw75] T. O. Hawkes. Two applications of twisted wreath products to finite soluble groups. *Trans. Amer. Math. Soc.*, 214:325–335, 1975.
- [Haw98] I. Hawthorn. The existence and uniqueness of injectors for Fitting sets of solvable groups. *Proc. Amer. Math. Soc.*, 126:2229–2230, 1998.
- [HB82a] B. Huppert and N. Blackburn. *Finite Groups II*, volume 242 of *Grundlehren der Mathematischen Wissenschaften*. Springer-Verlag, Berlin-Heidelberg-New York, 1982.
- [HB82b] B. Huppert and N. Blackburn. *Finite groups. III*, volume 243 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 1982.
- [Hei94] H. Heineken. Fitting classes of certain metanilpotent groups. *Glasgow Math. J.*, 36(2):185–195, 1994.
- [Hei97] H. Heineken. More metanilpotent Fitting classes with bounded chief factor ranks. *Rend. Sem. Mat. Univ. Padova*, 98:241–251, 1997.
- [HH84] K. L. Haberl and H. Heineken. Fitting classes defined by chief factor ranks. *J. London Math. Soc.*, 29:34–40, 1984.
- [Höl89] O. Hölder. Zurückführung einer beliebigen algebraischen Gleichung auf eine Kette von Gleichungen. *Math. Ann.*, pages 26–56, 1889.

- [Hun80] T. W. Hungerford. *Algebra*, volume 73 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1980. Reprint of the 1974 original.
- [Hup67] B. Huppert. *Endliche Gruppen I*. Springer-Verlag, Berlin, Heidelberg, New-York, 1967.
- [ILPM03] M. J. Iranzo, Julio P. Lafuente, and F. Pérez-Monazor. Preboundaries of perfect groups. *J. Group Theory*, 6(1):57–68, 2003.
- [ILPM04] M. J. Iranzo, J. P. Lafuente, and F. Pérez-Monazor. Preboundaries of perfect groups II. *J. Group Theory*, 7:113–125, 2004.
- [IMPM01] M. J. Iranzo, J. Medina, and F. Pérez-Monazor. On  $p$ -decomposable groups. *Siberian Math. J.*, 42:59–63, 2001.
- [IPM86] M. J. Iranzo and F. Pérez-Monazor. Fitting classes  $\mathfrak{F}$  such that all finite groups have  $\mathfrak{F}$ -injectors. *Arch. Math. (Basel)*, 46:205–210, 1986.
- [IPM88] M. J. Iranzo and F. Pérez-Monazor. Existencia de inyectoros en grupos finitos respecto de ciertas clases de Fitting. *Publ. Mat. Univ. Autònoma Barcelona*, 32:57–59, 1988.
- [IPMT90] M. J. Iranzo, F. Pérez-Monazor, and M. Torres. A criterion for the existence of injectors in finite groups. *Suppl. Rend. Circ. Mat. Palermo (2)*, 23:193–196, 1990.
- [Isa84] I. M. Isaacs. Characters of  $\pi$ -separable groups. *J. Algebra*, 86:98–128, 1984.
- [IT89] M. J. Iranzo and M. Torres. The  $p^*p$ -injectors of a finite group. *Rend. Sem. Mat. Uni. Padova*, 82:233–237, 1989.
- [Jor70] C. Jordan. *Traité des substitutions et des équations algébriques*. Gauthier-Villars, Paris, 1870.
- [JS96] P. Jiménez Seral. El teorema de O’Nan-Scott. Ph. D. course, 1996–97, Universidad de Zaragoza, 1996.
- [Kam92] S. F. Kamornikov. On a problem of Kegel. *Mat. Zametki*, 51(5):51–56, 157, 1992.
- [Kam93] S. F. Kamornikov. On some properties of the formation of quasinilpotent groups. *Mat. Zametki*, 53(2):71–77, 1993.
- [Kam94] S. F. Kamornikov. On two problems of L. A. Shemetkov. *Sibirsk. Mat. Zh.*, 35(4):801–812, ii, 1994. Russian. Translation in *Siberian Math. J.*, **35**, no. 4 (1994), pages 713–721.
- [Kam96] S. F. Kamornikov. Permutability of subgroups and  $\mathfrak{F}$ -subnormality. *Siberian Math. J.*, 37(5):936–949, 1996.
- [Kat77] U. Kattwinkel. Die größte untergruppenabgeschlossene Teilklasse einer Schunckklasse endlicher auflösbarer Gruppen. *Arch. Math.*, 29:337–343, 1977.
- [Keg65] O. H. Kegel. Zur Struktur mehrfach faktorisierter endlicher Gruppen. *Math. Z.*, 87:42–48, 1965.
- [Keg78] O. H. Kegel. Untergruppenverbände endlicher Gruppen, die den Subnormalteilerverband echt enthalten. *Arch. Math. (Basel)*, 30(3):225–228, 1978.
- [KL90] P. B. Kleidman and M. W. Liebeck. *The subgroup structure of the finite classical groups*, volume 129 of *London Math. Soc. Lecture Notes Series*. Cambridge Univ. Press, Cambridge, UK, 1990.
- [Kli77] A. A. Klimowicz.  $\mathfrak{X}$ -prefrattini subgroups of  $\pi$ -soluble groups. *Arch. Math. (Basel)*, 28:572–576, 1977.
- [Kov86] L. G. Kovács. Maximal subgroups in composite finite groups. *J. Algebra*, 99(1):114–131, 1986.

- [Kov88] L. G. Kovács. Primitive permutation groups of simple diagonal type. *Israel J. Math.*, 63:119–127, 1988.
- [Kov89] L. G. Kovács. Primitive subgroups of wreath products in product action. *Proc. London Math. Soc. (3)*, 58:306–332, 1989.
- [KS95] S. F. Kamornikov and L. A. Shemetkov. On coradicals of subnormal subgroups. *Algebra i Logika*, 34(5):493–513, 608, 1995.
- [KS03] S. F. Kamornikov and M. V. Sel'kin. *Subgroup functors and classes of finite groups*. Belaruskaya Nauka, Minsk, 2003.
- [KS04] H. Kurzweil and B. Stellmacher. *The theory of finite groups. An introduction*. Springer-Verlag, Berlin-Heidelberg-New York, 2004.
- [Kur89] H. Kurzweil. Die Praefrattinigruppe im Intervall eines Untergruppenverbandes. *Arch. Math. (Basel)*, 53(3):235–244, 1989.
- [Laf78] J. Lafuente. Homomorphs and formations of a given derived class. *Math. Proc. Cambridge Philos. Soc.*, 84:437–441, 1978.
- [Laf84a] J. Lafuente. Nonabelian crowns and Schunck classes of finite groups. *Arch. Math. (Basel)*, 42(1):32–39, 1984.
- [Laf84b] J. Lafuente. On restricted twisted wreath products of groups. *Arch. Math. (Basel)*, 43(3):208–209, 1984.
- [Laf89] J. Lafuente. Maximal subgroups and the Jordan-Hölder theorem. *J. Austral. Math. Soc. Ser. A*, 46(3):356–364, 1989.
- [Lau73] H. Lausch. On normal Fitting classes. *Math. Z.*, 130:67–72, 1973.
- [Loc71] P. Lockett. *On the theory of Fitting classes of finite soluble groups*. PhD thesis, University of Warwick, 1971.
- [LPS88] M. W. Liebeck, C. E. Praeger, and J. Saxl. On the O’Nan-Scott theorem for finite primitive permutation groups. *J. Austral. Math. Soc. (Ser. A)*, 44:389–396, 1988.
- [LS87] J. C. Lennox and S. E. Stonehewer. *Subnormal Subgroups of Groups*. Clarendon Press, Oxford, 1987.
- [LS91] M. W. Liebeck and J. Saxl. On point stabilizers in primitive permutation groups. *Comm. Algebra*, 19(10):2777–2789, 1991.
- [Lub63] U. Lubeseder. *Formationsbildungen in endlichen auflösbaren Gruppen*. Dissertation, Universität Kiel, Kiel, 1963.
- [Mak70] A. Makan. Another characteristic conjugacy class of subgroups of finite soluble groups. *J. Austral. Math. Soc. Ser. A*, 11:395–400, 1970.
- [Mak73] A. Makan. On certain sublattices of the lattice of subgroups generated by the prefrattini subgroups, the injectors and the formation subgroups. *Canad. J. Math. Soc.*, 25:862–869, 1973.
- [Man70] A. Mann.  $\mathfrak{F}$ -normalizers of a finite soluble group. *J. Algebra*, 14:312–325, 1970.
- [Man71] A. Mann. Injectors and normal subgroups of finite groups. *Israel J. Math.*, 56:554–558, 1971.
- [Men94] M. Menth. Examples of supersoluble Lockett sections. *Bull. Austral. Math. Soc.*, 94:325–332, 1994.
- [Men95a] M. Menth. Closure properties of supersoluble Fitting classes. In *Groups '93 Galway/St. Andrews, Vol. 2*, volume 212 of *London Math. Soc. Lecture Note Ser.*, pages 418–425. Cambridge Univ. Press, Cambridge, 1995.
- [Men95b] M. Menth. A family of Fitting classes of supersoluble groups. *Math. Proc. Cambridge Philos. Soc.*, 118(1):49–57, 1995.



