Chapter 4 Elementary Probability Theory

Section 4.1

1. Answers vary. Probability is a number between 0 and 1, inclusive, that expresses the likelihood that a specific event will occur. Three ways to find or assign a probability to an event are (1) through intuition (subjective probability), (2) by considering the long-term relative frequency of recurrence of an event in repeated independent trials (empirical probability), and (3) by computing the ratio of the number of favorable outcomes to the total number of possible outcomes, assuming all outcomes are equally likely (classical probability).

2. Answers vary. Probability in business: market research; in medicine: drug tests to determine if a new drug is more effective than the standard treatment; in social science: determining which characteristics to use in creating a profile to detect terrorists; in natural sciences: predicting the likely path and location of landfall for a hurricane.
   Statistics is the science of collecting, analyzing, and interpreting quantitative data in such a way that the reliability of the conclusions based on the data can be evaluated objectively. Probability is used in determining the reliability of the results.

3. These are not probabilities: (b) because it is greater than 1, (d) because it is less than zero (negative), (h) 150% = 1.50, because it is greater than 1.

4. Remember $0 \le$ probability of an event ≤ 1
   (a) $-0.41 < 0$
   (b) $1.21 > 1$
   (c) $120\% = 1.2 > 1$
   (d) yes, $0 \le 0.56 \le 1$

5. Answers vary. The result is a sample, although not necessarily a good one, showing the relative frequency of people able to wiggle their ears.

6. Answers vary. The results are one example (not necessarily a good one) of the relative frequency of occurrence of raising one eyebrow.

7. (a) $P(\text{no similar preferences}) = P(0) = \frac{15}{375}$, $P(1) = \frac{71}{375}$, $P(2) = \frac{124}{375}$, $P(3) = \frac{131}{375}$, $P(4) = \frac{34}{375}$
   (b) $\frac{15+71+124+131+34}{375} = \frac{375}{375} = 1$, yes
   Personality types were classified into 4 main preferences; all possible numbers of shared preferences were considered. The sample space is 0, 1, 2, 3, and 4 shared preferences.

8. (a) $P(\text{couple not engaged}) = \frac{200}{1000} = 0.20$, $P(\text{dated less than 1 year}) = \frac{240}{1000} = 0.24$, $P(\text{dated 1 to 2 years})$ $= \frac{210}{1000} = 0.21$, $P(\text{dated more than 2 years}) = \frac{350}{1000} = 0.35$, based on the number of "favorable outcomes divided by the total number of outcomes (1000 couples' engagement status)
   (b) $\frac{200+240+210+350}{1000} = \frac{1000}{1000} = 1$, yes
   They should add to 1 because all possible outcomes were considered. The sample space is never engaged, engaged less than 1 year, engaged 1 to 2 years, engaged more than 2 years.

9. **(a)** Note: "includes the left limit but not the right limit" means 6 A.M. $\le$ time $t <$ noon, noon $\le t <$ 6 P.M., 6 P.M. $\le t <$ midnight, midnight $\le t <$ 6 A.M. P(best idea 6 A.M.–12 noon) $= \frac{290}{966} \approx 0.30$; P(best idea 12 noon–6 P.M.) $= \frac{135}{966} \approx 0.14$; P(best idea 6 P.M.–12 midnight) $\frac{319}{966} \approx 0.33$; P(best idea from 12 midnight to 6 A.M.) $= \frac{222}{966} \approx 0.23$.

(b) The probabilities add up to 1. They should add up to 1 provided that the intervals do not overlap and each inventor chose only one interval. The sample space is the set of four time intervals.

10. **(a)** The sample space would be 1, 2, 3, 4, 5, 6 dots. If the die is fair, all outcomes would be equally likely.

(b) $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$ because the die faces are equally likely and there are 6 outcomes. The probabilities should and do add to 1 $\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{6}{6}=1\right)$ because all possible outcomes have been considered.

(c) P(number of dots < 5) $= P$(1 *or* 2 *or* 3 *or* 4 dots) $= P(1) + P(2) + P(3) + P(4) = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$ $= \frac{4}{6} = \frac{2}{3}$ or P(dots < 5) $= 1 - P$(5 *or* 6 dots) $= 1 - \frac{1}{3} = \frac{2}{3}$ (The applicable probability rule used here will be discussed in the next section of the text; rely on your common sense for now.)

