Chapter 6 Normal Distributions

Section 6.1

1. (a) not normal; left skewed instead of symmetric

   (b) not normal; curve touches and goes below
   x-axis instead of always being above the x-axis and being asymptotic to the x-axis in the tails

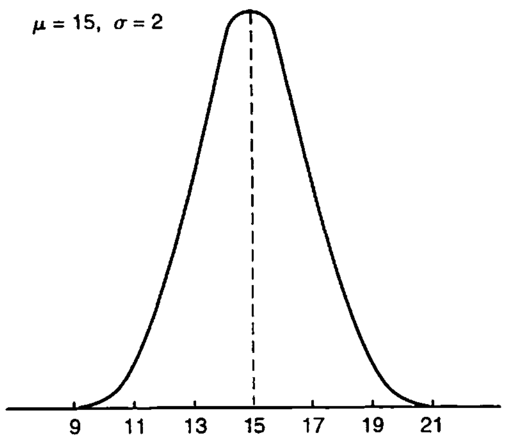
   (c) not normal; not bell-shaped, not unimodal

   (d) not normal; not a smooth curve

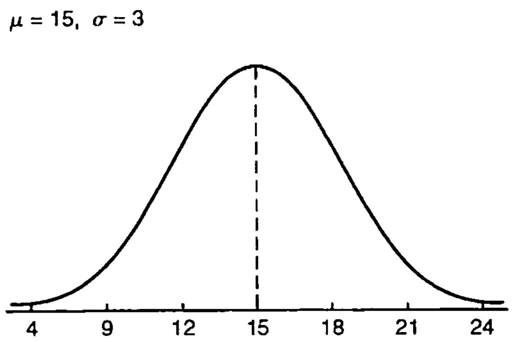
2. $\mu = 16$, $\sigma = 2$, $\mu + \sigma = 16 + 2 = 18$
   (The mean is located directly below the peak; one standard deviation from the mean is the x-value under the point of inflection [the transition point between the curve cupping upward and cupping downward].)

3. The mean is the x-value directly below the peak; in Figure 6-16, $\mu = 10$; in Figure 6-17, $\mu = 4$. Assuming the two figures are drawn on the same scale, Figure 6-16, being shorter and more spread out, has the larger standard deviation.

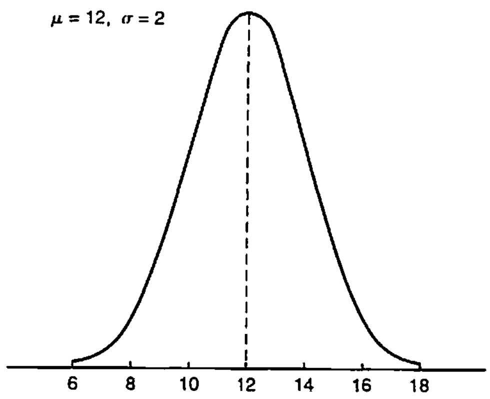
4. (a)



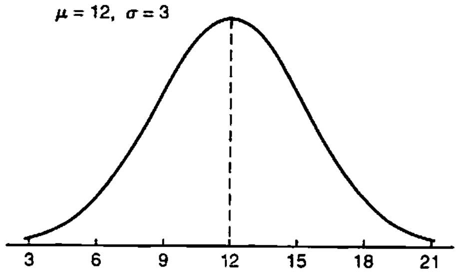
   (b)



(c)



(d)



(e) No; the mean μ and the standard deviation σ are independent of one another. If $\mu_1 > \mu_2$, then $\sigma_1 > \sigma_2$, $\sigma_1 = \sigma_2$, and $\sigma_1 < \sigma_2$ are all possible.

5. (a) 50%; the normal curve is symmetric about μ
(b) 68%
(c) 99.7%

6. (a) 50%; the normal curve is symmetric about μ
(b) 95%
(c) $\frac{1}{2}(100\% - 99.7\%) = \frac{1}{2}(0.3\%) = 0.15\%$,
99.7% lies between $\mu - 3\sigma$ and $\mu + 3\sigma$, so 0.3% lies in the tails, and half of that is in the upper tail.

7. (a) $\mu = 65$, so 50% are taller than 65 in.
(b) $\mu = 65$, so 50% are shorter than 65 in.
(c) $\mu - \sigma = 65 - 2.5 = 62.5$ in. and $\mu + \sigma = 65 + 2.5 = 67.5$ in. so 68% of college women are between 62.5 in. and 67.5 in. tall.
(d) $\mu - 2\sigma = 65 - 2(2.5) = 65 - 5 = 60$ in. and $\mu - 2\sigma = 65 + 2(2.5) = 65 + 5 = 70$ in. so 95% of college women are between 60 in. and 70 in. tall.

8. (a) $\mu - 2\sigma = 21 - 2(1) = 19$ days and $\mu + 2\sigma = 21 + 2(1) = 23$ days so 95% of 1000 eggs, or 950 eggs will hatch between 19 and 23 days of incubation.
(b) $\mu - \sigma = 21 - 1 = 20$ days and $\mu + \sigma = 21 + 1 = 22$ days so 68% of 1000 eggs, or 680 eggs, will hatch between 20 and 22 days of incubation.
(c) $\mu = 21$, so 50%, or 500, of the eggs will hatch in at most 21 days.

(d) $\mu - 3\sigma = 21 - 3(1) = 21 - 3 = 18$ days and $\mu + 3\sigma = 21 + 3(1) = 21 + 3 = 24$ days so 99.7%, or 997, eggs will hatch between 18 and 24 days of incubation.

9. (a) $\mu - \sigma = 1243 - 36 = 1207$ and $\mu + \sigma = 1243 + 36 = 1279$ so about 68% of the tree rings will date between 1207 and 1279 AD.

(b) $\mu - 2\sigma = 1243 - 2(36) = 1171$ and $\mu + 2\sigma = 1243 + 2(36) = 1315$ so about 95% of the tree rings will date between 1171 and 1315 AD.

(c) $\mu - 3\sigma = 1243 - 3(36) = 1135$ and $\mu + 3\sigma = 1243 + 3(36) = 1351$ so 99.7% (almost all) of the tree rings will date between 1135 and 1351 AD.

10. (a) $\mu + \sigma = 7.6 + 0.4 = 8.0$

Since 68% of the cups filled will fall into the $\mu \pm \sigma$ range, 100% – 68% = 32% will fall outside that range and $\frac{32\%}{2} = 16\% = 0.16$ will be over $\mu + \sigma = 8$ oz. Approximately 16% of the time, the cups will overflow.

(b) 100% – 16% = 84% = 0.84 so, by the complementary event rule, 84% of the time the cups will not overflow.

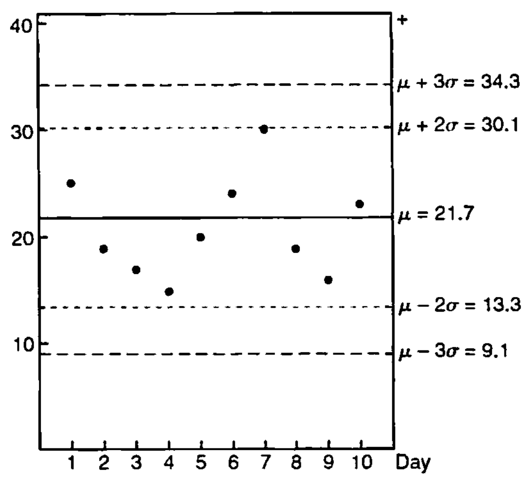
(c) Since 16% of 850 = 136, we can expect approximately 136 of the cups filled by this machine will overflow.

11. (a) $\mu - \sigma = 3.15 - 1.45 = 1.70$ and $\mu + \sigma = 3.15 + 1.45 = 4.60$ so 68% of the experimental group will have millamperes pain thresholds between 1.70 and 4.60 mA.

(b) $\mu - 2\sigma = 3.15 - 2(1.45) = 0.25$ and $\mu + 2\sigma = 3.15 + 2(1.45) = 6.05$ so 95% of the experimental group will have pain thresholds between 0.25 and 6.05 mA.

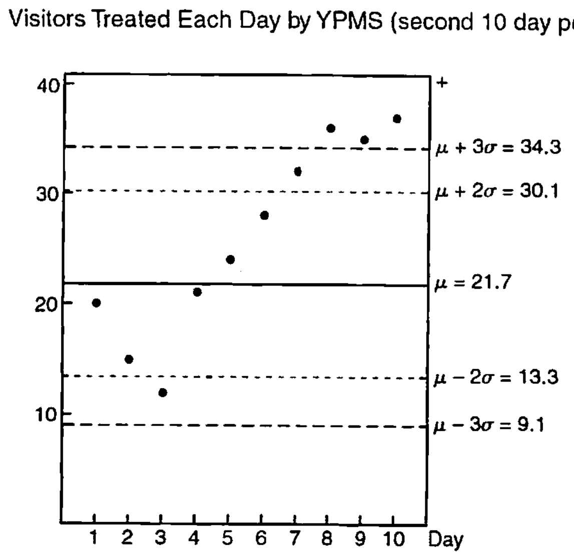
12. (a)

Visitors Treated Each Day by YPMS (first 10 day period)



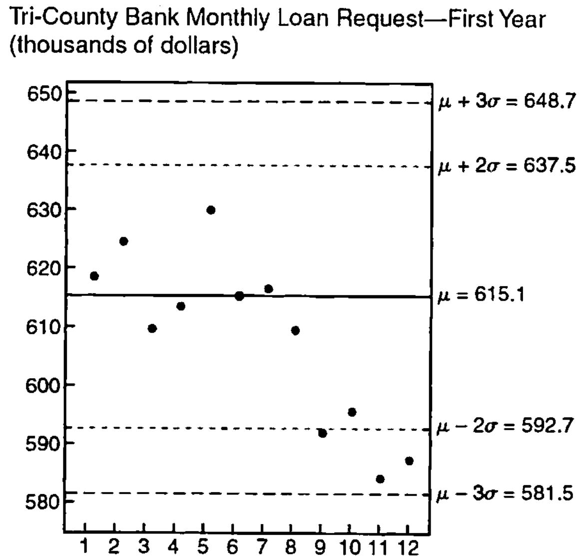
The data indicate the process is in control; none of the out-of-control warning signals are present.

(b)



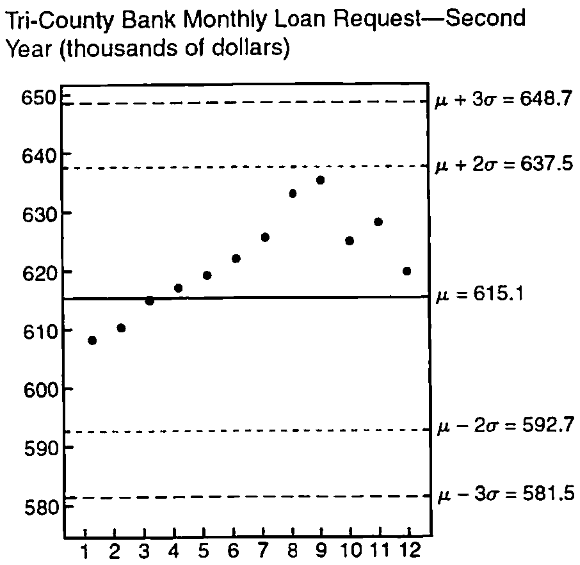
Three points fall beyond $\mu + 3\sigma = 34.3$. Four consecutive points lie beyond $\mu + 2\sigma = 30.1$. Out-of-control warning signals I and III are present; the data indicate the process is out-of-control. Under the conditions or time period (say, July 4) represented by the second 10-day period, YPMS probably needs (temporary) extra help to provide timely emergency health care for park visitors.

13. (a)



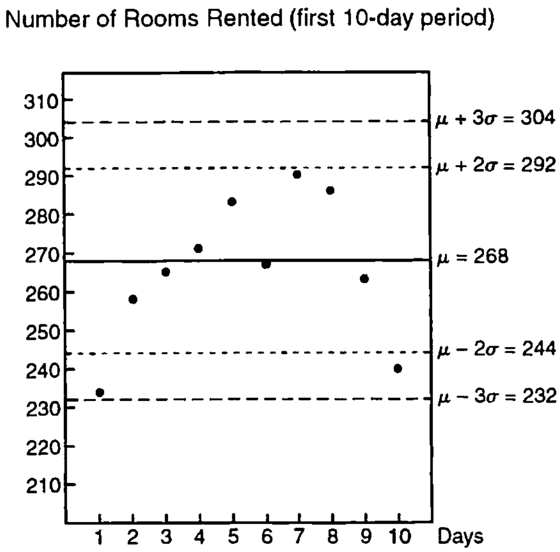
The economy would appear to be cooling off as evidenced by an overall downward trend. Out-of-control warning signal III is present: 2 of the last 3 consecutive points are below $\mu - 2\sigma = 592.7$.

(b)



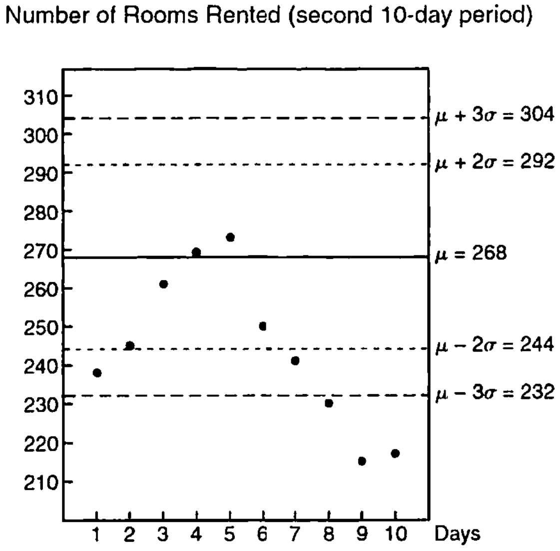
Here, it looks like the economy was heating up during months 1-9 and perhaps cooling off during months 10-12. Out-of-control warning signal II is present: there is a run of 9 consecutive points above $\mu = 615.1$.