- [Men96] M. Menth. A note on Hall closure of metanilpotent Fitting classes. *Bull. Austral. Math. Soc.*, 53(2):209–212, 1996.
- [MK84] V. D. Mazurov and E. I. Khukhro, editors. *Unsolved problems in Group Theory: The Kourovka Notebook*. Institute of Mathematics, Sov. Akad., Nauk SSSR, Siberian Branch, Novosibirsk, SSSR, 9 edition, 1984.
- [MK90] V. D. Mazurov and E. I. Khukhro, editors. *Unsolved problems in Group Theory: The Kourovka Notebook*. Institute of Mathematics, Sov. Akad., Nauk SSSR, Siberian Branch, Novosibirsk, SSSR, 11 edition, 1990.
- [MK92] V. D. Mazurov and E. I. Khukhro, editors. *Unsolved problems in Group Theory: The Kourovka Notebook*. Institute of Mathematics, Sov. Akad., Nauk SSSR, Siberian Branch, Novosibirsk, SSSR, 12 edition, 1992.
- [MK99] V. D. Mazurov and E. I. Khukhro, editors. *Unsolved problems in Group Theory: The Kourovka Notebook*. Institute of Mathematics, Sov. Akad., Nauk SSSR, Siberian Branch, Novosibirsk, SSSR, 14 edition, 1999.
- [MP92] A. Martínez-Pastor. *Classes inyectivas de grupos finitos*. PhD thesis, Facultat de Matemàtiques, Universitat de València, València, 1992.
- [Pen87] J. Pense. *Äußere Fittingpaare*. Dissertation, Johannes Gutenberg-Universität Mainz, Mainz, 1987.
- [Pen88] J. Pense. Outer Fitting pairs. *J. Algebra*, 119(1):34–50, 1988.
- [Pen90a] J. Pense. Allgemeines über äußere Fittingpaare. *J. Austral. Math. Soc. Ser. A*, 49(2):241–249, 1990.
- [Pen90b] J. Pense. Fittingmengen und Locketabschnitte. *J. Algebra*, 133(1):168–181, 1990.
- [Pen90c] J. Pense. Notiz über Injektoren. *Arch. Math. (Basel)*, 54(5):422–426, 1990.
- [Pen92] J. Pense. Ränder und Erzeugendensysteme von Fittingklassen. *Math. Nachr.*, 156:117–127, 1992.
- [Plo58] B. I. Plotkin. Generalized soluble and nilpotent groups. *Uspehi Mat. Nauk.*, 13:89–172, 1958. Translation in *Amer. Math. Soc. Translations* (2), 17, 29–115 (1961).
- [Rob02] D. J. S. Robinson. Minimality and Sylow-permutability in locally finite groups. *Ukr. Math. J.*, 54(6):1038–1049, 2002.
- [Sal] E. Salomon. A non-injective Fitting class. Private communication.
- [Sal85] E. Salomon. Über lokale und Baerlokale Formationen endlicher Gruppen. Master's thesis, Johannes Gutenberg-Universität, Mainz, 1983.
- [Sal87] E. Salomon. *Strukturerhaltende untergruppen, Schunkklassen und extreme klassen endlicher Gruppen*. Dissertation, Johannes Gutenberg-Universität, Mainz, 1987.
- [Sch66] H. Schunck. *Zur Konstruktion von Systemen konjugierter Untergruppen in endlichen auflösbaren Gruppen*. PhD thesis, Christian-Albrechts-Universität zu Kiel, 1966.
- [Sch67] H. Schunck.  $\mathfrak{S}$ -Untergruppen in endlichen auflösbaren Gruppen. *Math. Z.*, 97:326–330, 1967.
- [Sch74] P. Schmid. Lokale Formationen endlicher Gruppen. *Math. Z.*, 137:31–48, 1974.

- [Sch77] K.-U. Schaller. Über die maximale Formation in einem gesättigten Homomorph. *J. Algebra*, 45(453–464), 1977.
- [Sch78] P. Schmid. Every saturated formation is a local formation. *J. Algebra*, 51:144–148, 1978.
- [Sco80] L. Scott. Representations in characteristic  $p$ . In *Proc. Symp. Pure Math. The Santa Cruz Conf. on finite groups*, volume 37, page 327. AMS, 1980.
- [SE02] X. Soler-Escrivà. *On certain lattices of subgroups of finite groups. Factorizations*. PhD thesis, Nafarroako Unibertsitate publikoa-Universidad Pública de Navarra, 2002.
- [Sem92] V. N. Semenchuk. A characterization of  $\check{s}$ -formations. *Problems in Algebra*, 7:103–107, 1992. Russian.
- [She72] L. A. Shemetkov. Formation properties of finite groups. *Dokl. Akad. Nauk. SSSR*, 204(6):851–855, 1972.
- [She74a] L. A. Shemetkov. The complementability of the  $F$ -coradical and the properties of the  $F$ -hypercenter of a finite group. *Dokl. Akad. Nauk BSSR*, 18:204–206, 282, 1974.
- [She74b] L. A. Shemetkov. Graduated formations of groups. *Mat. Sb. (N.S.)*, 94(136):628–648, 656, 1974.
- [She75] L. A. Shemetkov. Two trends in the development of the theory of nonsimple finite groups. *Uspehi Mat. Nauk*, 30(2(182)):179–198, 1975.
- [She76] L. A. Shemetkov. Factorization of nonsimple finite groups. *Algebra i Logika*, 15(6):684–715, 744, 1976.
- [She78] L. A. Shemetkov. *Formations of finite groups*. Nauka, Moscow, 1978. Russian.
- [She84] L. A. Shemetkov. The product of formations. *Dokl. Akad. Nauk BSSR*, 28(2):101–103, 1984.
- [She92] L. A. Shemetkov. Some ideas and results in the theory of formations of finite groups. *Problems in Algebra*, 7:3–38, 1992.
- [She97] L. A. Shemetkov. Frattini extensions of finite groups and formations. *Comm. Algebra*, 25(3):955–964, 1997.
- [She00] L. A. Shemetkov. Radical and residual classes of finite groups. In *Proceedings of the International Algebraic Conference on the Occasion of the 90th. Birthday of A. G. Kurosh, Moscow, Russia, May 25–30, 1998*, pages 331–344, Berlin-New York, 2000. Yuri Bahturin, Walter de Gruyter.
- [She01] L. A. Shemetkov. On partially saturated formations and residuals of finite groups. *Comm. Algebra*, 29(9):4125–4137, 2001. Special issue dedicated to Alexei Ivanovich Kostrikin.
- [Ski90] A. N. Skiba. On a class of formations of finite groups. *Dokl. Akad. Nauk Belarus*, 34(11):982–985, 1990. Russian.
- [Ski97] A. N. Skiba. *Algebra formatsii*. Izdatel'stvo Belaruskaya Navuka, Minsk, 1997.
- [Ski99] A. N. Skiba. On factorizations of compositional formations. *Mat. Zametki*, 65(3):389–395, 1999.
- [SR97] A. N. Skiba and V. N. Ryzhik. Factorizations of  $p$ -local formations. In *Problems in algebra, No. 11 (Russian)*, pages 76–89. Gomel. Gos. Univ., Gomel', 1997.
- [SS89] L. A. Shemetkov and A. N. Skiba. On inherently non-decomposable formations. *Dokl. Akad. Nauk BSSR*, 37(7):581–583, 1989.

- [SS95] A. N. Skiba and L. A. Shemetkov. On partially local formations. *Dokl. Akad. Nauk Belarusi*, 39(3):9–11, 123, 1995.
- [SS99] A. N. Skiba and L. A. Shemetkov. Partially compositional formations of finite groups. *Dokl. Nats. Akad. Nauk Belarusi*, 43(4):5–8, 123, 1999.
- [SS00a] L. A. Shemetkov and A. N. Skiba. Multiply  $\omega$ -local formations and Fitting classes of finite groups [translation of *Mat. Tr.* **2** (1999), no. 2, 114–147;]. *Siberian Adv. Math.*, 10(2):112–141, 2000.
- [SS00b] A. N. Skiba and L. A. Shemetkov. Multiply  $\mathfrak{L}$ -composition formations of finite groups. *Ukr. Math. J.*, 52(6):898–913, 2000.
- [Suz82] M. Suzuki. *Group theory I*, volume 247 of *Grundlehren der Mathematischen Wissenschaften*. Springer-Verlag, Berlin-Heidelberg-New York, 1982.
- [Suz86] M. Suzuki. *Group theory. II*, volume 248 of *Grundlehren der Mathematischen Wissenschaften (Fundamental Principles of Mathematical Sciences)*. Springer-Verlag, New York, 1986.
- [SV84] V. N. Semenchuk and A. F. Vasil'ev. Characterization of local formations  $\mathfrak{F}$  by given properties of minimal non- $\mathfrak{F}$ -groups. In V. I. Sergienko, editor, *Investigation of the normal and subgroup structure of finite groups. Proceedings of the Gomel's seminar, Minsk 1984*, volume 224, pages 175–181, Minsk, 1984. Nauka i Tekhnika. Russian.
- [SW70] G. M. Seitz and C. R. B. Wright. On complements of  $\mathfrak{F}$ -residuals in finite solvable groups. *Arch. Math. (Basel)*, 21:139–150, 1970.
- [Tom75] M. J. Tomkinson. Prefrattini subgroups and cover-avoidance properties in  $\mathfrak{U}$ -groups. *Canadian J. Math.*, 27:837–851, 1975.
- [Tra98] G. Traustason. A note of supersoluble Fitting classes. *Arch. Math. (Basel)*, 70:1–8, 1998.
- [Vas87] A. F. Vasil'ev. On the problem of the enumeration of local formations with a given property. *Problems in algebra, No. 3*, 126:3–11, 1987. Russian.
- [Vas92] A. F. Vasil'ev. On the enumeration of local formations with the Kegel condition. *Problems in algebra, No. 7*, pages 86–93, 1992. Russian.
- [Ved88] V. A. Vedernikov. On some classes of finite groups. *Dokl. Akad. Nauk BSSR*, 32(10):872–875, 1988.
- [VK01] A. F. Vasil'ev and S. F. Kamornikov. On the functor method for studying lattices of subgroups of finite groups. *Sibirsk. Mat. Zh.*, 42(1):30–40, i, 2001.
- [VK02] A. F. Vasil'ev and S. F. Kamornikov. On the Kegel-Shemetkov problem on lattices of generalized subnormal subgroups of finite groups. *Algebra Logika*, 41(4):411–428, 510, 2002.
- [VKS93] A. F. Vasil'ev, S. F. Kamornikov, and V. N. Semenchuk. On lattices of subgroups of finite groups. In N. S. Chernikov, editor, *Infinite groups and related algebraic structures*, pages 27–54, Kiev, 1993. Institut Matematiki AN Ukrainy. Russian.
- [Vor93] N. T. Vorob'ev. On factorizations of nonlocal formations of finite groups. *Problems in Algebra, No. 6*, pages 21–24, 1993.
- [Wie39] H. Wielandt. Eine Verallgemeinerung der invarianten Untergruppen. *Math. Z.*, 45:209–244, 1939.
- [Wie57] H. Wielandt. Vertauschbare nachinvariante Untergruppen. *Abh. Math. Sem. Univ. Hamburg*, 21(1-2):55–62, 1957.

- [Wie58] H. Wielandt. Über die Existenz von Normalteilern in endlichen Gruppen. *Math. Nachr.*, 18:274–280, 1958.
- [Wie74] H. Wielandt. Kriterien für Subnormalität in endlichen Gruppen. *Math. Z.*, 138:199–203, 1974.
- [Wie94a] H. Wielandt. *Mathematische Werke-Mathematical Works*, volume 1: Group Theory. Walter de Gruyter, 1994. Edited by B. Huppert and H. Schneider.
- [Wie94b] H. Wielandt. Subnormale Untergruppen endlicher Gruppen. In *Mathematical Works* [Wie94a], pages 413–479. Vorlesung Univ. Tübingen, 1971.
- [Yen70] T. Yen. On  $\mathfrak{F}$ -normalizers. *Proc. Amer. Math. Soc.*, 26:49–56, 1970.