(d) Complementary event rule: $P(A) = 1 - P(\text{not } A)$
P(5 *or* 6 dots) $= 1 - P$(1 *or* 2 *or* 3 *or* 4 dots) $\frac{2}{3} = \frac{1}{3}$, or P(5 *or* 6) $= P(5) + P(6) = \frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$

11. **(a)** Since the responses of all 1000 people surveyed have been accounted for (770 + 160 + 70 = 1000), the 3 responses describe the sample space. P(left alone) $= \frac{770}{1000} = 0.77$, P(waited on) $= \frac{160}{1000} = 0.16$,
P(treated differently) $= \frac{70}{1000} = 0.07$
$0.77 + 0.16 + 0.07 = 1$
The probabilities do add to 1, which they should, because they are the sum of probabilities of all possible outcomes in the sample space.

(b) Complementary events: $P(\text{not } A) = 1 - P(A)$:
P(do not want to be left alone) $= 1 - P$(left alone) $= 1 - 0.77 = 0.23$
P(do not want to be waited on) $= 1 - P$(waited on) $= 1 - 0.16 = 0.84$

12. **(a)** $P(\text{germinate}) = \frac{\text{number germinated}}{\text{number planted}} = \frac{2430}{3000} = 0.81$

(b) $P(\text{not germinate}) = \frac{3000 - 2430}{3000} = \frac{570}{3000} = 0.19$

(c) The sample space is 2 outcomes, germinate and not germinate.
P(germinate) + P(not germinate) = 0.81 + 0.19 = 1
The probabilities of all the outcomes in the sample space should and do sum to 1.

(d) no; P(germinate) = 0.81, P(not germinate) = 0.19
If they were equally likely, each would have probability $\frac{1}{2} = 0.5$.

13. (a) Given: odds in favor of A are $n{:}m$ $\left(\text{i.e., } \frac{n}{m}\right)$.

Show $P(A) = \frac{n}{m+n}$

Proof: odds in favor of A are $\frac{P(A)}{P(not\ A)}$ by definition

$P(not\ A) = 1 - P(A)$ complementary events

$\frac{n}{m} = \frac{P(A)}{P(not\ A)} = \frac{P(A)}{1-P(A)}$ substitution

$n[1-P(A)] = m[P(A)]$ cross multiply

$$n - n[P(A)] = m[P(A)]$$
$$n = n[P(A)] + m[P(A)]$$
$$n = (n+m)[P(A)]$$

So $\frac{n}{n+m} = P(A)$ as was to be shown.

(b) Odds of a successful call = odds of sale are 2 to 15. 2 to 15 can be written as 2:15 or $\frac{2}{15}$ then from part (a): if the odds in favor of a sale are 2:15 (let $n = 2$, $m = 15$) then $P(\text{sale}) = \frac{n}{n+m} = \frac{2}{2+15} = \frac{2}{17} \approx 0.118$.

(c) Odds of free throw are 3 to 5, i.e., 3:5
Let $n = 3$ and $m = 5$ here then from part (a):
$P(\text{free throw}) = \frac{n}{n+m} = \frac{3}{3+5} = \frac{3}{8} = 0.375$.

14. (a) Given: odds against W are $a{:}b$ $\left(\text{or } \frac{a}{b}\right)$

Show: $P(not\ W) = \frac{a}{a+b}$

Proof: odds against W are $\frac{P(not\ W)}{P(W)}$ by definition

$P(W) = 1 - P(not\ W)$ complementary events

$\frac{P(not\ W)}{P(W)} = \frac{a}{b}$ substitution

$\frac{P(not\ W)}{1-P(not\ W)} = \frac{a}{b}$ substitution

$b[P(not\ W)] = a[1 - P(not\ W)]$ cross multiply

$$b[P(not\ W)] = a - a[P(not\ W)]$$
$$b[P(not\ W)] + a[P(not\ W)] = a$$
$$(a+b)[P(not\ W)] = a$$
$$P(not\ W) = \frac{a}{a+b}$$

$P(not\ W) = \frac{a}{a+b}$ as was to be shown.