14. (a)



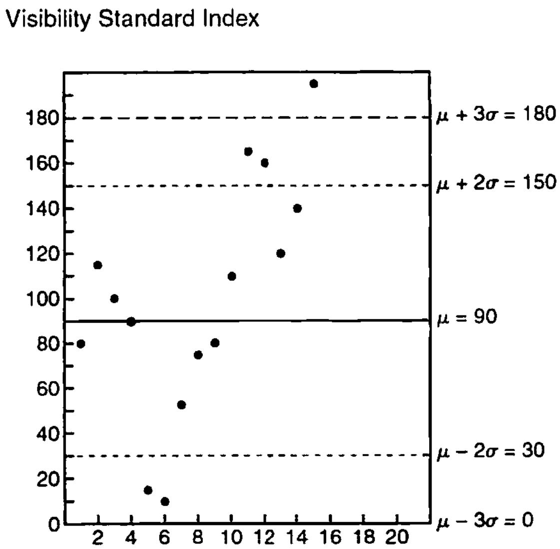
The room rentals are about what would be expected. None of the 3 out-of-control warning signals are present.

(b)



The room rentals are lower than what would be expected. Comparing (a) and (b), we see the same basic cyclical pattern (probably up on the weekends and down on the weekdays), but the pattern in (b) has been shifted down about $2\sigma = 24$ rooms rented. Out-of-control warning signals I and III are present: there are 3 points below $\mu - 3\sigma = 232$ rooms rented, and all 4 of the last 4 consecutive points are below $\mu - 2\sigma = 244$ rooms rented.

15.



Out-of-control warning signals I and III are present. Day 15's VSI exceeds $\mu + 3\sigma$. Two of 3 consecutive points (days 10, 11, 12 or days 11, 12, 13) are about $\mu + 2\sigma = 150$, and 2 of 3 consecutive points (days 4, 5, 6 or days 5, 6, 7) are below $\mu - 2\sigma = 30$. Days 10-15 all show above average air pollution levels: days 11, 12, and 15 triggered out-of-control signals, indicating pollution abatement procedures should be in place.

Section 6.2

1. (a) z-scores > 0 indicate the student scored above the mean: Robert, Jane, and Linda

(b) z-scores = 0 indicates the student scored at the mean: Joel

(c) z-scores < 0 indicate the student scored below the mean: John and Susan

(d) $z = \frac{x-\mu}{\sigma}$ so $x = \mu + z\sigma$

In this case, if the student's score is x, $x = 150 + z(20)$.

Robert: $x = 150 + 1.10(20) = 172$
Joel: $x = 150 + 0(20) = 150$
Jan: $x = 150 + 1.70(20) = 184$
John: $x = 150 - 0.80(20) = 134$
Susan: $x = 150 - 2.00(20) = 110$
Linda: $x = 150 + 1.60(20) = 182$

2. Use $z = \frac{x-\mu}{\sigma}$. In this case, $z = \frac{x-0.51}{0.25}$ with x expressed as a decimal or $z = \frac{x-51\%}{25\%}$ with x expressed in percent.

(a) $z = \frac{0.45-0.51}{0.25} = -0.24$

(b) $z = \frac{0.72-0.51}{0.25} = 0.84$

(c) $z = \frac{0.75-0.51}{0.25} = 0.96$

(d) $z = \frac{65\%-51\%}{25\%} = 0.56$

(e) $z = \frac{33\%-51\%}{25\%} = -0.72$

(f) $z = \frac{55\%-51\%}{25\%} = 0.16$

3. Use $z = \frac{x-\mu}{\sigma}$. In this case, $z = \frac{x-73}{5}$.

(a) $53^\circ\text{F} < x < 93^\circ\text{F}$

$\frac{53-73}{5} < \frac{x-73}{5} < \frac{93-73}{5}$ Subtract $\mu = 73^\circ$F from each piece; divide result by $\sigma = 5^\circ$F.

$-\frac{20}{5} < z < \frac{20}{5}$

$-4.00 < z < 4.00$

(b) $x < 65°\text{F}$

$x - 73 < 65 - 73$ Subtract $\mu = 73°\text{F}$.

$\frac{x-73}{5} < \frac{65-73}{5}$ Divide both sides by $\sigma = 5°\text{F}$.

$z < -1.6$

(c) $78°\text{F} < x$

$\frac{78-73}{5} < \frac{x-73}{5}$ Subtract $\mu = 73°\text{F}$ from each side; divide by $\sigma = 5°\text{F}$.

$\frac{5}{5} < z$

$1.00 < z$ (or $z > 1.00$)

Since $z = \frac{x-73}{5}$, $x = 73 + 5z$.

(d) $1.75 < z$

$5(1.75) < 5z$ Multiply both sides by $\sigma = 5°\text{F}$.

$73 + 5(1.75) < 73 + 5z$ Add $\mu = 73°\text{F}$ to both sides.

$81.75°\text{F} < x$ (or $x > 81.75$)

(e) $z < -1.90$

$5z < 5(-1.90)$ Multiply both sides by $\sigma = 5°\text{F}$.

$73 + 5z < 73 + 5(-1.90)$ Add $\mu = 73°\text{F}$ to both sides.

$x < 63.5°\text{F}$

(f) $-1.80 < z < 1.65$

$5(-1.80) < 5z < 5(1.65)$ Multiply each part by $\sigma = 5°\text{F}$.

$73 + 5(-1.80) < 73 + 5z < 73 + 5(1.65)$ Add $\mu = 73°\text{F}$ to each part of the inequality.

$64°\text{F} < x < 81.25°\text{F}$

4. $z = \frac{x-\mu}{\sigma}$; here, $z = \frac{x-27.2}{4.3}$

(a) $x < 30$ kg

$x - 27.2 < 30 - 27.2$ Subtract $\mu = 27.2$ kg from each side.

$\frac{x-27.2}{4.3} < \frac{30-27.2}{4.3}$ Divide both sides by $\sigma = 4.3$ kg.

$z < 0.65$ (rounded to 2 decimal places)

(b) $19 \text{ kg} < x$

$19 - 27.2 < x - 27.2$ Subtract $\mu = 27.2$ kg from each side.

$\frac{19-27.2}{4.3} < \frac{x-27.2}{4.3}$ Divide both sides by $\sigma = 4.3$ kg.

$-1.91 < z$ (rounded)

(c) $32 \text{ kg} < x < 35 \text{ kg}$

$32 - 27.2 < x - 27.2 < 35 - 27.2$ Subtract $\mu = 27.2$ kg from each part.

$\frac{32-27.2}{4.3} < \frac{x-27.2}{4.3} < \frac{35-27.2}{4.3}$ Divide each part by $\sigma = 4.3$ kg.

$1.12 < z < 1.81$ (rounded)

Since $z = \frac{x-27.2}{4.3}$, $x = 27.2 + 4.3z$ kg.

(d)
$$-2.17 < z$$
$$(4.3)(-2.17) < 4.3z \quad \text{Multiply both sides by } \sigma = 4.3 \text{ kg.}$$
$$27.2 + 4.3(-2.17) < 27.2 + 4.3z \quad \text{Add } \mu = 27.2 \text{ kg to each side.}$$
$$17.9 \text{ kg} < x, \text{ or } x > 17.9 \text{ kg} \quad \text{(rounded)}$$

(e)
$$z < 1.28$$
$$4.3z < 4.3(1.28) \quad \text{Multiply both sides by } \sigma = 4.3 \text{ kg.}$$
$$27.2 + 4.3z < 27.2 + 4.3(1.28) \quad \text{Add } \mu = 27.2 \text{ kg to both sides.}$$
$$x < 32.7 \text{ kg} \quad \text{(rounded)}$$

(f)
$$-1.99 < z < 1.44$$
$$4.3(-1.99) < 4.3z < 4.3(1.44) \quad \text{Multiply each part by } \sigma = 4.3 \text{ kg.}$$
$$27.2 + 4.3(-1.99) < 27.2 + 4.3z < 27.2 + 4.3(1.44) \quad \text{Add } \mu = 27.2 \text{ kg to each part.}$$
$$18.6 \text{ kg} < x < 33.4 \text{ kg} \quad \text{(rounded)}$$

(g) 14 kg is an unusually low weight for a fawn

$$z = \frac{x - 27.2}{4.3} = \frac{14 - 27.2}{4.3} = -3.07 \quad \text{(rounded)}$$

(note $\mu = 27.2$ kg)

(h) An unusually large fawn would have a large positive z, such as 3.

5. $z = \dfrac{x - \mu}{\sigma}$, here $z = \dfrac{x - 4400}{620}$

(a)
$$3300 < x$$
$$3300 - 4400 < x - 4400 \quad \text{Subtract } \mu = 4400 \text{ deer.}$$
$$\frac{3300 - 4400}{620} < \frac{x - 4400}{620} \quad \text{Divide by } \sigma = 620 \text{ deer.}$$
$$-1.77 < z$$

(b)
$$x < 5400$$
$$x - 4400 < 5400 - 4400 \quad \text{Subtract } \mu = 4400 \text{ deer.}$$
$$\frac{x - 4400}{620} < \frac{5400 - 4400}{620} \quad \text{Divide by } \sigma = 620 \text{ deer.}$$
$$z < 1.61$$

(c)
$$3500 < x < 5300$$
$$3500 - 4400 < x - 4400 < 5300 - 4400 \quad \text{Subtract } \mu = 4400.$$
$$\frac{3500 - 4400}{620} < \frac{x - 4400}{620} < \frac{5300 - 4400}{620} \quad \text{Divide by } \sigma = 620.$$
$$-1.45 < z < 1.45$$

Since $z = \dfrac{x - 4400}{620}$, $x = 4400 + 620z$ deer.

(d)
$$-1.12 < z < 2.43$$
$$620(-1.12) < 620z < 620(2.43) \quad \text{Multiply by } \sigma = 620.$$
$$4400 + 620(-1.12) < 4400 + 620z < 4400 + 620(2.43) \quad \text{Add } \mu = 4400 \text{ to each part.}$$
$$3706 \text{ deer} < x < 5907 \text{ deer (rounded)}$$

(e)
$$\begin{aligned} z &< 1.96 \\ 620z &< 620(1.96) && \text{Multiply by } \sigma. \\ 4400+620z &< 4400+620(1.96) && \text{Add } \mu. \\ x &< 5615 \text{ deer} \end{aligned}$$

(f)
$$\begin{aligned} 2.58 &< z \\ 620(2.58) &< 620z && \text{Multiply by } \sigma. \\ 4400+620(2.58) &< 4400+620z && \text{Add } \mu. \\ 6000 \text{ deer} &< x \end{aligned}$$

(g) If $x = 2800$ deer, $z = \dfrac{2800-4400}{620} = -2.58.$

This is a small z-value, so 2800 deer is quite low for the fall deer population.

If $x = 6300$ deer, $z = \dfrac{6300-4400}{620} = 3.06.$

This is a very large z-value, so 6300 deer would be an unusually large fall population size.

6. $z = \dfrac{x-\mu}{\sigma}$ so in this case, $z = \dfrac{x-7500}{1750}$.

(a)
$$\begin{aligned} 9000 &< x \\ \frac{9000-7500}{1750} &< \frac{x-7500}{1750} && \text{Subtract } \mu;\text{ divide by } \sigma. \\ 0.86 &< z \end{aligned}$$

(b)
$$\begin{aligned} x &< 6000 \\ \frac{x-7500}{1750} &< \frac{6000-7500}{1750} && \text{Subtract } \mu;\text{ divide by } \sigma. \\ z &< -0.86 \end{aligned}$$

(c)
$$\begin{aligned} 3500 < x &< 4500 \\ \frac{3500-7500}{1750} < \frac{x-7500}{1750} &< \frac{4500-7500}{1750} && \text{Subtract } \mu;\text{ divide by } \sigma. \\ -2.29 < z &< -1.71 \end{aligned}$$

Since $z = \dfrac{x-7500}{1750}$, $x = 7500+1750z$.

(d)
$$\begin{aligned} z &< 1.15 \\ 7500+1750z &< 7500+1750(1.15) && \text{Multiply by } \sigma;\text{ add } \mu. \\ x &< 9513 \end{aligned}$$

(e)
$$\begin{aligned} 2.19 &< z \\ 7500+1750(2.19) &< 7500+1750(z) && \text{Multiply by } \sigma;\text{ add } \mu. \\ 11{,}333 &< x \end{aligned}$$

(f)
$$\begin{aligned} 0.25 < z &< 1.25 \\ 7500+1750(0.25) < 7500+1750z &< 7500+1750(1.25) && \text{Multiply by } \sigma;\text{ add } \mu. \\ 7938 < x &< 9688 \end{aligned}$$

(g) Since $\mu = 7500$. $x = 2500$ is quite low.

$$z = \frac{x-\mu}{\sigma} = \frac{2500-7500}{1750} = -2.86 \text{ (a very small } z\text{)}$$

7. $z = \frac{x-\mu}{\sigma}$ so in this case, $z = \frac{x-4.8}{0.3}$.

(a)
$$4.5 < x$$
$$\frac{4.5-4.8}{0.3} < \frac{x-4.8}{0.3} \quad \text{Subtract } \mu\text{; divide by } \sigma.$$
$$-1.00 < z$$

(b)
$$x < 4.2$$
$$\frac{x-4.8}{0.3} < \frac{4.2-4.8}{0.3} \quad \text{Subtract } \mu\text{; divide by } \sigma.$$
$$z < -2.00$$

(c)
$$4.0 < x < 5.5$$
$$\frac{4.0-4.8}{0.3} < \frac{x-4.8}{0.3} < \frac{5.5-4.8}{0.3} \quad \text{Subtract } \mu\text{; divide by } \sigma.$$
$$-2.67 < z < 2.33$$

Since $z = \frac{x-4.8}{0.3}$, $x = 4.8 + 0.3z$.

