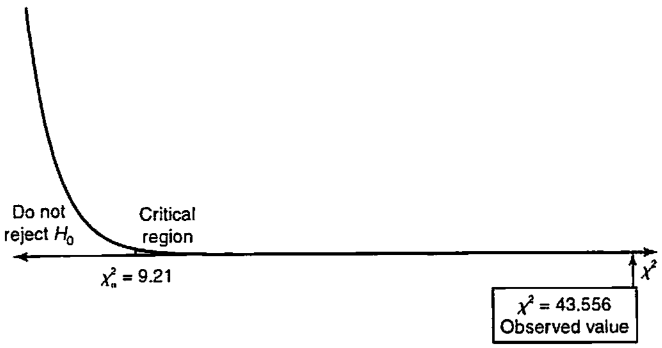
Chapter 11 Chi-Square and *F* Distributions

Section 11.1

1. H_0: Myers-Briggs preference and profession are independent.
   H_1: Myers-Briggs preference and profession are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(308-241.05)^2}{241.05} + \frac{(226-292.95)^2}{292.95} + \frac{(667-723.61)^2}{723.61} + \frac{(936-879.39)^2}{879.39} + \frac{(112-122.33)^2}{122.33} + \frac{(159-148.67)^2}{148.67}$$
$$= 43.5562$$

Since there are 3 rows and 2 columns, *d.f.* = (3 − 1)(2 − 1) = 2. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 9.21$.

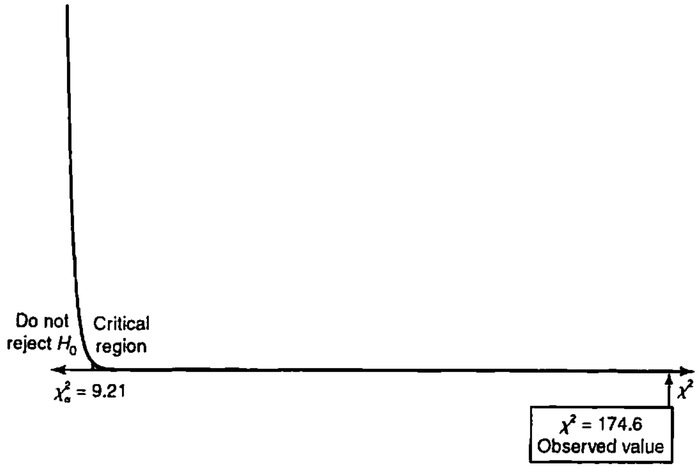


Since the sample statistic falls inside the critical region, we reject H_0. We conclude that Myers-Briggs preference and profession are not independent.

2. H_0: Myers-Briggs preference and profession are independent.
   H_1: Myers-Briggs preference and profession are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(114-238.39)^2}{238.39} + \frac{(420-295.61)^2}{295.61} + \frac{(785-715.62)^2}{715.62} + \frac{(818-887.37)^2}{887.37} + \frac{(176-120.98)^2}{120.98} + \frac{(95-150.02)^2}{150.02}$$
$$= 174.6$$

Since there are 3 rows and 2 columns, *d.f.* = (3 − 1)(2 − 1) = 2. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 9.21$.



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that Myers-Briggs preference and profession are not independent.

3. H_0: Site type and pottery type are independent.
   H_1: Site type and pottery type are not independent.

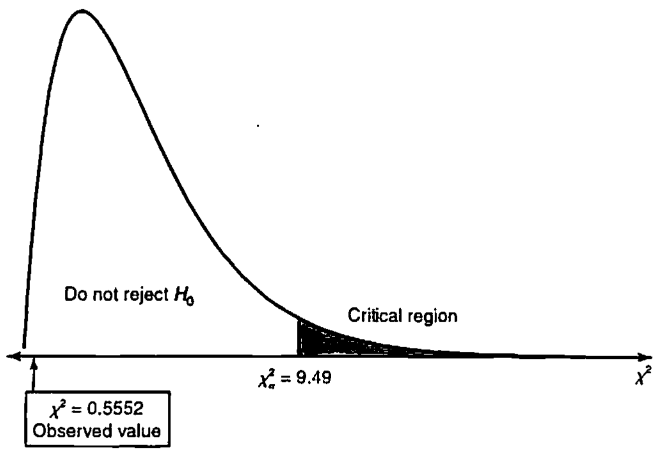
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(75-74.64)^2}{74.64} + \frac{(61-59.89)^2}{59.89} + \frac{(53-54.47)^2}{54.47} + \frac{(81-84.11)^2}{84.11} + \frac{(70-67.5)^2}{67.5}$$

$$+ \frac{(62-61.39)^2}{61.39} + \frac{(92-89.25)^2}{89.25} + \frac{(68-71.61)^2}{71.61} + \frac{(66-65.14)^2}{65.14}$$

$$= 0.5552$$

Since there are 3 rows and 3 columns, $d.f. = (3-1)(3-1) = 4$. For $\alpha = 0.05$, the critical value is $\chi^2_{0.05} = 9.49$.



Since the sample statistic falls outside the critical region, do not reject H_0. There is insufficient evidence to conclude that site type and pottery type are not independent.

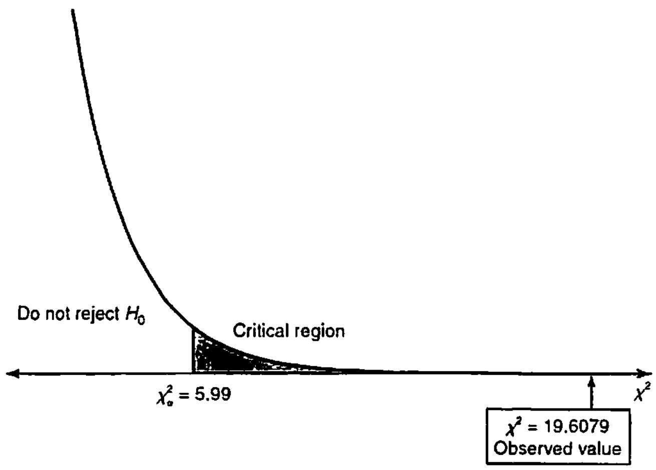
4. H_0: Ceremonial ranking and pottery type are independent.
   H_1: Ceremonial ranking and pottery type are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(242-242.61)^2}{242.61} + \frac{(26-25.39)^2}{25.39} + \frac{(658-636.41)^2}{636.41} + \frac{(45-66.59)^2}{66.59} + \frac{(371-391.98)^2}{391.98} + \frac{(62-41.02)^2}{41.02}$$

$$= 19.6079$$

Since there are 3 rows and 2 columns. $d.f. = (3-1)(2-1) = 2$. For $\alpha = 0.05$. the critical value is $\chi^2_{0.05} = 5.99$.



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that ceremonial ranking and pottery type are not independent.

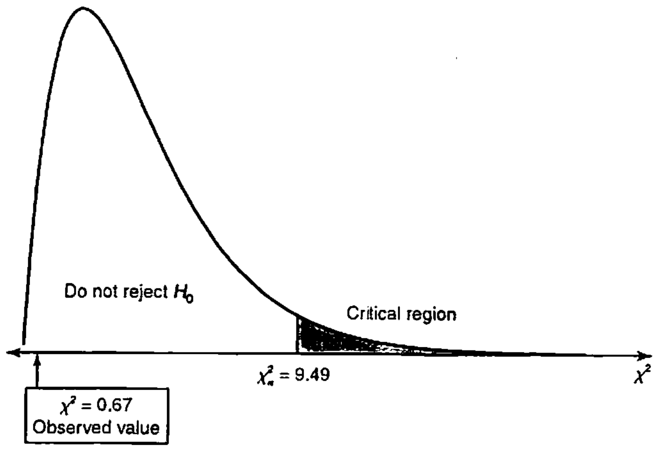
5. H_0: Age distribution and location are independent.
   H_1: Age distribution and location are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(13-14.08)^2}{14.08} + \frac{(13-12.84)^2}{12.84} + \frac{(15-14.08)^2}{14.08} + \frac{(10-11.33)^2}{11.33} + \frac{(11-10.34)^2}{10.34} + \frac{(12-11.33)^2}{11.33} + \frac{(34-31.59)^2}{31.59} + \frac{(28-28.82)^2}{28.82} + \frac{(30-31.59)^2}{31.59}$$

$$= 0.67$$

Since there are 3 rows and 3 columns, $d.f. = (3-1)(3-1) = 4$. For $\alpha = 0.05$, the critical value is $\chi^2_{0.05} = 9.49$.



Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that age distribution and location are not independent.

6. H_0: Type and career choice are independent.

   H_1: Type and career choice are not independent.

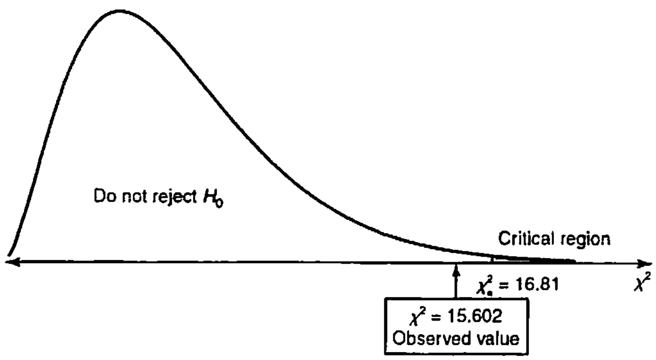
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(64-53.46)^2}{53.46} + \frac{(15-24.79)^2}{24.79} + \frac{(17-17.76)^2}{17.76} + \frac{(82-85.75)^2}{85.75} + \frac{(42-39.76)^2}{39.76} + \frac{(30-28.49)^2}{28.49}$$

$$+ \frac{(68-64.04)^2}{64.04} + \frac{(35-29.69)^2}{29.69} + \frac{(12-21.27)^2}{21.27} + \frac{(75-85.75)^2}{85.75} + \frac{(42-39.76)^2}{39.76} + \frac{(37-28.49)^2}{28.49}$$

$$= 15.602$$

Since there are 4 rows and 3 columns, $d.f. = (4-1)(3-1) = 6$. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 16.81$.

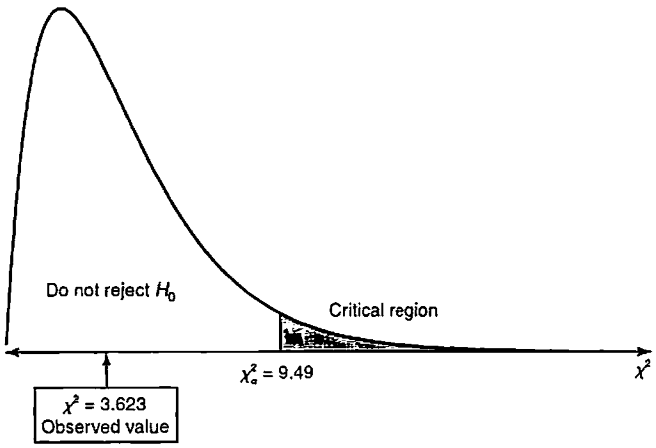


Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that type and career choice are not independent.

7. H_0: Ages of young adults and movie preferences are independent.
H_1: Ages of young adults and movie preferences are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(8-10.60)^2}{10.60} + \frac{(15-12.06)^2}{12.06} + \frac{(11-11.33)^2}{11.33} + \frac{(12-9.35)^2}{9.35} + \frac{(10-10.65)^2}{10.65} + \frac{(8-10.00)^2}{10.00} + \frac{(9-9.04)^2}{9.04} + \frac{(8-10.29)^2}{10.29} + \frac{(12-9.67)^2}{9.67}$$
$$= 3.623$$

Since there are 3 rows and 3 columns, $d.f. = (3-1)(3-1) = 4$. For $\alpha = 0.05$, the critical value is $\chi^2_{0.05} = 9.49$.

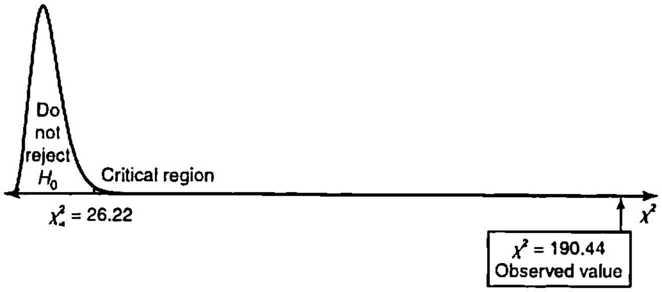


Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that ages of young adults and movie preferences are not independent.

8. H_0: Contribution and ethnic group are independent.
H_1: Contribution and ethnic group are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(310-441.42)^2}{441.42} + \frac{(715-569.96)^2}{569.96} + \frac{(201-244.61)^2}{244.61} + \frac{(105-86.87)^2}{86.87} + \frac{(42-30.13)^2}{30.13}$$
$$+ \frac{(619-501.86)^2}{501.86} + \frac{(511-648.01)^2}{648.01} + \frac{(312-278.10)^2}{278.10} + \frac{(97-98.77)^2}{98.77} + \frac{(22-34.26)^2}{34.26}$$
$$+ \frac{(402-439.17)^2}{439.17} + \frac{(624-567.06)^2}{567.06} + \frac{(217-243.36)^2}{243.36} + \frac{(88-86.43)^2}{86.43} + \frac{(35-29.98)^2}{29.98}$$
$$+ \frac{(544-492.54)^2}{492.54} + \frac{(571-635.97)^2}{635.97} + \frac{(309-272.93)^2}{272.93} + \frac{(79-96.93)^2}{96.93} + \frac{(29-33.62)^2}{33.62}$$
$$= 190.44$$

Since there are 4 rows and 5 columns, *d.f.* = (4 − 1)(5 − 1) = 12. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 26.22$.

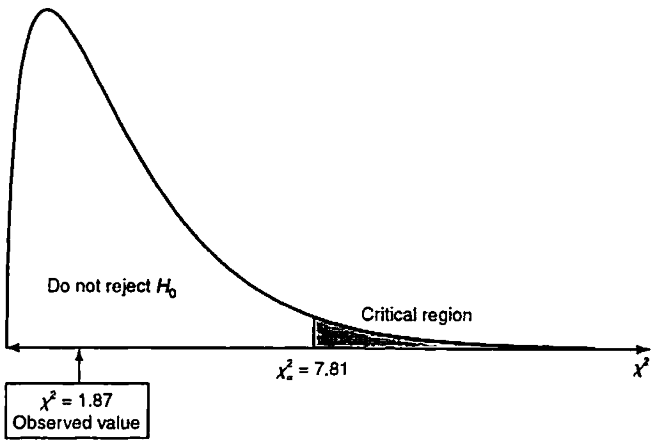


Since the sample statistic falls inside the critical region, we reject H_0. We conclude that contribution and ethnic group are not independent.

9. H_0: Ticket sales and type of billing are independent.
H_1: Ticket sales and type of billing are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(10-7.52)^2}{7.52} + \frac{(12-13.16)^2}{13.16} + \frac{(18-18.80)^2}{18.80} + \frac{(7-7.52)^2}{7.52} + \frac{(6-8.48)^2}{8.48} + \frac{(16-14.84)^2}{14.84} + \frac{(22-21.20)^2}{21.20} + \frac{(9-8.48)^2}{8.48}$$
$$= 1.87$$

Since there are 2 rows and 4 columns, *d.f.* = (2 − 1)(4 − 1) = 3. For $\alpha = 0.05$, the critical value is $\chi^2_{0.05} = 7.81$.



Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that ticket sales and type of billing are not independent.

10. H_0: Party affiliation and dollars spent are independent.

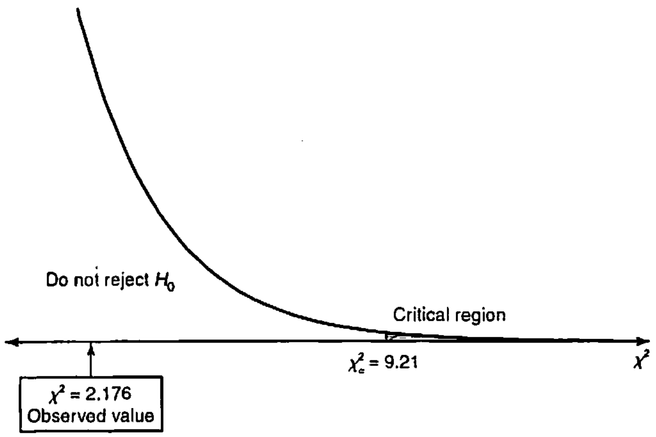
H_1: Party affiliation and dollars spent are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(8-9.78)^2}{9.78} + \frac{(15-16.63)^2}{16.63} + \frac{(22-18.59)^2}{18.59} + \frac{(12-10.22)^2}{10.22} + \frac{(19-17.37)^2}{17.37} + \frac{(16-19.41)^2}{19.41}$$

$$= 2.176$$

Since there are 2 rows and 3 columns, $d.f. = (2-1)(3-1) = 2$. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 9.21$.



Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that party affiliation and dollars spent are not independent.

11. H_0: Stone tool construction material and site are independent.

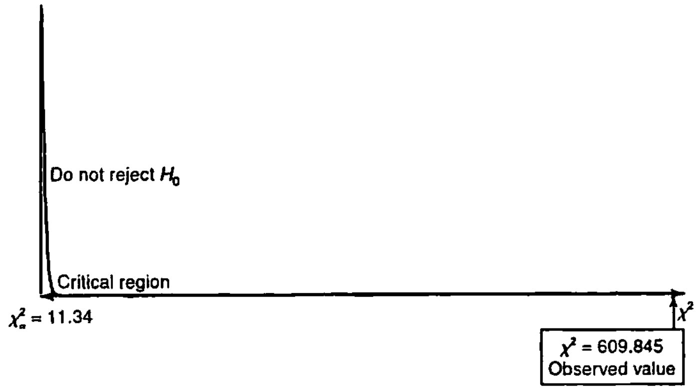
H_1: Stone tool construction material and site are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(3657-4099.96)^2}{4099.96} + \frac{(1238-795.04)^2}{795.04} + \frac{(497-473.23)^2}{473.23} + \frac{(68-91.77)^2}{91.77} + \frac{(3606-3214.64)^2}{3214.64} + \frac{(232-623.36)^2}{623.36} + \frac{(357-329.17)^2}{329.17} + \frac{(36-63.83)^2}{63.83}$$

$$= 609.845$$

Since there are 4 rows and 2 columns, *d.f.* = (4 − 1)(2 − 1) = 3. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 11.34$.



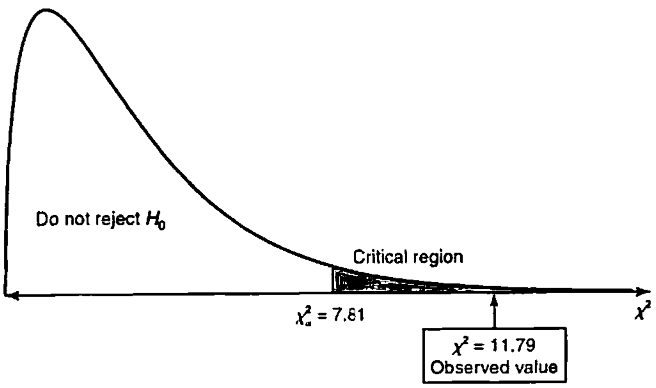
Since the sample statistic falls inside the critical region, we reject H_0. We conclude that stone tool construction material and site are not independent.

Section 11.2

1. H_0: The distributions are the same.
   H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(47-32.76)^2}{32.76} + \frac{(75-61.88)^2}{61.88} + \frac{(288-305.31)^2}{301.31} + \frac{(45-55.06)^2}{55.06}$$
$$= 11.79$$

d.f. = (number of *E* entries) − 1 = 4 − 1 = 3. The critical value is $\chi^2_{0.05} = 7.81$.

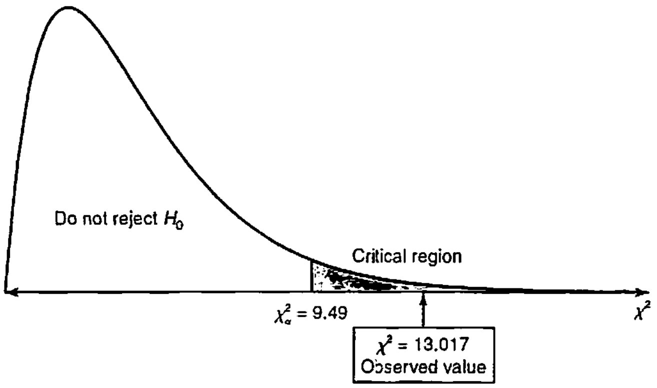


Since the sample statistic falls inside the critical region, we reject H_0. We conclude the distributions are different.

2. H_0: The distributions are the same.
H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(102-106.86)^2}{106.86} + \frac{(112-119.19)^2}{119.19} + \frac{(33-36.99)^2}{36.99} + \frac{(96-102.75)^2}{102.75} + \frac{(68-45.21)^2}{45.21}$$
$$= 13.017$$

$d.f.$ = (number of E entries) − 1 = 5 − 1 = 4. The critical value is $\chi^2_{0.05} = 9.49$.

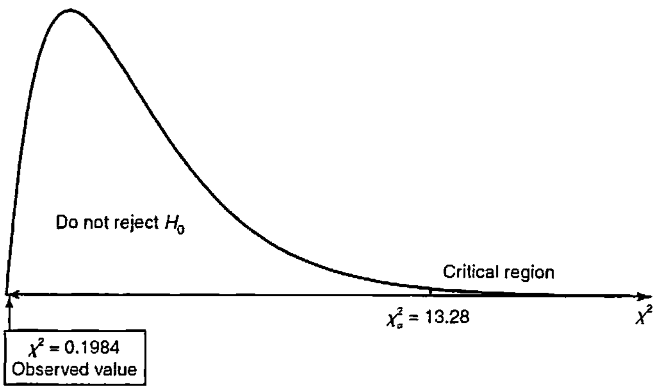


Since the sample statistic falls inside the critical region, we reject H_0. We conclude the distributions are different.

3. H_0: The distributions are the same.
H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(906-910.92)^2}{910.92} + \frac{(162-157.52)^2}{157.52} + \frac{(168-169.40)^2}{169.40} + \frac{(197-194.67)^2}{194.67} + \frac{(53-53.50)^2}{53.50}$$
$$= 0.1984$$

$d.f.$ = (number of E entries) − 1 = 5 − 1 = 4. The critical value is $\chi^2_{0.01} = 13.28$.

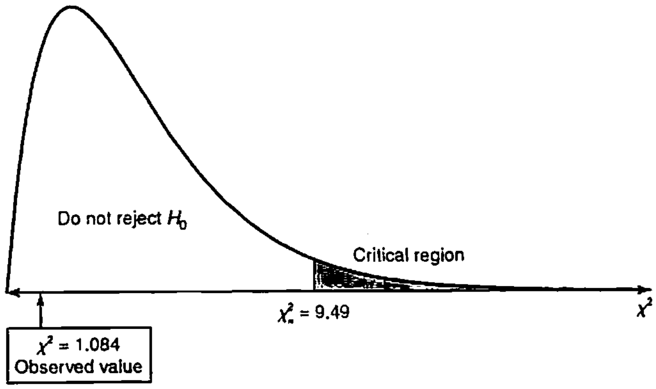


Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that the distributions are different.

4. H_0: The distributions are the same.
   H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(102-102.40)^2}{102.40} + \frac{(125-123.84)^2}{123.84} + \frac{(43-38.40)^2}{38.40} + \frac{(27-29.76)^2}{29.76} + \frac{(23-25.60)^2}{25.60}$$
$$= 1.084$$

d.f. = (number of *E* entries) − 1 = 5 − 1 = 4. The critical value is $\chi^2_{0.05} = 9.49$.

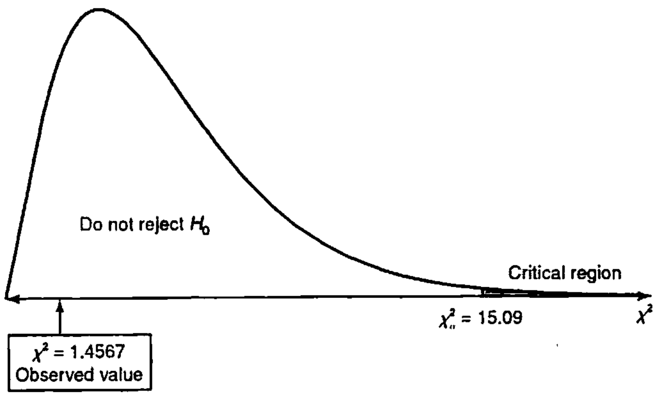


Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that the distributions are different.

5. (a) Essay.
   (b) H_0: The distributions are the same.
   H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(16-14.57)^2}{14.57} + \frac{(78-83.08)^2}{83.08} + \frac{(212-210.80)^2}{210.80} + \frac{(221-210.80)^2}{210.80}$$
$$+ \frac{(81-83.08)^2}{83.08} + \frac{(12-14.57)^2}{14.57}$$
$$= 1.4567$$

$d.f.$ = (number of E entries) − 1 = 6 − 1 = 5. The critical value is $\chi^2_{0.01} = 15.09$.



Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that the distribution is not normal.

6. (a) Essay.

(b) H_0: The distributions are the same.

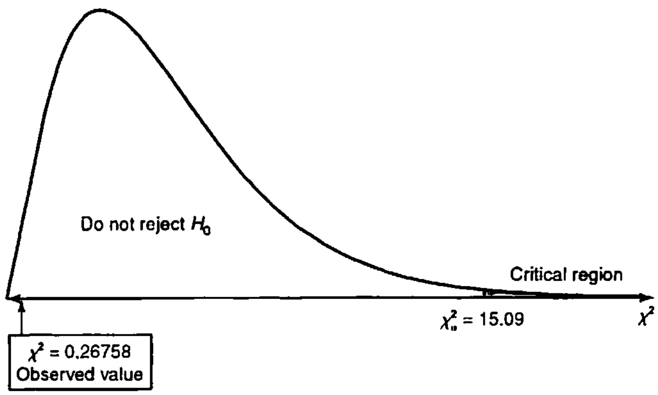
H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(14-14.57)^2}{14.57} + \frac{(86-83.08)^2}{83.08} + \frac{(207-210.80)^2}{210.80} + \frac{(215-210.80)^2}{210.80} + \frac{(83-83.08)^2}{83.08} + \frac{(15-14.57)^2}{14.57}$$

$$= 0.26758$$

$d.f.$ = (number of E entries) − 1 = 6 − 1 = 5. The critical value is $\chi^2_{0.01} = 15.09$.

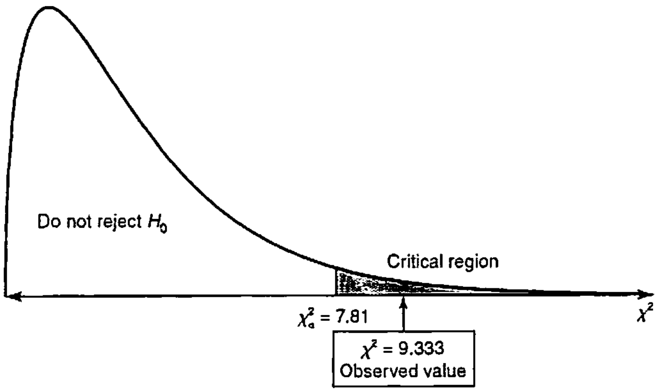


Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that the distribution is not normal.

7. H_0: The distributions are the same.
   H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(120-150)^2}{150} + \frac{(85-75)^2}{75} + \frac{(220-200)^2}{200} + \frac{(75-75)^2}{75}$$
$$= 9.333$$

d.f. = (number of *E* entries) − 1 = 4 − 1 = 3. The critical value is $\chi^2_{0.05} = 7.81$.

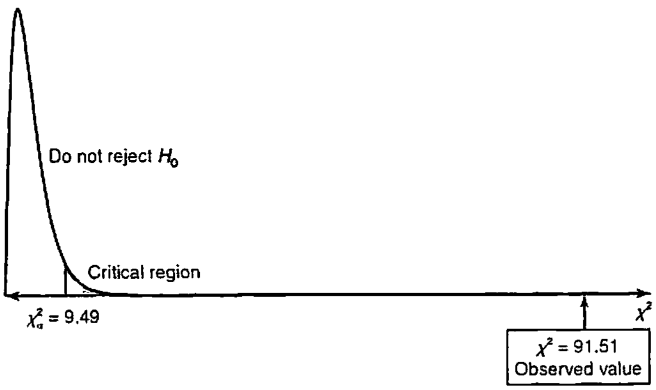


Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the fish distribution has changed.

8. H_0: The distributions are the same.
   H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(1210-1349.44)^2}{1349.44} + \frac{(956-1054.25)^2}{1054.25} + \frac{(940-843.40)^2}{843.40} + \frac{(814-632.55)^2}{632.55} + \frac{(297-337.36)^2}{337.36}$$
$$= 91.51$$

d.f. = (number of *E* entries) − 1 = 5 − 1 = 4. The critical value is $\chi^2_{0.05} = 9.49$.

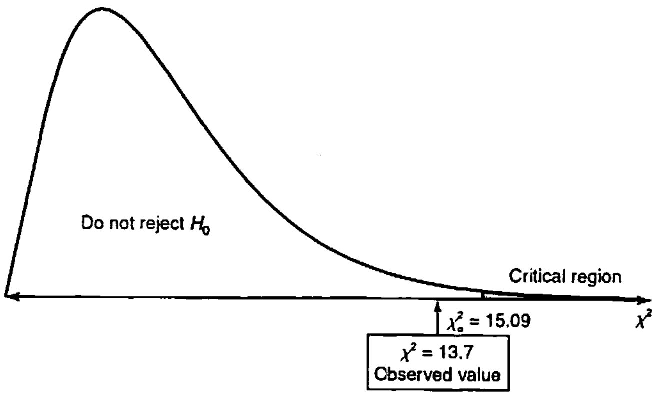


Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the distributions are different.

9. H_0: The distributions are the same.
H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(127-121.50)^2}{121.50} + \frac{(40-36.45)^2}{36.45} + \frac{(480-461.70)^2}{461.70}$$
$$+ \frac{(502-498.15)^2}{498.15} + \frac{(56-72.90)^2}{72.90} + \frac{(10-24.30)^2}{24.30}$$
$$= 13.7$$

$d.f.$ = (number of E entries) − 1 = 6 − 1 = 5. The critical value is $\chi^2_{0.01} = 15.09$.

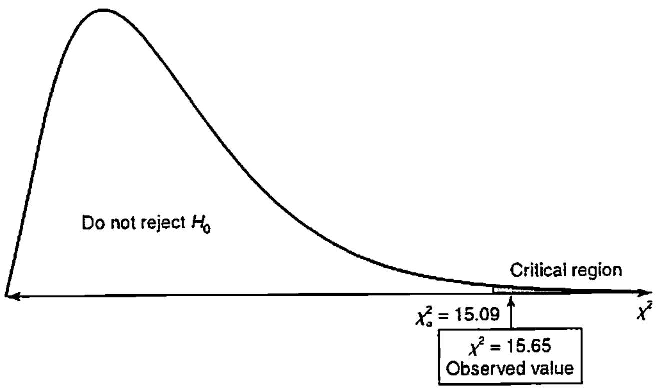


Since the sample statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that the distributions are different.

10. H_0: The distributions are the same.
H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
$$= \frac{(88-62.28)^2}{62.28} + \frac{(135-150.51)^2}{150.51} + \frac{(52-57.09)^2}{57.09}$$
$$+ \frac{(40-51.90)^2}{51.90} + \frac{(76-72.66)^2}{72.66} + \frac{(128-124.56)^2}{124.56}$$
$$= 15.65$$

$d.f.$ = (number of E entries) − 1 = 6 − 1 = 5. The critical value is $\chi^2_{0.01} = 15.09$.



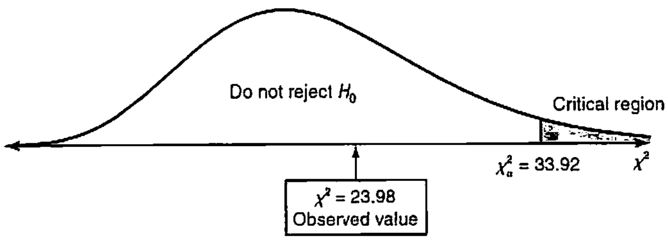
Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the distributions are different.

Section 11.3

1. H_0: $\sigma^2 = 42.3$
   H_1: $\sigma^2 > 42.3$

   Since > is in H_1, a right-tailed test is used.
   For $d.f. = 23 - 1 = 22$, the critical value is $\chi^2_{0.05} = 33.92$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(23-1)46.1}{42.3} = 23.98$$



Since the sample statistic falls outside the critical region, we do not reject H_0. We have insufficient evidence to conclude that the variance in the new section is greater than 42.3.

For $d.f. = 22$ and $\alpha = \frac{1-0.95}{2} = 0.025$, $\chi^2_U = 36.78$.

For $d.f. = 22$ and $\alpha = \frac{1+0.95}{2} = 0.975$, $\chi^2_L = 10.98$.

The 95% confidence interval for σ^2 is

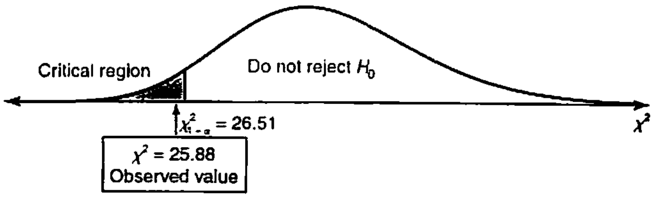
$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(23-1)46.1}{36.78} < \sigma^2 < \frac{(23-1)46.1}{10.98}$$

$$27.57 < \sigma^2 < 92.37$$

2. H_0: $\sigma^2 = 5.1$
H_1: $\sigma^2 < 5.1$
Since < is in H_1, a left-tailed test is used.
For $d.f. = 41 - 1 = 40$, the critical value is $\chi^2_{0.95} = 26.51$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(41-1)3.3}{5.1} = 25.88$$



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the current variance is less than 5.1.