(b) Point Given's betting odds is 9:5. Betting odds are based on the probability that the horse does not win, so odds against PG winning is $\frac{P(\text{not } PG \text{ wins})}{P(PG \text{ wins})}$.

Let $a = 9$ and $b = 5$ in part (a) formula. From part (a), $P(\text{not } PG \text{ wins}) = \frac{a}{a+b} = \frac{9}{9+5} = \frac{9}{14}$, but event *not PG* wins is the same as *PG* loses, so $P(\text{Point Given loses}) = \frac{9}{14} \approx 0.64$ and $P(PG \text{ wins the race})$ $= 1 - \frac{9}{14} = \frac{5}{14} \approx 0.36$.

(c) Betting odds for Monarchos is 6:1. Betting odds are based on the probability that the horse does not win, i.e., the horse loses.
Let W be the event that Monarchos wins. From part (a), if the events against W are given as $a{:}b$, the $P(\text{not } W) = \frac{a}{a+b}$. Let $a = 6$ and $b = 1$ in part (a) formula so

$$P(\text{not } W) = \frac{6}{6+1} = \frac{6}{7}$$

$$P(\text{not } W) = P(\text{Monarchos loses}) = \frac{6}{7} \approx 0.86$$

$$P(\text{Monarchos wins}) = P(W) = 1 - P(\text{not } W)$$

$$= 1 - \frac{6}{7} = \frac{1}{7} \approx 0.14.$$

(d) Invisible Ink was given betting odds of 30 to 1, i.e., odds against Invisible Ink winning were $\frac{30}{1}$. Let W denote the event Invisible Ink wins. Let $a = 30$, $b = 1$ in formula from part (a). Then, from part (a), $P(\text{not } W) = \frac{a}{a+b}$. $P(\text{not Invisible Ink wins}) = \frac{30}{30+1} = \frac{30}{31}$, i.e.,

$$P(\text{Invisible Ink loses}) = \frac{30}{31} \approx 0.97$$

$$P(\text{Invisible Ink wins}) = 1 - P(\text{Invisible Ink loses})$$

$$= 1 - \frac{30}{31} - \frac{1}{31} \approx 0.03.$$

15. Make a table showing the information known about the 127 people who walked by the store: [Example 6 in Section 4.2 uses this technique.]

	Buy	Did not buy	Row Total
Came into the store	25	58 − 25 = 33	58
Did not come in	0	69	127 − 58 = 69
Column Total	25	102	127

If 58 came in, 69 didn't; 25 of the 58 bought something, so 33 came in but didn't buy anything. Those who did not come in, couldn't buy anything. The row entries must sum to the row totals; the column entries must sum to the column totals; and the row totals, as well as the column totals, must sum to the overall total, i.e., the 127 people who walked by the store. Also, the four inner cells must sum to the overall total: 25 + 33 + 0 + 69 = 127.

This kind of problem relies on formula (2), $P(\text{event } A) = \frac{\text{number outcomes favorable to } A}{\text{total number of outcomes}}$. The "trick" is to decide what belongs in the denominator *first*. If the denominator is a row total, stay in that row. If the denominator is a column total, stay in that column. If the denominator is the overall total, the numerator can be a row total, a column total, or the number in any one of the four "cells" inside the table.

(a) total outcomes: people walking by, overall total, 127
favorable outcomes: enter the store, row total, 58 (that's all we know about them)
$P(A) = \frac{58}{127} \approx 0.46$

(b) total outcomes: people who walk into the store, row total 58
favorable outcomes: staying in the row, those who buy: 25
$P(A) = \frac{25}{58} \approx 0.43$

(c) total outcomes: people walking by, overall total 127
favorable outcomes: people coming in *and* buying, the cell at the *intersection* of the "coming in" row and the "buying" column (the upper left corner), 25 (Recall from set theory that "and" means both things happen, that the two sets *intersect*: >)
$P(A) = \frac{25}{127} \approx 0.20$

(d) total outcomes: people coming into the store, row total, 58
favorable outcomes: staying in the row, those who do not buy, 33
$P(A) = \frac{33}{58} \approx 0.57$
$\left(\text{alternate method: this is the complement to (b): } P(A) = 1 - \frac{25}{58} = \frac{33}{58} \approx 0.57\right)$