---

## List of symbols

$(G_1, \dots, G_n)$	87	$H^x$	$x^{-1}Hx$ , conjugate of $H$ by $x$
$(\mathcal{S})$	87	$H_G$	2
$(d, \mathbf{A})$	344	$K_G$	120
$A/B \ll C/D, C/D \gg A/B$	53	$R_{\mathbf{X}}$	64
$A \setminus B$	set/class difference of $A$ and $B$	$S^n$	8
$A \times B$	direct product of the (sub)groups $A$ and $B$	$UK\text{-}\mathfrak{F}\text{-sn } G$	236
$A^\varphi, A\phi, \phi(A)$	image of $A$ by $\phi$	$V \oplus W$	direct sum of the modules $V$ and $W$
$A^{(n)}$	$n$ th term of the derived series of $A$	$V \otimes W$	a tensor space over $K$ of the $K$ -vector spaces $V$ and $W$
$C^*/R_{\mathbf{X}}$	64	$V \otimes_A W$	a tensor product of the right $A$ -module $V$ and the left $A$ -module $W$
$C_m$	cyclic group of order $m$	$V_A$	the $G$ -module $V$ restricted to $A$
$E\lambda$	12	$V_{j_1} \otimes \dots \otimes V_{j_n}$	120
$E\lambda_E K$	210	$W \otimes K$	120
$E\lambda_T H$	230	$W^G$	induced module of the $A$ -module $W$ from $A$ up to $G$
$Eg$	11	$W^K$	120
$F$	canonical definition of the local formation $\mathfrak{F}$	$X =_U Y$	239
$F(p)$	131	$X =_{\cdot_G} Y$	239
$F_1 \cong_G F_2$	42	$X \sigma Y$	239
$G'$	derived subgroup of the group $G$	$X \sigma^\infty Y$	239
$G \cong H$	$G$ is isomorphic to $H$	$X \wr H$	8
$G^{\mathfrak{F}}$	95	$X \wr_\varphi H$	8
$G_{\mathfrak{F}}$	111	$X^{\mathfrak{h}}$	8
$G_{\Phi, p}$	5	$X_i : i \in \mathcal{I}_f$	207
$G_{\mathfrak{X}}(p)$	154	$[A/B \gg C/D]$	54
$G_{\mathcal{F}}$	114	$[H, K]$	commutator subgroup of $H$ and $K$
$G_{\omega d}$	163	$[H/K] * G$	45
$H \mathfrak{F}\text{-sn } G$	236	$[N]H$	semidirect product of the $H$ -group $N$ with $H$
$H \leq G$	$H$ is a subgroup of $G$		
$H \not\leq G$	$H$ is not a subgroup of $G$		
$H \trianglelefteq G$	$H$ is a normal subgroup of $G$		
$H^S$	352		

- $[N]_\alpha H$  semidirect product of the  $H$ -group  $N$  with  $H$  via the action  $\alpha$   
 $[a, b]$  commutator of  $a$  and  $b$ ,  $a^{-1}b^{-1}ab$   
 $\text{AGL}(n, p)$  5  
 $\text{A}_J(n_G)$  348  
 $\text{A}_p(G)$  5  
 $\mathfrak{A}$  87  
 $\text{Alt}(n)$  3  
 $\text{Aut}(G)$  group of automorphisms of the group  $G$   
 $\text{Aut}_G(H/K)$  41  
 $\mathfrak{B}_{\mathfrak{F}}$  281  
 $\text{BLF}(f)$  97  
 $\mathcal{B}(\mathfrak{F})$  288  
 $\text{C}^{\mathfrak{X}_p}(G)$  128  
 $\text{char } \mathfrak{X}$  88  
 $\text{Core}_G(H)$  2  
 $\text{Cosoc}(G)$  98  
 $\text{Cov}_{\mathfrak{S}^d(G)}$  226  
 $\text{Cov}_{\mathfrak{S}}(G)$  101  
 $\text{Crit}_{\mathfrak{S}}(\mathfrak{S})$  267  
 $\text{D}_J(\Gamma_T)$  349  
 $\text{D}_J(n_G)$  348  
 $\Delta_K(G)$  123  
 $\mathfrak{E}$  87  
 $\mathfrak{E}(\pi)$  231  
 $\mathfrak{E}(n)$  91  
 $\mathfrak{E}(p)$  135  
 $\text{E}(G)$  98  
 $\text{E}(q|p^e)$  333  
 $\text{E}_{\mathfrak{F}}(G)$  113  
 $\mathfrak{F} \circ \mathfrak{G}$  95  
 $\mathfrak{F} \times \mathfrak{G}$  95  
 $\text{F}'(G)$  78  
 $\text{F}(G)$  Fitting subgroup of the group  $G$   
 $\text{F}^*(G)$  97  
 $\text{Fit } \mathfrak{S}$  110  
 $\text{GF}(q)$  finite field of  $q$  elements  
 $\text{GL}(n, q)$  general linear group of dimension  $n$  over  $\text{GF}(q)$   
 $\text{Hall}_\pi(G)$  99  
 $\text{Hom}(U, A)$  set of homomorphisms between  $U$  and  $A$   
 $\text{Hom}_{KG}(V, W)$  set of  $KG$ -homomorphisms from  $V$  to  $W$   
 $\text{I}(G)$  258  
 $\text{I}_K(E)$  223  
 $\text{Inj}_{\mathfrak{F}}(G)$  114  
 $\text{Inj}_{\mathcal{F}}(G)$  114  
 $\text{Inn}(G)$  group of inner automorphisms of the group  $G$   
 $\text{Irr}(M)$  293  
 $\mathfrak{J}$  87  
 $\text{J}(KG)$  5  
 $\text{K}_G(X)$  239  
 $\text{K}_n(G)$  93  
 $\mathfrak{K}(\mathfrak{X})$  324  
 $\text{Ker}(d, \mathbf{A})$  344  
 $\text{Ker}(f)$  kernel of the homomorphism  $f$   
 $\text{Ker}(x \text{ on } U)$  kernel of the action of the element  $x \in G$  on the  $KG$ -module  $U$   
 $\text{LF}(f)$  97  
 $\text{LF}_{\mathfrak{X}}(f)$  126  
 $\mathfrak{L}(\mathfrak{X})$  142  
 $\text{L}(G)$  197  
 $\text{L}_{\mathfrak{S}}(G)$  197  
 $\text{Locksec}(\mathfrak{X})$  112  
 $\mathfrak{M}(G)$  120  
 $\mathfrak{M}_p$  295  
 $\text{Max}(G)$  52  
 $\text{Max}(G)_{\mathfrak{M}}^n$  197  
 $\text{Max}^*(G)$  53  
 $\text{Max}^*(G)_{\mathfrak{S}}^a$  192  
 $\text{Max}^*(G)_{\mathfrak{M}}^a$  197  
 $\text{Max}_{\mathfrak{S}^d}(G)$  225  
 $\text{Max}_{\mathfrak{S}}(G)$  101  
 $\text{Mod}_{\mathfrak{F}}(U)$  293  
 $\mathbb{N}$ , set of all natural numbers  
 $\mathfrak{N}$  87  
 $\mathfrak{N}_c$  91  
 $\text{Nor}_{\mathfrak{S}}(G)$  171  
 $\text{N}_G(\Sigma)$  183  
 $\text{N}_G(\mathbf{X}(\Sigma))$  85  
 $\text{O}^{p'}(G)$  smallest normal subgroup of  $G$  of  $p'$ -index  
 $\text{O}^p(G)$  smallest normal subgroup of  $G$  of  $p$ -index  
 $\text{O}_J(n_G)$  348  
 $\text{O}_{\mathfrak{M}}(G)$  126  
 $\text{O}_\pi(G)$  largest normal  $\pi$ -subgroup of  $G$   
 $\text{O}_{p',p}(G)$  largest  $p$ -nilpotent normal subgroup of the group  $G$   
 $\text{O}_p(G)$  largest normal  $p$ -subgroup of  $G$

- $\text{PGL}(n, q)$  projective general linear group of dimension  $n$  over  $\text{GF}(q)$   
 $\mathfrak{P}$  87  
 $\mathfrak{P}_1$  87  
 $\mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3$  87  
 $\mathfrak{P}_2$  87  
 $\mathfrak{P}_3$  87  
 $\text{PSL}(n, q)$  projective special linear group of dimension  $n$  over  $\text{GF}(q)$   
 $\text{Para}(G)$  200  
 $\text{Para}_{\mathbf{X}}(G)$  199  
 $\Phi(G \bmod N)$  the subgroup  $S$  of  $G$  with  $S/N = \Phi(G/N)$   
 $\Phi(G)$  Frattini subgroup of  $G$   
 $\Phi_{\mathbf{X}}(G)$  52  
 $\text{Pref}_{\mathbf{X}_L}(G)$  193  
 $\text{Pref}_{\mathbf{X}}(G)$  192  
 $\mathbb{P}$  87  
 $\text{Pro}(G)$  200  
 $\text{Pro}_{\mathbf{X}}(G)$  199  
 $\text{Proj}_{\mathfrak{H}^d}(G)$  225  
 $\text{Proj}_{\mathfrak{H}}(G)$  101  
 $\Omega$  97  
 $\mathfrak{R}$  140  
 $\mathfrak{R}_{\mathfrak{X}}$  134  
 $\text{R}(G)$  197  
 $\text{R}_{\mathfrak{H}}(G)$  197  
 $\text{Rad } T$  radical of the module  $T$   
 $\text{SL}(n, q)$  special linear group of dimension  $n$  over  $\text{GF}(q)$   
 $\text{Sec}(\mathfrak{X}, \mathfrak{Y})$  111  
 $\Sigma N/N$  184  
 $\text{Soc}(G)$  socle of  $G$   
 $\text{Soc}(G \bmod \Phi(G))$  the subgroup  $S$  of  $G$  with  $S/\Phi(G) = \text{Soc}(G/\Phi(G))$   
 $\mathfrak{S}$  87  
 $\mathfrak{S}^{(d)}$  91  
 $\mathfrak{S}_{\pi}$  100  
 $\text{Supp}(f)$  296  
 $\text{Syl}_p(G)$  set of all Sylow  $p$ -subgroups of the group  $G$   
 $\text{T}(G)$  281  
 $\text{T}_G(H; \mathfrak{F})$  245  
 $\mathfrak{T}(1, \mathfrak{M})$  120, 292, 297  
 $\mathfrak{T}(1, r, \mathfrak{P}, \mathfrak{X}_1, \mathfrak{X}_2)$  297  
 $\mathfrak{T}(\mathfrak{M}(K, \mathcal{P}, \mathcal{X}))$  123  
 $\mathfrak{T}_K(G)$  120, 292  
 $\mathfrak{T}_{\pi}$  233  
 $\text{Tr}_{\mathfrak{F}}(G)$  114  
 $\mathfrak{U}$  87  
 $\mathcal{W}(X, \mathfrak{F})$  288  
 $\text{W}(G, \mathbf{X})$  192  
 $\mathfrak{X}'$  126  
 $\mathfrak{X}\mathfrak{Y}$  88  
 $\mathfrak{X} \cdot \mathfrak{Y}$  111  
 $\mathfrak{X} \circ \mathfrak{Y}$  111  
 $\mathfrak{X} \subseteq \mathfrak{Y}$  87  
 $\mathfrak{X}^*$  111  
 $\mathfrak{X}^n$  88  
 $\mathfrak{X}^b$  112  
 $\mathfrak{X}_*$  112, 148  
 $\mathfrak{Y}'$  126  
 $\mathfrak{Y}^{\mathfrak{G}}$  158  
 $\mathfrak{Y}_p$  126  
 $\mathbb{Z}$  set of all integer numbers  
 $\mathbb{Z}_m$  set of all integer numbers modulo  $m$   
 $\mathfrak{Z}_f$  260  
 $\bar{f}$  135  
 $\text{bform}(\mathfrak{Y})$  128  
 $\bigoplus_{i \in I} V_i$  direct sum of the modules  $V_i$   
 $\prod_{i \in I} A_i$  direct product of the groups  $A_i, i \in I$   
 $\prod_{i \in I} \mathfrak{F}_i$  96  
 $\bar{\mathfrak{b}}(\mathfrak{X})$  112  
 $\text{b}(\mathfrak{H})$  (for a class of groups) 102  
 $\text{b}(\mathfrak{X})$  (for a Fitting class) 112  
 $\text{b}_3(\mathfrak{F})$  295  
 $\text{b}_n(\mathfrak{F})$  306  
 $\text{b}_p(\mathfrak{X})$  112  
 $\text{b}_{\mathfrak{X}}(\mathfrak{F})$  134  
 $\text{b}_m(G)$  113  
 $C_G^*(H/K)$  41  
 $C_G(H)$  centraliser of  $H$  in  $G$   
 $Z(G)$  centre of the group  $G$   
 $Z_{\mathfrak{F}}(G)$  179  
 $Z_{\mathfrak{F}}(G \bmod \Phi(G))$  the subgroup  $S$  of  $G$  with  $S/\Phi(G) = Z_{\mathfrak{F}}(G/\Phi(G))$   
 $\chi_f$  263  
 $\text{AB}, \text{BA}$  90  
 $\text{C}$  89  
 $\text{C} = \langle A_{\lambda} : \lambda \in \Lambda \rangle$  90  
 $\text{C}^2$  89  
 $\text{C}_1 \leq \text{C}_2$  89  
 $\text{D}_0(\mathfrak{F}, S)$  92  
 $\text{D}_0 \mathfrak{X}$  89  
 $\text{E}_{\Phi} \mathfrak{X}$  89  
 $\text{K} \mathfrak{X}$  88