For $d.f. = 40$ and $\alpha = \frac{1-0.90}{2} = 0.05$, $\chi^2_U = 55.76$.

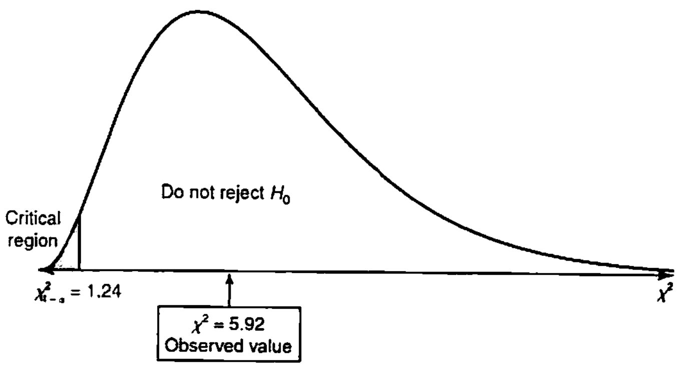
For $d.f. = 40$ and $\alpha = \frac{1+0.90}{2} = 0.95$. $\chi^2_L = 26.51$.

The 90% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$
$$\frac{(41-1)3.3}{55.76} < \sigma^2 < \frac{(41-1)3.3}{26.51}$$
$$2.37 < \sigma^2 < 4.98$$

3. H_0: $\sigma^2 = 136.2$
H_1: $\sigma^2 < 136.2$
Since < is in H_1. a left-tailed test is used.
For $d.f. = 8 - 1 = 7$, the critical value is $\chi^2_{0.99} = 1.24$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(8-1)115.1}{136.2} = 5.92$$



Since the sample statistic falls outside the critical region, we do not reject H_0. We have insufficient evidence to conclude that the recent variance for number of mountain-climber deaths is less than 136.1.

For $d.f. = 7$ and $\alpha = \frac{1-0.90}{2} = 0.05$, $\chi^2_U = 14.07$.

For $d.f. = 7$ and $\alpha = \frac{1+0.90}{2} = 0.95$, $\chi^2_L = 2.17$.

The 90% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(8-1)115.1}{14.07} < \sigma^2 < \frac{(8-1)115.1}{2.17}$$

$$57.26 < \sigma^2 < 371.29$$

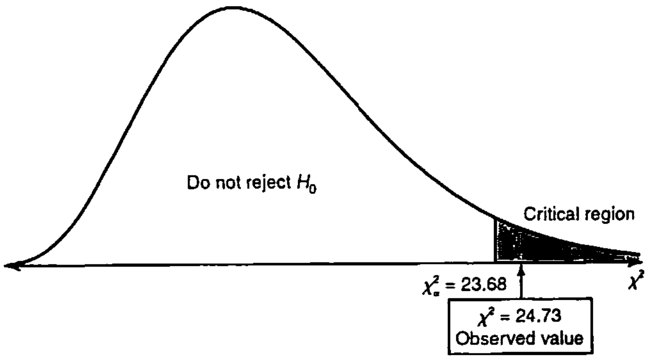
4. H_0: $\sigma^2 = 47.1$

H_1: $\sigma^2 > 47.1$

Since > is in H_1, a right-tailed test is used.

For $d.f. = 15 - 1 = 14$, the critical value is $\chi^2_{0.05} = 23.68$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(15-1)83.2}{47.1} = 24.73$$



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the variance for colleges and universities in Kansas is greater than 47.1.

For $d.f. = 14$ and $\alpha = \frac{1-0.95}{2} = 0.025$, $\chi^2_U = 26.12$.

For $d.f. = 14$ and $\alpha = \frac{1+0.95}{2} = 0.975$, $\chi^2_L = 5.63$.

The 95% confidence interval for σ^2 is

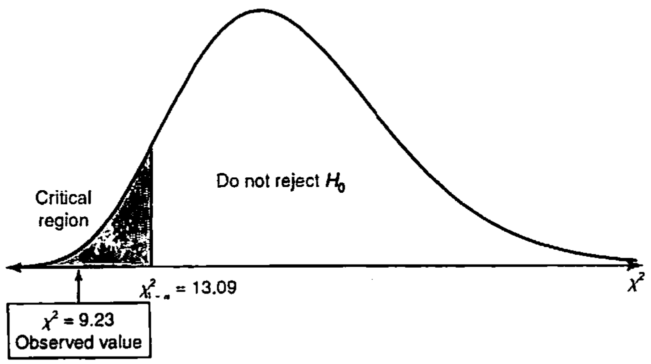
$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(15-1)83.2}{26.12} < \sigma^2 < \frac{(15-1)83.2}{5.63}$$

$$44.59 < \sigma^2 < 206.89$$

5. H_0: $\sigma^2 = 9$
H_1: $\sigma^2 < 9$
Since < is in H_1, a left-tailed test is used.
For $d.f. = 24 - 1 = 23$, the critical value is $\chi^2_{0.95} = 13.09$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(1.9)^2}{3^2} = 9.23$$



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the new typhoid shot has a smaller variance of protection times.

For $d.f. = 23$ and $\alpha = \frac{1-0.90}{2} = 0.05$, $\chi^2_U = 35.17$.

For $d.f. = 23$ and $\alpha = \frac{1+0.90}{2} = 0.95$, $\chi^2_L = 13.09$.

The 90% confidence interval for σ is

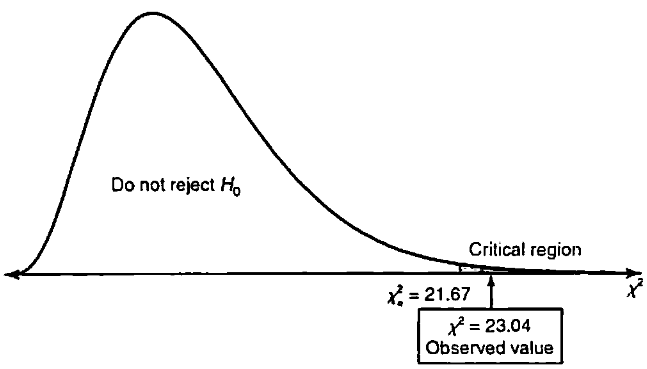
$$\sqrt{\frac{(n-1)s^2}{\chi^2_U}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(24-1)(1.9)^2}{35.17}} < \sigma < \sqrt{\frac{(24-1)(1.9)^2}{13.09}}$$

$$1.54 < \sigma < 2.52$$

6. H_0: $\sigma^2 = 225$
H_1: $\sigma^2 > 225$
Since > is in H_1, a right-tailed test is used.
For $d.f. = 10 - 1 = 9$, the critical value is $\chi^2_{0.01} = 21.67$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)(24)^2}{(15)^2} = 23.04$$



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that the variance is larger than that stated in his journal.

For $d.f. = 9$ and $\alpha = \frac{1-0.95}{2} = 0.025$, $\chi_U^2 = 19.02$.

For $d.f. = 9$ and $\alpha = \frac{1+0.95}{2} = 0.975$, $\chi_L^2 = 2.70$.

The 95% confidence interval for σ is

$$\sqrt{\frac{(n-1)s^2}{\chi_U^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

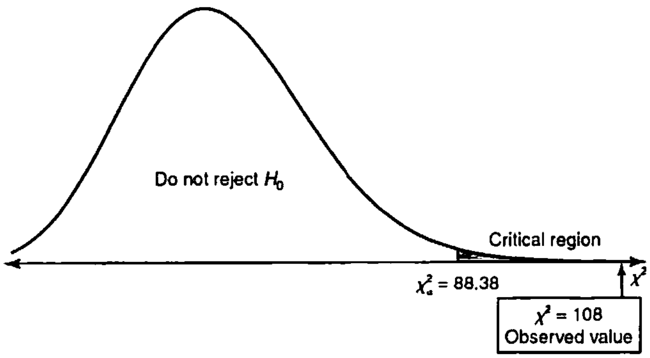
$$\sqrt{\frac{(10-1)(24)^2}{19.02}} < \sigma < \sqrt{\frac{(10-1)(24)^2}{2.70}}$$

$$16.5 < \sigma < 43.8$$

7. H_0: $\sigma^2 = 0.15$
   H_1: $\sigma^2 > 0.15$

   Since > is in H_1, a right-tailed test is used.
   For $d.f. = 61 - 1 = 60$, the critical value is $\chi_{0.01}^2 = 88.38$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(61-1)0.27}{0.15} = 108$$



Since the sample statistic falls inside the critical region, we reject H_0. We conclude that all the engine blades must be replaced (i.e., the variance exceeds 0.15).

For $d.f. = 60$ and $\alpha = \frac{1-0.90}{2} = 0.05$, $\chi_U^2 = 79.08$.

For $d.f. = 60$ and $\alpha = \frac{1+0.90}{2} = 0.95$, $\chi_L^2 = 43.19$.

The 90% confidence interval for σ is

$$\sqrt{\frac{(n-1)s^2}{\chi_U^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

$$\sqrt{\frac{(61-1)0.27}{79.08}} < \sigma < \sqrt{\frac{(61-1)0.27}{43.29}}$$

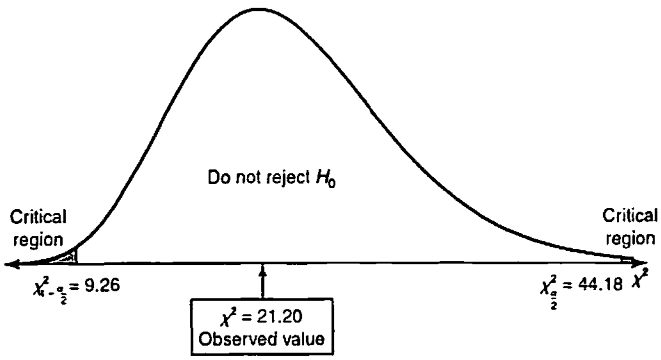
$$0.45 < \sigma < 0.61$$

8. (a) $\left.\begin{array}{l}H_0\colon \sigma^2 = 5625\\ H_1\colon \sigma^2 \neq 5625\end{array}\right\}$ these hypotheses are equivalent to $\left\{\begin{array}{l}H_0\colon \sigma = 75\\ H_1\colon \sigma \neq 75\end{array}\right.$

Since ≠ is in H_1, a two-tailed test is used.

For *d.f.* = 24 − 1 = 23, the critical values are $\chi^2_{0.005} = 44.18$ and $\chi^2_{0.995} = 9.26$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(72)^2}{(75)^2} = 21.20$$



Since the sample statistic falls outside both critical regions, do not reject H_0. There is insufficient evidence to conclude that the population standard deviation for the new examination is different from 75 ($\sigma^2 = 75^2 = 5625$).

(b) For *d.f.* = 23 and $\alpha = \frac{1-0.99}{2} = 0.005$, $\chi^2_U = 44.18$.

For *d.f.* = 23 and $\alpha = \frac{1+0.99}{2} = 0.995$, $\chi^2_L = 9.26$.

The 99% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(24-1)(72)^2}{44.18} < \sigma^2 < \frac{(24-1)(72)^2}{9.26}$$

$$2698.78 < \sigma^2 < 12{,}876.03$$

(c) The 99% confidence interval for σ is

$$\sqrt{\frac{(n-1)s^2}{\chi^2_U}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{2698.78} < \sigma < \sqrt{12{,}876.03}$$

$$51.95 < \sigma < 113.47$$

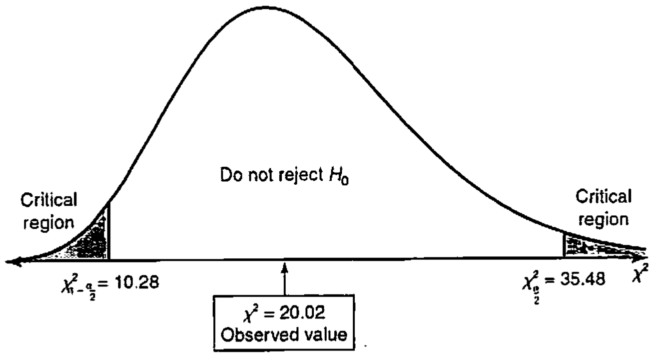
9. (a) H_0: $\sigma^2 = 15$

H_1: $\sigma^2 \neq 15$

Since $\neq$ is in H_1, a two-tailed test is used.

For $d.f. = 22 - 1 = 21$, the critical values are $\chi^2_{0.025} = 35.48$ and $\chi^2_{0.975} = 10.28$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(22-1)(14.3)}{15} = 20.02$$



Since the sample statistic falls outside both critical regions, do not reject H_0. There is insufficient evidence to conclude that the population variance is different from 15.

(b) For $d.f. = 21$ and $\alpha = \frac{1-0.90}{2} = 0.05$, $\chi^2_U = 32.67$.

For $d.f. = 21$ and $\alpha = \frac{1+0.90}{2} = 0.95$, $\chi^2_L = 11.59$.

The 90% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_U} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{(22-1)(14.3)}{32.67} < \sigma^2 < \frac{(22-1)(14.3)}{11.59}$$

$$9.19 < \sigma^2 < 25.91$$

(c) The 90% confidence interval for σ is

$$\sqrt{\frac{(n-1)s^2}{\chi^2_U}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{9.19} < \sigma < \sqrt{25.91}$$

$$3.03 < \sigma < 5.09$$

Section 11.4

1. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 > \sigma_2^2$

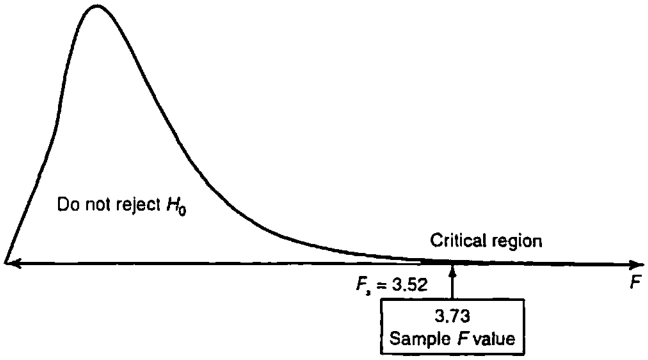
   Since > is in H_1, a right-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 0.332$ is larger than $s^2 \approx 0.089$, we designate Population I as the first plot.

$$F = \frac{s_1^2}{s_2^2} = \frac{0.332}{0.089} = 3.73$$

   Using $d.f._N = n_1 - 1 = 16 - 1 = 15$, and $d.f._D = n_2 - 1 = 16 - 1 = 15$, and $\alpha = 0.01$, the critical F value is 3.52.



   Since the sample F statistic falls inside the critical region, we reject H_0. We conclude that the variance in annual wheat production from the first plot is greater than that from the second plot.

2. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 \neq \sigma_2^2$

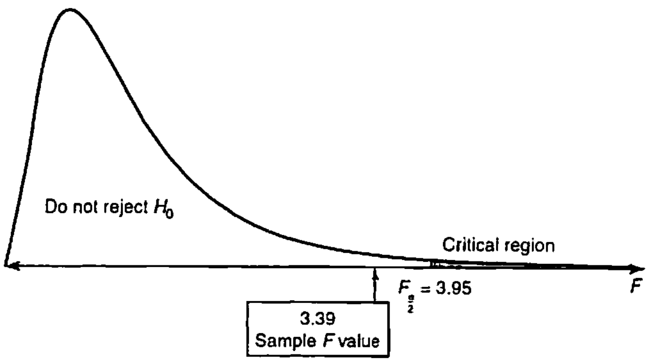
   Since ≠ is in H_1, a two-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 1.078$ is larger than $s^2 \approx 0.318$, we designate Population I as the second plot.

$$F = \frac{s_1^2}{s_2^2} = \frac{1.078}{0.318} = 3.39$$

   Using $d.f._N = n_1 - 1 = 8 - 1 = 7$, and $d.f._D = n_2 - 1 = 11 - 1 = 10$, and $\alpha = 0.05$, the critical $F_{\frac{\alpha}{2}}$ value is 3.95.



Since the sample F statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that the variance in annual wheat straw production from the first plot is different from the variance of wheat straw production from the second plot.

3. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 \neq \sigma_2^2$

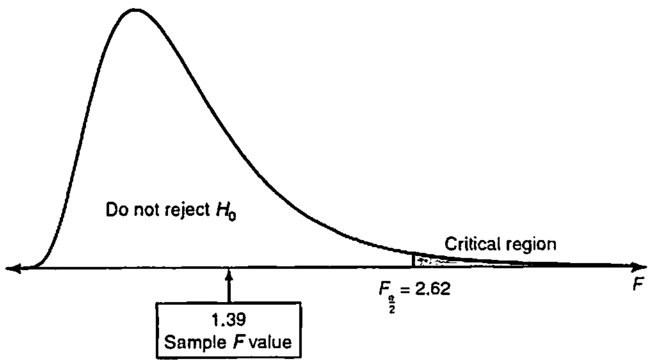
   Since $\neq$ is in H_1, a two-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 1.786$ is larger than $s^2 \approx 1.285$, we designate Population 1 as France.

$$F = \frac{s_1^2}{s_2^2} = \frac{1.786}{1.285} = 1.390$$

   Using $d.f._N = n_1 - 1 = 21 - 1 = 20$, and $d.f._D = n_2 - 1 = 18 - 1 = 17$, and $\alpha = 0.05$, the critical $F_{\frac{\alpha}{2}}$ value is 2.62.



   Since the sample F statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude a significant difference exists in the population variances. There is no significant difference in the volatility of corporate productivity of large companies in France and in Germany.

4. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 > \sigma_2^2$

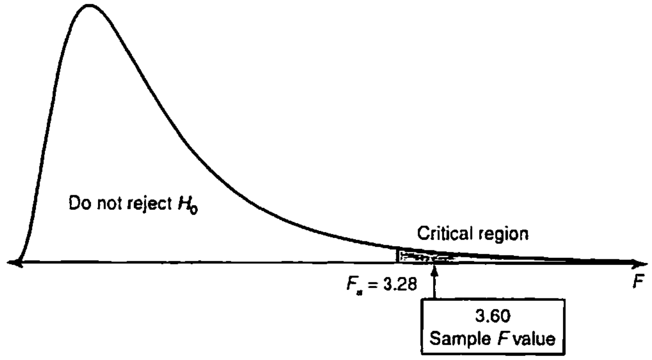
   Since > is in H_1, a right-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 2.247$ is larger than $s^2 \approx 0.624$, we designate Population 1 as South Korea.

$$F = \frac{s_1^2}{s_2^2} = \frac{2.247}{0.624} = 3.60$$

   Using $d.f._N = n_1 - 1 = 13 - 1 = 12$, and $d.f._D = n_2 - 1 = 9 - 1 = 8$, and $\alpha = 0.05$, the critical F value is 3.28.



Since the sample F statistic falls inside the critical region, we reject H_0. We conclude the population variance of percentage yield on assets for South Korean companies is higher than for companies in Sweden. The volatility of corporate productivity of large companies is greater for South Korea than for Sweden.

5. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 > \sigma_2^2$

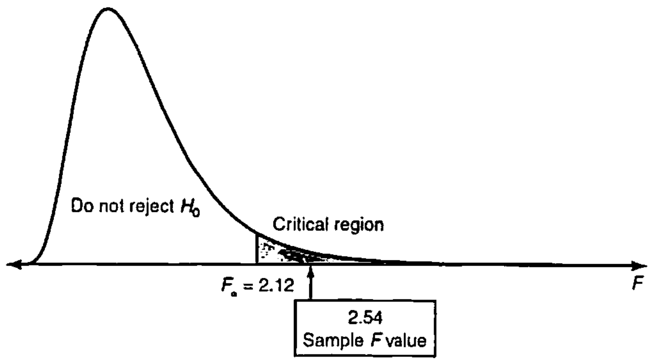
   Since > is in H_1, a right-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 348.43$ is larger than $s^2 \approx 137.31$, we designate Population I as aggressive growth.

$$F = \frac{s_1^2}{s_2^2} = \frac{348.43}{137.31} = 2.54$$

   Using $d.f._N = n_1 - 1 = 21 - 1 = 20$, and $d.f._D = n_2 - 1 = 21 - 1 = 20$, and $\alpha = 0.05$, the critical F value is 2.12.



   Since the sample F statistic falls inside the critical region, we reject H_0. We conclude the population variance for mutual funds holding aggressive growth small stocks is larger than that for funds holding value stocks. The smaller variance for funds holding value stocks implies they are more reliable investments.

6. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 \neq \sigma_2^2$

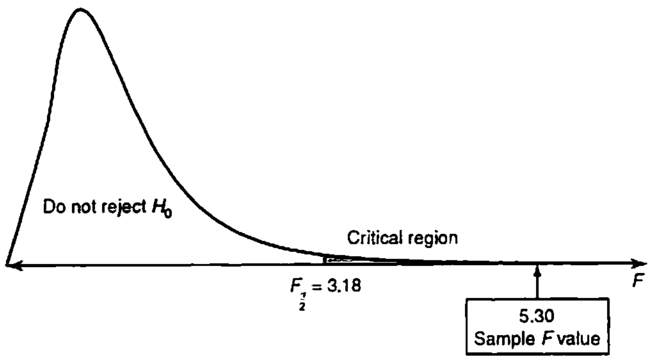
   Since $\neq$ is in H_1, a two-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 72.06$ is larger than $s^2 \approx 13.59$, we designate Population I as intermediate-term bonds.

$$F = \frac{s_1^2}{s_2^2} = \frac{72.06}{13.59} = 5.30$$

   Using $d.f._N = n_1 - 1 = 16 - 1 = 15$, and $d.f._D = n_2 - 1 = 13 - 1 = 12$, and $\alpha = 0.05$, the critical $F_{\frac{\alpha}{2}}$ value is 3.18.



   Since the sample F statistic falls inside the critical region, we reject H_0. We conclude that the population variance for annual percentage return of mutual funds holding short-term government bonds is different from the population variance for mutual funds holding intermediate-term corporate bonds. That the variances are different says only that the reliability of returns differs fund to fund. If we resort to data snooping, we would reach the conclusion, based on the sample variances, that government bond returns are more reliable.

7. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 \neq \sigma_2^2$

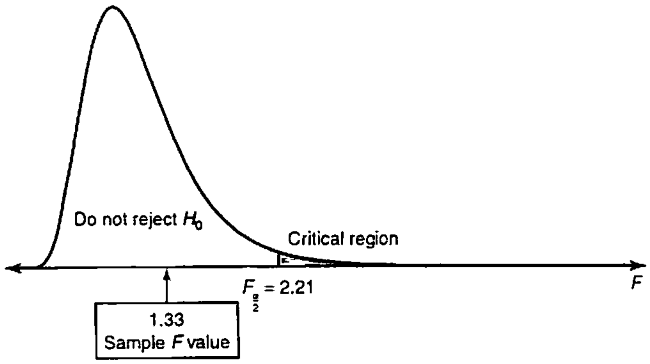
   Since $\neq$ is in H_1, a two-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 51.4$ is larger than $s^2 \approx 38.6$, we designate Population I as the new system.

$$F = \frac{s_1^2}{s_2^2} = \frac{51.4}{38.6} = 1.33$$

   Using $d.f._N = n_1 - 1 = 31 - 1 = 30$, and $d.f._D = n_2 - 1 = 25 - 1 = 24$, and $\alpha = 0.05$, the critical $F_{\frac{\alpha}{2}}$ value is 2.21.



Since the sample F statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude there is a difference in the population variance of gasoline consumption for the two injection systems. There is no difference in fuel consumption consistency for the two systems.

8. H_0: $\sigma_1^2 = \sigma_2^2$

   H_1: $\sigma_1^2 > \sigma_2^2$

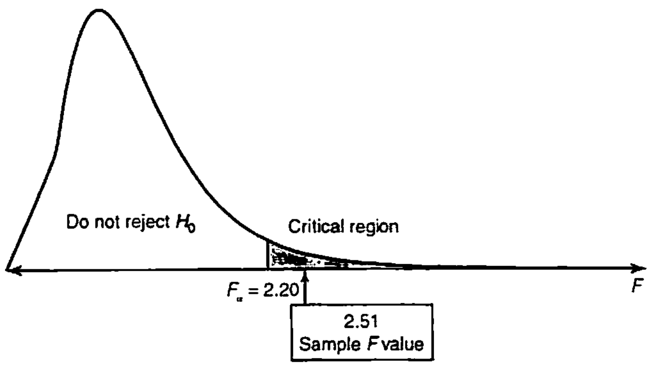
   Since > is in H_1, a right-tailed test is used.

   The populations follow independent normal distributions. The samples are random samples from each population.

   Since $s^2 \approx 12.8$ is larger than $s^2 \approx 5.1$, we designate Population I as the old thermostat.

$$F = \frac{s_1^2}{s_2^2} = \frac{12.8}{5.1} = 2.51$$

   Using $d.f._N = n_1 - 1 = 16 - 1 = 15$, and $d.f._D = n_2 - 1 = 21 - 1 = 20$, and $\alpha = 0.05$, the critical F value is 2.20.



Since the sample F statistic falls inside the critical region, we reject H_0. We conclude that the population variance of the old thermostat temperature readings is larger than that for the new thermostat. The temperature readings from the old thermostat are less dependable.

Section 11.5

1. H_0: $\mu_1 = \mu_2 = \mu_3$

   H_1: Not all the means are equal.

Site I	Site II	Site III
$n = 7$	$n = 4$	$n = 6$
$\sum x_1 = 286$	$\sum x_2 = 164$	$\sum x_3 = 176$
$\sum x_1^2 = 15{,}312$	$\sum x_2^2 = 8354$	$\sum x_3^2 = 7450$
$SS_1 = 3626.857$	$SS_2 = 1630$	$SS_3 = 2287.33\overline{3}$

$$\sum x_{TOT} = 286 + 164 + 176 = 626$$
$$\sum x_{TOT}^2 = 15{,}312 + 8354 + 7450 = 31{,}116$$
$$N = 7 + 4 + 6 = 17$$
$$k = 3$$
$$SS_{TOT} = \sum x_{TOT}^2 - \frac{(\sum x_{TOT})^2}{N} = 31{,}116 - \frac{(626)^2}{17} = 8064.470$$
$$SS_{BET} = \sum_{\text{all groups}} \left(\frac{(\sum x_i)^2}{n_i} \right) - \frac{(\sum x_{TOT})^2}{N}$$
$$= \frac{(286)^2}{7} + \frac{(164)^2}{4} + \frac{(176)^2}{6} - \frac{(626)^2}{17} = 520.280$$
$$SS_W = SS_1 + SS_2 + SS_3 = 3626.857 + 1630 + 2287.333 = 7544.190$$

Check that $SS_{TOT} = SS_{BET} + SS_W$: $8064.470 = 520.280 + 7544.190$

$$d.f._{BET} = k - 1 = 3 - 1 = 2$$
$$d.f._W = N - k = 17 - 3 = 14$$
$$d.f._{TOT} = N - 1 = 17 - 1 = 16$$
$$MS_{BET} = \frac{SS_{BET}}{d.f._{BET}} = \frac{520.280}{2} = 260.14$$
$$MS_W = \frac{SS_W}{d.f._W} = \frac{7544.190}{14} = 538.87$$
$$F = \frac{MS_{BET}}{MS_W} = \frac{260.14}{538.87} = 0.48$$

For $d.f._N = 2$ and $d.f._D = 14$, the critical value is $F_{0.01} = 6.51$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the means are equal.

Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	520.280	2	260.14	0.48	6.51	Do not reject H_0
Within groups	7544.190	14	538.87			
Total	8064.470	16				

2. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_1: Not all the means are equal.

Site I	Site II	Site III	Site IV
$n = 5$	$n = 6$	$n = 4$	$n = 6$
$\sum x_1 = 76$	$\sum x_2 = 104$	$\sum x_3 = 112$	$\sum x_4 = 107$
$\sum x_1^2 = 1336$	$\sum x_2^2 = 2374$	$\sum x_3^2 = 3558$	$\sum x_4^2 = 2251$
$SS_1 = 180.8$	$SS_2 = 571.33\overline{3}$	$SS_3 = 422$	$SS_4 = 342.83\overline{3}$

$$\sum x_{TOT} = 76 + 104 + 112 + 107 = 399$$
$$\sum x_{TOT}^2 = 1336 + 2374 + 3558 + 2251 = 9519$$
$$N = 5 + 6 + 4 + 6 = 21$$
$$k = 4$$
$$SS_{TOT} = \sum x_{TOT}^2 - \frac{(\sum x_{TOT})}{N} = 9519 - \frac{(399)^2}{21} = 1938$$
$$SS_{BET} = \sum_{\text{all groups}} \left(\frac{(\sum x_i)^2}{n_i} \right) - \frac{(\sum x_{TOT})^2}{N}$$
$$= \frac{(76)^2}{5} + \frac{(104)^2}{6} + \frac{(112)^2}{4} + \frac{(107)^2}{6} - \frac{(399)^2}{21} = 421.033$$
$$SS_W = SS_1 + SS_2 + SS_3 + SS_4 = 180.8 + 571.333 + 422 + 342.833 = 1516.967$$

Check that $SS_{TOT} = SS_{BET} + SS_W : 1938 = 421.033 + 1516.967$

$$d.f._{BET} = k - 1 = 4 - 1 = 3$$
$$d.f._W = N - k = 21 - 4 = 17$$
$$d.f._{TOT} = N - 1 = 21 - 1 = 20$$
$$MS_{BET} = \frac{SS_{BET}}{d.f._{BET}} = \frac{421.033}{3} = 140.344$$
$$MS_W = \frac{SS_W}{d.f._W} = \frac{1516.967}{17} = 89.233$$
$$F = \frac{MS_{BET}}{MS_W} = \frac{140.344}{89.233} = 1.573$$

For $d.f._N = 3$ and $d.f._D = 17$, the critical value is $F_{0.05} = 3.20$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the means are equal.

Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	421.033	3	140.344	1.573	3.20	Do not reject H_0
Within groups	1516.967	17	89.233			
Total	1938.000	20				

3. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
   H_1: Not all the means are equal.
   See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.
   For $d.f._N = 3$ and $d.f._D = 18$, the critical value is $F_{0.05} = 3.16$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the means are equal.

   Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	89.637	3	29.879	0.846	3.16	Do not reject H_0
Within groups	635.827	18	35.324			
Total	725.464	21				

4. $H_0: \mu_1 = \mu_2 = \mu_3$
   H_1: Not all the means are equal.
   See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.
   For $d.f._N = 2$ and $d.f._D = 15$, the critical value is $F_{0.01} = 6.36$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the means are equal.

   Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	215.680	2	107.840	0.816	6.36	Do not reject H_0
Within groups	1981.725	15	132.115			
Total	2197.405	17				

5. $H_0: \mu_1 = \mu_2 = \mu_3$
   H_1: Not all the means are equal.

   See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

   For $d.f._N = 2$ and $d.f._D = 9$. the critical value is $F_{0.05} = 4.26$. Since the observed F ratio is inside the critical region, we reject H_0. We conclude that not all the means are equal.

   Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	1303.167	2	651.58	5.005	4.26	Reject H_0
Within groups	1171.750	9	130.19			
Total	2474.917	11				

6. H_0: $\mu_1 = \mu_2 = \mu_3$
H_1: Not all the means are equal.

See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

For $d.f._N = 2$ and $d.f._D = 18$, the critical value is $F_{0.05} = 3.55$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the means are equal.

Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	2.442	2	1.2208	2.95	3.55	Do not reject H_0
Within groups	7.448	18	0.4138			
Total	9.890	20				

7. H_0: $\mu_1 = \mu_2 = \mu_3$
H_1: Not all the means are equal.

See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

For $d.f._N = 2$ and $d.f._D = 11$. the critical value is $F_{0.01} = 7.21$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the means are equal.

Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	2.042	2	1.021	0.336	7.21	Do not reject H_0
Within groups	33.428	11	3.039			
Total	35.470	13				

8. H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_1: Not all the means are equal.

See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

For $d.f._N = 3$ and $d.f._D = 13$, the critical value is $F_{0.05} = 3.41$. Since the observed F ratio is inside the critical region, we reject H_0. We conclude that not all the means are equal.

Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	18.965	3	6.322	14.910	3.41	Reject H_0
Within groups	5.517	13	0.424			
Total	24.482	16				

9. $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
   H_1: Not all the means are equal.

   See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

   For $d.f._N = 3$ and $d.f._D = 15$, the critical value is $F_{0.05} = 3.29$. Since the observed F ratio is inside the critical region, we reject H_0. We conclude that not all the means are equal.

   Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	238.225	3	79.408	4.611	3.29	Reject H_0
Within groups	258.340	15	17.223			
Total	496.565	18				

Section 11.6

1. There are two factors. One factor is *walking device* with 3 levels and the other factor is *task* with two levels. The data table has 6 cells.

2. There are two factors. One factor is *rank* with 4 levels and the other factor is *institution* with 2 levels. The data table has 8 cells.

3. Since the P value is less than 0.01, there is a significant difference in mean cadence according to the factor *walking device used.* The critical value is $F_{0.01} = 6.01$. Since the sample $F = 30.94$ is greater than $F_{0.01}$, F lies in the critical region and we reject H_0.

4. (a) There are two factors. One factor is *education level* with 4 levels and the other factor is *media type* with 5 levels.