Section 4.2

1. (a) Orange and blue are mutually exclusive because each M&M candy is only 1 color.
$P(\text{orange } or \text{ blue}) = P(\text{orange}) + P(\text{blue}) = 10\% + 10\% = 20\%$
(b) Yellow and red are mutually exclusive, again, because each candy is only one color, and if the candy is yellow, it can't be red, too.
$P(\text{yellow } or \text{ red}) = P(\text{yellow}) + P(\text{red}) = 20\% + 20\% = 40\%$
(c) It is faster here to use the complementary event rule than to add up the probabilities of all the colors except brown.
$P(not \text{ brown}) = 1 - P(\text{brown}) = 1 - 0.30 = 0.70$, or 70%

2. (a) Orange and blue are mutually exclusive because each M&M candy is only 1 color.
$P(\text{orange } or \text{ blue}) = P(\text{orange}) + P(\text{blue}) = 10\% + 20\% = 30\%$
(b) Yes, mutually exclusive colors
$P(\text{yellow } or \text{ red}) = P(\text{yellow}) + P(\text{red}) = 20\% + 20\% = 40\%$
(c) $P(not \text{ brown}) = 1 - P(\text{brown}) = 1 - 0.20 = 0.80 = 80\%$
Since the color distributions differ for plain and peanut M&Ms (see brown and blue percentages), if the answers were the same, it would only be by coincidence.

3. (a) Mutually exclusive. Notice the color orange is not available in almond M&Ms.
$P(\text{orange } or \text{ blue}) = P(\text{orange}) + P(\text{blue}) = 0\% + 20\% = 20\%$
(b) Mutually exclusive: $P(\text{yellow } or \text{ red}) = P(\text{yellow}) + P(\text{red}) = 20\% + 20\% = 40\%$
(c) $P(not \text{ brown}) = 1 - P(\text{brown}) = 1 - 0.20 = 0.80 = 80\%$
Since the color distributions differ for plain and almond M&Ms, if the answers were the same, it would only be by coincidence.

4. The total number of arches tabled is 288. Arch heights are mutually exclusive because if the height is 12 feet, it can't be 42 feet as well.
(a) $P(3 \text{ to } 9) = \frac{111}{288}$

(b) $P(30 \text{ } or \text{ taller}) = P(30 \text{ to } 49) + P(50 \text{ to } 74) + P(75 \text{ and higher}) = \frac{30}{288} + \frac{33}{288} + \frac{18}{288} = \frac{81}{288}$

(c) $P(3 \text{ to } 49) = P(3-9) + P(10-29) + P(30-49) = \frac{111}{288} + \frac{96}{288} + \frac{30}{288} = \frac{237}{288}$

(d) $P(10 \text{ to } 74) = P(10-29) + P(30-49) + P(50-74) = \frac{96}{288} + \frac{30}{288} + \frac{33}{288} = \frac{159}{288}$

(e) $P(75 \text{ } or \text{ taller}) = \frac{18}{288}$

Hint for Problems 5–8: Refer to Figure 4–1 if necessary. (Without loss of generality, let the red die be the first die and the green die be the second die in Figure 4–1.) Think of the outcomes as an (*x*, *y*) ordered pair. Then, without loss of generality, (1, 6) means 1 on the red die and 6 on the green die. (We are "ordering" the dice for convenience only – which is first and which is second have no bearing on this problem.) The only important fact is that they are distinguishable outcomes, so that (1 on red, 2 on green) is different from (2 on red, 1 on green).

5. **(a)** Yes, the outcome of the red die does not influence the outcome of the green die.

(b) $P(5 \text{ on green } and \text{ 3 on red}) = P(5 \text{ on green}) \cdot P(3 \text{ on red}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36} \approx 0.028$ because they are independent.

(c) $P(3 \text{ on green } and \text{ 5 on red}) = P(3 \text{ on green}) \cdot P(5 \text{ on red}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36} \approx 0.028$

(d) $P[(5 \text{ on green } and \text{ 3 on red}) \text{ } or \text{ } (3 \text{ on green } and \text{ 5 on red})]$
$= P(5 \text{ on green } and \text{ 3 on red}) + P(3 \text{ on green } and \text{ 5 on red})]$
$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18} \approx 0.056$ [because they are mutually exclusive outcomes]

6. **(a)** Yes, the outcome of the red die does not influence the outcome of the green die.