- $N_0 \mathfrak{X}$  89  
 $PQ\mathfrak{X}$  102  
 $Q\mathfrak{X}$  89  
 $R_0\mathfrak{X}$  89  
 $S(X)$  205  
 $s\mathfrak{X}$  89  
 $S_n(G)$  206  
 $S_n\mathfrak{X}$  89  
 $S_p(G)$  206  
 $S_{\mathfrak{X}}(G)$  142  
 $d(G)$  205  
 $\det(x \text{ on } U)$  determinant of the element  $x \in G$  acting on the  $KG$ -module  $U$   
 $\text{diag}(n_1, \dots, n_r)$  diagonal matrix with elements  $n_1, \dots, n_r$  in its diagonal  
 $\emptyset$  87  
 $\eta(G, M)$  191  
 $e \leq f$  206  
 $r''(G)$  207  
 $r'(G)$  207  
 $r(G)$  207  
 $\text{form}_{\mathfrak{X}}(\mathfrak{Y})$  128  
 $f$  128  
 $\bar{t}''(G)$  207  
 $t'(G)$  207  
 $t(G)$  207  
 $h(\mathfrak{Y})$  (for a Fitting class) 112  
 $h(\mathfrak{Y})$  (for a class of groups) 102  
 $h_{\pi}(G)$  232  
 $\text{id}, \text{id}_K$  identity automorphism (of  $K$ )  
 $\text{Im}(f)$  image of the homomorphism  $f$   
 $\kappa(G)$  330  
 $\langle X^G \rangle$  normal closure of  $X$  in  $G$   
 $\langle A, B \rangle$  90  
 $\langle A \rangle$  90  
 $\langle a_1, a_2, \dots \rangle$  subgroup generated by the elements  $a_1, a_2, \dots$   
 $\text{lform}(\mathfrak{Y})$  128  
 $|G : H|$  index of the subgroup  $H$  in the group  $G$   
 $|X|$  cardinal/order of  $X$   
 $\mathbf{X}^{\pi}$  192  
 $\mathbf{A}$  344  
 $\mathbf{A}_J$  348  
 $\mathbf{D}_J$  348  
 $\mathbf{D}_J(\Gamma, \mathfrak{F}/\mathfrak{G})$  348  
 $\mathbf{G}$  344  
 $\mathbf{S}(\Sigma)$  83  
 $\mathbf{X}$  40  
 $\mathbf{X}/N$  52  
 $\mathbf{X}^{\varphi}$  52  
 $\mathbf{X}^g$  74  
 $\mathbf{X}_M$  80  
 $\mathbf{X}_p$  192  
 $\mathbf{X}_5^g$  191  
 $\mathbf{X}_5^g$  191  
 $\mathbf{Y}(F, N, T)$  75  
 $\mathbf{Y}^{\varphi^{-1}}$  74  
 $\mathcal{E}_i$  63  
 $\mathcal{F}_H$  114  
 $\mathcal{H}d^{-1}$  347  
 $\mathcal{P}$  87  
 $\mathcal{P}(\mathfrak{H})$  108  
 $\mathcal{R}(Z, \mathfrak{F})$  303  
 $\mathcal{W}(\mathfrak{F})$  303  
 $\mathfrak{M}(K, \mathcal{P}, \mathcal{X})$  121  
 $\mathfrak{C}(\mathfrak{M})$  123  
 $\mathfrak{M}(K, \mathcal{P}, \mathcal{X})$  121  
 $\max S$  maximum of the set  $S$   
 $\min S$  minimum of the set  $S$   
 $N(E)$  113  
 $N_G(H)$  normaliser of  $H$  in  $G$   
 $\pi(G)$  88  
 $\pi(\mathfrak{X})$  88  
 $\pi_S$  9  
 $\prod_{j \in S} X_j$  product of the subgroups  $X_j$  with  $j \in S$   
 $\psi^B$  16  
 $\psi^G$  26  
 $S_{n\mathfrak{F}}(G)$  236  
 $S_{nK-\mathfrak{F}}(G)$  236  
 $S_G(X)$  239  
 $S_G(X; \mathfrak{F})$  239  
 $S_G(X; K-\mathfrak{F})$  239  
 $a^*$  26  
 $a^{\varphi}, a\phi, \phi(a)$  image of  $a$  by  $\phi$   
 $d_J^{J, \mathfrak{F}/\mathfrak{G}}$  348  
 $f(\mathfrak{H})$  106  
 $f|_K$  restriction of  $f$  to  $K$   
 $f^*$  130  
 $f_1(\mathfrak{H})$  106  
 $f_1 \leq f_2$  128  
 $sV$  direct sum of  $s$  copies of the module  $V$   
 $v \otimes w$  a generator of a tensor product  
 $V \otimes_A W$



---

## Index of authors

- Alejandre, M. J. 281, 282  
Anderson, W. 114, 116, 340  
Arroyo-Jordá, M. 264, 265  
Arroyo-Jordá, P. 346, 349, 352  
Aschbacher, M. 22, 24
- Baer, R. VIII, 1, 3, 4, 89, 96, 97, 119,  
125, 127, 133, 144, 148, 155, 161,  
250, 296
- Ballester-Bolinches, A. IX, 93, 119,  
132, 133, 143, 146–148, 154,  
157, 160, 161, 163–166, 170, 171,  
189–192, 198, 244, 245, 247, 248,  
250, 258, 260, 264, 265, 268, 269,  
272–277, 279, 281, 282, 284, 286,  
301, 306, 307, 346, 349, 352
- Barnes, D. W. 40, 92, 93  
Bartels, D. 239  
Bechtell, H. VIII, 197  
Beidleman, J. C. 171  
Bender, H. 97, 328  
Berger, T. R. 123, 329  
Berkovich, Y. 281  
Birkhoff, G. 53  
Blackburn, N. 97, 98, 297, 328  
Blessenohl, D. 118, 315, 344  
Bolado-Caballero, A. 118  
Brandis, A. 49, 170, 193  
Brewster, B. 171  
Bryant, R. M. 93  
Bryce, R. A. 93, 118, 119, 329, 345
- Calvo, C. 132, 133, 146–148, 154, 161,  
164–166
- Cameron, P. J. 24  
Carocca, A. 204  
Carter, R. W. 40, 99, 100, 114, 169,  
171, 188, 268  
Chambers, G. A. 170  
Clifford, A. H. 286  
Cossey, J. VIII, 118–121, 123, 286, 300,  
329, 345
- Dade, E. C. 315  
Dark, R. 329, 330, 340, 342  
Dark, R. S. 110  
De Cervantes Saavedra, M. VII  
Deskings, W. E. 191  
Doerk, K. VII, VIII, 1, 40, 78, 84, 87,  
90, 91, 95, 97, 100, 101, 103, 104,  
106, 111, 114–116, 118–120, 123,  
128–130, 133, 140, 141, 146, 153,  
157, 161, 166, 171, 173–177, 179,  
181, 182, 188, 208, 235, 244, 247,  
248, 250, 254, 258, 264, 266–268,  
272, 284, 292, 315, 319, 329, 332,  
333, 340, 343, 345
- Erickson, R. P. 100, 170  
Esteban-Romero, R. VIII, 132, 133,  
146–148, 154, 161, 164–166, 268  
Ezquerro, L. M. IX, 119, 181, 190–192,  
198, 204, 284, 286, 307
- Feit, W. 219  
Feldman, A. VIII, 340, 342  
Fischer, B. 40, 109, 110, 114, 116, 268,  
309, 315, 337, 338