   (b) For education,
   H_0: No difference in population mean index according to education level.
   H_1: At least two education levels have different mean indices.

$$F_{\text{education}} = \frac{MS_{\text{education}}}{MS_{\text{error}}} = \frac{320}{108} = 2.963$$

   For $d.f._N = 3$ and $d.f._D = 12$, the critical value is $F_{0.05} = 3.49$.
   Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0.
   The data do not indicate any differences in population mean index according to education level.

   (c) For media,
   H_0: No difference in population mean index by media type.
   H_1: At least two types of media have different population mean indices.

$$F_{\text{media}} = \frac{MS_{\text{media}}}{MS_{\text{error}}} = \frac{1}{108} = 0.0093$$

   (Minitab results are often rounded to just a few digits. For example, MS for media is 5/4 = 1.25 which has been rounded to 1.)
   For $d.f._N = 4$ and $d.f._D = 12$, the critical value is $F_{0.05} = 3.26$.
   Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0.
   The data do not indicate any differences in population mean index according to media type.

5. (a) There are two factors. One factor is *income level* with 4 levels and the other factor is *media type* with 5 levels.

(b) For income,

H_0: There is no difference in population mean index based on income level.
H_1: At least two income levels have different population mean indices.

$$F_{\text{income}} = \frac{MS_{\text{income}}}{MS_{\text{error}}} = \frac{308}{160} = 1.925$$

For $d.f._N = 3$ and $d.f._D = 12$, the critical value is $F_{0.05} = 3.49$.

Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0.

The data do not indicate any differences in population mean index according to income level.

(c) For media,

H_0: No difference in population mean index by media type.
H_1: At least two media types have different population mean indices.

$$F_{\text{media}} = \frac{MS_{\text{media}}}{MS_{\text{error}}} = \frac{11}{160} = 0.069$$

For $d.f._N = 4$ and $d.f._D = 12$, the critical value is $F_{0.05} = 3.26$.

Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0.

The data do not indicate any differences in population mean index according to media type.

6. (a) There are two factors. One factor is *class* with 4 levels and the other factor is *gender* with 2 levels.

(b) H_0: No interaction between factors.
H_1: Some interaction between factors.

$$F_{\text{interaction}} = \frac{MS_{\text{interaction}}}{MS_{\text{error}}} = \frac{0.090}{0.221} = 0.4072$$

For $d.f._N = 3$ and $d.f._D = 24$, the critical value is $F_{0.05} = 3.01$.

Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0.

The data do not indicate any interaction between factors.

(c) H_0: No difference in population mean GPA based on class.
H_1: At least two classes have different population mean GPAs.

$$F_{\text{class}} = \frac{MS_{\text{class}}}{MS_{\text{error}}} = \frac{0.718}{0.221} = 3.2489$$

For $d.f._N = 3$ and $d.f._D = 24$, the critical value is $F_{0.05} = 3.01$.

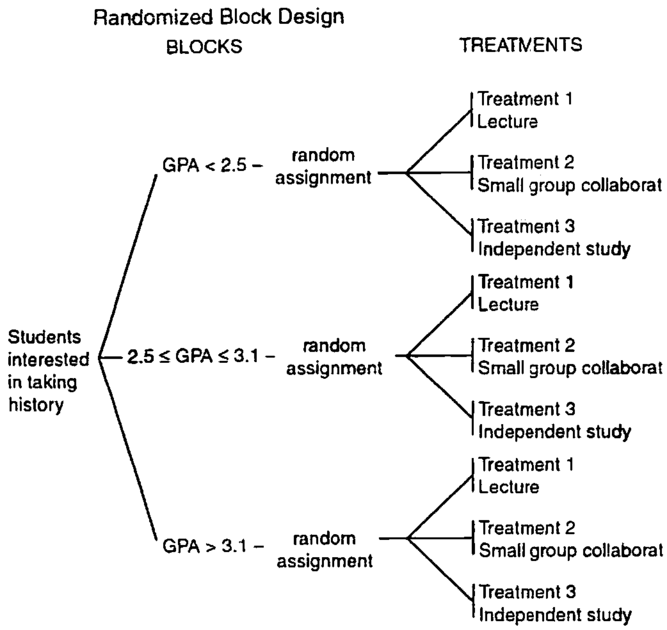
Since the observed F ratio is greater than the critical value, F lies in the critical region and we reject H_0. We conclude at least two classes have different population mean GPAs.

(d) H_0: No difference in population mean GPA based on gender.
H_1: Some difference in population mean GPA based on gender.

$$F_{\text{gender}} = \frac{MS_{\text{gender}}}{MS_{\text{error}}} = \frac{0.300}{0.221} = 1.3575$$

For $d.f._N = 1$ and $d.f._D = 24$, the critical value is $F_{0.05} = 4.26$. Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0. The data do not indicate any difference in population mean GPA based on gender.

7.



Yes; the design fits the model for randomized block design.

Chapter 11 Review

1. One-way ANOVA

   H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

   H_1: Not all the means are equal.

   See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

   For $d.f._N = 3$ and $d.f._D = 16$, the critical value is $F_{0.05} = 3.24$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the packaging mean sales are equal.

   Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	F Ratio	F Critical Value	Test Decision
Between groups	6150	3	2050	2.63	3.24	Do not reject H_0
Within groups	12,455	16	778			
Total	18,605	19				

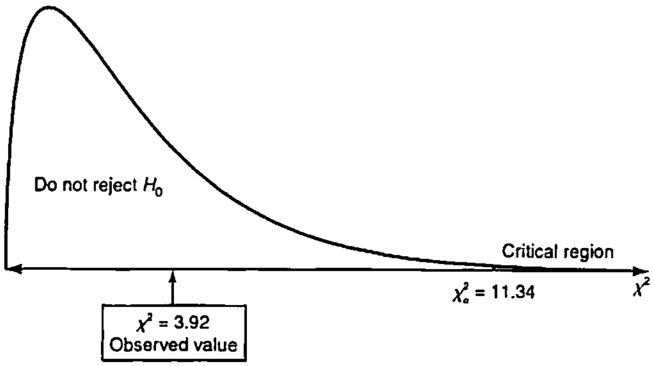
2. Chi-square test of independence
   H_0: Time to do a test and test score are independent.
   H_1: Time to do a test and test score are not independent.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(23-18.93)^2}{18.93} + \frac{(42-42.60)^2}{42.60} + \frac{(65-71.00)^2}{71.00} + \frac{(12-9.47)^2}{9.47} + \frac{(17-21.07)^2}{21.07} + \frac{(48-47.40)^2}{47.40} + \frac{(85-79.00)^2}{79.00} + \frac{(8-10.53)^2}{10.53}$$

$$= 3.92$$

   Since there are 2 rows and 4 columns, $d.f. = (2-1)(4-1) = 3$. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 11.34$.



   Since the sample statistic falls outside the critical region. we do not reject H_0. There is insufficient evidence to conclude that time to do a test and test score are not independent.

3. (a) Chi-square for testing σ^2

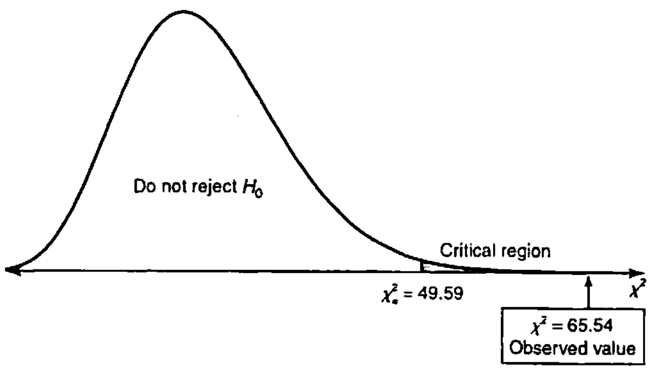
   H_0: $\sigma^2 = 810.000$

   H_1: $\sigma^2 > 810.000$

   Since > is in H_1, a right-tailed test is used.

   For $d.f. = 30 - 1 = 29$, the critical value is $\chi^2_{0.01} = 49.59$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(30-1)(1353)^2}{(900)^2} = 65.54$$



   Since the sample statistic falls in the critical region, we reject H_0. We conclude that the variance of blow-out pressures is more than Soap Stone claims it is.

(b) For $d.f. = 29$ and $\alpha = \frac{1-0.95}{2} = 0.025$, $\chi_U^2 = 45.72$.

For $d.f. = 29$ and $\alpha = \frac{1+0.95}{2} = 0.975$, $\chi_L^2 = 16.05$.

The 95% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi_U^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

$$\frac{(30-1)(1353)^2}{45.72} < \sigma^2 < \frac{(30-1)(1353)^2}{16.05}$$

$$1.161.147.4 < \sigma^2 < 3.307,642.4 \text{ square foot-pounds}$$

4. One-way ANOVA

$H_0: \mu_1 = \mu_2 = \mu_3$

H_1: Not all the means are equal.

See Section 11.5 or solutions to problems 1-2 for examples of calculations from formulas.

For $d.f._N = 2$ and $d.f._D = 9$, the critical value is $F_{0.01} = 8.02$. Since the observed F ratio is outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that not all the mean times to execute the programs are equal.

Summary of ANOVA results

Source of Variation	Sum of Squares	Degrees of Freedom	MS	*F* Ratio	*F* Critical Value	Test Decision
Between groups	1.002	2	0.501	0.443	8.02	Fail to reject H_0
Within groups	10.165	9	1.129			
Total	11.167	11				

5. Chi-square test of independence

H_0: Student grade and teacher rating are independent.

H_1: Student grade and teacher rating are not independent.

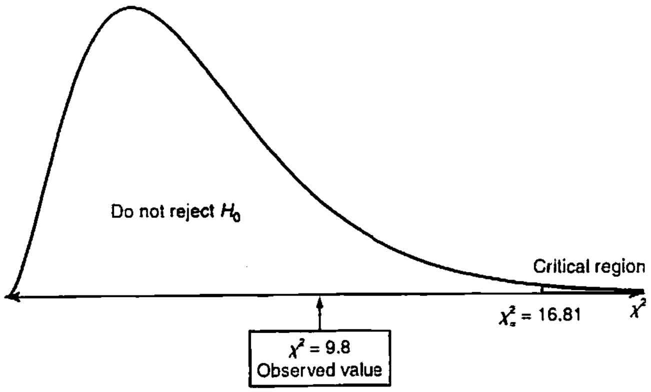
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(14-10.00)^2}{10.00} + \frac{(18-13.33)^2}{13.33} + \frac{(15-21.67)^2}{21.67} + \frac{(3-5.00)^2}{5.00} + \frac{(25-30.00)^2}{30.00} + \frac{(35-40.00)^2}{40.00}$$

$$+ \frac{(75-65.00)^2}{65.00} + \frac{(15-15.00)^2}{15.00} + \frac{(21-20.00)^2}{20.00} + \frac{(27-26.67)^2}{26.67} + \frac{(40-43.33)^2}{43.33} + \frac{(12-10.00)^2}{10.00}$$

$$= 9.8$$

Since there are 3 rows and 4 columns, $d.f. = (3-1)(4-1) = 6$. For $\alpha = 0.01$, the critical value is $\chi^2_{0.01} = 16.81$.



Since the sample statistic falls outside the critical region, we do not reject H_0. We have insufficient evidence to conclude that student grade and teacher rating are not independent.

6. Chi-square for testing σ^2

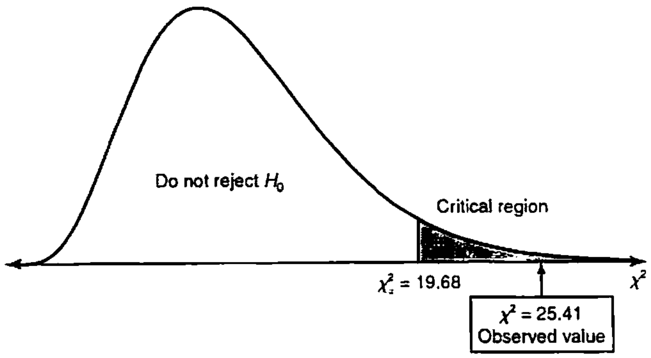
H_0: $\sigma^2 = 0.0625$

H_1: $\sigma^2 > 0.0625$

Since > is in H_1, a right-tailed test is used.

For $d.f.$ = 12 − 1 = 11, the critical value is $\chi^2_{0.05} = 19.68$.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(12-1)(0.38)^2}{(0.25)^2} = 25.41$$



Since the sample statistic falls in the critical region, we reject H_0. We conclude that the machine needs to be adjusted (i.e., the variance has increased, $\sigma^2 > 0.25^2 > 0.0625$).

7. Chi-square goodness of fit

H_0: The distributions are the same.

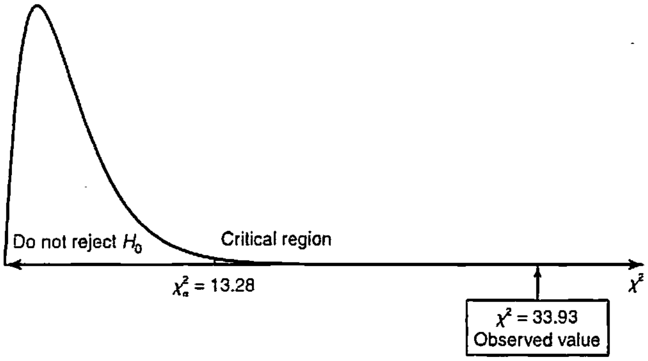
H_1: The distributions are different.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(15-42.0)^2}{42.0} + \frac{(25-31.5)^2}{31.5} + \frac{(70-63.0)^2}{63.0} + \frac{(80-52.5)^2}{52.5} + \frac{(20-21.0)^2}{21.0}$$

$$= 33.93$$

$d.f.$ = (number of E entries) − 1 = 5 − 1 = 4. The critical value is $\chi^2_{0.01} = 13.28$.



Since the sample statistic falls inside the critical region. we reject H_0. We conclude that the age distribution has changed.

8. F test for equality of two variances

H_0: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 \neq \sigma_2^2$

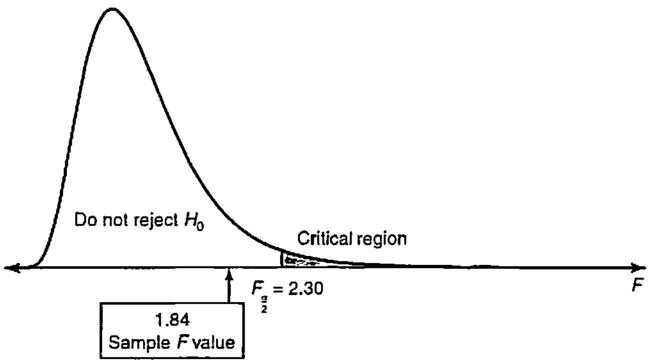
Since $\neq$ is in H_1, a two-tailed test is used.

The populations follow independent normal distributions. The samples are random samples from each population.

Since $s^2 = 0.235$ is larger than $s^2 = 0.128$, we designate Population I as the old process.

$$F = \frac{s_1^2}{s_2^2} = \frac{0.235}{0.128} = 1.84$$

Using $d.f._N = n_1 - 1 = 21 - 1 = 20$, and $d.f._D = n_2 - 1 = 26 - 1 = 25$, and $\alpha = 0.05$, the critical $F_{\frac{\alpha}{2}}$ value is 2.30.



Since the sample F statistic falls outside the critical region, we do not reject H_0. There is insufficient evidence to conclude that there is a difference in the population variances for the old and the new manufacturing processes.

9. F test for the equality of two variances

H_0: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 > \sigma_2^2$

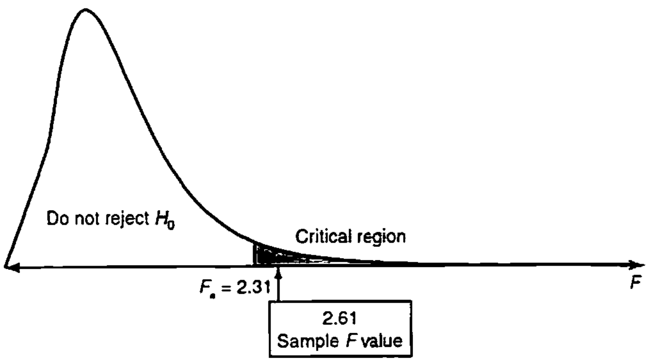
Since $>$ is in H_1, a right-tailed test is used.

The populations follow independent normal distributions. The samples are random samples from each population.

Since $s^2 = 135.24$ is larger than $s^2 = 51.87$, we designate Population 1 as the new process.

$$F = \frac{s_1^2}{s_2^2} = \frac{135.24}{51.87} = 2.61$$

Using $d.f._N = n_1 - 1 = 16 - 1 = 15$, and $d.f._D = n_2 - 1 = 18 - 1 = 17$, and $\alpha = 0.05$, the critical F value is 2.31.



Since the sample F statistic falls inside the critical region, we reject H_0. We conclude that the population variance of life times for bulbs made by the new process is greater than that for bulbs made by the old process.