(b) $P(1 \text{ on green } and \text{ 2 on red}) = P(1 \text{ on green}) \cdot P(2 \text{ on red}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$

(c) $P(2 \text{ on green } and \text{ 1 on red}) = P(2 \text{ on green}) \cdot P(1 \text{ on red}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$

(d) $P[(1 \text{ on green } and \text{ 2 on red}) \text{ } or \text{ } (2 \text{ on green } and \text{ 1 on red})]$
$= P(1 \text{ on green } and \text{ 2 on red}) + P(2 \text{ on green } and \text{ 1 on red})]$
$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$ [because they are mutually exclusive outcomes]

7. **(a)** $1 + 5 = 6, 2 + 4 = 6, 3 + 3 = 6, 4 + 2 = 6, 5 + 1 = 6$
$P(\text{sum} = 6) = P[(1, 5) \text{ } or \text{ } (2, 4) \text{ } or \text{ } (3 \text{ on red, } 3 \text{ on green}) \text{ } or \text{ } (4, 2) \text{ } or \text{ } (5, 1)]$
$= P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1)$
since the (red, green) outcomes are mutually exclusive
$= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$
because the red die outcome is independent of the green die outcome
$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{5}{36}$

(b) $1 + 3 = 4, 2 + 2 = 4, 3 + 1 = 4$

$P(\text{sum is } 4) = P[(1, 3) \text{ or } (2, 2) \text{ or } (3, 1)]$

$= P(1, 3) + P(2, 2) + P(3, 1)$

because the (red, green) outcomes are mutually exclusive

$= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

because the red die outcome is independent of the green die outcome

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$

(c) Since a sum of six can't simultaneously be a sum of 4, these are mutually exclusive events;

$P(\text{sum of 6 } or \text{ 4}) = P(\text{sum of 6}) + P(\text{sum of 4}) = \frac{5}{36} + \frac{3}{36} = \frac{8}{36} = \frac{2}{9}$

8. (a) $1 + 6 = 7, 2 + 5 = 7, 3 + 4 = 7, 4 + 3 = 7, 5 + 2 = 7, 6 + 1 = 7$

$P(\text{sum is } 7) = P[(1, 6) \text{ or } (2, 5) \text{ or } (3, 4) \text{ or } (4, 3) \text{ or } (5, 2) \text{ or } (6, 1)]$

$= P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1)$

because the (red, green) outcomes are mutually exclusive

$= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

because the red die outcome is independent of the green die outcome

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$

(b) $5 + 6 = 11, 6 + 5 = 11$

$P(\text{sum is } 11) = P[(5, 6) \text{ or } (6, 5)]$

$= P(5, 6) + P(6, 5)$

because the (red, green) outcomes are mutually exclusive

$= \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$

because the red die outcome is independent of the green die outcome

$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$

(c) Since a sum of can't be both 7 and 11, they are mutually exclusive

$P(\text{sum is 7 } or \text{ 11}) = P(\text{sum is 7}) + P(\text{sum is 11}) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$

9. (a) No, the key idea here is "without replacement," which means the draws are dependent, because the outcome of the second card drawn depends on what the first card drawn was. Let the card draws be represented by an (x, y) ordered pair. For example, (K, 6) means the first card drawn was a king and the second card drawn was a 6. Here the order of the cards is important.

(b) $P(\text{ace on 1st } and \text{ king on second}) = P(\text{ace, king}) = \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = \frac{16}{2652} = \frac{4}{663}$

There are 4 aces and 4 kings in the deck. Once the first card is drawn and not replaced, there are only 51 cards left to draw from, but all the kings are still there.

(c) $P(\text{king, ace}) = \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = \frac{16}{2652} = \frac{4}{663}$

There are 4 kings and 4 aces in the deck. Once the first card is drawn and not replaced, there are only 51 cards left to draw from, but all the aces are still there.