(d)
$$z < -1.44$$
$$0.3z < 0.3(-1.44) \quad \text{Multiply by } \sigma.$$
$$4.8 + 0.3z < 4.8 + 0.3(-1.44) \quad \text{Add } \mu.$$
$$x < 4.4$$

(e)
$$1.28 < z$$
$$0.3(1.28) < 0.3z \quad \text{Multiply by } \sigma.$$
$$4.8 + 0.3(1.28) < 4.8 + 0.3z \quad \text{Add } \mu.$$
$$5.2 < x$$

(f)
$$-2.25 < z < -1.00$$
$$0.3(-2.25) < 0.3z < 0.3(-1.00) \quad \text{Multiply by } \sigma.$$
$$4.8 + 0.3(-2.25) < 4.8 + 0.3z < 4.8 + 0.3(-1.00) \quad \text{Add } \mu.$$
$$4.1 < x < 4.5$$

(g) If the RBC was 5.9 or higher, that would be an unusually high red blood cell count.

$$x \geq 5.9$$
$$\frac{x-4.8}{0.3} \geq \frac{5.9-4.8}{0.3}$$
$$z \geq 3.67 \text{ (a very large } z\text{-value)}$$

8. (a) $z = \dfrac{x-\mu}{\sigma}$

Site 1: $z_1 = \dfrac{x_1 - \mu_1}{\sigma_1}$

$z_1 = \dfrac{x_1 - 1272}{35}$

so for $x_1 = 1250$

$z_1 = \dfrac{1250-1272}{35} = -0.63$

Site 2: $z_2 = \dfrac{x_2 - \mu_2}{\sigma_2}$

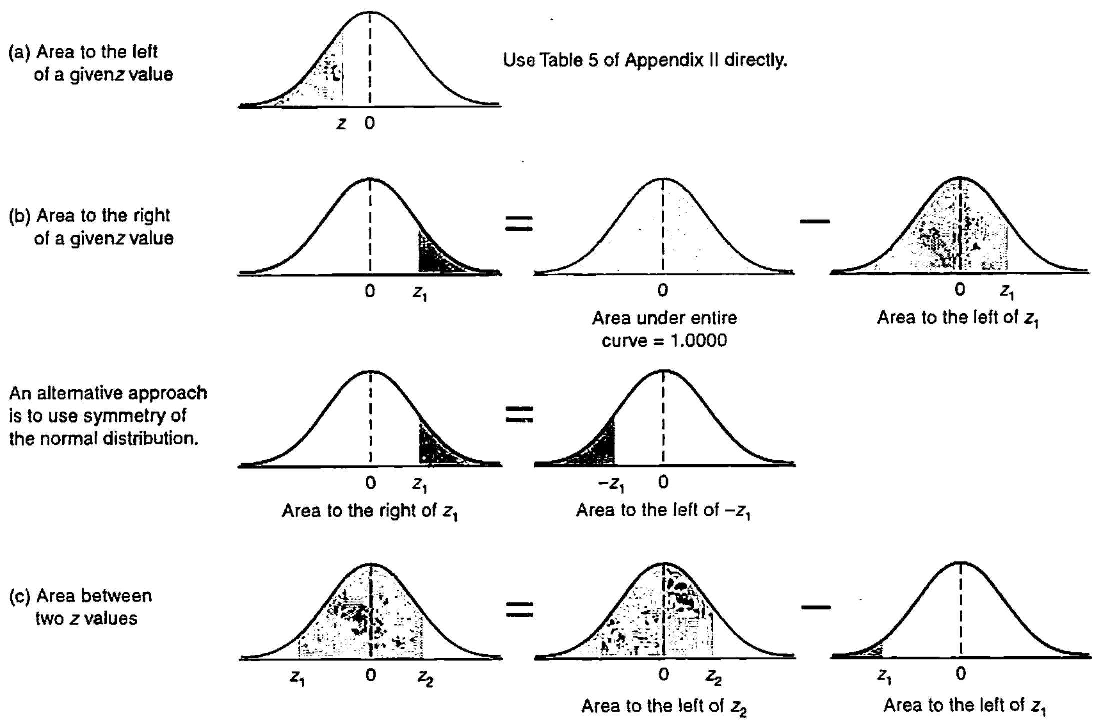
$z_2 = \dfrac{x_2 - 1122}{40}$

so for $x_2 = 1234$

$z_2 = \dfrac{1234-1122}{40} = 2.80$

(b) x_2, the object dated 1234 AD, is more unusual at its site, since $z_2 = 2.8$ vs. $z_1 = -0.63$.

For problems 9–48, refer to the following sketch patterns for guidance in calculations



Using the left-tail style standard normal distribution table (see figures above)

(a) For areas to the *left* of a specified z value, use the table entry directly.

(b) For areas to the *right* of a specified z value, look up the table entry for z and subtract the table value from 1. (This is the complementary event rule as applied to area as probability.)

OR: Use the fact that the normal curve is symmetric about the mean, 0. The area in the right tail above a z-value is the same as the area in the left tail below the value of $-z$. So, to find the area to the right of z, look up the table value for $-z$.

(c) For areas *between* two z-values, z_1 and z_2, where $z_1 < z_2$, subtract the tabled value for z_1, from the tabled value for z_2.

These sketches and rules for finding the area for probability from the standard normal table apply for *any* z: $-\infty < z < +\infty$.

Student sketches should resemble those indicated with negative z-values to left of 0 and positive z-values to the right of zero.

9. Refer to figure (b).
The area to the right of $z = 0$ is 1 − area to left of $z = 0$, or 1 − 0.5000 = 0.5000.

10. Refer to figure (a).
The area to the left of $z = 0$ is 0.5000 (direct read).

11. Refer to figure (a).
The area to the left of $z = -1.32$ is 0.0934.

12. Refer to figure (a).
The area to the left of $z = -0.47$ is 0.3192.

13. Refer to figure (a).
The area to left of $z = 0.45$ is 0.6736.

14. Refer to figure (a).
The area to left of $z = 0.72$ is 0.7642.

15. Refer to figure (b).
The area to right of $z = 1.52$ is 1 − 0.9357 = 0.0643.

16. Refer to figure (b).
The area to right of $z = 0.15$ is 1 − 0.5596 = 0.4404.

17. Refer to figure (b).
The area to right of $z = -1.22$ is 1 − 0.1112 = 0.8888.

18. Refer to figure (b).
The area to right of $z = -2.17$ is 1 − 0.0150 = 0.9850.

19. Refer to figure (c).
The area between $z = 0$ and $z = 3.18$ is 0.9993 − 0.5000 = 0.4993.

20. Refer to figure (c).
The area between $z = 0$ and $z = 2.92$ is 0.9982 − 0.5000 = 0.4982.

21. Refer to figure (c).
The area between $z = 0$ and $z = -2.01$ is 0.5000 − 0.0222 = 0.4778.

22. Refer to figure (c).
The area between $z = 0$ and $z = -1.93$ is 0.5000 − 0.0268 = 0.4732.

23. Refer to figure (c).
The area between $z = -2.18$ and $z = 1.34$ is 0.9099 − 0.0146 = 0.8953.

24. Refer to figure (c).
The area between $z = -1.40$ and $z = 2.03$ is $0.9788 - 0.0808 = 0.8980$.

25. Refer to figure (c).
The area between $z = 0.32$ and $z = 1.92$ is $0.9726 - 0.6255 = 0.3471$.

26. Refer to figure (c).
The area between $z = 1.42$ and $z = 2.17$ is $0.9850 - 0.9222 = 0.0628$.

27. Refer to figure (c).
The area between $z = -2.42$ and $z = -1.77$ is $0.0384 - 0.0078 = 0.0306$.

28. Refer to figure (c).
The area between $z = -1.98$ and $z = -0.03$ is $0.4880 - 0.0239 = 0.4641$.

29. Refer to figure (a).
$P(z \le 0) = -0.5000$

30. Refer to figure (b).
$P(z \ge 0) = 1 - P(z < 0) = 1 - 0.5000 = 0.5000$

31. Refer to figure (a).
$P(z \le -0.13) = 0.4483$ (direct read)

32. Refer to figure (a).
$P(z \le -2.15) = 0.0158$

33. Refer to figure (a).
$P(z \le 1.20) = 0.8849$

34. Refer to figure (a).
$P(z \le 3.20) = 0.9993$

35. Refer to figure (b).
$P(z \ge 1.35) = 1 - P(z < 1.35) = 1 - 0.9115 = 0.0885$

36. Refer to figure (b).
$P(z \ge 2.17) = 1 - P(z < 2.17) = 1 - 0.9850 = 0.0150$

37. Refer to figure (b).
$P(x \ge -1.20) = 1 - P(z < -1.20) = 1 - 0.1151 = 0.8849$

38. Refer to figure (b).
$P(z \ge -1.50) = 1 - P(z < -1.50) = 1 - 0.0668 = 0.9332$

39. Refer to figure (c).
$P(-1.20 \le z \le 2.64) = P(z \le 2.64) - P(z < -1.20) = 0.9959 - 0.1151 = 0.8808$

40. Refer to figure (c).
$P(-2.20 \le z \le 1.04) = P(z \le 1.04) - P(z < -2.20) = 0.8508 - 0.0139 = 0.8369$

41. Refer to figure (c).
$P(-2.18 \le z \le -0.42) = P(z \le -0.42) - P(z < -2.18) = 0.3372 - 0.0146 = 0.3226$

42. Refer to figure (c).
$P(-1.78 \le z \le -1.23) = P(z \le -1.23) - P(z < -1.78) = 0.1093 - 0.0375 = 0.0718$

43. Refer to figure (c).
$P(0 \le z \le 1.62) = P(z \le 1.62) - P(z < 0) = 0.9474 - 0.5000 = 0.4474$

44. Refer to figure (c).
$P(0 \le z \le 0.54) = P(z \le 0.54) - P(z < 0) = 0.7054 - 0.5000 = 0.2054$

45. Refer to figure (c).
$P(-0.82 \le z \le 0) = P(z \le 0) - P(z < -0.82) = 0.5000 - 0.2061 = 0.2939$

46. Refer to figure (c).
$P(-2.37 \le z \le 0) = P(z \le 0) - P(z < -2.37) = 0.5000 - 0.0089 = 0.4911$

47. Refer to figure (c).
$P(-0.45 \le z \le 2.73) = P(z \le 2.73) - P(z < -0.45) = 0.9968 - 0.3264 = 0.6704$

48. Refer to figure (c).
$P(-0.73 \le z \le 3.12) = P(z \le 3.12) - P(z < -0.73) = 0.9991 - 0.2327 = 0.7664$

Section 6.3

1. We are given $\mu = 4$ and $\sigma = 2$. Since $z = \dfrac{x-\mu}{\sigma}$, we have $z = \dfrac{x-4}{2}$.

$P(3 \le x \le 6)$
$= P(3-4 \le x-4 \le 6-4)$ Subtract $\mu = 4$ from each part of the inequality.
$= P\left(\dfrac{3-4}{2} \le \dfrac{x-4}{2} \le \dfrac{6-4}{2}\right)$ Divide each part by $\sigma = 2$.
$= P\left(-\dfrac{1}{2} \le z \le \dfrac{2}{2}\right)$
$= P(-0.5 \le z \le 1)$
$= P(z \le 1) - P(z < -0.5)$ Refer to sketch (c) in the solutions for Section 6.2.
$= 0.8413 - 0.3085$
$= 0.5328$

2. We are given $\mu = 15$ and $\sigma = 4$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-15}{4}$.

$P(10 \le x \le 26)$
$= P(10-15 \le x-15 \le 26-15)$ Subtract $\mu = 15$.
$= P\left(\frac{10-15}{4} \le \frac{x-15}{4} \le \frac{26-15}{4}\right)$ Divide each part of the inequality by $\sigma = 4$.
$= P(-1.25 \le z \le 2.75)$
$= P(z \le 2.75) - P(z < -1.25)$ Refer to sketch (c) in solutions for Section 6.2.
$= 0.9970 - 0.1056$
$= 0.8914$

3. We are given $\mu = 40$ and $\sigma = 15$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-40}{15}$.

$P(50 \le x \le 70)$
$= P(50-40 \le x-40 \le 70-40)$ Subtract $\mu = 40$.
$= P\left(\frac{50-40}{15} \le \frac{x-40}{15} \le \frac{70-40}{15}\right)$ Divide by $\sigma = 15$.
$= P(0.67 \le z \le 2)$
$= P(z \le 2) - P(z < 0.67)$
$= 0.9772 - 0.7486 = 0.2286$

4. We are given $\mu = 5$ and $\sigma = 1.2$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-5}{1.2}$.

$P(7 \le x \le 9)$
$= P(7-5 \le x-5 \le 9-5)$ Subtract $\mu = 5$.
$= P\left(\frac{7-5}{1.2} \le \frac{x-5}{1.2} \le \frac{9-5}{1.2}\right)$ Divide by $\sigma = 1.2$.
$= P(1.67 \le z \le 3.33)$
$= P(z \le 3.33) - P(z < 1.67)$
$= 0.9996 - 0.9525 = 0.0471$

5. We are given $\mu = 15$ and $\sigma = 3.2$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-15}{3.2}$.

$P(8 \le x \le 12)$
$= P(8-15 \le x-15 \le 12-15)$ Subtract $\mu = 15$.
$= P\left(\frac{8-15}{3.2} \le \frac{x-15}{3.2} \le \frac{12-15}{3.2}\right)$ Divide by $\sigma = 3.2$.
$= P(-2.19 \le z \le -0.94)$
$= P(z \le -0.94) - P(z < -2.19)$
$= 0.1736 - 0.0143 = 0.1593$

6. We are given $\mu = 50$ and $\sigma = 15$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-50}{15}$.

$P(40 \le x \le 47)$
$= P(40-50 \le x-50 \le 47-50)$ Subtract $\mu = 50$.
$= P\left(\frac{40-50}{15} \le \frac{x-50}{15} \le \frac{47-50}{15}\right)$ Divide by $\sigma = 15$.
$= P(-0.67 \le z \le -0.20)$
$= P(z \le -0.20) - P(z < -0.67)$
$= 0.4207 - 0.2514 = 0.1693$

7. We are given $\mu = 20$ and $\sigma = 3.4$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-20}{3.4}$.

$P(x \ge 30)$
$= P(x-20 \ge 30-20)$ Subtract $\mu = 20$.
$= P\left(\frac{x-20}{3.4} \ge \frac{30-20}{3.4}\right)$ Divide by $\sigma = 3.4$.
$= P(z \ge 2.94)$
$= 1 - P(z < 2.94)$ Refer to sketch (b) in Section 6.2.
$= 1 - 0.9984 = 0.0016$

8. We are given $\mu = 100$ and $\sigma = 15$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-100}{15}$.

$P(x \ge 120)$
$= P(x-100 \ge 120-100)$ Subtract μ.
$= P\left(\frac{x-100}{15} \ge \frac{120-100}{15}\right)$ Divide by σ.
$= P(z \ge 1.33)$ Refer to sketch (b) in Section 6.2.
$= 1 - P(z < 1.33)$
$= 1 - 0.9082 = 0.0918$

9. We are given $\mu = 100$ and $\sigma = 15$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-100}{15}$.

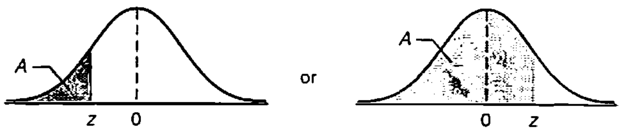
$P(x \ge 90)$
$= P\left(\frac{x-100}{15} \ge \frac{90-100}{15}\right)$ Subtract μ; divide by σ.
$= P(z \ge -0.67)$
$= 1 - P(z < -0.67) = 1 - 0.2514 = 0.7486$

10. We are given $\mu = 3$ and $\sigma = 0.25$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-3}{0.25}$.

$P(x \ge 2)$
$= P\left(\frac{x-3}{0.25} \ge \frac{2-3}{0.25}\right)$ Subtract μ; divide by σ.
$= P(z \ge -4)$
$= 1 - P(z < -4) \approx 1 - 0 = 1$

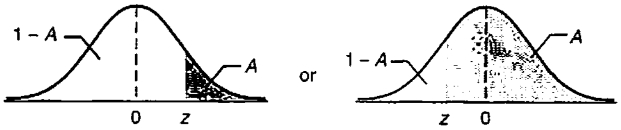
For problems 11–20, refer to the following sketch patterns for guidance in calculation.