10. Two-way ANOVA

(a) There are two factors. One factor is *day* with 2 levels and the other factor is *section* with 3 levels.

(b) H_0: No interaction between day and section.
H_1: Some interaction between day and section.

$$F_{\text{interaction}} = \frac{MS_{\text{interaction}}}{MS_{\text{error}}} = \frac{19.0}{18.5} = 1.027$$

For $d.f._N = 2$ and $d.f._D = 30$, the critical value is $F_{0.01} = 5.39$.

Since the observed F ratio is less than the critical value, F lies outside the critical region and we do not reject H_0. The data do not indicate any interaction between day and section.

(c) H_0: No difference in population mean number of responses according to day.
H_1: At least two population means are different among the days.

$$F_{\text{day}} = \frac{MS_{\text{day}}}{MS_{\text{error}}} = \frac{1024.0}{18.5} = 55.35$$

For $d.f._N = 1$ and $d.f._D = 30$, the critical value is $F_{0.01} = 7.56$.

Since the observed F ratio is greater than the critical value, F lies in the critical region and we reject H_0. We conclude that the two population means differ by day.

(d) H_0: No difference in population mean number of responses according to section.
H_1: At least two population means are different among the sections.

$$F_{\text{section}} = \frac{MS_{\text{section}}}{MS_{\text{error}}} = \frac{786.8}{18.5} = 42.53$$

For $d.f._N = 2$ and $d.f._D = 30$, the critical value is $F_{0.01} = 5.39$. Since the observed F ratio is greater than the critical value, F lies in the critical region and we reject H_0. We conclude that at least two population means are different among the sections.

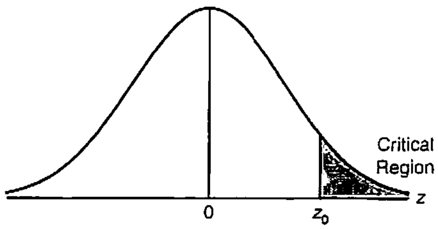
Chapter 12 Nonparametric Statistics

Section 12.1

For part (c) in each of the following problems, refer to sketches (a), (b) or (c) shown below.

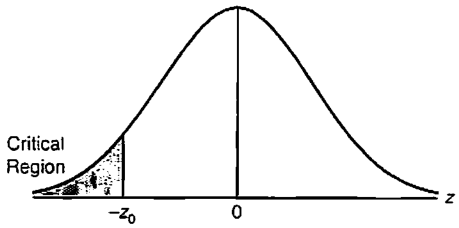
(a)

$H_0: p = 0.5$
$H_1: p > 0.5$



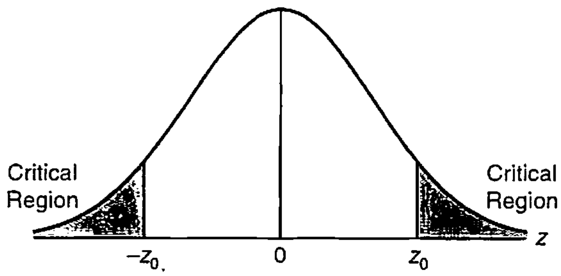
(b)

$H_0: p = 0.5$
$H_1: p < 0.5$



(c)

$H_0: p = 0.5$
$H_1: p \neq 0.5$



1.

Pen	New Tip	Old Tip	Sign of Difference, New – Old
1	52	50	+
2	47	55	–
3	56	51	+
4	48	45	+
5	51	57	–
6	59	54	+
7	47	46	+
8	57	53	+
9	56	52	+
10	46	40	+
11	56	49	+
12	47	51	–

(a) $H_0: p = \frac{1}{2}$; $H_1: p > \frac{1}{2}$. i.e., H_0: the distributions of writing life for the old and new pen tips are the same; H_1: the distribution of the writing life for the new pen tip is shifted to the right of that for the old pen tip; since the alternative is $p > \frac{1}{2}$, this will be a right-tailed test at level of significance $\alpha = 0.05$.

(b) Sign test with number of nonzero pairs $n = 12 \geq 12$. so use the normal approximation with $\mu = p = \frac{1}{2} = 0.5. \sigma = \sqrt{pq/n} = \sqrt{(0.5)(0.5)/12} = \sqrt{0.25/12} = 0.1443$; the critical value is $z_0 = 1.645$.

(c) Refer to Figure (a) above with $z_0 = 1.645$.

(d) $r = 9$ plus signs, $n = 12$ nonzero pairs. $x = r/n = 9/12 = 0.75$

$z = \frac{x-\mu}{\sigma} = \frac{0.75-0.5}{0.1443} = 1.73$. which is to the right of $z_0 = 1.645$, in the critical region. in the figure for part (c).

(e) Reject H_0: the data do not support the null hypothesis. but rather indicate the writing life of the new pen tip is longer than that of the old pen tip.

2.

Student	Pulse Rate Before Exam	Pulse Rate Before Ordinary Class	Sign of Difference, Exam – Ordinary Class
1	88	81	+
2	77	77	0
3	72	75	–
4	74	79	–
5	81	79	+
6	70	68	+
7	75	77	–
8	80	73	+
9	68	71	–
10	75	73	+
11	82	76	+
12	61	66	–
13	77	68	+
14	64	60	+

(a) H_0: the distribution of pulse rates is the same before an exam as it is before an ordinary class, H_1: the distribution of pulse rates before an exam is shifted to the right compared to that before an ordinary class, or H_0: $p = 0.5$. H_1:$p > 0.5$. Because the alternative is $p > 0.5$, this will be a right-tailed sign test at significance level $\alpha = 0.05$.

(b) The number of nonzero difference is $n = 13 \geq 12$. so we will use the normal approximation with $\mu = p = 0.5$. and $\sigma = \sqrt{pq/n} = \sqrt{0.5(0.5)/13} = \sqrt{0.25/13} = 0.1387$. The critical value cuts off an area of $\alpha = 0.05$ in the upper tail: $z_0 = 1.645$.

(c) Refer to Figure (a) above with $z_0 = 1.645$.

(d) $r = 8$ plus signs, $n = 13$ plus and minus signs, $x = r/n = 8/13 = 0.6154$

$z = \frac{x-\mu}{\sigma} = \frac{0.6154-0.5}{0.1387} = 0.83$, which is between 0 and $z_0 = 1.645$. in the figure for part (c); z is not in the critical region.

(e) Because z is not in the critical region, we fail to reject H_0. There is insufficient evidence to support the claim that student pulse rates increase before an exam.

3.

Student	Sign of Difference After – Before
1	–
2	+
3	+
4	+
5	–
6	0
7	–
8	+
9	–
10	+
11	+
12	+
13	–
14	+
15	–
16	+
17	0
18	+

(a) H_0: the score distributions for student awareness of current events are the same, before and after lectures, H_1: the score distributions before lectures and after lectures are different, or H_0: $p = 0.5$, H_1: $p \neq 0.5$. Because the alternative is $p \neq 0.5$, this will be a two-tailed sign test at level of significance $\alpha = 0.05$.

(b) Because the number of nonzero difference is $n = 16 \geq 12$, we will use the normal approximation with $\mu = p = 0.5$, and $\sigma = \sqrt{pq/n} = \sqrt{(0.5)(0.5)/16} = 0.125$. The critical value cut off a total area of $\alpha = 0.05$, half in each tail, $\pm z_0 = \pm 1.96$.

(c) Refer to Figure (c) above with $\pm z_0 = \pm 1.96$.

(d) $r = 10$ plus signs, $n = 16$ plus or minus signs, $x = r/n = 10/16 = 0.625$

$z = \dfrac{x-\mu}{\sigma} = \dfrac{0.625-0.5}{0.125} = 1.00$ which is between 0 and $z_0 = 1.96$; z is not in a critical region.

(e) Since z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to support the claim that lectures on current events change student awareness of current events.

4.

Day	Sign of Difference, Line A – Line B
1	–
2	–
3	+
4	0
5	+
6	–
7	+
8	+
9	+
10	+
11	–
12	–
13	+
14	+
15	–

(a) H_0: the score distributions of defective lens filiters are the same for both production lines. H_1: the distribution of defective lens filters for line A is different from that for line B, or H_0: $p = 0.5$, H_1: $p \neq 0.5$. Because the alternative is $p \neq 0.5$, this will be a two-tailed sign test at level of significance $\alpha = 0.01$.

(b) Because the number of nonzero difference is $n = 14 \geq 12$, we will use the normal approximation with $\mu = p = 0.5$, and $\sigma = \sqrt{pq/n} = \sqrt{0.25/14} = 0.1336$. The critical values cut off a total area of $\alpha = 0.01$, half in each tail, $\pm z_0 = \pm 2.58$.

(c) Refer to Figure (c) above with $\pm z_0 = \pm 2.58$.

(d) $r = 8$ plus signs, $n = 14$ plus and minus signs. $x = r/n = 8/14 = 0.5714$

$z = \frac{x-\mu}{\sigma} = \frac{0.5714 - 0.5}{0.1336} = 0.53$, which is between 0 and $z_0 = 2.58$; z is not in a critical region.

(e) Since z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to support the claim that employee experience makes a difference in the number of defective lens filters produced.

5.

Twin Pair	Sign of Difference, School A – School B
1	+
2	+
3	–
4	–
5	+
6	+
7	–
8	+
9	+
10	+
11	–
12	–

(a) H_0: the distributions of reading achievement scores are the same for both schools; H_1: the distributions are different, or H_0: $p = 0.5$, H_1: $p \neq 0.5$. Because the alternative is $p \neq 0.5$, this will be a two-tailed sign test at level of significance $\alpha = 0.05$.

(b) Because the number of nonzero differences is $n = 12 \geq 12$, we will use the normal approximation with $\mu = p = 0.5$, and $\sigma = \sqrt{0.25/12} = 0.1443$. The critical values cut off a total area of $\alpha = 0.05$, half in each tail, $\pm z_0 = \pm 1.96$.

(c) Refer to Figure (c) above with $\pm z_0 = \pm 1.96$.

(d) $r = 7$ plus signs, $n = 12$ plus and minus signs, $x = r/n = 7/12 = 0.5833$

$z = \frac{x-\mu}{\sigma} = \frac{0.5833 - 0.5}{0.1443} = 0.58$, which is between 0 and $z_0 = 1.96$ in the figure for part (c), z is not in a critical region.

(e) Because z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to support the claim that the schools are not equally effective at teaching reading skills.

6.

Bakery	Sign of Difference, A – B
1	–
2	+
3	0
4	–
5	–
6	–
7	+
8	–
9	–
10	+
11	+
12	–
13	+
14	+
15	–
16	–
17	+
18	–
19	–
20	+

(a) H_0: the shelf life distributions are the same for preservatives A and B, H_1: the shelf life distribution for preservative B is shifted to the right of that for preservative A, or $H_0: p = 0.5, H_1: p < 0.5$. (Note that the difference is specified as A – B, so under H_1, there should be less than half plus signs.) Since the alternative is $p < 0.5$, this will be a left-tailed sign test at level of significance $\alpha = 0.05$.

(b) Because the number of nonzero differences is $n = 19 \geq 12$, we can use the normal approximation with $\mu = p = 0.5$, and $\sigma = \sqrt{pq/n} = \sqrt{0.25/19} = 0.1147$. The critical value cuts off an area of $\alpha = 0.05$, in the lower tail; $-z_0 = -1.645$.

(c) Refer to Figure (b) above with $-z_0 = -1.645$.

(d) $r = 8$ plus signs, $n = 19$ plus and minus signs, $x = r/n = 8/19 = 0.4211$.

$$z = \frac{x - \mu}{\sigma} = \frac{0.4211 - 0.5}{0.1147} = -0.69,$$

which is between $-z_0 = -1.645$ and 0; z is not in the critical region.

(e) Because z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to support the claim that bread baked with preservative B will stay fresh longer.

7.

Bakery	Sign of Difference, A – B
1	0
2	–
3	–
4	0
5	+
6	–
7	–
8	+
9	–
10	–
11	–
12	–
13	+
14	–
15	–
16	–
17	–
18	–

(a) H_0: the distribution of cigarettes smoked is the same, H_1: the distribution of cigarettes smoked after hypnosis is shifted to the left of the distribution for cigarettes smoked before hypnosis, or $H_0: p = 0.5$. $H_1: p < 0.5$. Because the alternative is $p < 0.5$, this will be a left-tailed sign test, at significance level $\alpha = 0.01$.

(b) Because the number of nonzero differences is $n = 16 \geq 12$, we will use the normal approximation with $\mu = p = 0.5$, and $\sigma = \sqrt{pq/n} = \sqrt{0.25/16} = 0.125$. The critical value cuts off an area of $\alpha = 0.01$ in the lower tail: $-z_0 = -2.33$.

(c) Refer to Figure (b) above with $-z_0 = -2.33$.

(d) $r = 3$ plus signs, $n = 16$ plus or minus signs, $x = r/n = 3/16 = 0.1875$.

$z = \frac{x-\mu}{\sigma} = \frac{0.1875 - 0.5}{0.125} = -2.50$, which is to the left of $-z_0 = -2.33$ and in the critical region.

(e) Because z is in the critical region, we reject H_0. The data do not support the null hypothesis: hypnosis appear to help smokers cut back on the number of cigarettes smoked.

8.

Student	Sign of Difference, After – Before
1	–
2	–
3	–
4	–
5	0
6	+
7	–
8	–
9	–
10	+
11	–
12	–
13	–
14	–
15	–
16	+

(a) H_0: the distribution of mosquito bites is the same before as after eating garlic, H_1: the distribution of mosquito bites after eating garlic is shifted to the left of that before eating garlic, or H_0: $p = 0.5$, H_1: $p < 0.5$. Because the alternative is $p < 0.5$, this will be a left-tailed sign test, at level of significance $\alpha = 0.05$.

(b) Because the number of nonzero differences is $n = 15 \geq 12$, we can use the normal approximation to the distribution of p with $\mu = p = 0.5$, and $\sigma = \sqrt{pq/n} = \sqrt{0.25/15} = 0.1291$. The critical value cuts off an area of $\alpha = 0.05$ in the lower tail; $-z_0 = -1.645$.

(c) Refer to Figure (b) above with $-z_0 = -1.645$.

(d) $r = 3$ plus signs, $n = 15$ plus and minus signs, $x = r/n = 3/15 = 0.2$.

$z = \frac{x-\mu}{\sigma} = \frac{0.2-0.5}{0.1291} = -2.32$, which is below $-z_0 = -1.645$, and in the critical region.

(e) Because z is in the critical region, we reject H_0. The data do not support the null hypothesis; eating garlic appears to repel mosquitoes.

9.

Baby	Sign of Difference, After – Before
1	+
2	+
3	+
4	–
5	0
6	+
7	0
8	–
9	–
10	+
11	+
12	–
13	+
14	0
15	+
16	–
17	+

(a) H_0: the distribution of baby pulse rates is the same 24 hours before labor begins as it is 24 hours after birth, H_1: the distributions are different, or H_0: $p = 0.5$, H_1: $p \neq 0.5$. Because the alternative is $p \neq 0.5$, this will be a two-tailed test, at level of significance $\alpha = 0.01$.

(b) Because the number of nonzero differences is $n = 14 \geq 12$, we can use the normal approximation with mean $\mu = p = 0.5$, and standard deviation $\sigma = \sqrt{pq/n} = \sqrt{0.25/14} = 0.1336$. The critical value cuts off a total area of $\alpha = 0.01$, evenly in both tails: $\pm z_0 = \pm 2.58$.

(c) Refer to Figure (c) above with $\pm z_0 = \pm 2.58$.

(d) $r = 9$ plus signs, $n = 14$ plus and minus signs, $x = r/n = 9/14 = 0.6429$.

$z = \frac{x-\mu}{\sigma} = \frac{0.6429-0.5}{0.1336} = 1.07$, which is between 0 and $z_0 = 2.58$, and not in either critical region.

(e) Because z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to support the claim that babies' pulse rates are different before and after birth.

10.

Salesperson	Sign of Difference, After − Before
1	+
2	+
3	−
4	−
5	+
6	0
7	+
8	+
9	+
10	−
11	+
12	+
13	0
14	−
15	+

(a) H_0: the distribution of the number of magazines sold is the same before TV ads aired as it was after the ads aired, H_1: the distributions are different, or $H_0: p = 0.5, H_1: p \neq 0.5$. Because the alternative is $p \neq 0.5$, this will be a two-tailed sign test, with level of significance $\alpha = 0.05$.

(b) Because the number of nonzero differences is $n = 13 \geq 12$, we can use the normal approximation with mean $\mu = p = 0.5$, and standard deviation $\sigma = \sqrt{pq/n} = \sqrt{0.25/13} = 0.1387$. The critical values cut off a total area of $\alpha = 0.05$, evenly in both tails; $\pm z_0 = \pm 1.96$.

(c) Refer to Figure (c) above with $\pm z_0 = \pm 1.96$.