(d) $P(\text{ace } \textit{and} \text{ king in either order})$
$= P[(\text{ace, king}) \textit{ or } (\text{king, ace})]$
$= P(\text{ace, king}) + P(\text{king, ace})$ because these two outcomes are mutually exclusive
$= \frac{16}{2652} + \frac{16}{2652} = \frac{32}{2652} = \frac{8}{663}$

10. (a) No, the key idea here is "without replacement," which means the draws are dependent, because the outcome of the second card drawn depends on what the first card drawn was. Let the card draws be represented by an (x, y) ordered pair. For example, (K, 6) means the first card drawn was a king and the second card drawn was a 6. Here the order of the cards *is* important.

(b) $P(3, 10) = P[(3 \text{ on 1st}) \textit{ and } (10 \text{ on 2nd, } \textit{given } 3 \text{ on 1st})]$
$= P(3 \text{ on 1st}) \cdot P(10 \text{ on 2nd, } \textit{given } 3 \text{ on 1st})$
$= \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = \frac{16}{2652} = \frac{4}{663} \approx 0.006$

(c) $P(10, 3) = P[(10 \text{ on 1st}) \textit{ and } (3 \text{ on 2nd, } \textit{given } 10 \text{ on 1st})]$
$= P(10 \text{ on 1st}) \cdot P(3 \text{ on 2nd, } \textit{given } 10 \text{ on 1st})$
$= \left(\frac{4}{52}\right)\left(\frac{4}{51}\right) = \frac{16}{2652} = \frac{4}{663} \approx 0.006$

(d) $P[(3, 10) \textit{ or } (10, 3)] = P(3, 10) + P(10, 3)$ since these 2 outcomes are mutually exclusive
$= \frac{4}{663} + \frac{4}{663} = \frac{8}{663} \approx 0.012$

11. (a) Yes; the key idea here is "with replacement." When the first card drawn is replaced, the sample space is the same when the second card is drawn as it was when the first card was drawn and the second card is in no way influenced by the outcome of the first draw; in fact, it is possible to draw the same card twice. Let the card draws be represented by an (x, y) ordered pair; for example (K, 6) means a king was drawn first, replaced, and then the second card, a "6," was drawn independently of the first.

(b) $P(A, K) = P(A) \cdot P(K)$ because they are independent
$= \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = \frac{16}{2704} = \frac{1}{169}$

(c) $P(K, A) = P(K) \cdot P(A)$ because they are independent
$= \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = \frac{16}{2704} = \frac{1}{169}$

(d) $P[(A, K) \textit{ or } (K, A)] = P(A, K) + P(K, A)$ since the 2 outcomes are mutually exclusive when we consider the order
$= \frac{1}{169} + \frac{1}{169} = \frac{2}{169}$

12. (a) Yes; the key idea here is "with replacement." When the first card drawn is replaced, the sample space is the same when the second card is drawn as it was when the first card was drawn and the second card is in no way influenced by the outcome of the first draw; in fact, it is possible to draw the same card twice. Let the card draws be represented by an (x, y) ordered pair; for example (K, 6) means a king was drawn first, replaced, and then the second card, a "6," was drawn independently of the first.

(b) $P(3, 10) = P(3) \cdot P(10)$ because draws are independent
$= \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = \frac{16}{2704} = \frac{1}{169} \approx 0.0059$

(c) $P(10, 3) = P(10) \cdot P(3)$ because of independence
$= \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = \frac{16}{2704} = \frac{1}{169} \approx 0.0059$

(d) $P[(3, 10)\ or\ (10, 3)] = P(3, 10) + P(10, 3)$ because these outcomes are mutually exclusive

$$= \frac{1}{169} + \frac{1}{169} = \frac{2}{169} \approx 0.0118$$

13. (a) $P(6\ or\ \text{older}) = P[(6\ \text{to}\ 9)\ or\ (10\ \text{to}\ 12)\ or\ (13\ \text{and over})]$

$= P(6-9) + P(10-12) + P(13+)$ because they are mutually exclusive age groups - no child is both 7 and 11 years old.