- Fitting, H. VIII, 78, 90, 97, 99,  
109–114, 116–121, 123, 126, 134,  
135, 153, 181, 213, 248, 250, 252,  
254, 257, 259, 260, 264, 265, 282,  
284–288, 292, 293, 295–297, 300,  
301, 303, 306, 307, 309, 314–333,  
335–341, 344–349, 351–353
- Förster, P. 32, 42, 46, 68, 69, 100, 101,  
103, 104, 109, 125, 126, 134, 138,  
140, 141, 144, 147, 153, 170, 206,  
208, 315, 324, 327
- Frattini, G. 5, 40, 41, 44, 52, 54, 56–60,  
62, 66, 70, 92, 96, 119, 125, 139,  
144, 147, 148, 153, 169, 174, 175,  
183, 194, 196–199, 273
- Frobenius, G. 217, 221, 277, 281
- Gajendragadkar, D. 120
- Galois, É. 5
- Gaschütz, W. 344
- Gaschütz, W. VIII, 62, 69, 73, 95–97,  
100, 101, 104, 110, 111, 114, 116,  
118, 125, 144, 148, 152, 169, 170,  
192, 309, 319, 337, 344
- Gillam, J. D. 181, 182
- Griess, R. L. 175, 273
- Gross, F. 1, 10, 13, 18, 21, 309
- Haberl, K. L. 123, 300
- Hall, P. 73, 83, 84, 89, 98–100, 118, 153,  
163, 169–172, 182–187, 195, 218,  
231, 235, 250, 257, 268, 328, 339,  
340
- Harris, M. E. 115
- Hartley, B. 93, 110, 114, 116, 309
- Hauck, P. 332
- Hawkes, T. O. VII, VIII, 1, 40, 50, 78,  
84, 87, 90, 91, 95, 97, 100, 103, 104,  
106, 111, 114–116, 118–120, 123,  
128–130, 133, 140, 141, 146, 153,  
157, 161, 166, 169–171, 173–177,  
179, 181, 182, 188, 202, 208, 235,  
247, 250, 254, 266–268, 272, 284,  
292, 300, 315, 319, 329, 332, 333,  
340, 343, 345
- Hawthorn, I. 114, 116
- Heineken, H. 123, 300
- Higman, G. 188
- Hölder, O. L. VIII, 40, 41, 52, 53, 61, 73
- Huppert, B. 2, 5, 7, 93, 96–98, 166, 217,  
219, 221, 231, 268, 277, 297, 309,  
328
- Iranzo, M. J. VIII, 315–318, 327, 329
- Isaacs, I. M. 120
- Itô, N. 277
- Jacobson, N. 5
- Jiménez-Seral, P. VIII, 25
- Jordan, C. VIII, 40, 41, 52, 53, 61, 73
- Kamornikov, S. F. 248, 253, 257, 260,  
262–264, 269, 272, 286, 296, 301,  
306, 307
- Kanes, C. 119–121, 123, 300
- Kattwinkel, U. 106
- Kegel, O. H. 92, 93, 236, 248, 283, 284,  
301
- Khukhro, E. I. 157, 158, 248, 265, 268,  
284
- Klimowicz, A. A. 170
- Kovács, L. G. 1, 10, 13, 18, 21, 25, 29,  
31, 309
- Kurzweil, H. 1, 27, 97, 98, 170,  
193–195, 354
- Lacasa-Esteban, C. VIII
- Lafuente, J. VIII, 6, 34, 40–42, 53, 61,  
62, 103, 106, 315, 317, 318
- Laue, H. 315
- Lausch, H. 345
- Lennox, J. C. 235
- Liebeck, M. W. 25
- Lizasoain, I. VIII
- Lockett, P. 111, 112, 116, 257, 319,  
320, 330, 339, 344
- Lubeseder, U. VIII, 96, 97, 125, 144,  
148, 152
- Maier, R. 204
- Makan, A. 170
- Mann, A. 84, 169, 171, 176, 315, 325
- Martínez-Pastor, A. 260, 284, 329
- Martínez-Verdusch, J. R. 118
- Mazurov, V. D. 157, 158, 248, 265,  
268, 284
- Medina, J. 329
- Menth, M. 329–331, 339

- O'Nan, M. 1, 24, 25, 28, 34, 39  
Ormerod, E. O. 123
- Pedraza, T. VIII  
Pedraza-Aguilera, M. C. VIII, 268,  
284, 301, 306, 307  
Pense, J. 345, 346, 348, 351–353  
Pérez-Monazor, F. VIII, 315–318, 327,  
329  
Pérez-Ramos, M. D. 93, 157, 160, 244,  
245, 247, 248, 250, 258, 260, 264,  
265, 268, 269, 272–277, 279, 301,  
306, 307, 346, 349, 352  
Plotkin, B. I. 89  
Praeger, C. 25
- Robinson, D. J. S. 268
- Salomon, E. VIII, 133, 134, 138, 140,  
153–155, 166, 205, 206, 208, 220,  
309, 329  
Saxl, J. 25  
Schaller, K.-U. 106  
Schmid, P. VIII, 96, 97, 100, 125, 133,  
144, 148, 152, 170, 175, 180, 189,  
273  
Schmidt, O. J. 268, 269, 272–274, 281,  
282  
Schreier, O. 27, 354  
Schubert, H. 100  
Schunck, H. VIII, 99–110, 112, 118,  
123, 169–171, 177, 192, 193, 197,  
202, 205, 206, 224, 226–231  
Schur, I. 156, 231  
Scott, L. 1, 22, 24, 25, 28, 34, 39
- Sel'kin, M. V. 264  
Semenchuk, V. N. 248, 253, 257, 265,  
268  
Shemetkov, L. A. 96, 97, 133, 152, 153,  
157, 158, 160–165, 171, 189, 248,  
268, 269, 286, 296, 315  
Skiba, A. N. 157, 158, 161, 163, 164,  
269  
Soler-Escrivà, X. 204  
Stellmacher, B. 1, 27, 97, 98, 354  
Stonehewer, S. E. 235  
Suzuki, M. 156, 309  
SyLOW, P. L. M. 3, 98–100, 166, 170,  
176, 187, 189, 206, 219, 237, 247,  
273, 301, 330, 346, 349, 350
- Thompson, J. G. 219  
Tomkinson, M. J. 170, 203  
Torres, M. 315, 316, 327, 329  
Traustason, G. 330
- Vasil'ev, A. F. 248, 253, 257, 260,  
262–264, 268, 283, 284  
Vedernikov, V. A. 157  
Vorob'ev, N. T. 157
- Wieladt, H. 170  
Wielandt, H. VII, VIII, 98, 170, 215,  
217, 221, 235, 243, 244, 247, 268,  
285–288, 295–297, 300, 301, 306
- Yen, T. 183
- Zassenhaus, H. 231

---

## Index

- ACAP 194, 195  
action 2, 8, 12, 14, 16, 17, 23, 25–28, 34, 39, 41, 92, 93, 96, 99, 224, 286, 348, 349, *see also* representation  
induced 17, 26  
regular 31  
scalar 350, 351  
transitive 310, 311  
automorphism 16, 22, 332–335, 348  
group of *see* group, automorphism  
inner 7, 25, 41, 97, 223, 224, 230, 309, 345–347  
outer 26, 309, 310, 345, 353, 354  
power 330  
Baer function 97, 119, 127, 296  
Baer-local formation 96, 97, 125, 127, 133, 153, 155, 161, 269, 272, *see also* solubly saturated formation  
defined by  $f$  97  
block 1, 23, 25, 27, 28, 34, 39, 96  
non-trivial 2  
trivial 1  
 $U$ -invariant 28  
boundary 101, 102, 103, 104, 106, 107, 112, 113, 155, 174, 180, 226–231, 266, 275, 277, 286, 288, 293, 295, 303, 304, 306  
 $\mathfrak{X}$ -wide 134, 135, 136, 138–140  
 $\mathfrak{X}\mathfrak{G}$ -free 155, 156  
centre 7, 17, 22, 98, 113, 140, 146, 153, 156, 174, 309, 330, 349  
chain  
of classes of groups 329  
of critical subgroups 171, 172, 180  
of crucial critical subgroups 176  
of subgroups 177, 207, 235, 236  
character  
 $\pi$ -factorable 120  
 $\pi$ -special 120  
characteristic 88, 125–132, 134–138, 141, 142, 144, 146–149, 152–157, 159, 160, 164–166, 169, 235, 244, 250, 257–260, 264, 265, 267, 282, 284, 307, 308  
of a field 120, 121, 123, 291  
chief factor 40, 41, 42, 44, 45, 49, 50, 52–55, 61, 62, 66, 70, 74, 78–80, 96, 97, 126, 128, 134, 135, 139, 140, 147, 200, 342, 348–351  
abelian 45, 74, 75, 120, 191  
avoided by a subgroup 79, 99, 169, 170, 173, 181, 182, 186, 194, 196  
central 99  
complemented 44, 45, 49, 62, 74, 75, 92, 123, 169, 191, 200  
composition type 152  
covered by a subgroup 79, 99, 169, 170, 173–176, 178, 181, 182, 186, 194, 196, 208, 213, 214, 342  
Frattini 40, 41, 44, 92, 139, 169, 199  
 $G$ -connected 42, 42, 46, 48, 57, 62–66, 70, 71  
 $G$ -isomorphic 40, 41, 42, 57, 62, 63, 65, 66, 74, 75, 126, 140, 291  
 $H$ -central 106