(a) **Left-tail case:**
The given area A is to the left of z.



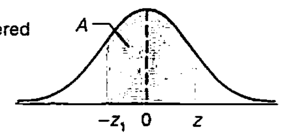
For the left-tail case, look up the number A in the body of the table and use the corresponding z value.

(b) **Right-tail case:**
The given area A is to the right of z.



For the right-tail case, look up the number $1 - A$ in the body of the table and use the corresponding z value.

(c) **Center case:**
The given area A is symmetric and centered above $z = 0$. Half of A lies to the left and half lies to the right of $z = 0$.



For the center case, look up the number $\frac{1-A}{2}$ in the body of the table and use the corresponding $\pm z$ value.

Student sketches should resemble the figures above, with negative z-values to the left of zero and positive z-values to the right of zero, and A written as a decimal.

11. Refer to figure (a).
Find z so that the area A to the left of z is 6% = 0.06. Since $A = 0.06$ is less than 0.5000, look for a negative z value. A to left of −1.55 is 0.0606 and A to left of −1.56 is 0.0594. Since 0.06 is in the middle of 0.0606 and 0.0594, for our z-value we will use the average of −1.55 and −1.56:

$$\frac{-1.55 + (-1.56)}{2} = -1.555.$$

12. Refer to figure (a).
Find z so that the area A to the left of z is 5.2% = 0.052. Since $A = 0.052 < 0.5000$, look for a negative z value. A to the left of −1.63 is 0.0516, which is closer to 0.052 than is A to the left of −1.62 (0.0526), so $z = -1.63$.

13. Refer to figure (a).
Find z so that the area A to the left of z is 55% = 0.55. Since $A = 0.55 > 0.5000$, look for a positive z-value. The area to the left of 0.13 is 0.5517, so $z = 0.13$.

14. Refer to figure (a).
Find z so that the area A to the left of z is 97.5% = 0.975. Since $A = 0.975 > 0.5000$, look for a positive z. A to left of $z = 1.96$ is 0.9750.

15. Refer to figure (b).
Find z so that the area A to the right of z is 8% = 0.08. Since A to the right of z is 0.08, $1 - A = 1 - 0.08 = 0.92$ is to the left of z-value. The area to the left of 1.41 is 0.9207.

16. Refer to figure (b).
Find z so that the area A to the right of z is 5% = 0.05. Since A to the right of z is 0.05, $1 - A = 1 - 0.05 = 0.95$ is to the left of z. Since $1 - A = 0.95 > 0.5000$, look for a positive z-value. The area to the left of 1.64 is 0.9495, and the area to the left of 1.65 is 0.9505. Since 0.95 is halfway between 0.9495 and 0.9505, we average the two z values.

$$\frac{1.64+1.65}{2} = 1.645$$

17. Refer to figure (b).
Find z so that the area A to the right of z is 82% = 0.82. Since A to the right of z, $1 - A = 1 - 0.82 = 0.18$ is to the left of z. Since $1 - A = 0.18 < 0.5000$, look for a negative z value. The area to the left of $z = -0.92$ is 0.1788.

18. Refer to figure (b).
Find z so that the area A to the right of z is 95% = 0.95. Since A to the right of z is 0.95, $1 - A = 1 - 0.95 = 0.05$ is to the left of z. Because $1 - A = 0.05 < 0.5000$, look for a negative z value. The area to the left of −1.64 is 0.0505. The area to the left of −1.65 is 0.0495. Since 0.05 is halfway between these two area values we average the two z-values.

$$\frac{-1.64+(-1.65)}{2} = -1.645$$

19. Refer to figure (c).
Find z such that the area A between $-z$ and z is 98% = 0.98. Since A is between $-z$ and z, $1 - A = 1 - 0.98 = 0.02$ lies in the tails, and since we need $\pm z$, half of $1 - A$ lies in each tail. The area to the left of $-z$ is $\frac{1-A}{2} = \frac{0.02}{2} = 0.01$. The area to the left of −2.33 is 0.0099. Thus $-z = -2.33$ and $z = 2.33$.

20. Refer to figure (c).
Find z such that the area A between $-z$ and z is 95% = 0.95. If A between $-z$ and $z = 0.95$, then $1 - A = 1 - 0.95 = 0.05$ is the area in the tails, and that is split evenly between the two tails. Thus, the area to the left of $-z$ is $\frac{1-A}{2} = \frac{0.05}{2} = 0.025$. The area to the left of −1.96 is 0.0250, so $-z$ is −1.96 and $z = 1.96$.

21. x is approximately normal with $\mu = 85$ and $\sigma = 25$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-85}{25}$.

(a) $P(x > 60)$

$$= P\left(\frac{x-85}{25} > \frac{60-85}{25}\right) = P(z > -1)$$
$$= 1 - P(z \le -1) = 1 - 0.1587 = 0.8413$$

(b) $P(x < 110) = P\left(\frac{x-85}{25} < \frac{110-85}{25}\right) = P(z < 1) = 0.8413.$

(c) $P(60 < x < 110)$

$$= P(-1 < z < 1) \quad \text{using (a) and (b)}$$
$$= P(z < 1) - P(z \le -1) = 0.8413 - 0.1587 = 0.6826$$

(i.e., approximately 68% of the blood glucose measurements lie within $\mu \pm \sigma$)

(d) $P(x > 140)$
$= P\left(\frac{x-85}{25} > \frac{140-85}{25}\right) = P(z > 2.2)$
$= 1 - P(z \le 2.2) = 1 - 0.9861 = 0.0139$

22. x is approximately normally distributed with $\mu = 38$ and $\sigma = 12$. Since $z = \frac{x-\mu}{\sigma}$, we have $z = \frac{x-38}{12}$.

(a) $P(x < 60) = P\left(\frac{x-38}{12} < \frac{60-38}{12}\right) = P(z < 1.83) = 0.9664$

(b) $P(x > 16) = P\left(\frac{x-38}{12} > \frac{16-38}{12}\right) = P(z > -1.83) = 1 - P(z \le -1.83) = 1 - 0.0336 = 0.9664$

(c) $P(16 < x < 60)$
$= P(-1.83 < z < 1.83)$ using (a) and (b)
$= P(z < 1.83) - P(z \le -1.83) = 0.9664 - 0.0336$
$= 0.9328$

(d) $P(x > 60)$
$= 1 - P(x \le 60)$ complementary event rule
$= 1 - 0.9664$ from (a)
$= 0.0336$

23. SAT scores, x, are normal with $\mu_x = 500$ and $\sigma_x = 100$. Since $z = \frac{x-\mu_x}{\sigma_x}$, we have $x = \frac{x-500}{100}$.

(a) $P(x > 675) = P\left(\frac{x-500}{100} > \frac{675-500}{100}\right) = P(z > 1.75) = 1 - P(z \le 1.75) = 1 - 0.9599 = 0.0401$

(b) $P(x < 450) = P\left(\frac{x-500}{100} < \frac{450-500}{100}\right) = P(z < -0.5) = 0.3085$

(c) $P(450 \le x \le 675)$
$= P(-0.5 \le z \le 1.75)$ using (a), (b)
$= P(z \le 1.75) - P(z < -0.5) = 0.9599 - 0.3085$
$= 0.6514$ using work in (a), and (b)

ACT scores, y, are normal with $\mu_y = 18$ and $\sigma_y = 6$. Since $z = \frac{y-\mu_y}{\sigma_y}$, we have $z = \frac{y-18}{6}$.

(d) $P(y > 28) = P\left(\frac{y-18}{6} > \frac{28-18}{6}\right) = P(z > 1.67) = 1 - P(z \le 1.67) = 1 - 0.9525 = 0.0475$

(e) $P(y > 12) = P\left(\frac{y-18}{6} > \frac{12-18}{6}\right) = P(z > -1) = 1 - P(z \le -1) = 1 - 0.1587 = 0.8413$

(f) $P(12 \le y \le 28)$
$= P(-1 \le z \le 1.67)$ using (a), (b)
$= P(z \le 1.67) - P(z < -1) = 0.9525 - 0.1587 = 0.7938$

24. SAT scores, x, are normal with $\mu_x = 500$ and $\sigma_x = 100$; ACT scores, y, are normal with $\mu_y = 18$ and $\sigma_x = 6$. Since $z_0 = \frac{x_0 - \mu_x}{\sigma_x}$, a little algebra shows $x_0 = \mu_x + z_0\sigma_x$ and, similarly, $y_0 = \mu_y + z_0\sigma_y$.

(a) Find the SAT score, x_0, such that $P(x \geq x_0) = 10\% = 0.10$.

$$P(x \geq x_0) = P\left(\frac{x-500}{100} \geq \frac{x_0 - 500}{100}\right) = P(z \geq z_0) = 0.10$$

(that is, find the value z_0 such that 10% of the standard normal curve lies to the right of z_0). Since 10% is to the right of z_0, $1 - 0.10 = 0.90 = 90\%$ is to the left of z_0. Because $0.90 > 0.5000$, z_0 will be a positive number.

$$P(z \leq 1.28) = 0.8997, \text{ so } z_0 = 1.28$$

$x_0 = \mu_x + z_0\sigma_x$ so here, $x_0 = 500 + 1.28(100) = 628$ students scoring 628 points or more on the SAT math exam are in the top 10%.

Similarly, find y_0 such that $P(y \geq y_0) = 0.10$. Since 10% of the standard normal curve is to the right of z_0, $100\% - 10\% = 90\% = 0.90$ is to the left of z_0. $P(z \leq 1.28) = 0.8997$, so $z_0 = 1.28$. Then $y_0 = \mu_y + z_0\sigma_y = 18 + 1.28(6) = 25.68 \approx 26$. Students scoring 26 or more points on the ACT math test are in the top 10%.

(b) Find the SAT score, x_0, and the ACT score, y_0, such that $P(x \geq x_0) = P(y \geq y_0) = 20\% = 0.20$.
First, find z_0 such that $P(z \geq z_0) = 0.20$, or $P(z < z_0) = 1 - 0.20 = 0.80$. $P(z < 0.84) = 0.7995$, so $z_0 = 0.84$. Then $x_0 = \mu_x + z_0\sigma_x = 500 + 0.84(100) = 584$ and $y_0 = \mu_y + z_0\sigma_y = 18 + 0.84(6) = 23.04 \approx 23$. Students scoring at least 584 on the SAT math test, or at least 23 on the ACT math test, are in the top 20%.

(c) Find x_0, y_0, and z_0 such that $P(x \geq x_0) = P(y \geq y_0) = P(z \geq z_0) = 60\% = 0.60$.
First, z_0: $P(z < z_0) = 1 - 0.60 = 0.40$. $P(z < -0.25) = 0.4013$, so $z_0 = -0.25$. Then $x_0 = \mu_x + z_0\sigma_x = 500 + (-0.25)(100) = 475$ and $y_0 = \mu_y + z_0\sigma_y = 18 + (-0.25)(6) = 16.5$. So students scoring at least 475 on the SAT test or at least 16.5 on the ACT test are in the top 60%.

25. Pot shard thickness, x, is approximately normally distributed with $\mu = 5.1$ and $\sigma = 0.9$ millimeters.

(a) $P(x < 3.0) = P\left(\frac{x-5.1}{0.9} < \frac{3.0-5.1}{0.9}\right) = P(z < -2.33) = 0.0099$

(b) $P(x > 7.0) = P\left(\frac{x-5.1}{0.9} > \frac{7.0-5.1}{0.9}\right) = P(z > 2.11) = 1 - P(z \leq 2.11) = 1 - 0.9826 = 0.0174$

(c) $P(3.0 \leq x \leq 7.0)$
$= P(-2.33 \leq z \leq 2.11)$ using (a), (b)
$= P(z \leq 2.11) - P(z < -2.33) = 0.9826 - 0.0099$
$= 0.9727$

26. Response time, x, is normally distributed with $\mu = 8.4$ and $\sigma = 1.7$ minutes.

(a) $P(5 \leq x \leq 10)$
$= P\left(\frac{5-8.4}{1.7} \leq \frac{x-8.4}{1.7} \leq \frac{10-8.4}{1.7}\right) = P(-2 \leq z \leq 0.94)$
$= P(z \leq 0.94) - P(z < -2) = 0.8264 - 0.0228 = 0.8036$

(b) $P(x < 5) = P(z < -2) = 0.0228$ using (a)

(c) $P(x > 10) = P(z > 0.94) = 1 - P(z \le 0.94) = 1 - 0.8264 = 0.1736$ using (a)

27. Fuel consumption, x, is approximately normal with $\mu = 3213$ and $\sigma = 180$ gallons per hour.

(a) $P(3000 \le x \le 3500)$

$$= P\left(\frac{3000-3213}{180} \le \frac{x-3213}{180} \le \frac{3500-3213}{180}\right)$$
$$= P(-1.18 \le z \le 1.59) = P(z \le 1.59) - P(z < -1.18)$$
$$= 0.9441 - 0.1190 = 0.8251$$

(b) $P(x < 3000) = P(z < -1.18) = 0.1190$ using (a)

(c) $P(x > 3500) = P(z > 1.59) = 1 - P(z \le 1.59) = 1 - 0.9441 = 0.0559$ using (a)

28. Temperature, x, is normally distributed with $\mu = 22$ and $\sigma = 10°$.

(a) $P(x \ge 42) = P\left(\frac{x-22}{10} \ge \frac{42-22}{10}\right) = P(z \ge 2) = 1 - P(z < 2) = 1 - 0.9772 = 0.0228$

(b) $P(x \le 15) = P\left(\frac{x-22}{10} \le \frac{15-22}{10}\right) = P(z \le -0.70) = 0.2420$

(c) $P(29 \le x \le 40)$

$$= P\left(\frac{29-22}{10} \le \frac{x-22}{10} \le \frac{40-22}{10}\right)$$
$$= P(0.7 \le z \le 1.8) = P(z \le 1.8) - P(z < 0.7)$$
$$= 0.9641 - 0.7580 = 0.2061$$

29. Lifetime, x, is normally distributed with $\mu = 45$ and $\sigma = 8$ months.

(a) $P(x \le 36) = P\left(\frac{x-45}{8} \le \frac{36-45}{8}\right) = P(z \le -1.125) \approx P(z \le -1.13) = 0.1292$

The company will have to replace approximately 13% of its batteries.