$= 27\% + 14\% + 22\% = 63\% = 0.63$

(b) $P(12\ \text{or younger}) = 1 - P(13\ \text{and over}) = 1 - 0.22 = 0.78$

(c) $P(\text{between 6 and 12}) = P[(6\ \text{to}\ 9)\ or\ (10\ \text{to}\ 12)]$

$= P(6\ \text{to}\ 9) + P(10\ \text{to}\ 12)$ because the age groups are mutually exclusive

$= 27\% + 14\% = 41\% = 0.41$

(d) $P(\text{between 3 and 9}) = P[(3\ \text{to}\ 5)\ or\ (6\ \text{to}\ 9)]$

$= P(3\ \text{to}\ 5) + P(6\ \text{to}\ 9)$ because age categories are mutually exclusive

$= 22\% + 27\% = 49\% = 0.49$

Answers vary; however, category 10–12 years covers only 3 years while 13 and over covers many more years and many more people, including adults who buy toys for themselves.

14. What we know: $P(\text{seniors get flu}) = 0.14$,
$P(\text{younger people get flu}) = 0.24$
$P(\text{senior}) = 0.125$
Let S denote seniors, so *not* S denotes younger people. Let F denote flu and *not* F denote did not get the flu. So $P(F,\ given\ S) = 0.14$, $P(F,\ given\ not\ S) = 0.24$ and $P(S) = 0.125$ so $P(not\ S) = 1 - 0.125 = 0.875$. Note the phrases 14% <u>of</u> seniors, i.e., they were already seniors, so this is a given condition; and 24% <u>of</u> people under 65, i.e., these people were already under 65, so under 65 (younger) is a given condition

(a) $P(\text{person is senior}\ and\ \text{will get flu}) = P(S\ and\ F)$

$= P(S) \cdot P(F,\ given\ S) = (0.125)(0.14) = 0.0175$

conditional probability rule

(b) $P(\text{person is}\ not\ \text{senior}\ and\ \text{will get flu}) = P[(not\ S)\ and\ F]$

$= P(not\ S) \cdot P(F,\ given\ not\ S) = 0.875(0.24) = 0.21$

(c) Here, $P(S) = 0.95$ so $P(not\ S) = 1 - 0.95 = 0.05$
  (a) $P(S\ and\ F) = P(S) \cdot P(F,\ given\ S) = (0.95)(0.14) = 0.133$
  (b) $P(not\ S\ and\ F) = P(not\ S) \cdot P(F,\ given\ not\ S) = (0.05)(0.24) = 0.012$

(d) Here, $P(S) = P(not\ S) = 0.50$
  (a) $P(S\ and\ F) = P(S) \cdot P(F,\ given\ S) = 0.50(0.14) = 0.07$
  (b) $P(not\ S\ and\ F) = P(not\ S) \cdot P(F,\ given\ not\ S) = 0.50(0.24) = 0.12$

15. What we know: $P(\text{polygraph says "lying" when person is lying}) = 72\%$
$P(\text{polygraph says "lying" when person is not lying}) = 7\%$
Let L denote that the polygraph results show lying and *not* L denote that the polygraph results show the person is not lying. Let T denote that the person is telling the truth and let *not* T denote that the person is not telling the truth, so $P(L,\ given\ not\ T) = 72\%$

$P(L,\ given\ T) = 7\%$.

We are told whether the person is telling the truth or not; what we know is what the polygraph results are, given the case where the person tells the truth, and given the situation where the person is not telling the truth.

(a) $P(T) = 0.90$ so $P(not\ T) = 0.10$

$P(\text{polygraph says lying and person tells truth})$

$= P(L\ and\ T) = P(T) \cdot P(L,\ given\ T)$

$= (0.90)(0.07) = 0.063 = 6.3\%$

(b) $P(\textit{not } T) = 0.10$ so $P(T) = 0.90$
P(polygraph says lying and person is not telling the truth)
$= P(L \textit{ and not } T) = P(\textit{not } T) \cdot P(L, \textit{given not } T)$
$= (0.10)(0.72) = 0.072 = 7.2\%$

(c) $P(T) = P(\textit{not } T) = 0.50$
(a) $P(L \textit{ and } T) = P(T) \cdot P(L, \textit{ given } T)$
$= (0.50)(0.07) = 0.035 = 3.5\%$
(b) $P(L \textit{ and not } T) = P(\textit{not } T) \cdot P(L, \textit{given not } T)$
$= (0.50)(0.72) = 0.36 = 36\%$