- $\mathfrak{H}$ -central 106, 108, 173, 174, 176, 178, 181, 182, 186, 293
- $\mathfrak{H}$ -eccentric 106, 108, 173–175, 181, 182, 186, 189, 293
- supplemented 44, 45, 48, 52, 53, 63–65, 74, 78, 174, 176, 179
- X**-complemented 52, 60, 65, 194–196, 198–201
- X**-Frattini 52, 54, 56–60, 62, 66, 70, 194, 196
- X**-related 57
- X**-supplemented 52, 54–62, 64–66, 70, 71, 73, 75, 191, 193, 200, 201
- chief series 40, 53, 54, 58, 64–66, 69–71, 73, 126, 178, 181, 348–351
- class
  - nilpotency 91
- class of groups VII, VIII, 1, 87, 88–91, 96–98, 100–102, 106–112, 125, 126, 128, 129, 132–135, 138, 139, 142, 144, 147, 148, 152, 153, 156–158, 161, 163–166, 174, 191, 200, 205, 225, 231, 233, 252, 260, 264, 265, 267–269, 272, 281, 285, 286, 288, 294, 295, 300, 303, 306, 307, 314, 315, 318, 324, 338
- closed for a closure operation 89, 90
- d-projective 226, 227
- $D_0$ -closed 103
- extreme 268
- Fitting *see* Fitting class
- injective 115
- Lockett *see* Lockett class
- power 88
- product *see* product, class; product, Fitting; product, formation
- projective 100, 101, 103, 110, 170
- Q-closed 91, 126, 140, 337, 338
- $R_0$ -closed 91, 103, 111, 338
- residually closed 90
- saturated 90, 91, 95, 101, 337
- Schunck *see* Schunck class
- $S_n$ -closed *see* class of groups, subnormal subgroup-closed
- subgroup-closed 90, 95, 96, 111, 118, 119, 142, 144, 263, 268, 337
- subnormal subgroup-closed 90, 110
- subnormally independent 112
- trivial 349, 352
- closure
  - $\mathfrak{F}$ -subnormal 239
  - $\mathfrak{N}$ -subnormal 244
  - normal 239, 243
  - subnormal 239, 243
- closure operation 89, 90, 91, 103, 109, 110, 112, 141, 142
  - generated by a set of operations 90
  - idempotent 90
- complement 5, 38, 44, 45–50, 52, 60, 62, 70, 71, 73–75, 169, 170, 187–189, 210, 220–222, 224, 225, 230, 231, 309, 310
- component 23, 34, 96, 98, 113, 315, 324
  - for a Fitting class 113, 315, 316
- composition factor 40, 65, 88, 97, 120, 123, 125, 126, 163, 164, 211, 214, 219, 269, 291–293, 328
- composition formation 97, 152, 153, *see also* Baer-local formation
- composition series 40, 62, 65, 235
- conjecture
  - Schreier 27, 354
- conjugacy class 14, 18, 25, 98–100, 110, 116–118, 169, 170, 175, 179, 180, 182, 183, 187–189, 191, 194, 196, 228, 230–232, 241, 315, 323, 325, 326, 328, 347, 350, 351, 353
  - characteristic 169, 315
  - of complements 45, 46
  - of maximal subgroups 3, 5, 18, 37
- conjugation 16, 23, 25, 26, 32, 41, 93, 99, 316, 330, 341
- core 2, 17, 25, 26, 44–64, 70, 71, 73, 74, 80, 103, 108, 176, 179, 182, 191, 197, 198, 225, 236, 303
- core-relation 73, 84
- cosocle 98, 112
- cover-avoidance property 170, 173, 175, 179, 181, 182, 191, 194, 202
- covering subgroup *see* subgroup, covering
- $(C_p)$ -local formation 133, 157, *see also*  $(C_p)$ -saturated formation
- $(C_p)$ -saturated formation 150, 152, 162, *see*  $(C_p)$ -local formation
- crown VIII, 62, 63, 65–67, 73, 75, 103, 169, 170
- $C_{\mathfrak{N}}$ -satellite 152

- d-Schunck class 226, 227–229, 231
- Duality Principle 53
- element
  - conjugate 220, 250
- embedding 344, 352
  - normal 344, 345
- $\mathfrak{E}_\pi$ -projector 231
- $\mathfrak{E}(\pi)$ -projector 233
- $\mathfrak{E}(\pi)$ -t-projector 231
- epimorphism 12, 74, 75, 89, 93, 99, 128, 310, 312, 319
- extension 11, 330, 335
  - field 120, 292, 295
  - Frattini *see* Frattini, extension
  - induced 1, 10, 13, 15–18, 21, 26, 309–312
  - non-split 21, 96, 225, 309, 312
  - pull-back 11
  - split 18, 188, 189, 200, 311
- $\mathfrak{F}$ -Fitting class 265
- $\mathfrak{F}$ -hypercentre 178, 179, 182, 189, 197
- $\mathcal{F}$ -injector 114, *see also* injector
- f-join 207
- $\mathcal{F}$ -subgroup 114
- factorisation
  - solubly saturated 154
- Fischer class 337, 338
- Fischer  $\mathfrak{F}$ -subgroup *see* subgroup, Fischer
- Fitting class VIII, 90, 97, 99, 109, 110, 111–114, 118, 126, 248, 259, 260, 264, 265, 300, 309, 315–327, 329, 330, 332–335, 337–340, 344–346, 348, 349, 351–353
  - dominant 118, 257
  - formation *see* Fitting formation
  - generated by a class of groups 110
  - Hall-closed 339
  - Hall- $\pi$ -closed 339
  - injective 309, 315, 317, 322–325, 327–329, 350
  - metanilpotent 329, 331
  - non-injective VIII, 110, 309, 314, 323
  - normal 118, 252, 254, 321–323, 344, 345
  - repellent 332
  - soluble 332
  - subgroup-closed 111
  - supersoluble 329–332, 335–339
- Fitting family of modules 119, 120, 123, 292, 293, 295, 297
- Fitting formation 91, 110, 118, 119, 121, 123, 134, 135, 252, 284–288, 292, 295, 297, 300, 314
  - defined by a Fitting family of modules 119, 120, 121, 123, 292, 293, 295, 297
- extensible 328
  - non-saturated 119
  - saturated 118, 119, 250, 254, 282, 301, 306, 307, 328, *see also* saturated formation
- soluble 119, 296
- solubly saturated 119, 293, 295, *see also* solubly saturated formation
- subgroup-closed 118, 119, 296, 301, 303, 306, 307
- Fitting pair 344, 345, 346, 351
  - chief factor product 348
  - kernel 344
  - outer 339, 345, 346, 348, 352, 353
    - equivalent 348
    - induced 347, 353
- Fitting pairs 346
  - outer 348
- Fitting set 114, 117, 339–341, 345–347, 349, 352, 353
  - dominant 348
  - injective 347, 348
  - $p$ -supersoluble 348
- Fitting sets pair 345, 346–348
  - chief factor product 349
  - outer 345, 346–349, 352
    - equivalent 346
    - induced 353
- formation 90, 91, 92–97, 100, 101, 105–111, 118, 123, 125, 127, 128, 132, 134, 137, 139, 140, 142, 148, 149, 152–154, 156–158, 160–166, 171, 177, 202, 235, 237, 239, 247, 248, 253, 260, 262, 263, 265, 267–269, 272, 277, 281, 283, 284, 286, 301, 302
- Baer-local *see* Baer-local formation

- closed under taking triple factorisations *see* formation, with the Kegel property
- composition *see* composition formation
- $(C_p)$ -local *see*  $(C_p)$ -local formation
- $(C_p)$ -saturated *see*  $(C_p)$ -saturated formation
- Fitting *see* Fitting formation
- function *see* formation function
- generated by a class of groups 91
- K-lattice 248, 250, 251, 262, 305
- largest contained in a Schunck class 106, 107
- lattice 247–254, 257–265, 268, 301–303, 305, 306
- local *see* local formation
- maximal contained in a class of groups 138, 140
- $\omega$ -local *see*  $\omega$ -local formation
- $\omega$ -saturated *see*  $\omega$ -saturated formation
- $p$ -local *see*  $p$ -local formation
- $p$ -saturated *see*  $p$ -saturated formation
- saturated 272, 273, *see* saturated formation
- $S_n$ -closed *see* formation, subnormal subgroup-closed
- soluble 119, 260, 268, 272, 301, 306
- solubly saturated *see* solubly saturated formation
- subgroup-closed 96, 141, 237, 244, 248, 252, 254, 260, 263–266, 268, 269, 272, 274, 277, 279, 281, 284, 286, 301, 303, 304, 306–308
- subnormal subgroup-closed 95, 97, 254, 257, 260, 283, 284
- totally nonsaturated 198
- with the generalised Wielandt property for residuals 301, 303, 305–307
- with the Kegel property 283, 284
- with the Kegel-Wielandt property for residuals 301, 305
- with the Shemetkov property 268, 269, 272–275, 277, 279, 281, 283, 284
- with the Wielandt property for residuals 285, 286, 296, 297, 300
- $\mathfrak{X}$ -local *see*  $\mathfrak{X}$ -local formation
- $\mathfrak{X}$ -saturated *see*  $\mathfrak{X}$ -saturated formation
- $\mathfrak{X}_\omega$ -saturated *see*  $\mathfrak{X}_\omega$ -saturated formation
- formation function 97, 127, 140, 166, 190, 265–267, 274, 275, 277, 296, 307
- Frattini
  - argument 5, 183
  - extension 96, 148, 153, 163
  - maximal 5, 119, 174, 273
  - module 175
  - subgroup 96, 144–148, 153, 161, 163, 197
- Frattini-like subgroup *see also*  $\mathfrak{X}$ -Frattini subgroup
- functor *see* subgroup functor
- G**-embedding 345, 347
  - normal 345, 346
- $G$ -isomorphism 41, 74
- $G$ -set 1, 25, *see also* representation
  - transitive 1, 2, *see also* representation, transitive
- Gaschütz class 101, 104
- group algebra 5, 146
- group theoretical class *see* class of groups
- group theoretical property 87
- group(s)
  - $\pi$ -soluble 171
  - $p$ -soluble 5
  - abelian 93, 96, 119, 120, 125, 132, 134, 147, 150, 153, 157, 164, 165
  - class of 87, 91, 107, 111, 133, 260, 263
  - homocyclic 333, 335
  - affine general linear 5
  - almost simple 7, 25, 28, 29, 31, 34, 275
  - alternating 3, 21, 34, 39, 129, 133, 141, 142, 144, 146, 153, 161, 166, 172, 174–176, 179, 192, 211, 220, 225, 230, 236, 237, 244, 250, 253, 273, 274, 286, 309, 310, 312, 314, 346

- automorphism 5, 7–9, 16, 22, 23, 25, 26, 29, 32, 41, 79, 96
  - induced by conjugation 41, 79, 97, 174
- base 8, 33, 140, 210, 211, 224, 230, 353
- class *see* class of groups
- comonolithic 98, 112, 113, 215, 217, 221, 315, 352, 353
- critical for a class of groups 252, 254, 267–269, 272, 274, 281
- cyclic 5, 31, 39, 70, 126, 140, 200, 219, 224, 252, 254, 268, 269, 272, 274, 295, 333, 350
  - class of 88
- d-primitive 225, 226, 227
- dihedral 31
- elementary abelian 147, 150, 161
- $\mathfrak{F}$ -constrained 325, 326, 327
  - class of 324, 325
- factorised 53, 75, 77, 218, 220–222, 265, 284, 307, 330
- finite 96, 98, 119, 133, 152, 344, 346
  - class of 87, 349, 352
- Frobenius 217, 219, 221
- Frobenius-Wielandt 217, 221
- general linear 5, 309, 335, 349
  - induced 17
- infinite 344, 345
- Lausch 345
- meta- $\mathfrak{X}$  88
- metabelian 336
- metanilpotent 284
- monolithic 91, 126, 128, 147, 252,
  - see also* group(s), primitive, monolithic
- $\mathfrak{N}$ -constrained 315, 325
  - class of 325
- nilpotent 91, 93, 95, 97, 99, 100, 111, 250, 260, 268, 272, 283, 307, 331, 336, 337
  - class of 87, 100, 110, 118, 169, 172, 235, 236, 239, 263–265, 281, 327
- non-abelian 91, 92, 98, 134, 140, 146, 147, 156, 164
- non-soluble 96, 101, 170, 171, 190, 194
- $\omega$ -separable 165, 166
- $p'$ -perfect
  - class of 88
- $p$ -constrained 327, 328, 348, 350, 351
- $p$ -decomposable
  - class of 329
- $p$ -nilpotent 277, 350
  - class of 247, 265, 268, 277, 281, 307, 327, 350
- $p$ -quasinilpotent
  - class of 329
- $p$ -soluble 119, 192, 193, 215, 218, 225, 231, 350
  - $p$ -length 350
- perfect 98, 113, 215, 221, 315, 317, 352, 353
- permutation
  - transitive 29
- $\pi$ -perfect
  - class of 118
- $\pi$ -soluble 170
- primitive VIII, 1, 2, 3–5, 8, 24, 29, 31, 44, 45, 62, 91, 100, 101, 103, 107, 108, 194, 208, 220, 284
  - associated with a chief factor 45
  - class of 1, 87
  - monolithic 4, 45, 215, 222, 225, 226, 254, 266
  - of type 1 4, 5, 6, 44, 180, 215, 220, 221, 224, 225
  - of type 2 4, 5–7, 10, 24, 25, 28–34, 37–39, 44, 60, 66, 104, 107, 119, 143, 144, 147, 200, 220, 224, 225, 254, 273
  - of type 3 4, 5–7, 42, 44, 46, 61, 63, 224, 228
  - soluble 5
    - with small maximal subgroups 32
- projective general linear 309, 310
- projective special linear 309, 310, 314
- quasinilpotent 97, 198, 209, 213, 224, 226, 228
  - class of 97, 297, 315, 325–327, 329
- quasisimple 98, 113
  - class of 324
- quaternion 219, 346
- $r'$ -primitive 225
- $r$ -soluble 120, 123, 292
- $\mathfrak{S}$ -constrained
  - class of 325