(b) Find x_0 such that $P(x \le x_0) = 10\% = 0.10$. First, find z_0 such that $P(z \le z_0) = 0.10$.
$P(z \le -1.28) = 0.1003$, so $z_0 = -1.28$. Then $x_0 = \mu + z_0\sigma = 45 + (-1.28)(8) = 34.76 \approx 35$.
The company should guarantee the batteries for 35 months.

30. Lifetime, x, is normally distributed with $\mu = 28$ and $\sigma = 5$ months.

(a) 2 years = 24 months

$$P(x \le 24) = P\left(z \le \frac{24-28}{5}\right) = P(z \le -0.8) = 0.2119$$

The company should expect to replace about 21.2% of its watches.

(b) Find x_0 such that $P(x \le x_0) = 12\% = 0.12$. First, find z_0 such that $P(z \le z_0) = 0.12$.
$P(z \le -1.17) = 0.1210$ and $P(z \le -1.18) = 0.1190$. Since 0.12 is halfway between 0.1210 and 0.1190, we will average the z-values: $z_0 = \frac{-1.17 + (-1.18)}{2} = -1.175$.
So $x_0 = \mu + z_0\sigma = 28 + (-1.175)(5) = 22.125$.
The company should guarantee its watches for 22 months.

31. Age at replacement, x, is approximately normal with $\mu = 8$ and range = 6 years.

(a) The empirical rule says that about 95% of the data are between $\mu - 2\sigma$ and $\mu + 2\sigma$, or about 95% of the data are in a $(\mu + 2\sigma) - (\mu - 2\sigma) = 4\sigma$ range (centered around μ). Thus, the range $\approx 4\sigma$, or $\sigma \approx$ range/4. Here, we can approximate σ by 6/4 = 1.5 years.

(b) $P(x > 5) = P\left(z > \frac{5-8}{1.5}\right) = P(z > -2)$ using the estimate of σ from (a)
$= 1 - P(z \le -2) = 1 - 0.0228 = 0.9772$

(c) $P(x < 10) = P\left(z < \frac{10-8}{1.5}\right) = P(z < 1.33) = 0.9082$

(d) Find x_0 so that $P(x \le x_0) = 10\% = 0.10$. First, find z_0 such that $P(z \le z_0) = 0.10$.
$P(z \le -1.28) = 0.1003$, so $z_0 = -1.28$. Then $x_0 = \mu + z_0\sigma = 8 + (-1.28)(1.5) = 6.08$.
The company should guarantee their TVs for about 6.1 years.

32. Age at replacement, x, is approximately normal with $\mu = 14$ and a (95%) range from 9 to 19 years.

(a) From Problem 31(a), range $\approx 4\sigma$, or $\sigma = \frac{\text{range}}{4}$. Here range = 19 − 9 = 10 years, so
$\sigma \approx 10/4 = 2.5$ years.

(b) $P(x < 11) = P\left(z < \frac{11-14}{2.5}\right) = P(z < -1.2) = 0.1151$

(c) $P(x > 18) = P\left(z > \frac{18-14}{2.5}\right) = P(z > 1.6) = 1 - P(z \le 1.6) = 1 - 0.9452 = 0.0548$

(d) Find x_0 so that $P(x < x_0) = 5\% = 0.05$. First, find z_0 so that $P(z < z_0) = 0.05$.
$P(z < -1.64) = 0.0505$ and $P(z < -1.65) = 0.0495$.
Since 0.05 is halfway between 0.0495 and 0.0505, we will average the z-values to get
$z_0 = \frac{-1.64 + (-1.65)}{2} = -1.645$. Then $x_0 = \mu + z_0\sigma = 14 + (-1.645)(2.5) = 9.8875$.
The company should guarantee its refrigerator for about 9.9 years.

33. Resting heart rate, x, is approximately normal with $\mu = 46$ and (95%) range from 22 to 70 bpm.

(a) From Problem 31(a), range $\approx 4\sigma$, or $\sigma \approx$ range/4. Here range = 70 − 22 = 48, so
$\sigma \approx 48/4 = 12$ bpm.

(b) $P(x < 25) = P\left(z < \frac{25-46}{12}\right) = P(z < -1.75) = 0.0401$

(c) $P(x > 60) = P\left(z > \frac{60-46}{12}\right) = P(z > 1.17) = 1 - P(z \le 1.17) = 1 - 0.8790 = 0.1210$

(d) $P(25 \le x \le 60) = P(-1.75 \le z \le 1.17)$ using (b), (c)
$= P(z \le 1.17) - P(z < -1.75)$
$= 0.8790 - 0.0401$
$= 0.8389$

(e) Find x_0 such that $P(x > x_0) = 10\% = 0.10$. First, find z_0 such that $P(z > z_0) = 0.10$.
$P(z \le z_0) = 1 - 0.10 = 0.90$
$P(z \le 1.28) = 0.8997 \approx 0.90$, so let $z_0 = 1.28$.
When $x_0 = \mu + z_0\sigma = 46 + 1.28(12) = 61.36$, so horses with resting rates of 61 bpm or more may need treatment.

34. Kitten weight x is approximately normally distributed with $\mu = 24.5$ and (95%) range from 14 to 35 oz.

(a) From Problem 31(a), $\sigma \approx$ range $\div$ 4. Here range = 35 – 14 = 21 oz, so $\sigma = \dfrac{21}{4} \approx 5.25$ oz.

(b) $P(x < 14) = P\left(z < \dfrac{14 - 24.5}{5.25}\right) = P(z < -2) = 0.0228$

(c) $P(x > 33) = P\left(z > \dfrac{33 - 24.5}{5.25}\right) = P(z > 1.62) = 1 - P(z \le 1.62) = 1 - 0.9474 = 0.0526$

(d) $P(14 \le x \le 33) = P(-2 \le z \le 1.62) = P(z \le 1.62) - P(z < -2) = 0.9474 - 0.0228 = 0.9246$

(e) Find x_0 such that $P(x \le x_0) = 10\% = 0.10$. First, find z_0 such that $P(z \le z_0) = 0.10$.
$P(z \le -1.28) = 0.1003 \approx 0.10$, so let $z_0 = -1.28$.
Since $z_0 = \dfrac{x_0 - \mu}{\sigma}$, $-1.28 = \dfrac{x_0 - 24.5}{5.25}$, so
$x_0 = 24.5 + (-1.28)(5.25) = 17.78$

The cutoff point is about 17.8 oz.

35. Life expectancy x is normal with $\mu = 90$ and $\sigma = 3.7$ months.

(a) The insurance company wants 99% of the microchips to last <u>longer</u> than x_0. Saying this another way: the insurance company wants to pay the \$50 million at most 1% of the time. So, find x_0 such that $P(x \le x_0) = 1\% = 0.01$. First, find z_0 such that $P(z \le z_0) = 0.01$. $P(z \le -2.33) = 0.0099 \approx 0.01$, so let $z_0 = -2.33$. Since $z_0 = \dfrac{x_0 - \mu}{\sigma}$, $x_0 = \mu + z_0\sigma = 90 + (-2.33)(3.7) = 81.379 \approx 81$ months.

(b) $P(x \le 84) = P\left(z \le \dfrac{84 - 90}{3.7}\right) = P(z \le -1.62) = 0.0526 \approx 5\%$.

(c) The "expected loss" is 5.26% [from (b)] of the \$50 million, or 0.0526(50,000,000) = \$2,630,000.

(d) Profit is the difference between the amount of money taken in (here, \$3 million), and the amount paid out (here, \$2.63 million, from (c)). So the company expects to profit 3,000,000 – 2,630,000 = \$370,000.

36. (Questions 1–6 in the text will be labeled (a)-(f) below.) Daily attendance, x, is normally distributed with $\mu = 8000$ and $\sigma = 500$ people.

(a) $P(x < 7200) = P\left(z < \dfrac{7200 - 8000}{500}\right) = P(z < -1.6) = 0.0548$

(b) $P(x > 8900) = P\left(z > \dfrac{8900 - 8000}{500}\right) = P(z > 1.8) = 1 - P(z \le 1.8) = 1 - 0.9641 = 0.0359$

(c) $P(7200 \le x \le 8900) = P(-1.6 \le z \le 1.8) = P(z \le 1.8) - P(z < -1.6) = 0.9641 - 0.0548 = 0.9093$

Arrival times are normal with $\mu = 3$ hours, 48 minutes and $\sigma = 52$ minutes after the doors open. Convert μ to minutes: $(3 \times 60) + 48 = 228$ minutes.

(d) Find x_0 such that $P(x \le x_0) = 90\% = 0.90$. First, find z_0 such that $P(z \le z_0) = 0.90$.
$P(z \le 1.28) = 0.8997 \approx 0.90$, so let $z_0 = 1.28$. Since $z_0 = \frac{x_0 - \mu}{\sigma}$, $x_0 = \mu + z_0\sigma = 228 + (1.28)(52)$
= 294.56 minutes, or 294.56/60 = 4.9093 ≈ 4.9 hours after the doors open.

(e) Find x_0 such that $P(x \le x_0) = 15\% = 0.15$. First, find z_0 such that $P(z \le z_0) = 0.15$.
$P(z \le -1.04) = 0.1492 \approx 0.15$, so let $z_0 = -1.04$. Then $x_0 = \mu + z_0\sigma = 228 + (-1.04)(52)$
$= 173.92$ minutes, or $173/60 = 2.899 \approx 2.9$ hours after the doors open.

(f) Answers vary. Most people have Saturday off, so many may come early in the day. Most people work Friday, so most people would probably come after 5 P.M. There is no reason to think weekday and weekend arrival times would have the same distribution.

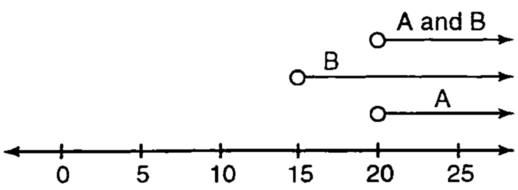
37. Waiting time, x, is approximately normal with $\mu = 18$ and $\sigma = 4$ minutes.

(a) Let A be the event that $x > 20$, and B be the event that $x > 15$. We want to find P(A, given B). Recall

$$P(\text{A, given B}) = \frac{P(\text{A and B})}{P(\text{B})}.$$

$$P(\text{A and B}) = P(x > 20 \text{ and } x > 15) = P(x > 20)$$

Use a number line to find where both events occur simultaneously.



The number 20 is not included in "both A and B" because A says x is strictly greater than 20. The intervals $(15, \infty)$ and $(20, \infty)$ intersect at $(20, \infty)$.

$$P(x > 20) = P\left(z > \frac{20-18}{4}\right) = P(z > 0.5) = 1 - P(z \le 0.5) = 1 - 0.6915 = 0.3085$$

$$P(x > 15) = P\left(z > \frac{15-18}{4}\right) = P(z > -0.75) = 1 - P(z \le -0.75) = 1 - 0.2266 = 0.7734$$

$$\begin{aligned} P(x > 20, \text{ given } x > 15) &= \frac{P(x > 20) \text{ and } x > 15}{P(x > 15)} \\ &= \frac{P(x > 20)}{P(x > 15)} \\ &= \frac{0.3085}{0.7734} \\ &= 0.3989 \end{aligned}$$

(b) $$P(x > 25, \text{ given } x > 18) = \frac{P(x > 25 \text{ and } x > 18)}{P(x > 18)}$$
$$= \frac{P(x > 25)}{P(x > 18)}$$
$$= \frac{P\left(z > \frac{25-18}{4}\right)}{P\left(z > \frac{18-18}{4}\right)}$$
$$= \frac{P(z > 1.75)}{P(z > 0)}$$
$$= \frac{1 - P(z \le 1.75)}{1 - P(z \le 0)}$$
$$= \frac{(1 - 0.9599)}{(1 - 0.5000)}$$
$$= \frac{0.0401}{0.5}$$
$$= 0.0802$$

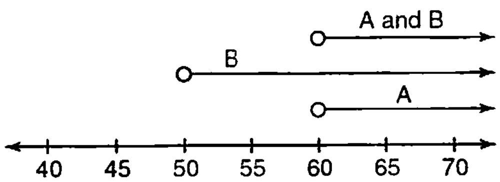
38. Cycle time, x, is approximately normal with $\mu = 45$ and $\sigma = 12$ minutes.

(a) $P(x > 60, \text{ given } x > 50)$

Let event A be $x > 60$, and event B be the $x > 50$.

The problem asks P(A, given B). Recall P(A, given B) = $\frac{P(\text{A and B})}{P(\text{B})}$.

Note that x is both greater than 60 and greater than 50 when x is greater than 60:



In this case, P(A and B) is the same as P(A).