(d) $r = 9$ plus signs, $n = 13$ plus and minus signs, $x = r/n = 9/13 = 0.6923$.

$z = \frac{x-\mu}{\sigma} = \frac{0.6923-0.5}{0.1387} = 1.39$, which is between 0 and $z_0 = 1.96$, and not in one of the critical regions.

(e) Because z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to show TV ads for magazines affected magazine sales.

11.

Month	Sign of Difference, Madison − Juneau
January	−
February	−
March	−
April	+
May	+
June	+
July	+
August	+
September	+
October	+
November	+
December	−

(a) H_0: the temperature distribution is the same in Madison, Wisconsin as it is in Juneau, Alaska, H_1: the temperature distributions are different, or $H_0: p = 0.5, H_1: p \neq 0.5$. Because the alternative is $p \neq 0.5$, this will be a two-tailed sign test, with level of significance $\alpha = 0.05$.

(b) Because the number of nonzero differencess is $n = 12 \geq 12$, we can use the normal approximation with $\mu = p = 0.5$, and $\sigma = \sqrt{pq/n} = \sqrt{0.25/12} = 0.1443$. The critical values cut off a total area of $\alpha = 0.05$, evenly in each tail: $\pm z_0 = \pm 1.96$.

(c) Refer to Figure (c) above with $\pm z_0 = \pm 1.96$.

(d) $r = 8$ plus signs, $n = 12$ plus and minus signs, $x = r/n = 8/12 = 0.6667$.
$z = \dfrac{x-\mu}{\sigma} = \dfrac{0.6667-0.5}{0.1443} = 1.16$, which is between 0 and $z_0 = 1.96$, and not in either critical region.

(e) Because z is outside the critical regions, we fail to reject H_0. There is insufficient evidence to support the claim that Madison and Juneau have different temperature distributions.
Note: Whereas it is true that there is no statistically significant difference in the distributions, the sign test misses one very important point: the differences are cyclical: Madison is warmer from December to March (i.e., "winter"). A "runs test" might detect this obvious pattern, whereas the sign test merely counts the number of plus signs, which would be the same if the data were ordered July to June, or April to March, etc.

Section 12.2

For problems 1-5, H_0: the distributions of the two populations are the same, H_1: the population distributions are different. Use the Wilcoxon rank sum test, two-tailed, with level of significance $\alpha = 0.05$. When n_1 and n_2 are ≥ 8, use the normal approximation with $\mu_R = \dfrac{n_1(n_1+n_2+1)}{2}$ and $\sigma_R = \sqrt{\dfrac{n_1 n_2(n_1+n_2+1)}{12}}$.

Let $\Sigma R_A, \Sigma R_B$ be the sum of the ranks of Groups A and B, statistic R be the sum of the ranks of the smaller sample $n_1 \leq n_2$. The critical values are $\pm z_0 = \pm 1.96$. Begin by ordering and ranking the combined data:

1.

Student Score	Group	Rank
44	A	1
45	B	2
50	A	3
55	B	4
60	A	5
63	B	6
65	A	7
66	B	8
69	B	9
70	A	10
71	A	11
73	A	12
75	B	13
77	B	14
81	A	15
84	B	16
85	B	17
88	A	18
90	B	19

$n_1 = 9$ (group A), $n_2 = 10$: both are ≥ 8; use normal approximation R = sum of ranks of As = $\sum R_A = 82$.
(As a check: sum of ranks of Bs = $\sum R_B = 108$, sum of ranks of all data = 190
This should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 190$ ✓)

$$\mu_R = \frac{9(9+10+1)}{2} = \frac{9(20)}{2} = 90$$

$$\sigma_R = \sqrt{\frac{9(10)(9+10+1)}{12}} = \sqrt{150} = 12.2474 \approx 12.25$$

$$z \approx \frac{R-\mu_R}{\sigma_R} = \frac{82-90}{12.2474} = -0.65$$

Since z is not in either critical region, we fail to reject H_0. The data do not support the claim that the distributions of student test scores are different.

2.

Boredom Tolerance Score	Group	Rank
35	B	1
41	A	2
50	A	3
54	B	4
66	A	5
68	A	6
69	B	7
72	B	8
73	A	9
75	A	10
77	B	11
85	B	12
88	A	13
92	A	14
99	B	15
100	B	16
103	A	17
111	A	18
115	A	19
120	A	20
135	B	21
150	B	22

Group A: $n_2 = 12, \sum R_A = 136$
Group B: $n_1 = 10, \sum R_B = 117$ (Note: Group B is smaller)
[Check: $\sum R_A + \sum R_B = 136 + 117 = 253$; this should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 22(23)/2 = 253$ ✓]
$R = \sum R_B = 117$. Since n_1 and n_2 are each ≥ 8, use the normal approximation with

$$\mu_R = 10(10+12+1)/2 = 115$$

$$\sigma_R = \sqrt{10(12)(10+12+1)/12} = \sqrt{230} = 15.1658 \approx 15.17$$

$$z \approx \frac{R-\mu_R}{\sigma_R} = \frac{117-115}{15.1658} = 0.13,$$ which is not in either critical region.

Fail to reject H_0; the data do not support the claim that the distributions of boredom tolerance are different.

3.

Number of Sessions	Method	Rank
19	A	1
20	B	2
24	B	3
25	A	4
26	B	5
28	A	6
31	A	7
33	B	8
34	B	9
35	A	10
37	A	11
38	A	12
39	B	13
40	A	14
41	A	15
42	B	16
44	B	17
46	B	18
48	B	19

Method A: $n_1 = 9, \sum R_A = 80$

Method B: $n_2 = 10, \sum R_B = 110$

[Check: $\sum R_A + \sum R_B = 80 + 110 = 190$; this should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 19(20)/2 = 190$ ✓]

Since n_1 and n_2 are each ≥8, use the normal approximation with $\mu_R = 9(20)/2 = 90$.

$\sigma_R = \sqrt{(9)(10)(20)/12} = \sqrt{150} = 12.2474$. $z \approx \dfrac{R - \mu_R}{\sigma_R} = \dfrac{80 - 90}{12.2474} = -0.82$, which is not in either rejection region. Fail to reject H_0; there is insufficient evidence to support the claim that the distribution of the number of horse training sessions with a lead horse differs from that with no lead horse.

4.

Time	Method	Rank
15	B	$\frac{1+2}{2} = 1.5$
15	A	$\frac{1+2}{2} = 1.5$
18	A	3
19	B	4
20	A	5
22	A	6
25	A	7
28	B	8
29	B	9
30	A	$\frac{10+1}{2} = 10.5$
30	B	$\frac{10+1}{2} = 10.5$
33	A	12
40	B	13
41	A	14
44	A	15
46	B	16
55	B	17
56	A	18
58	B	19
63	B	20

(remembering to assign ties at 15 and at 30 the average of the ranks)

Method A: $n_1 = 10, \sum R_A = 92$

Method B: $n_2 = 10, \sum R_B = 118$

[Check: $\sum R_A + \sum R_B = 92 + 118 = 210$; this should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 20(21)/2 = 210$ ✓]

Since n_1 and $n_2 = 10 \geq 8$, use the normal approximation with $\mu_R = 10(21)/2 = 105$.

$\sigma_R = \sqrt{10(10)(21)/12} = \sqrt{175} = 13.2288$ and $R = \sum R_A = 92$. (Since $n_1 = n_2$, we could use $R = \sum R_B$ instead.)

$z \approx \dfrac{R - \mu_R}{\sigma_R} = \dfrac{92 - 105}{13.2288} = -0.98$, which is not in either critical region. Fail to reject H_0; there is insufficient evidence to support the claim that the method of teaching French verbs makes the distributions different.

(Note: using $R = \sum R_B$, we would get $z \approx \dfrac{R - \mu_R}{\sigma_R} = \dfrac{118 - 105}{13.2288} = +0.98$)

5.

Test Score	Group	Rank
7	A	1
8	A	2
9	B	3
10	A	4
11	B	5
12	A	6
13	A	7
14	B	8
15	A	9
16	B	10
17	A	11
18	A	12
19	B	13
22	A	14
24	B	15
27	B	16
28	B	17
29	B	18
30	B	19
31	B	20
33	B	21

Group A: $n_1 = 9, \sum R_A = 66$

Group B: $n_2 = 12, \sum R_B = 165$

[Check: $\sum R_A + \sum R_B = 66 + 165 = 231$; which should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 21(22)/2 = 231$ ✓]

Since $n_1, n_2 \geq 8$, use the normal approximation with $\mu_R = 9(9 + 12 + 1)/2 = 99$.

$\sigma_R = \sqrt{9(12)(22)/12} = \sqrt{198} = 14.0712$, and $R = \sum R_A = 66$. $z \approx \dfrac{R - \mu_R}{\sigma_R} = \dfrac{66 - 99}{14.0712} = -2.35$, which is in the (lower/left) critical region. Reject H_0; there is sufficient evidence to support the claim that the competitive and noncompetitive settings' distributions are different.

For problems 6-10: H_0: the population distributions are the same, H_1: the population distributions are different. Use a two-tailed Wilcozon rank sum test at level of significance $\alpha = 0.01$. When the sample sizes are each ≥8, use the normal approximation with $\mu_R = n_1(n_1 + n_2 + 1)/2$ and $\sigma_R = \sqrt{n_1(n_2)(n_1 + n_2 + 1)/12}$. Let $\Sigma R_A, \Sigma R_B$ be the sums of the ranks for data set A and data set B. Let n_1 denote the smaller sample size, and let R = sum of ranks of that group in the test statistic. The critical values are $\pm z_0 = \pm 2.58$. Begin by ranking the combined data:

6.

Test Scores	Group	Rank
42	B	1
44	B	2
52	A	3
57	B	4
60	A	$\frac{5+6}{2} = 5.5$
60	B	$\frac{5+6}{2} = 5.5$
62	B	7
65	B	8
67	A	9
68	A	10
76	A	11
79	A	12
81	A	13
82	B	14
85	A	15
86	B	16
88	A	17
90	A	18
91	B	19
93	A	20
96	B	21

Group A: $n_2 = 11, \Sigma R_A = 133.5$

Group B: $n_1 = 10, \Sigma R_B = 97.5$

[Check: $\Sigma R_A + \Sigma R_B = 133.5 + 97.5 = 231$; this should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 21(22)/2 = 231$ ✓]

Since both n_1 and n_2 are ≥ 8, use the normal approximation with $\mu_R = 10(22)/2 = 110$.

$\sigma_R = \sqrt{10(11)(22)/12} = 14.2009$, and $R = \Sigma R_A = 97.5$. $z \approx \frac{R - \mu_R}{\sigma_R} = \frac{97.5 - 110}{14.2009} = -0.88$, which is not in either critical region. Fail to reject H_0; there is insufficient evidence to support the claim that the distributions of test scores by diet type differ.

7.

Days Sick	Group	Rank
8	B	1
9	B	2
10	A	3
11	B	4
12	A	5
14	A	6
15	B	7
16	A	8
17	B	9
18	B	10
19	A	11
20	A	12
21	A	13
22	B	14
24	B	15
25	A	16
26	B	17
28	A	18
31	B	19

Group A: $n_1 = 9, \sum R_A = 92$

Group B: $n_2 = 10. \sum R_B = 98$

(Check: $\sum R_A + \sum R_B = 190$; this should be equal to $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 19(20)/2 = 190$ ✓)

Since both n_1 and n_2 are ≥ 8. use the normal approximation with $\mu_R = 9(20)/2 = 90$.

$\sigma_R = \sqrt{9(10)(20)/12} = \sqrt{150} = 12.2474$, and $R = \sum R_A = 92$. $z \approx \frac{R - \mu_R}{\sigma_R} = \frac{92-90}{12.2474} = 0.16$, which is not in either critical region. Fail to reject H_0; there is insufficient evidence to support the claim that the distributions of duration of colds differ.

8.

Time	Group	Rank
30	B	1
31	B	2
33	A	3
36	B	4
37	B	5
38	A	6
39	B	7
40	A	8
41	A	9
42	B	10
44	B	11
45	A	12
47	A	13
50	B	14
52	A	15
55	A	16
58	A	17
61	B	18

Group A: $n_1 = 9, \sum R_A = 99$

Group B: $n_2 = 9, \sum R_B = 72$

[Check: $\sum R_A + \sum R_B = 99 + 72 = 171$; this should be equal to $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 18(19)/2 = 171$ ✓]

Since both n_1 and $n_2 \geq 8$, we will use the normal approximation with $\mu_R = 9(19)/2 = 85.5$.

$\sigma_R = \sqrt{(9)(9)(19)/12} = \sqrt{128.25} = 11.3248$. Since $n_1 = n_2$, it doesn't matter which sample size is called n_1 or whether $R = \sum R_A$ or $R = \sum R_B$, so let $R = \sum R_A$ and $n_A = n_1$. $z \approx \dfrac{R - \mu_R}{\sigma_R} = \dfrac{99 - 85.5}{11.3248} = 1.19$, which is not in either critical region. Fail to reject H_0; the data do not support the claim that the distribution of skiers' times with Teflon ski bottoms is different from that of skiers' times for those who used traditional wax on their ski bottoms.

[Note: if $R = \sum R_B = 72, z \approx \dfrac{72 - 85.5}{11.3248} = -1.19$]

9.

Spelling Score	Group	Rank
61	A	1
62	B	2
63	B	3
69	A	4
70	B	5
72	B	6
75	B	7
77	A	8
79	A	9
80	B	10
81	B	11
83	A	12
85	A	13
90	B	14
92	A	15
95	A	16

Group A: $n_1 = 8, \sum R_A = 78$

Group B: $n_2 = 8, \sum R_B = 58$

[Check: $\sum R_A + \sum R_B = 78 + 58 = 136$; this should be equal to $(n_1 + n_2)(n_1 + n_2 + 1)/2 = 16(17)/2 = 136$✓]

Since $n_i = 8 \geq 8$, we will use the normal approximation with $\mu_R = 8(8 + 8 + 1)/2 = 68$, and $\sigma_R = \sqrt{8(8)(17)/12} = \sqrt{90.6667} = 9.5219$. Since the sample sizes are the same, it doesn't matter which is n_1 or which sum of ranks is labeled R. Without loss of generality let $n_1 = n_A$ and let $R = \sum R_A$.

$z \approx \dfrac{R - \mu_R}{\sigma_R} = \dfrac{78 - 68}{9.5219} = 1.05$, which does not fall in either critical region. Fail to reject H_0; the data do not support the claim that the distribution of spelling scores for students taught by the phonetic method differs from that of students taught spelling by memorization.

[Note: if $R = \sum R_B$, then $z \approx \dfrac{58 - 68}{9.5219} = -1.05$.]

10.

Cement Setting Time	Group	Rank
1.4	B	1
1.6	A	2.5
1.6	B	2.5
1.8	B	4
1.9	A	5
2.2	B	6
2.4	A	7
2.5	B	8
2.7	A	9
2.8	B	10
2.9	A	11
3.4	A	12
3.6	A	13
3.8	B	14
4.0	B	15
4.1	A	16

For the ties at time = 1.6, their rank is the average of the ranks they would otherwise have received: $\frac{2+3}{2} = 2.5$.

Group A: $n_1 = 8, \Sigma R_A = 75.5$

Group B: $n_2 = 8, \Sigma R_B = 60.5$

[Check: $\Sigma R_A + \Sigma R_B = 136$; which should equal $(n_1 + n_2)(n_1 + n_2 + 1)/2 = (8+8)(8+8+1)/2 = 136$ ✓]

Since $n_i = 8 \geq 8$, we can use the normal approximation with $\mu_R = 8(8+8+1)/2 = 68$, and $\sigma_R = \sqrt{8(8)(8+8+1)/12} = \sqrt{90.6667} = 9.5219$. Since the sample sizes are equal, we can chose either sample size n_1; similarly, R can equal either ΣR_A or ΣR_B. Without loss of generality let $n_1 = n_A$ and $R = \Sigma R_A$.

$z \approx \frac{R - \mu_R}{\sigma_R} = \frac{75.5 - 68}{9.5219} = 0.79$, which does not fall in either critical region. Fail to reject H_0; there is insufficient evidence to support the claim that the distribution of cement setting times for cement with no catalyst added differs from that for cement with catalyst added.

[Note: if $R = \Sigma R_B$, then $z \approx \frac{60.5 - 68}{9.5219} = -0.79$.]

Section 12.3

1.

Person	Class Rank, x	Sales Rank, y	$d = x - y$	d^2
1	6	4	2	4
2	8	9	−1	1
3	11	10	1	1
4	2	1	1	1
5	5	6	−1	1
6	7	7	0	0
7	3	8	−5	25
8	9	11	−2	4
9	1	3	−2	4
10	10	5	5	25
11	4	2	2	4
Sum	66	66	0	70

[Note: since there are n = 11 persons. $\sum R_x = \sum R_y = \frac{n(n+1)}{2} = \frac{11(12)}{2} = 66$ and the $\sum d$ always = 0; these can be used as checks on the calculations so far.]