- $S$ -perfect
  - class of 175
- Schmidt 268, 269, 272–274, 281, 282
- semisimple 208, 222
- simple 7, 21, 34, 41, 96, 97, 125, 129,
  - 132, 133, 138, 144, 147, 148, 152,
  - 153, 156, 157, 161, 163–166, 170,
  - 208, 213, 214, 223, 269, 272, 295,
  - 310, 311, 314, 317, 348, 349, 352,
  - 354
- class of 87, 125, 126, 142, 144, 152
- non-abelian 5, 7–9, 22–25, 28, 29,
  - 31, 32, 41, 62, 91, 92, 98, 103, 106,
  - 108, 110, 146, 147, 156, 164, 196,
  - 198, 200, 208, 210–214, 218, 219,
  - 224, 225, 230, 233, 253, 266, 275,
  - 295
- soluble VII, VIII, 5, 27, 40, 45, 62,
  - 73, 74, 78, 83–85, 87, 91, 96–101,
  - 109, 110, 112, 114–117, 119–121,
  - 123, 133, 153, 166, 169–177,
  - 179–188, 190–196, 198, 200–202,
  - 204, 205, 208, 212, 214, 215, 220,
  - 221, 224, 225, 230, 231, 233, 235,
  - 236, 244, 245, 248, 250, 253,
  - 257–260, 262–269, 272, 274,
  - 281–284, 295, 297, 301, 303, 307,
  - 309, 315, 323, 325, 328, 332, 337,
  - 339–343, 345, 353, 354
- class of 87, 100, 110, 139, 268, 323,
  - 325, 328, 345
- soluble  $p$ -nilpotent
  - class of 295
- special linear 140, 146, 153, 166
- strictly semisimple 208, 209, 211,
  - 215, 222
- supersoluble 329–331
  - class of 87, 91, 111, 268, 329
- symmetric 9, 12, 14, 22, 25, 39, 58,
  - 60, 68, 69, 172, 211, 220, 230, 237,
  - 247, 250, 286, 301, 309–311, 346
- $t$ -primitive 225, 230, 231
- triply factorised 283, 284
- $\mathfrak{X}$ -dense 134
- $\mathfrak{Y}$ -perfect
  - class of 102, 106
- GWP-formation *see* formation, with
  - the generalised Wielandt property
  - for residuals
- $\mathfrak{H}$ -covering subgroup 101
- $\mathfrak{H}$ - $d$ -covering subgroup 226, 227–232
- $\mathfrak{H}$ - $d$ -projector 225, 226–233
- $\mathfrak{H}$ -projector 101, *see also* projector
- $\mathfrak{H}$ - $t$ -projector 230
- Hall subgroup *see* subgroup, Hall
- Hall system 73, 83, 84, 99, 169–172,
  - 182–187, 195, 235
  - reducing into a subgroup 73, 84,
  - 169, 172, 184, 186, 235
- head 98, 352, 353
- homomorph 90, 101–103, 111, 112,
  - 226, 227, 260
- homomorphism 10–12, 16, 18, 26, 32,
  - 41, 52, 285, 310–312, 346–349, 352
  - trivial 346
- inductivity 103, *see also* subgroup
  - functor, inductive
- injector 99, 109, 110, 114, 115–118,
  - 257–259, 265, 309, 314–317, 323,
  - 325–328, 337, 339–343, 347, 350,
  - 351, 353
  - normal 118
  - $p$ -nilpotent 328, 350, 351
- inneriser 41, 46, 97
- involution 220, 230, 309–311
- kernel 10, 12, 14, 17, 25, 26, 41, 74,
  - 93, 119, 140, 146, 175, 254, 297,
  - 310–312, 319, 344, 345, 349, 350
  - Frobenius-Wielandt 217
- KW-formation *see* formation, with
  - the Kegel-Wielandt property for
  - residuals
- lattice 207
  - modular 53, 61, 62
- layer 98, 113, 325
- length
  - derived 91
  - nilpotent 100, 250, 265, 268, 301
- local definition *see* formation function
  - canonical 130
  - maximal 133, 134, 190
- local formation 96, 97, 125, 127, 129,
  - 132–134, 139–141, 152, 157, 160,
  - 161, 163, 331
- local function *see* formation function
- Lockett class 112, 330, 339

- metanilpotent 339
- supersoluble 339
- M-set 61, 62
- module 62, 65, 120, 140, 175, 290, 293
  - absolutely irreducible 297
  - completely reducible 62, 65, 290
  - faithful 143, 144, 150, 166, 233, 247, 273
  - homogeneous 62, 65, 297
  - induced 17, 291
  - irreducible 120, 121, 123, 140, 143, 144, 146, 150, 166, 233, 247, 273, 286, 291–293, 297
  - $\mathcal{P}$ -factorable 120, 121
  - $\pi$ -factorable 121
  - $\pi$ -special 120, 121
  - socle 175
  - trivial 120
- monomorphism 10–12, 310, 311, 344–346
- normaliser
  - associated with a saturated formation 169–171, 174–183, 187–190, 204
  - associated with a Schunck class VIII, 78, 169, 170, 171, 172–179, 203
  - associated with a system of maximal subgroups 172, 173, 175, 203, 235
  - crucial 176
- $\omega$ -local formation 161, 163–167
  - satellite 163, 164
    - canonical 164
    - minimal 164
- $\omega$ -saturated formation 161, 162–164, 166
- operation 89, 90
  - closure *see* closure operation
  - idempotent 89
  - product 89
- operator
  - Wielandt *see* Wielandt operator
- orbit 1, 217
- $p^*$ -group 329
  - class of 329
- $p^*$  $p$ -group
  - class of 328, 329
- $p$ -chief factor 119, 120, 132, 150
- $p$ -group 237, 281, 284, 289–291, 332
  - class of 158, 162
- $p$ -local formation 163, *see also*
  - $p$ -saturated formation
- $p$ -rank 119
- $p$ -saturated formation 161–163, 165, *see also*  $p$ -local formation
- $p$ -subgroup
  - strongly closed 114, 115, 116
- partially saturated formation VIII, 125
- partition 2, 22, 23, 25, 28, 39, 120, 121, 254, 257, 262, 264, 297
  - trivial 22
  - $U$ -invariant 23, 25, 27, 28, 34
- $\pi$ -complement 308
- $\pi$ -group 244, 245, 250, 279
  - class of 118, 328
  - soluble 257–259
    - class of 253
- preboundary 112, 113, 317
- precrown 45, 45, 48, 52, 63
- primitive group *see* group, primitive
- primitive pair 3, 29, 31, 32
  - of diagonal type 39
  - of type 2 32
    - with product action 31, 34, 35
    - with simple diagonal action 29, 31, 34
    - with twisted wreath product action 32, 35
- product
  - central 98, 330
  - class 88, 95, 111
  - direct 5, 7–10, 16, 21–23, 28, 31, 34, 39, 41, 62, 65, 90, 106, 108, 111, 112, 128, 140, 156, 175, 208, 212–214, 225, 254, 314, 317, 329, 349–352
    - restricted 348
  - Fitting 111, 324, 326, 327
  - formation 95, 111, 153, 158, 160, 162, 163
  - Gaschütz *see* product, formation
  - mutually permutable 329

- semidirect 6, 8, 17, 21, 44–46, 70, 93, 140, 143, 146, 166, 210, 221, 233, 247, 332, 333, 335
- subdirect 90, 112, 214
- totally permutable 329
- wreath 8, 16, 22, 26, 29, 31, 140, 210
  - natural 348
  - regular 39, 140, 200, 210, 211, 224, 230, 331, 332
  - twisted 17, 18, 32, 35, 224
- projector VIII, 99, 100, 101, 103–105, 109, 110, 118, 171, 176, 177, 179, 180, 182–187, 205, 206, 224, 229–231, 233, 235
- property of groups *see* class of groups
- pull-back 11, 12, 309
- $r''$ -subgroup *see* subgroup functor,  $r''$
- $r'$ -subgroup *see* subgroup functor,  $r'$
- $r$ -subgroup *see* subgroup functor,  $r$
- radical 111, 114, 118, 248, 250, 295, 351
  - $\mathfrak{E}\mathfrak{X}$ -radical 147
  - $\mathfrak{E}\mathfrak{J}$ -radical 126
  - Jacobson 5
  - soluble 96, 125, 147, 148
- representation
  - faithful 1–3, 8
  - modular 96
  - permutation 1, 8, 140, 210
  - primitive 2, 3
  - transitive 1–3, 12, 25, 27, 99, 210
- residual 94, 95, 171, 177, 179–188, 244, 245, 250, 269, 281, 285–288, 295–297, 300, 301, 306
  - nilpotent 281, 285, 287
  - soluble 201
- right coset 1, 12, 14, 31, 41, 140, 210
- right transversal 2, 11, 12, 17, 26, 310, 311
- root of unity
  - primitive 330
- $\check{S}$ -formation *see* formation, with the Shemetkov property
- satellite 96
- saturated formation VIII, 95–97, 99–101, 106, 107, 109, 118, 119, 123, 137, 144, 148, 153, 166, 169–171, 174–180, 182, 183, 188, 190, 192, 197, 198, 203, 204, 235, 236, 239, 244, 248–254, 257, 259–261, 263–269, 272–275, 277, 279, 281–284, 286, 296, 301, 303, 305–308, *see also* Fitting formation, saturated
- maximal contained in a class of groups 166
- subgroup-closed 244, 277, 281
- Schunck class VIII, 99, 100, 101, 102–110, 112, 118, 123, 169–171, 177, 192, 193, 197, 202, 205, 206, 224, 228–230
- section 5, 34, 41, 53, 62, 79, 111, 211, 220
  - complemented 62
  - key 330
  - Lockett 111, 112, 319, 320, 322, 323, 344
  - $S$ -head 352
  - simple 208, 213, 214
- series
  - central 330
- set
  - injective 115
  - JH-solid 52, 52, 53, 55, 57, 58, 60–66, 68–70, 73, 74, 194
  - solid VIII, 191, 192, 194, 195, 197–202
  - w-solid 190, 191, 192, 194–196, 202–204
  - weakly solid *see* set, w-solid
- socle 3, 5–8, 23–29, 31–35, 37–39, 45, 46, 61, 64, 65, 78, 98, 119, 126, 128, 134, 143, 144, 146, 147, 155, 156, 159, 163, 175, 191, 200, 215, 220–222, 224, 225, 231, 236, 252, 254, 273, 275, 277, 284, 288–290, 292, 303, 306
- solubly saturated formation VIII, 96, 97, 119, 125, 153, 154, 165, 166, 264, 272, 292, 293, 295–297, *see also* Baer-local formation; Fitting formation, solubly saturated
- stabiliser 1, 26, 29, 85, 99
- subformation 158, 162
- subgroup
  - CAP 194, 196, 340
  - Carter 99, 100, 114
  - chain *see* chain of subgroups