The problem has been reduced to P(A, given B) = P(A)/P(B).

$$P(x > 60, \text{ given } x > 50) = \frac{P(x > 60)}{P(x > 50)}$$
$$= \frac{P\left(z > \frac{60-45}{12}\right)}{P\left(z > \frac{50-45}{12}\right)}$$
$$= \frac{P(z > 1.25)}{P(z > 0.42)}$$
$$= \frac{1 - P(z \le 1.25)}{1 - P(z \le 0.42)}$$
$$= \frac{1 - 0.8944}{1 - 0.6628}$$
$$= \frac{0.1056}{0.3372}$$
$$= 0.3132$$

(b) $P(x > 55, \text{ given } x > 40) = \dfrac{P(x > 55 \text{ and } x > 40)}{P(x > 40)}$

$$= \frac{P(x > 55)}{P(x > 40)}$$
$$= \frac{P\left(z > \frac{55-45}{12}\right)}{P\left(z > \frac{40-45}{12}\right)}$$
$$= \frac{P(z > 0.83)}{P(z > -0.42)}$$
$$= \frac{1 - P(z \le 0.83)}{1 - P(z \le -0.42)}$$
$$= \frac{1 - 0.7967}{1 - 0.3372}$$
$$= \frac{0.2033}{0.6628}$$
$$= 0.3067$$

39. Maintenance cost, x, is approximately normal with $\mu = 615$ and $\sigma = 42$ dollars.

(a) $P(x > 646) = P\left(z > \dfrac{646-615}{42}\right) = P(z > 0.74) = 1 - P(z \le 0.74) = 1 - 0.7704 = 0.2296$

(b) Find x_0 such that $P(x > x_0) = 0.10$.
But, if the actual cost exceeds the budgeted amount 10% of the time, the actual cost must be <u>within</u> the budgeted amount 90% of the time. The problem can be rephrased as how much should be budgeted so that the probability the actual cost is less than or equal to the budgeted amount is 0.90, or find x_0 such that $P(x \le x_0) = 0.90$. First, find z_0 such that $P(z \le z_0) = 0.90$. $P(z \le 1.28) = 0.8997$, so let $z_0 = 1.28$. Since $z_0 = \frac{x_0 - \mu}{\sigma}$, $x_0 = \mu + z_0\sigma = 615 + 1.28(42) = \$668.76 \approx \$669$.

Section 6.4

Answers may vary slightly due to rounding.

1. Previously, $p = 88\% = 0.88$; now, $p = 9\% = 0.09$; $n = 200$; $r = 50$
Let a success be defined as a child with a high blood-lead level.

(a) $P(r \ge 50) = P(50 \le r) = P(49.5 \le x)$
$np = 200(0.88) = 176$; $nq = n(1-p) = 200(0.12) = 24$
Since both np and nq are greater than 5, we will use the normal approximation to the binomial with $\mu = np = 176$ and $\sigma = \sqrt{npq} = \sqrt{200(0.88)(0.12)} = \sqrt{21.12} = 4.60$.
So, $P(r \ge 50) = P(49.5 \le x) = P\left(\dfrac{49.5-176}{4.6} \le z\right) = P(-27.5 \le z)$.
Almost every z value will be greater than or equal to –27.5, so this probability is approximately 1. It is almost certain that 50 or more children a decade ago had high blood-lead levels.

(b) $P(r \geq 50) = P(50 \leq r) = P(49.5 \leq x)$

In this case, $np = 200(0.09) = 18$ and $nq = 200(0.91) = 182$, so both are greater than 5. Use the normal approximation with $\mu = np = 18$ and $\sigma = \sqrt{npq} = \sqrt{200(0.09)(0.91)} = \sqrt{16.38} = 4.05$.

So $P(49.5 \leq x) = P\left(\frac{49.5 - 18}{4.05} \leq z\right) = P(7.78 \leq z)$.

Almost no z values will be larger than 7.78, so this probability is approximately 0. Today, it is almost impossible that a sample of 200 children would include at least 50 with high blood-lead levels.

2. We are given $p = 0.40$ and $n = 128$. Let a success be defined as an insurance claim inflated (padded) to cover the deductible.

(a) $\frac{1}{2}(128) = 64$

$P(r \geq 64) = P(64 \leq r) = P(63.5 \leq x)$ 64 is a left endpoint $np = 128(0.4) = 51.2$ and

$nq = 128(0.6) = 76.8$ are both greater than 5, so we will use the normal approximation to the binomial

with $\mu = np = 51.2$ and $\sigma = \sqrt{npq} = \sqrt{128(0.4)(0.6)} = \sqrt{30.72} = 5.54$.

$$P(63.5 \leq x) = P\left(\frac{63.5 - 51.2}{5.54} \leq z\right) = P(2.22 \leq z) = P(z \geq 2.22) = 1 - P(z < 2.22) = 1 - 0.9868 = 0.0132$$

(b) $P(r < 45) = P(r \leq 44) = P(x \leq 44.5)$ 44 is a right endpoint.

$$= P\left(z \leq \frac{44.5 - 51.2}{5.54}\right) = P(z \leq -1.21) = 0.1131$$

(c) $P(40 \leq r \leq 64) = P(39.5 \leq x \leq 64.5)$

$$\begin{aligned}
&= P\left(\frac{39.5 - 51.2}{5.54} \leq z \leq \frac{64.5 - 51.2}{5.54}\right) \\
&= P(-2.11 \leq z \leq 2.40) \\
&= P(z \leq 2.40) - P(z < -2.11) \\
&= 0.9918 - 0.0174 \\
&= 0.9744
\end{aligned}$$

(d) More than 80 *not* padded = 81 or more *not* padded, i.e., 128 − 81 = 47 or fewer *are padded.*
Method 1:

$$P(r \leq 47) = P(x \leq 47.5) = P\left(z \leq \frac{47.5 - 51.2}{5.54}\right) = P(z \leq -0.67) = 0.2514$$

Method 2:
Success is now *redefined* to mean an insurance claim that has not been padded, and p is not
1 − 0.40 = 0.60.

$P(r \geq 81) = P(81 \leq r) = P(80.5 \leq x)$. 81 is a left endpoint. The normal approximation is still valid, since what was np in (a) is now nq and vice versa. The standard deviation is still the same, but now $\mu = np = 128(0.60) = 76.8$. So,

$$P(80.5 \leq x) = P\left(\frac{80.5 - 76.8}{5.54} \leq z\right) = P(0.67 \leq z) = P(z \geq 0.67) = 1 - P(z < 0.67) = 1 - 0.7486 = 0.2514.$$

3. We are given $n = 125$ and $p = 17\% = 0.17$. Let a success be defined as the police receiving enough information to locate and arrest a fugitive within 1 week.

(a) $P(r \geq 15) = P(15 \leq r) = P(14.5 \leq x)$. 15 is a left endpoint. $np = 125(0.17) = 21.25$ and $nq = 125(1-0.17) = 125(0.83) = 103.75$, which are both greater than 5, so we can use the normal approximation with $\mu = np = 21.25$ and $\sigma = \sqrt{npq} = \sqrt{125(0.17)(0.83)} = \sqrt{17.6375} = 4.20$. So $P(14.5 \leq x) = P\left(\frac{14.5-21.25}{4.2} \leq z\right) = P(-1.61 \leq z) = P(z \geq -1.61) = 1 - P(z < -1.61) = 1 - 0.0537 = 0.9463$.

(b) $P(r \geq 28) = P(28 \leq r)$ 28 is a left endpoint.

$$
\begin{aligned}
&= P(27.5 \leq x) \\
&= P\left(\frac{27.5-21.25}{4.2} \leq z\right) \\
&= P(1.49 \leq z) \\
&= P(z \geq 1.49) \\
&= 1 - P(z < 1.49) \\
&= 1 - 0.9319 \\
&= 0.0681
\end{aligned}
$$

(c) Remember, r "between" a and b is $a \leq r \leq b$.

$P(15 \leq r \leq 28) = P(14.5 \leq x \leq 28.5)$ 15 is a left endpoint and 28 is a right endpoint.

$$
\begin{aligned}
&= P\left(\frac{14.5-21.25}{4.2} \leq z \leq \frac{28.5-21.25}{4.2}\right) \\
&= P(-1.61 \leq z \leq 1.73) \\
&= P(z \leq 1.73) - P(z < -1.61) \\
&= 0.9582 - 0.0537 \\
&= 0.9045.
\end{aligned}
$$

(d) $n = 125$, $p = 0.17$, $q = 1 - p = 1 - 0.17 = 0.83$.

np and nq are both greater than 5, so the normal approximation is appropriate.

4. $n = 316, p = 11\% = 0.11$; a success occurs when the book sold is a romance novel.

(a) $P(r < 40) = P(r \leq 39) = P(x \leq 39.5)$ 39 is a right endpoint $np = 316(0.11) = 34.76$. $nq = n(1-p) = 316(1-0.11) = 316(0.89) = 281.24$, both of which are greater than 5, so we can apply the normal approximation with $\mu = np = 34.76$ and $\sigma = \sqrt{npq} = \sqrt{316(0.11)(0.89)} = \sqrt{30.9364} = 5.56$.

$P(x \leq 39.5) = P\left(z \leq \frac{39.5-34.76}{5.56}\right) = P(z \leq 0.85) = 0.8023$

(b) $P(r \geq 25) = P(25 \leq r)$ 25 is a left endpoint.

$$
\begin{aligned}
&= P(24.5 \leq x) \\
&= P\left(\frac{24.5-34.76}{5.56} \leq z\right) \\
&= P(-1.85 \leq z) \\
&= P(z \geq -1.85) \\
&= 1 - P(z < -1.85) \\
&= 1 - 0.0322 \\
&= 0.9678
\end{aligned}
$$

(c) $P(25 \le r \le 40) = P(24.5 \le x \le 40.5)$

$$= P\left(\frac{24.5-34.76}{5.56} \le z \le \frac{40.5-34.76}{5.56}\right)$$
$$= P(-1.85 \le z \le 1.03)$$
$$= P(z \le 1.03) - P(z < -1.85)$$
$$= 0.8485 - 0.0322$$
$$= 0.8163$$

(d) $n = 316$, $p = 0.11$, $q = 1 - p = 0.89$

np and nq are both greater than 5, so the normal approximation to the binomial is appropriate. (See (a) above.)

5. We are given $n = 753$ and $p = 3.5\% = 0.035$; $q = 1 - p = 1 - 0.035 = 0.965$.
Let a success be a person living past age 90.

(a) $P(r \ge 15) = P(15 \le r) = P(14.5 \le x)$ 15 is a left endpoint.

Here, $np = 753(0.035) = 26.355$, and $nq = 753(0.965) = 726.645$, both of which are greater than 5; the normal approximation is appropriate, using $\mu = np = 26.355$ and

$\sigma = \sqrt{npq} = \sqrt{753(0.035)(0.965)} = \sqrt{25.4326} = 5.0431.$

$$P(14.5 \le x) = P\left(\frac{14.5-26.355}{5.0431} \le z\right)$$
$$= P(-2.35 \le z)$$
$$= P(z \ge -2.35)$$
$$= 1 - P(z < -2.35)$$
$$= 1 - 0.0094$$
$$= 0.9906$$

(b) $P(r \ge 30) = P(30 \le r)$

$$= P(29.5 \le x)$$
$$= P\left(\frac{29.5-26.355}{5.0431} \le z\right)$$
$$= P(0.62 \le z)$$
$$= P(z \ge 0.62)$$
$$= 1 - P(z < 0.62)$$
$$= 1 - 0.7324$$
$$= 0.2676$$

(c) $P(25 \le r \le 35) = P(24.5 \le x \le 35.5)$

$$= P\left(\frac{24.5-26.355}{5.0431} \le z \le \frac{35.5-26.355}{5.0431}\right)$$
$$= P(-0.37 \le z \le 1.81)$$
$$= P(z \le 1.81) - P(z < -0.37)$$
$$= 0.9649 - 0.3557$$
$$= 0.6092$$

(d) $P(r > 40) = P(r \geq 41)$
$= P(41 \leq r)$
$= P(40.5 \leq x)$
$= P\left(\frac{40.5 - 26.355}{5.0431} \leq z\right)$
$= P(2.80 \leq z)$
$= P(z \geq 2.80)$
$= 1 - P(z < 2.80)$
$= 1 - 0.9974$
$= 0.0026$

6. $n = 24$, $p = 44\% = 0.44$, $q = 1 - p = 1 - 0.44 = 0.56$
A success occurs when a billfish striking the line is caught.

(a) $P(r \leq 12) = P(x \leq 12.5)$
$np = 24(0.44) = 10.56$ and $nq = 24(0.56) = 13.44$, both of which are greater than 5, so the normal approximation is appropriate. Here, $\mu = np = 10.56$ and

$\sigma = \sqrt{npq} = \sqrt{24(0.44)(0.56)} = \sqrt{5.9136} = 2.4318$.

$P(x \leq 12.5) = P\left(z \leq \frac{12.5 - 10.56}{2.4318}\right) = P(z \leq 0.80) = 0.7881$

(b) $P(r \geq 5) = P(5 \leq r)$
$= P(4.5 \leq x)$
$= P\left(\frac{4.5 - 10.56}{2.4318} \leq z\right)$
$= P(-2.49 \leq z)$
$= P(z \geq -2.49)$
$= 1 - P(z < -2.49)$
$= 1 - 0.0064$
$= 0.9936$

(c) $P(5 \leq r \leq 12) = P(4.5 \leq x \leq 12.5)$
$= P(-2.49 \leq z \leq 0.80)$
$= P(z \leq 0.80) - P(z < -2.49)$
$= 0.7881 - 0.0064$
$= 0.7817$

(d) $n = 24$, $p = 0.44$, $q = 0.56$
Both np and $nq > 5$, so the normal approximation to the binomial is appropriate.

7. $n = 66$, $p = 80\% = 0.80$, $q = 1 - p = 1 - 0.80 = 0.20$
A success is when a new product fails within 2 years.

(a) $P(r \ge 47) = P(47 \le r) = P(46.5 \le x)$
$np = 66(0.80) = 52.8$, and $nq = 66(0.20) = 13.3$. Both exceed 5. so the normal approximation with $\mu = np = 52.8$ and $\sigma = \sqrt{npq} = \sqrt{66(0.8)(0.2)} = \sqrt{10.56} = 3.2496$ is appropriate.

$$\begin{aligned}P(46.5 \le x) &= P\left(\frac{46.5-52.8}{3.2496} \le z\right)\\ &= P(-1.94 \le z)\\ &= P(z \ge -1.94)\\ &= 1-P(z<-1.94)\\ &= 1-0.0262\\ &= 0.9738\end{aligned}$$

(b) $P(r \le 58) = P(x \le 58.5) = P\left(z \le \frac{58.5-52.8}{3.2496}\right) = P(z \le 1.75) = 0.9599$

For (c) and (d), note we are interested now in products succeeding, so a success is redefined to be a new product staying on the market for 2 years. Here, $n = 66$. p is now 0.20 with q is now 0.80 (p and q above have been switched. Now $np = 13.2$ and $nq = 52.8$. $\mu = 13.2$. and σ stays equal to 3.2496.