H_0: $\rho_s = 0$ (there is no monotone relationship between x and y)

H_1: $\rho_s \neq 0$ (there is a monotonic relationship between x and y)

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(70)}{11(11^2-1)} = 1 - \frac{420}{11(120)} = 0.6818 \approx 0.682$$

$\alpha = 0.05$, two-tailed test, n = 11, from Table 9, the critical values, call them $\pm r_0$, are ± 0.619. Since $r_s = 0.682$ is in the upper critical region, we reject H_0 and conclude that there is a monotonic relationship between x and y, the person's class rank and sales rank.

2.

Stock	Cost Rank, x	Earnings Rank, y	$d = x - y$	d^2
1	5	5	0	0
2	2	13	−11	121
3	4	1	3	9
4	7	10	−3	9
5	11	7	4	16
6	8	3	5	25
7	12	14	−2	4
8	3	6	−3	9
9	13	4	9	81
10	14	12	2	4
11	10	8	2	4
12	1	2	−1	1
13	9	11	−2	4
14	6	9	−3	9
Sum	105	105	0	296

[Check: n = 14. ΣR_x and ΣR_y should be $n(n+1)/2 = 14(15)/2 = 105$ ✓ and $\sum d$ should be 0✓]

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(296)}{14(196-1)} = 1 - \frac{1776}{2730} = 0.349.$$

H_0: $\rho_s = 0$ (there is no monotonic relationship between x and y)

H_1: $\rho_s \neq 0$ (there is a monotonic relationship)

$\alpha = 0.01$, $n = 14$, two-tailed test: critical values, call them $\pm r_0$, are ± 0.680.

Since $r_s = 0.349$ is not in either critical region, fail to reject H_0: the data do not support the claim of a monotonic relationship between cost rank and earnings rank.

3.

Rat Colony	Population Density Rank, x	Violence Rank, y	$d = x - y$	d^2
1	3	1	2	4
2	5	3	2	4
3	6	5	1	1
4	1	2	−1	1
5	8	8	0	0
6	7	6	1	1
7	4	4	0	0
8	2	7	−5	25
Sum	36	36	0	36

$n = 8$

[Check: $\Sigma R_x = \Sigma R_y = n(n+1)/2 = 8(9)/2 = 36$ ✓ and $\Sigma d = 0$ ✓]

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(36)}{8(64-1)} = 0.571.$$

H_0: $\rho_s = 0$ (there is no monotonic relationship.)

H_1: $\rho_s > 0$ (The relationship between x and y is monotone increasing; the higher population density ranks are as associated with the higher violence ranks.)

$\alpha = 0.05$, one tailed test, $n = 8$, so $r_0 = 0.620$.

Since $r_s = 0.571$ is not in the critical region, we fail to reject H_0; the data do not support the claim of a monotone-increasing relationship between the population density rankings and the violence rankings.

4. (a)

Student	Rank Order Of Finish, x	Score	Score Rank, y	$d = x - y$	d^2
1	5	73	8	−3	9
2	7	90	2	5	25
3	3	82	3.5	−0.5	0.25
4	1	95	1	0	0
5	6	65	9	−3	9
6	2	82	3.5	−1.5	2.25
7	8	78	6	2	4
8	4	75	7	−3	9
9	10	80	5	5	25
10	9	55	10	−1	1
Sum	55	-	55	0	84.5

$n = 10, n(n+1)/2 = 10(11)/2 = 55$

[Check: $\Sigma R_x = \Sigma R_y = n(n+1)/2 = 55$ ✓; $\Sigma d = 0$ ✓]

(Since there are two scores at 82, at rank 3 and rank 4, their ranks are $\frac{3+4}{2} = 3.5$.)

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(84.5)}{10(99)} = 0.488.$$

(b) H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s > 0$ (there is a monotone-increasing relationship)

(Here, the longer it takes a student to finish the exam, the worse his or her score.)

$\alpha = 0.05$, upper/right tailed test, $n = 10$, $r_0 = 0.564$.

Since $r_s = 0.488$ is not in the critical region, we fail to reject H_0; there is insufficient evidence to support the claim that there is a monotone-increasing relationship between order of finish and rank of exam score.

5. (a)

Soldier	Humor Test Score x	Humor Test Rank, x	Aggressiveness Test Score	Aggressiveness Rank, y	$d = x - y$	d^2
1	60	5	78	1	4	16
2	85	3	42	7	−4	16
3	78	4	68	3	1	1
4	90	2	53	5	−3	9
5	93	1	62	4	−3	9
6	45	7	50	6	1	9
7	51	6	76	2	4	1
Sum	-	28	-	28	0	68

$n = 7, n(n+1)/2 = 7(8)/2 = 28$

[Check: $\sum R_x = \sum R_y = n(n+1)/2 = 28$ ✓; $\sum d = 0$ ✓]

(b) H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s < 0$ (there is a monotone-decreasing relationship between x and y)(Here, soldiers with a greater sense of humor, (smaller rank number) have lower aggression scores (large rank numbers)).

$\alpha = 0.05, n = 7$, left/lower tailed test, $r_0 = -0.715$.

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(68)}{7(48)} = 1 - 1.214 = -0.214,$$

which is outside the critical region; fail to reject H_0; there is insufficient evidence to support the claim that humor rankings and aggressiveness rankings have a monotone-decreasing relationship.

6. (a)

System	Quality Rank, x	Price	Price Rank, y	$d = x - y$	d^2
1	4	690	5	−1	1
2	8	175	8	0	0
3	5	1200	1	4	16
4	2	970	2	0	0
5	7	225	7	0	0
6	6	785	4	2	4
7	1	470	6	−5	25
8	3	850	3	0	0
Sum	36	-	36	0	46

$n = 8, n(n+1)/2 = 8(9)/2 = 36$

[Check: $\Sigma R_x = \Sigma R_y = n(n+1)/2 = 36$ ✓; $\sum d = 0$ ✓]

(b) $$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(46)}{8(64-1)} = 0.452$$

H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s > 0$ (there is a monotone relationship)

$\alpha = 0.05, n = 8$, two-tailed test, $\pm r_0 = \pm 0.715$.

Since $r_s = 0.452$ is outside both critical regions, we fail to reject H_0; there is insufficient evidence to show that a monotonic relationship exists between the quality ranking and the price ranking.

7. (a)

Cadet	Aptitude Score	Aptitude Rank, x	Performance Rank, y	$d = x - y$	d^2
1	720	8	7	1	1
2	390	1	1	0	0
3	710	7	8	−1	1
4	480	3	4	−1	1
5	970	11	10	1	1
6	480	3	2	1	1
7	517	5	5	0	0
8	830	9	11	−2	4
9	690	6	6	0	0
10	850	10	9	1	1
11	480	3	3	0	0
Sum	-	66	66	0	10

The three aptitude scores of 480 will receive the average of the ranks they would otherwise receive: $\frac{2+3+4}{3} = \frac{9}{3} = 3$.

$n = 11, \frac{n(n+1)}{2} = \frac{11(12)}{2} = 66$

[Check: $\Sigma R_x = \Sigma R_y = 66$ ✓; $\Sigma d = 0$ ✓]

(b) $r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(10)}{11(121-1)} = 0.955$

H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s > 0$ (there is a monotone-increasing relationship between aptitude rand and performance rank)

$\alpha = 0.005$, upper/right-tailed test, $n = 11$, $r_0 = 0.764$.

Because $r_s = 0.955$ falls in the critical region, we reject H_0; the data support the claim that there is a monotone-increasing relationship between aptitude rank and performance rank.

8. (a)

Secretary	Manager A rank, x	Manager B rank, y	$d = x - y$	d^2
1	3	1	2	4
2	5	3	2	4
3	2	6	−4	16
4	1	2	−1	1
5	6	5	1	1
6	4	4	0	0
Sum	21	21	0	26

$n = 6, \frac{n(n+1)}{2} = \frac{6(7)}{2} = 21$

[Check: $\Sigma R_x = \Sigma R_y = 21$ ✓: $\Sigma d = 0$ ✓]

$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(26)}{6(35)} = 0.257$

H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s > 0$ (there is a monotone-increasing relationship)

$\alpha = 0.05 = 0.257$ upper/right-tailed test, $n = 6, r_0 = 0.829$.

Since $r_s = 0.257$ is outside the critical region, we fail to reject H_0: the data do not support the claim that there is an increasing monotone relationship between the two managers' rankings.

9. (a)

City	Insurance Sales Rank, x	Per Capita Income	Income Rank, y	$d = x - y$	d^2
1	6	17	5	1	1
2	7	18	4	3	9
3	1	19	2.5	−1.5	2.25
4	8	11	8	0	0
5	3	16	6	−3	9
6	2	20	1	1	1
7	5	15	7	−2	4
8	4	19	2.5	1.5	2.25
Sum	36	-	36	0	28.5

$n = 8, \frac{n(n+1)}{2} = \frac{8(9)}{2} = 36$

[Check: $\sum R_x = \sum R_y = 36$ ✓; $\sum d = 0$ ✓]

$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(28.5)}{8(64-1)} = 0.661$

H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s \neq 0$ (there is a monotone relationship between x and y)

$\alpha = 0.01$, two-tailed test, $n = 8$, $\pm r_0 = \pm 0.881$.

Because $r_s = 0.661$ falls outside both critical regions, we fail to reject H_0; there is insufficient evidence to support the claim of a monotone relationship between volume of insurance sales ranking and per capita income ranking.

Chapter 12 Review

No sketches of the critical region(s) will be shown in (d) below.

1. (a) Wilcoxon rank-sum test (2 independent samples)

   (b) H_0: the population distributions are the same
   H_1: the distributions of the populations are different.

   (c) $\alpha = 0.05$, two-tailed test, $\pm z_0 = \pm 1.96$

(d)

Viscosity Index	Rank	Group
1.1	1	A
1.5	2	B
1.6	3	A
1.8	4	A
1.9	5	B
2.2	6	B
2.4	7	B
2.5	8	A
2.8	9	B
2.9	10	A
3.2	11	A
3.3	12	B
3.5	13	B
3.6	14	B
3.7	15	A
3.8	16	A
3.9	17	B
4.0	18	B
4.2	19	A
4.4	20	A
4.6	21	B

(where Group = A if the catalyst was used. and Group B if no catalyst was used)

$n_1 = n_A = 10, n_2 = n_B = 11, (n_1+n_2)\frac{(n_1+n_2+1)}{2} = \frac{21(22)}{2} = 231$

$\Sigma R_A = 107,\ \Sigma R_B = 124,\ \Sigma R_A + \Sigma R_B = 107 + 124 = 231$

$R = \Sigma R_A = 107$

Since n_1, n_2 both ≥ 8, use the normal approximation with

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{10(10+11+1)}{2} = 110$$

$$\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{10(11)(22)}{12}} = \sqrt{201.6667} = 14.2009.$$

Then

$$z = \frac{R-\mu_R}{\sigma_R} = \frac{107-110}{14.2009} = -0.21$$

(e) Because $z = -0.21$ falls outside both critical regions. fail to reject H_0.

(f) There is insufficient evidence to support the claim that the presence of the catalyst affects the viscosity. The distributions appear to be the same.

2. (a) Paired data (before and after memory course); sign test.

(b) H_0: the population distributions are the same

H_1: the population distribution of the last exam scores is shifted to the right of the population distribution of the first exam scores.

(c) $\alpha = 0.05$, one-tailed test, $z_0 = 1.645$

(d)

Student	Sign of Difference, Last Exam – First Exam
1	+
2	+
3	0
4	+
5	+
6	–
7	–
8	+
9	+
10	+
11	+
12	+
13	+
14	+
15	–

(d) Number of nonzero differences $n = 14 \geq 12$, so use normal approximation with $\mu = p = 0.5$ and $\sigma = \sqrt{pq/n} = \sqrt{0.25/n} = \sqrt{0.25/14} = 0.1336$. $r = 11$ plus signs. $x = r/n = 11/14 = 0.7857$.

$$z = \frac{x-\mu}{\sigma} = \frac{0.79-0.5}{0.1336} = 2.17$$

(e) Since $z = 2.17$ is in the critical region, reject H_0.

(f) There is sufficient evidence to conclude that taking the memory course improves memory exam scores, i.e., that the last exam's population distribution is shifted to the right of that for the first exam.

3. (a) Paired data, before and after; sign test.

(b) H_0: the population distributions are the same.
H_1: the population distribution of sales after mailing out advertising pamphlets is shifted to the right of the "before pamphlet" sales distribution.

(c) $\alpha = 0.01$, right-tailed test, $z_0 = 2.33$

(d)

City	Sign of Difference, After – Before
1	+
2	–
3	+
4	+
5	+
6	+
7	+
8	+
9	+
10	0
11	+
12	+
13	0
14	–
15	–

n = number of non-zero differences, 13
r = number of + signs, 10
$x = r/n = 10/13 = 0.7692 \approx 0.77$
Since $n = 13 \geq 12$, we will use the normal approximation with $\mu = p = 0.5$ and
$\sigma = \sqrt{pq/n} = \sqrt{0.25/13} = 0.1387$.

$$z = \frac{x-\mu}{\sigma} = \frac{0.77-0.5}{0.1387} = 1.95$$

(e) Since $z = 1.95$ is outside the critical region, fail to reject H_0.

(f) The data do not support the claim that advertising improved sales, i.e., there is insufficient evidence to support the claim that the "after advertising" distribution is shifted to the right of that for "before advertising."

4. (a) Two independent samples, Wilcoxon rank-sum.

(b) H_0: the population distributions are the same.
H_1: the population distributions are different.

(c) $\alpha = 0.05$, two-tailed test, $\pm z_0 = \pm 1.96$.

(d)

Number of Sessions	Group	Rank
8	A	1
9	A	2
10	A	3
11	B	4
12	A	5
13	B	6
14	A	7
15	A	8
16	A	9
17	A	10
18	B	11
19	B	12
20	A	13
21	B	14
22	B	15
23	A	16
24	B	17
25	B	18
28	B	19

where Group = A if the dog was rewarded, and Group = B otherwise.
Group A: $n_2 = 10, \Sigma R_A = 74$
Group B: $n_1 = 9, \Sigma R_B = 116$

[Check: $(n_1 + n_2)(n_1 + n_2 + 1)/2 = (9+10)(9+10+1)/2 = 190$
which should equal $\Sigma R_A + \Sigma R_B = 74 + 116 = 190$ ✓]

Since $n_i \geq 8$, we can use the normal approximation with

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{9(9+10+1)}{2} = 90 \text{ and } \sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}} = \sqrt{\frac{9(10)(9+10+1)}{12}} = \sqrt{150} = 12.2474.$$

$R = \Sigma R_B = 116$

$$z = \frac{R-\mu_R}{\sigma_R} = \frac{116-90}{12.2474} = 2.12$$

(e) Since $z = 2.12$ is in the upper critical area, reject H_0:

(f) There is sufficient evidence to show that the distribution of the number of dog training sessions with rewards is different from that for "no rewards."

5. (a) Relationship between rankings: Spearman's rank correlation test.

(b) H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s > 0$ (there is a monotone-increasing relationship)

(c) $\alpha = 0.05$, right-tailed test, $n = 9$ pairs, $r_0 = 0.600$

(d)

Employee	Training Program Rank, x	Rank on the Job, y	$d = x - y$	d^2
1	8	9	−1	1
2	9	8	1	1
3	7	6	1	1
4	3	7	−4	16
5	6	5	1	1
6	4	1	3	9
7	1	3	−2	4
8	2	4	−2	4
9	5	2	3	9
Sum	45	45	0	46

[Check: $\sum R_x = \sum R_y = 45$ which should equal $n(n+1)/2 = 9(10)/2 = 45$ ✓, $\sum d = 0$ ✓]

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(46)}{9(81-1)} = 0.617$$

(e) Since $r_s = 0.617$ exceeds $r_0 = 0.600$, it is in the critical region and we reject H_0.

(f) There is sufficient evidence to show that there is a monotone-increasing relationship between the training program ranking and the on-the-job performance ranking.

6. (a) (Monotonic) relationship between ranks: Spearman's rank correlation test.

(b) H_0: $\rho_s = 0$ (there is no monotonic relationship)

H_1: $\rho_s \neq 0$ (there is a monotone-increasing relationship)

(c) $\alpha = 0.10$, two-tailed test, $n = 5$ pairs, $\pm r_0 = \pm 0.900$.

Student	Chef Pierre Rank, x	Chef André Rank, y	$d = x - y$	d^2
1	4	4	0	0
2	2	1	1	1
3	3	2	1	1
4	1	3	−2	4
5	5	5	0	0
Sum	15	15	0	6

[Check: $\sum R_x = \sum R_y = 15$ which should equal $n(n+1)/2 = 5(6)/2 = 15$ ✓, $\sum d = 0$ ✓]

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(6)}{5(25-1)} = 0.700$$

(e) Since $r_s = 0.7$ is outside both critical regions, fail to reject H_0.

(f) The evidence is insufficient to conclude that there is a monotone relationship between Chef Pierre's and Chef André's rankings.