- characteristic 95, 145, 148
- conjugate 1, 5, 15, 18, 27, 39, 84, 98–100, 110, 170, 183, 188, 189, 194–196, 220, 230–232, 239, 284, 341, 342
- covering VIII, 100, 101, 103, 104, 110, 169, 180, 205, 206, 230, 337
- d-maximal 225
  - core-free 225
- derived 307
- diagonal 22, 31, 211, 214
- $\mathfrak{F}$ -Dnormal 264
- $\mathfrak{F}$ -hypercentral 178, 179
- $\mathcal{F}$ -maximal 114, 118
- $\mathfrak{F}$ -normal 236, 264
- $\mathfrak{F}$ -subnormal VIII, 235, 236, 237, 239, 244, 247–250, 254, 258–260, 262, 264–266, 272, 284, 300, 301, 303, 306, 307
- Fischer 110, 337
- Fitting 78, 98, 105, 139, 153, 176, 181, 182, 250, 287, 315, 330
  - generalised 97, 98, 104, 213, 224, 239, 315, 324, 325
- Frattini *see* Frattini subgroup
- Frattini-like 125, 144, 147, 148, *see*  $\mathfrak{X}$ -Frattini subgroup
- full diagonal 22, 23, 28, 31, 32, 34, 39
- functor *see* subgroup functor
- $\mathfrak{H}$ -d-maximal 225, 228
- $\mathfrak{H}$ -maximal 101, 104, 105, 118, 176, 179, 257, 265, 315, 322, 328, 350, 351
- $H$ -prefrattini 170
- $\mathfrak{H}$ -prefrattini 192, 193
- Hall 99, 100, 101, 110, 118, 153, 163, 218, 231, 233, 250, 257, 258, 268, 328, 339, 340
- K- $\mathfrak{F}$ -subnormal 236, 239, 248, 250, 262, 301, 305, 307
- $L$ -prefrattini 193, 195
- maximal 3, 18, 24, 25, 27–29, 31–35, 39–41, 44, 45, 50, 52–55, 57, 58, 60–63, 66, 70, 73, 75, 78, 79, 84, 101, 103, 108, 169, 174, 175, 179, 181, 191, 196–198, 200, 204, 236, 264, 268, 284, 328, 340, 343
  - conjugate subsystems 74, 84, 85
  - core-free 2–7, 25, 27–32, 34, 37, 39, 44, 61, 175, 176, 194, 224, 288
  - critical 78, 79–81, 83, 107, 108, 171, 204
  - crucial 176
  - frequent 37
  - $\mathfrak{H}$ -abnormal 108, 169, 175, 179, 186, 197
  - $\mathfrak{H}$ -critical 108, 169, 171–174, 176, 180
  - $\mathfrak{H}$ -normal 108, 191, 195, 203, 235, 236, 247
  - JH-solid set *see* set, JH-solid
  - monolithic 4, 52, 55, 57, 60–66, 68–70, 73, 74, 78–80, 108, 175, 176, 191, 192, 197, 198, 200
  - of diagonal type 39
  - of type 1 44, 74
  - of type 2 50, 74
  - of type 3 44, 50, 62
  - small 32, 34, 60
  - solid set *see* set, solid
  - subsystem 73, 74, 75, 77, 80, 84, 191–195
  - system VIII, 73, 74, 77, 78, 81, 83–85, 171–173, 175, 190–193, 195, 197, 198, 202–204
  - w-solid set *see* set, w-solid
  - weakly solid set *see* subgroup, maximal, w-solid set
- maximal normal 98, 200, 210, 217, 289
- minimal normal 3, 5–7, 9, 21, 25, 40–42, 44, 46, 61–64, 66, 91, 108, 126, 139, 147, 156, 174–176, 180, 205, 210, 211, 215, 220–222, 224, 225, 227, 254, 266, 288, 303
  - abelian 3, 4
  - complemented 3, 5, 6, 24, 31, 33, 38, 40, 199, 215
  - non-abelian 3, 4, 7
  - self-centralising 3, 5
- $\mathfrak{N}$ -subnormal 236
- normal
  - parafattini 198
  - profrattini 198
  - $\mathbf{X}$ -parafattini 198, 199, 202
  - $\mathbf{X}$ -profrattini 198, 199
- normally embedded 115, 343

- NTL-functor *see* subgroup
- NTL-functor
- of prefrattini type 73, 169, 191, 192, 194, 204
- of soluble type 205, 206, 208, 214, 215, 224
- $p$ -prefrattini 193
- prefrattini VIII, 73, 169, 170, 190, 191, 192, 194, 196, 197, 202, 204
- pronormal 84, 340–343
- strongly conjugate 239
- subnormal 98, 112, 113, 118, 206–209, 214, 215, 219, 223, 224, 235–237, 239, 243, 244, 247, 285–288, 295, 303, 315, 325, 329, 340–342, 352
- supplemented 3, 6
- Sylow 3, 5, 98, 100, 110, 166, 176, 187, 189, 206, 219, 237, 247, 273, 301, 328, 330, 349, 350
- $U$ -invariant 23, 25, 27, 28, 210, 211
- $\mathfrak{X}$ -Frattini 125, 147, 148
  - Förster 125, 144, 145–148
- $\mathbf{X}$ -Frattini 52, 197, 198
- $\mathbf{X}$ -parafprattini 199
- $\mathbf{X}$ -prefrattini 192, 195, 197, 202
- $\mathbf{X}$ -profrattini 199
- $(X, g)$ -pronormal 341, 342
- subgroup functor 206, 207, 208, 210–212, 215, 221, 224, 226, 229, 230, 236, 263, 264
  - inductive 211, 212, 229
  - inherited 206, 207, 210, 230, 236
  - NTL *see* subgroup NTL-functor
- $r$  206, 208, 211, 212, 214, 215, 222, 224–226, 230
- $r'$  206, 208, 211–213, 222, 224–226, 231
- $r''$  208, 211, 212, 214, 215
- $s$  205, 206, 208, 230, 231, 233, 266, 281
- $s_n$  206, 207, 224, 236, 237
- $s_p$  206
- $t$  206, 208, 210, 211, 214, 215, 217–221, 224–226, 230–233
- $t'$  206, 208, 210–213, 215, 222–224, 230, 231
- $t''$  208, 210–212
  - w-inherited 206, 207, 210–212, 219, 237, 263
  - weakly inductive 211, 212, 224
- subgroup NTL-functor 263, 264
- submodule 121, 291
- supplement 3, 5, 6, 18, 21, 22, 25, 27, 37, 38, 40, 41, 44, 45, 49, 50, 52, 53, 55, 63, 64, 70, 71, 73, 74, 78, 79, 108, 171, 179, 181, 220
  - monolithic 45, 49, 191
- Sylow subgroup *see* subgroup, Sylow
- system normaliser 99, 100, 169, 183, 185
  - absolute 183, 187
- $t''$ -subgroup *see* subgroup functor,  $t''$
- $t'$ -Schunck class 230
- $t'$ -subgroup *see* subgroup functor,  $t'$
- $t$ -Schunck class 230, 231
- $t$ -subgroup *see* subgroup functor,  $t$
- theorem
  - Baer VIII, 3, 4, 7, 125, 144, 148
  - Clifford 286
  - Frobenius 277
  - Gaschütz-Lubeseder-Schmid VIII, 96, 125, 144, 148
  - Jordan-Hölder 40, 41, 52, 53, 60, 61, 73
  - O’Nan-Scott 24, 25, 34
  - Odd Order 219
  - Orbit-Stabiliser 1
  - Schur-Zassenhaus 231
- trace 114, 345, 346, 349
- transversal *see* right transversal
- Wielandt operator 285
- $\mathfrak{X}$ -by- $\mathfrak{Y}$ -groups 88
- $\mathfrak{X}$ -chief factor 126
- $\mathbf{X}$ -crossing 54
- $\mathfrak{X}$ -formation function 126, 127–136, 138–140, 154, 269, 274, 293, 297, *see also*  $\mathfrak{X}$ -local definition
- $\mathfrak{X}$ -group 87
- $\mathfrak{X}$ -local definition 127, 129, 130, 133, 154, 295
  - canonical 132, 140, 141, 153, 166
  - full 130, 131, 134
  - integrated 130, 131, 132, 135, 154

- maximal 133, 134, 136, 138–140
- maximal integrated 131
- minimal *see*  $\mathfrak{X}$ -local definition, smallest
- smallest 128, 129
- $\mathfrak{X}$ -local formation 125, 126, 127–138, 140–143, 148, 149, 151, 153–160, 163, 165, 166, 269, 272, *see also*  $\mathfrak{X}$ -saturated formation
- generated by a class of groups 128, 154, 156, 159, 160
- smallest containing a class of groups 154
- $\mathbf{X}$ -precrown 52, *see also* precrown
- $\mathfrak{X}$ -saturated formation 125, 148, 149, 151–153, 161, 164–166, *see also*  $\mathfrak{X}$ -local formation
  - (F) 148, 149
  - (N) 148
- $\mathfrak{X}_\omega$ -saturated formation 161
- $\mathfrak{X}_p$ -chief factor 126, 128, 154
- $\mathfrak{Y}$ -chief factor 126, 152
- $\mathfrak{Y}_p$ -chief factor 126