(c) $$\begin{aligned}P(r \ge 15) &= P(15 \le r)\\ &= P(14.5 \le x)\\ &= P\left(\frac{14.5-13.2}{3.2496} \le z\right)\\ &= P(0.40 \le z)\\ &= P(z \ge 0.40)\\ &= 1-P(z<0.40)\\ &= 1-0.6554\\ &= 0.3446\end{aligned}$$

(d) $P(r<10) = P(r \le 9) = P(x \le 9.5) = P\left(z \le \frac{9.5-13.2}{3.2496}\right) = P(z \le -1.14) = 0.1271$

8. $n = 63$, $p = 64\% = 0.64$, $q = 1-p = 1-0.64 = 0.36$
A success is when the murder victim knows the murderer.

(a) $P(r \ge 35) = P(35 \le r) = P(34.5 \le x)$
$np = 63(0.64) = 40.32$ and $nq = 63(0.36) = 22.68$
Since both np and nq are greater than 5. the normal approximation is appropriate. Use $\mu = np = 40.32$ and $\sigma = \sqrt{npq} = \sqrt{63(0.64)(0.36)} = \sqrt{14.5152} = 3.8099$. So

$$P(34.5 \le x) = P\left(\frac{34.5-40.32}{3.8099} \le z\right) = P(-1.53 \le z) = 1-P(z<-1.53) = 1-0.0630 = 0.9370$$

(b) $P(r \le 48) = P(x \le 48.5) = P\left(z \le \frac{48.5-40.32}{3.8099}\right) = P(z \le 2.15) = 0.9842$

(c) If fewer than 30 victims, i.e., 29 or fewer did not know their murderer, then 63 – 29 = 34 or more victims did know their murderer.

$$\begin{aligned}P(r \geq 34) &= P(34 \leq r)\\ &= P(33.5 \leq x)\\ &= P\left(\frac{33.5-40.32}{3.8099} \leq z\right)\\ &= P(-1.79 \leq z)\\ &= 1-P(z<-1.79)\\ &= 1-0.0367\\ &= 0.9633\end{aligned}$$

(d) If more than 20, i.e., 21 or more, victims did not know their murdered, then 63 – 21 = 42 or fewer victims did know their murdered.

$$P(r \leq 42) = P(x \leq 42.5) = P\left(z \leq \frac{42.5-40.32}{3.8099}\right) = P(z \leq 0.57) = 0.7157$$

9. $n = 430, p = 70\% = 0.70, q = 1 - p = 1 - 0.70 = 0.30$
A success is finding the address or lost acquaintances.

(a) $P(r > 280) = P(r \geq 281) = P(281 \leq r) = P(280.5 \leq x)$
$np = 430(0.7) = 301$ and $nq = 430(0.3) = 129$
Since both np and nq are greater than 5, the normal approximation with $\mu = np = 301$ and $\sigma = \sqrt{npq} = \sqrt{430(0.7)(0.3)} = \sqrt{90.3} = 9.5026$ is appropriate.

$$P(280.5 \leq x) = P\left(\frac{280.5-301}{9.5026} \leq z\right) = P(-2.16 \leq z) = 1 - P(z < -2.16) = 1 - 0.0154 = 0.9846$$

(b)

$$\begin{aligned}P(r \geq 320) &= P(320 \leq r)\\ &= P(319.5 \leq x)\\ &= P\left(\frac{319.5-301}{9.5026} \leq z\right)\\ &= P(1.95 \leq z)\\ &= 1-P(z>1.95)\\ &= 1-0.9744\\ &= 0.0256\end{aligned}$$

(c)

$$\begin{aligned}P(280 \leq r \leq 320) &= P(279.5 \leq x \leq 320.5)\\ &= P\left(\frac{279.5-301}{9.5026} \leq z \leq \frac{320.5-301}{9.5026}\right)\\ &= P(-2.26 \leq z \leq 2.05)\\ &= P(z \leq 2.05) - P(z < -2.26)\\ &= 0.9798-0.0119\\ &= 0.9679\end{aligned}$$

(d) $n = 430, p = 0.7, q = 0.3$
Both np and nq are greater than 5 so the normal approximation is appropriate, See (a).

10. $n = 8641, p = 61\% = 0.61, q = 1 - p = 0.39$
A success is when a pottery shard is Santa Fe black on white.

(a) $P(r < 5200) = P(r \le 5199) = P(x \le 5199.5)$
$np = 8641(0.61) = 5271.01$ and $nq = 3369.99$
Since both np and nq are greater than 5, we can use the normal approximation with
$\mu = np = 5271.01$ and $\sigma = \sqrt{npq} = \sqrt{8641(0.61)(0.39)} = \sqrt{2055.6939} = 45.3398$

$$P(x \le 5199.5) = P\left(z \le \frac{5199.5 - 5271.01}{45.3398}\right) = P(z \le -1.58) = 0.0571$$

(b)
$$\begin{aligned} P(r > 5400) &= P(r \ge 5401) \\ &= P(5401 \le r) \\ &= P(5400.5 \le x) \\ &= P\left(\frac{5400.5 - 5271.01}{45.3398} \le z\right) \\ &= P(2.86 \le z) \\ &= 1 - P(z < 2.86) \\ &= 1 - 0.9979 \\ &= 0.0021 \end{aligned}$$

(c)
$$\begin{aligned} P(5200 \le r \le 5400) &= P(5199.5 \le x \le 5400.5) \\ &= P\left(-1.58 \le z \le \frac{5400.5 - 5271.01}{45.3398}\right) \\ &= P(-1.58 \le z \le 2.86) \\ &= P(z \le 2.86) - P(z < -1.58) \\ &= 0.9979 - 0.0571 \\ &= 0.9408 \end{aligned}$$

(d) $n = 8641, p = 0.61, q = 0.39, np = 5271.01, nq = 3369.99.$

11. $n = 850, p = 57\% = 0.57, q = 0.43$
Success = pass Ohio bar exam

(a) $P(r \ge 540) = P(540 \le r) = P(539.5 \le x)$

$np = 484.5, nq = 365.5, \mu = np = 484.5, \sigma = \sqrt{npq} = \sqrt{208.335} = 14.4338$

Since both np and nq are greater than 5, use normal approximation with μ and σ as above.

$$P(539.5 \le x) = P\left(\frac{539.5 - 484.5}{14.4338} \le z\right) = P(3.81 \le z) \approx 0$$

(b) $$P(r \le 500) = P(x \le 500.5) = P\left(z \le \frac{500.5 - 484.5}{14.4338}\right) = P(z \le 1.11) = 0.8665$$

(c)
$$\begin{aligned} P(485 \le r \le 525) &= P(484.5 \le x \le 525.5) \\ &= P\left(0 \le z \le \frac{525.5 - 484.5}{14.4338}\right) \\ &= P(0 \le z \le 2.84) \\ &= P(z \le 2.84) - P(z < 0) \\ &= 0.9977 - 0.5 \\ &= 0.4977 \end{aligned}$$

12. $n = 5000$. $p = 3.2\% = 0.032$, $q = 0.968$
Success = coupon redeemed

(a) $P(100 < r) = P(101 \le r) = P(100.5 \le x)$
$np = 160$, $nq = 4840$, $\sigma = \sqrt{npq} = \sqrt{154.88} = 12.4451$
Since both np and nq are greater than 5, use normal approximation with $\mu = np$ and σ as shown.
$$P(100.5 \le x) = P\left(\frac{100.5 - 160}{12.4451} \le z\right) = P(-4.78 \le z) \approx 1$$

(b) $$\begin{aligned} P(r < 200) &= P(r \le 199) \\ &= P(x \le 199.5) \\ &= P\left(z \le \frac{199.5 - 160}{12.4451}\right) \\ &= P(z \le 3.17) \\ &= 0.9992 \end{aligned}$$

(c) $$\begin{aligned} P(100 \le r \le 200) &= P(99.5 \le x \le 200.5) \\ &= P\left(\frac{99.5 - 160}{12.4451} \le z \le \frac{200.5 - 160}{12.4451}\right) \\ &= P(-4.86 \le z \le 3.25) \\ &\approx P(z \le 3.25) \\ &= 0.9994 \end{aligned}$$

13. $n = 317$, $P(\text{buy, given sampled}) = 37\% = 0.37$, $P(\text{sampled}) = 60\% = 0.60 = p$ so $q = 0.40$

(a) $P(180 < r) = P(181 \le r) = P(180.5 \le x)$
$np = 190.2$, $nq = 126.8$, $\sigma = \sqrt{npq} = \sqrt{76.08} = 8.7224$
Since both np and np are greater than 5, use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq}$.
$$P(180.5 \le x) = P\left(\frac{180.5 - 190.2}{8.7224} \le z\right) = P(-1.11 \le z) = 1 - P(z < -1.11) = 1 - 0.1335 = 0.8665$$

(b) $$P(r < 200) = P(r \le 199) = P(x \le 199.5) = P\left(z \le \frac{199.5 - 190.2}{8.7224}\right) = P(z \le 1.07) = 0.8577$$

(c) Let A be the event buy product; let B be the event tried free sample, Thus $P(\text{A, given B}) = 0.37$ and $P(\text{B}) = 0.60$. Since $P(\text{A and B}) = P(\text{B}) \cdot P(\text{A, given B}) = 0.60(0.37) = 0.222$,
$P(\text{sample and buy}) = 0.222$.

(d) Let a success be sample and buy. Then $p = 0.222$ from (c), and $q = 0.778$.
$$P(60 \le r \le 80) = P(59.5 \le x \le 80.5)$$
Here, $np = 317(0.222) = 70.374$ and $nq = 246.626$, so use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq} = \sqrt{317(0.222)(0.778)} = \sqrt{54.750972} = 7.3994$.
$$\begin{aligned} P(59.5 \le x \le 80.5) &= P\left(\frac{59.5 - 70.374}{7.3994} \le z \le \frac{80.5 - 70.374}{7.3994}\right) \\ &= P(-1.47 \le z \le 1.37) \\ &= P(z \le 1.37) - P(z < -1.47) \\ &= 0.9147 - 0.0708 \\ &= 0.8439 \end{aligned}$$

14. $n = 175$, $P(\text{vanilla}) = 25\% = 0.25$, $P(\text{chocolate}) = 9\% = 0.09$

(a) Success = buy vanilla ice cream, so $p = 0.25$ and $q = 0.75$.

$$P(50 \le r) = P(49.5 \le x)$$

$np = 43.75$, $nq = 131.25$, $\sqrt{npq} = \sqrt{32.8125} = 5.7282 = \sigma$

Since both np and nq area greater than 5, use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq}$.

$$P(49.5 \le x) = P\left(\frac{49.5 - 43.75}{5.7282} \le z\right) = P(1.00 \le z) = 1 - P(z < 1) = 1 - 0.8413 = 0.1587$$

(b) Success = buy chocolate ice cream, so $p = 0.09$ and $q = 0.91$.

$$P(12 \le r) = P(11.5 \le x)$$

Since $np = 15.75 > 5$ and $nq = 175(0.91) = 159.25 > 5$, so use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq} = \sqrt{14.3325} = 3.7858$.

$$P(11.5 \le x) = P\left(\frac{11.5 - 15.75}{3.7858} \le z\right) = P(-1.12 \le z) = 1 - P(z < -1.12) = 1 - 0.1314 = 0.8686$$

(c) Let V be the even the person buys vanilla ice cream and C be the event the person buys chocolate ice cream. V and C are <u>not</u> mutually exclusive, since a person buying vanilla can also buy chocolate ice cream. Given V and C are independent, i.e., $P(\text{V and C}) = P(\text{V}) \cdot P(\text{C})$.

$$\begin{aligned} P(\text{C or V}) &= P(\text{C}) + P(\text{V}) - P(\text{C and V}) \\ &= 0.09 + 0.25 - (0.09)(0.25) \quad \text{using (a) and (b)} \\ &= 0.3175 \end{aligned}$$

(d) Success = buy chocolate or vanilla, so $p = 0.3175$ from (c) and $q = 0.6825$.

$$P(50 \le r \le 60) = P(49.5 \le x \le 60.5)$$

Since $np = 175(0.3175) = 55.5625 > 5$ and $nq = 175(0.6825) = 119.4375 > 5$, use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq} = \sqrt{175(0.3175)(0.6825)} = \sqrt{37.9214} = 6.1580$.

$$\begin{aligned} P(49.5 \le x \le 60.5) &= P\left(\frac{49.5 - 55.5625}{6.158} \le z \le \frac{60.5 - 55.5625}{6.158}\right) \\ &= P(-0.98 \le z \le 0.80) \\ &= P(z \le 0.80) - P(z < -0.98) \\ &= 0.7881 - 0.1635 \\ &= 0.6246 \end{aligned}$$

15. $n = 267$ reservations, $P(\text{show}) = 1 - 0.06 = 0.94 = p$ so $q = 0.06$.

(a) $p = 0.94$

(b) Success = show up for flight (with a reservation) seat available for all who show up means the number showing up must be $\le$ 255 actual plane seats. Thus, $P(r \le 255)$.

(c) $P(r \le 255) = P(x \le 255.5)$

Since $np = 267(0.94) = 250.98 > 5$ and $nq = 267(0.06) = 16.02 > 5$, use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq} = \sqrt{267(0.94)(0.06)} = \sqrt{15.0588} = 3.8806$.

$$P(x \le 255.5) = P\left(z \le \frac{255.5 - 250.98}{3.8806}\right) = P(z \le 1.16) = 0.8770$$

16. Answers vary.
The normal approximation to the binomial is appropriate (reasonably accurate) when np and nq are both greater than 5. (If $n \le 25$, say, tables of the exact binomial distribution can be used.) In this case, the normal distribution approximating the binomial has mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$. Because the normal distribution is continuous whereas the binomial is discrete, the accuracy of the approximation is improved by using the continuity correction. The interval around the number of successes is first written as a closed interval (such as $[a, b]$ instead of (c, d) where $c + 1 = a$ and $d - 1 = b$); then the left endpoint is decreased by 0.5 and the right endpoint is increased by 0.5. (In the case of half closed intervals, such as $r \le 14$ or $r \ge 7$, only the one endpoint needs to be adjusted.)

Chapter 6 Review

1. (a) $P(0 \le z \le 1.75) = P(z \le 1.75) - P(z < 0) = 0.9599 - 0.5 = 0.4599$

(b) $P(-1.29 \le z \le 0) = P(z \le 0) - P(z < -1.29) = 0.5 - 0.0985 = 0.4015$

(c) $P(1.03 \le z \le 1.21) = P(z \le 1.21) - P(z < 1.03) = 0.8869 - 0.8485 = 0.0384$

(d) $P(z \ge 2.31) = 1 - P(z < 2.31) = 1 - 0.9896 = 0.0104$

(e) $P(z \le -1.96) = 0.0250$

(f) $P(z \le 1) = 0.8413$

2. (a) $P(0 \le z \le 0.75) = P(z \le 0.75) - P(z < 0) = 0.7734 - 0.5 = 0.2734$

(b) $P(-1.50 \le z \le 0) = P(z \le 0) - P(z < -1.50) = 0.5 - 0.0668 = 0.4332$

(c) $P(-2.67 \le z \le -1.74) = P(z \le -1.74) - P(z < -2.67) = 0.0409 - 0.0038 = 0.0371$

(d) $P(z \ge 1.56) = 1 - P(z < 1.56) = 1 - 0.9406 = 0.0594$

(e) $P(z \le -0.97) = 0.1660$

(f) $P(z \le 2.01) = 0.9778$

3. x is normal with $\mu = 47$ and $\sigma = 6.2$

(a) $P(x \le 60) = P\left(z \le \frac{60-47}{6.2}\right) = P(z \le 2.10) = 0.9821$

(b) $P(x \ge 50) = P\left(z \ge \frac{50-47}{6.2}\right) = P(z \ge 0.48) = 1 - P(z < 0.48) = 1 - 0.6844 = 0.3156$

(c) $P(50 \le x \le 60) = P(0.48 \le z \le 2.10) = P(z \le 2.10) - P(z < 0.48) = 0.9821 - 0.6844 = 0.2977$

4. x is normal with $\mu = 110$, $\sigma = 12$

(a) $P(x \le 120) = P\left(z \le \frac{120-110}{12}\right) = P(z \le 0.83) = 0.7967$

(b) $P(x \ge 80) = P\left(z \ge \frac{80-110}{12}\right) = P(z \ge -2.5) = 1 - P(z < -2.5) = 1 - 0.0062 = 0.9938$

(c) $P(108 \le x \le 117) = P\left(\frac{108-110}{12} \le z \le \frac{117-110}{12}\right) = P(-0.17 \le z \le 0.58)$
$= P(z \le 0.58) - P(z < -0.17) = 0.7190 - 0.4325 = 0.2865$

5. Find z_0 such that $P(z \geq z_0) = 5\% = 0.05$. Same as find z_0 such that $P(z < z_0) = 0.95$
$P(z < 1.645) = 0.95$, so $z_0 = 1.645$

6. Find z_0 such that $P(z \leq z_0) = 1\% = 0.01$.
$P(z \leq -2.33) = 0.0099$, so $z_0 = -2.33$

7. Find z_0 such that $P(-z_0 \leq z \leq +z_0) = 0.95$
Same as 5% of area outside $[-z_0,\ +z_0]$; split in half:
$P(z \leq -z_0) = 0.05/2 = 0.025$
$P(z \leq -1.96) = 0.0250$ so $-z_0 = -1.96$ and $+z_0 = 1.96$

8. Find z_0 so $P(-z_0 \leq z \leq +z_0) = 0.99$
Same as 1% outside $[-z_0,\ +z_0]$; divide in half.
$P(z \leq -z_0) = 0.01/2 = 0.005$.
$P(z \leq -2.575) = 0.0050$, so $\pm z_0 = \pm 2.575$, or ± 2.58

9. $\mu = 79$, $\sigma = 9$

(a) $z = \dfrac{x-\mu}{\sigma} = \dfrac{87-79}{9} = 0.89$

(b) $z = \dfrac{79-79}{9} = 0$

(c) $P(x > 85) = P\left(z > \dfrac{85-79}{9}\right) = P(z > 0.67) = 1 - P(z \leq 0.67) = 1 - 0.7486 = 0.2514$

10. $\mu = 270$, $\sigma = 35$

(a) $z = \dfrac{x-\mu}{\sigma}$ so $x = \mu + z\sigma$
here, $x = 270 + 1.9(35) = 336.5$

(b) $x = \mu + z\sigma$; here, $x = 270 + (-0.25)(35) = 261.25$

(c) $P(200 \leq x \leq 340) = P\left(\dfrac{200-270}{35} \leq z \leq \dfrac{340-270}{35}\right) = P(-2 \leq z \leq 2)$
$= P(z \leq 2) - P(z < -2) = 0.9772 - 0.0228 = 0.9544$

11. Binomial with $n = 400$, $p = 0.70$, and $q = 0.30$.
Success = can recycled

(a) $P(r \geq 300) = P(300 \leq r) = P(299.5 \leq x)$

$np = 280 > 5$, $nq = 120 > 5$, $\sqrt{npq} = \sqrt{84} = 9.1652$

Use normal approximation with $\mu = np$ and $\sigma = \sqrt{npq}$.

$P(299.5 \leq x) = P\left(\dfrac{299.5-280}{9.1652} \leq z\right) = P(2.13 \leq z) = 1 - P(z < 2.13) = 1 - 0.9834 = 0.0166$

(b) $P(260 \le r \le 300) = P(259.5 \le x \le 300.5)$

$$= P\left(\frac{259.5-280}{9.1652} \le z \le \frac{300.5-280}{9.1652}\right)$$
$$= P(-2.24 \le z \le 2.24)$$
$$= P(z \le 2.24) - P(z < -2.24)$$
$$= 0.9875 - 0.0125$$
$$= 0.9750$$

12. Lifetime x is normally distributed with $\mu = 5000$ and $\sigma = 450$ hours.

(a) $P(x \le 5000) = P(z \le 0) = 0.5000$

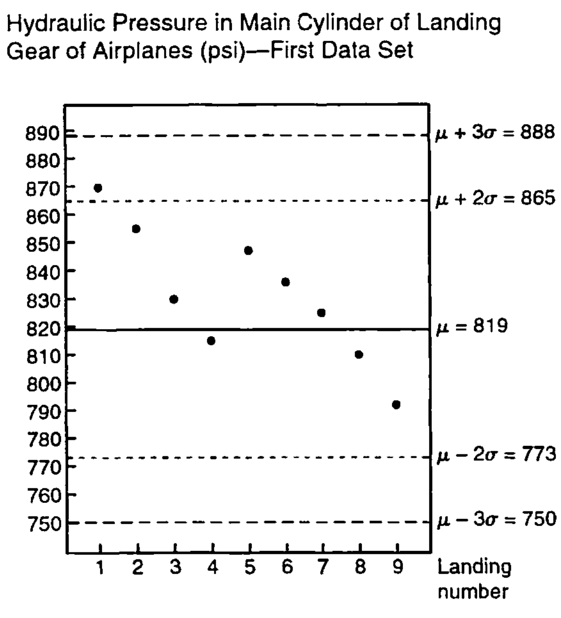
(b) Find x_0 such that $P(x \le x_0) = 0.05$. First, find z_0 so that $P(z \le z_0) = 0.05$.
$P(z \le -1.645) = 0.05$, so $z_0 = -1.645$
$x_0 = \mu + z_0\sigma = 5000 + (-1.645)(450) = 4259.75$
Guarantee the CD player for 4260 hours.

13. Delivery time x is normal with $\mu = 14$ and $\sigma = 2$ hours.

(a) $P(x \le 18) = P\left(z \le \frac{18-14}{2}\right) = P(z \le 2) = 0.9772$

(b) Find x_0 such that $P(x \le x_0) = 0.95$.
Find z_0 so that $P(z \le z_0) = 0.95$.
$P(z \le 1.645) = 0.95$, so $z_0 = 1.645$.
$x_0 = \mu + z_0\sigma = 14 + 1.645(2) = 17.29 \approx 17.3$ hours

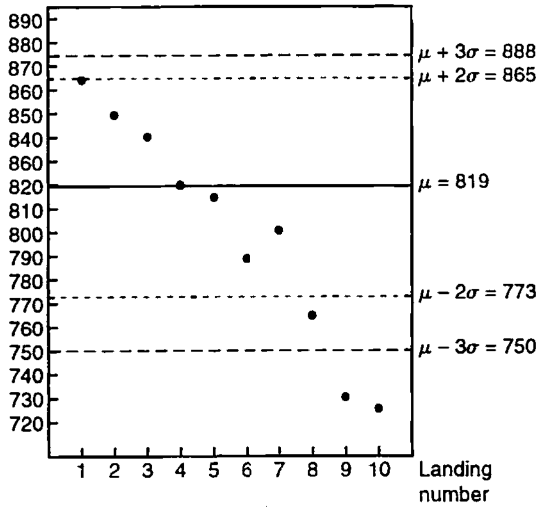
14. (a)



The pressure is "in control;" none of the 3 warning signals is present.

(b)

Hydraulic Pressure in Main Cylinder of Landing Gear of Airplanes (psi)—Second Data Set



The last 2 points are below $\mu - 3\sigma$. The last 3 (consecutive) points are all below $\mu - 2\sigma$. Since warning signals I and III are present, the pressure is "out of control."

15. Scanner price errors in the store's favor are mound-shaped with $\mu = \$2.66$ and $\sigma = \$0.85$.

(a) 68% of the errors should be in the range $\mu \pm 1\sigma$, approximately, or $2.66 \pm 1(0.85)$ which is \$1.81 to \$3.51.

(b) Approximately 95% of the errors should be in the range $\mu \pm 2\sigma$, or $2.66 \pm 2(0.85)$, which is \$0.96 to \$4.36.

(c) Almost all (99.7%) of the errors should lie in the range $\mu \pm 3\sigma$, or $2.66 \pm 3(0.85)$, which is \$0.11 to \$5.21.

16. Time spent on a customer's complaint, x, is normally distributed with $\mu = 9.3$ and $\sigma = 2.5$ minutes.

(a) $P(x < 10) = P\left(z < \frac{10 - 9.3}{2.5}\right) = P(z < 0.28) = 0.6103$

(b) $P(x > 5) = P\left(z > \frac{5 - 9.3}{2.5}\right) = P(z > -1.72) = 1 - P(z \le -1.72) = 1 - 0.0427 = 0.9573$

(c)
$$\begin{aligned} P(8 \le x \le 15) &= P\left(\frac{8 - 9.3}{2.5} \le z \le \frac{15 - 9.3}{2.5}\right) \\ &= P(-0.52 \le z \le 2.28) \\ &= P(z \le 2.28) - P(z < -0.52) \\ &= 0.9887 - 0.3015 \\ &= 0.6872 \end{aligned}$$

17. Response time, x, is normally distributed with $\mu = 42$ and $\sigma = 8$ minutes.

(a) $$\begin{aligned} P(30 \le x \le 45) &= P\left(\frac{30-42}{8} \le z \le \frac{45-42}{8}\right) \\ &= P(-1.5 \le z \le 0.375) \\ &= P(z \le 0.38) - P(z < -1.5) \\ &= 0.6480 - 0.0668 \\ &= 0.5812 \end{aligned}$$

(b) $P(x < 30) = P(z < -1.5) = 0.0668$

(c) $P(x > 60) = P\left(z > \frac{60-42}{8}\right) = P(z > 2.25) = 1 - P(z \le 2.25) = 1 - 0.9878 = 0.0122$

18. Success = unlisted phone number
$n = 150, p = 68\% = 0.68, q = 1 - p = 0.32$
$np = 150(0.68) = 102, nq = 48, npq = 150(0.68)(0.32) = 32.64$

(a) $P(r \ge 100) = P(100 \le r) = P(99.5 \le x)$

Since np and nq are both greater than 5, we can use the normal approximation with $\mu = np = 102$ and $\sigma = \sqrt{npq} = \sqrt{32.64} = 5.7131$.

$P(99.5 \le x) = P\left(\frac{99.5-102}{5.7131} \le z\right) = P(-0.44 \le z) = 1 - P(z < -0.44) = 1 - 0.3300 = 0.6700$

(b) $P(r < 100) = P(r \le 99) = P(x \le 99.5) = P\left(z \le \frac{99.5-102}{5.7131}\right) = P(z \le -0.44) = 0.3300$

(c) Success is redefined to be a listed phone number, so $n = 150$, $p = 0.32$, $q = 0.68$, $np = 48$, $nq = 102$, and $\sqrt{npq} = \sqrt{32.64} = 5.7131 = \sigma$; μ is now 48; normal approximation is still appropriate.

$$\begin{aligned} P(50 \le r \le 65) &= P(49.5 \le x \le 65.5) \\ &= P\left(\frac{49.5-48}{5.7131} \le z \le \frac{65.5-48}{5.7131}\right) \\ &= P(0.26 \le z \le 3.06) \\ &= P(z \le 3.06) - P(z < 0.26) \\ &= 0.9989 - 0.6026 \\ &= 0.3963 \end{aligned}$$

19. Success = having blood type AB
$n = 250, p = 3\% = 0.03, q = 1 - p = 0.97, np = 7.5, nq = 242.5, npq = 7.275$

(a) $P(5 \le r) = P(4.5 \le x)$

$np > 7.5$ and $\sigma = \sqrt{npq} = \sqrt{7.275} = 2.6972$

$P(4.5 \le x) = P\left(\frac{4.5-7.5}{2.6972} \le z\right) = P(-1.11 \le z) = 1 - P(z < -1.11) = 1 - 0.1335 = 0.8665$

(b) $$\begin{aligned} P(5 \le r \le 10) &= P(4.5 \le x \le 10.5) \\ &= P\left(-1.11 \le z \le \frac{10.5-7.5}{2.6972}\right) \\ &= P(-1.11 \le z \le 1.11) \\ &= 1 - 2P(z < -1.11) \\ &= 1 - 2(0.1335) \\ &= 0.7330 \end{aligned}$$