(d) $P(T) = 0.15$ so $P(\textit{not } T) = 1 - P(T) = 1 - 0.15 = 0.85$
(a) $P(L \textit{ and } T) = P(T) \cdot P(L, \textit{ given } T)$
$= (0.15)(0.07) = 0.0105 = 1.05\%$
(b) $P(L \textit{ and not } T) = P(\textit{not } T) \cdot P(L, \textit{ given not } T)$
$= (0.85)(0.72) = 0.612 = 61.2\%$

16. What we know: P(polygraph says "lying" when person is lying) = 72%
P(polygraph says "lying" when person is not lying) = 7%
Let L denote that the polygraph results show lying and *not* L denote that the polygraph results show the person is not lying. Let T denote that the person is telling the truth and let *not* T denote that the person is not telling the truth, so $P(L, \textit{given not } T) = 72\%$
$P(L, \textit{given } T) = 7\%$
We are told whether the person is telling the truth or not; what we know is what the polygraph results are, given the case where the person tells the truth, and given the situation where the person is not telling the truth.

(a) P(polygraph reports "lying") = $P(L) = 30\%$
We want to find P(person is lying) = $P(\textit{not } T)$
There are two possibilities when the polygraph says the person is lying: either the polygraph is right, or the polygraph is wrong. If the polygraph is right, the polygraph results show "lying" and the person is not telling the truth, i.e., $P(L \textit{ and not } T)$. If the polygraph is wrong, then the polygraph results show "lying" but, in fact, the person is telling the truth, i.e., $P(L \textit{ and } T)$. (This is the basic "trick" to this problem, and the idea comes directly from set theory.)

So $P(L) = P(L \textit{ and not } T) + P(L \textit{ and } T)$
$= [P(\textit{not } T) \cdot P(L, \textit{ given not } T)] + [P(T) \cdot P(L, \textit{ given } T)]$
using conditional probability rules
$= [P(\textit{not } T) \cdot P(L, \textit{ given not } T)] + \{[1 - P(\textit{not } T)] \cdot P(L, \textit{ given } T)\}$
using the complementary event rule to rewrite $P(T)$ as $1 - P(\textit{not } T)$
$0.30 = [P(\textit{not } T)] \cdot (0.72)] + [(1 - P(\textit{not } T)] \cdot (0.07)$
substituting in the known values as given in # 15, and as given above
$= (0.72) \cdot P(\textit{not } T) + [0.07 - (0.07) \cdot P(\textit{not } T)]$
$0.30 - 0.07 = (0.72) \cdot P(\textit{not } T) - (0.07) \cdot P(\textit{not } T)$
$0.23 = P(\textit{not } T)(0.72 - 0.07) = P(\textit{not } T)(0.65)$
$\frac{0.23}{0.65} = P(\textit{not } T)$, or $P(\textit{not } T) \approx 0.354 = 35.4\%$

(b) Here, $P(\text{L}) = 70\% = 0.70$

This is the same as (a) except for the new $P(\text{L})$. Starting from the step in (a) just before we substituted in the numerical values we knew:

$$P(\text{L}) = [P(\textit{not}\,\text{T}) \cdot P(\text{L},\ \textit{given not}\ \text{T})] + \{[1 - P(\textit{not}\,\text{T})] \cdot P(\text{L},\ \textit{given}\,\text{T})\}$$
$$0.70 = P(\textit{not}\,\text{T}) \cdot (0.72) + [1 - P(\textit{not}\ \text{T})] \cdot (0.07)$$
$$0.70 = (0.72) \cdot P(\textit{not}\ \text{T}) + [0.07 - 0.07 \cdot P(\textit{not}\,\text{T})]$$
$$0.70 - 0.07 = (0.72 - 0.07) \cdot P(\textit{not}\ \text{T})$$
$$0.63 = 0.65P(\textit{not}\ \text{T})$$

so $P(\textit{not}\ \text{T}) = \dfrac{0.63}{0.65} \approx 0.969 = 96.9\%$.

17. Let E denote eyeglasses, C denote contact lenses, W denote women, and M denote men.
Then we have $P(\text{E}) = 56\%$, $P(\text{C}) = 3.6\%$, $P(\text{W},\ \textit{given}\ \text{E}) = 55.4\%$, $P(\text{M},\ \textit{given}\ \text{E}) = 44.6\%$, $P(\text{W},\ \textit{given}\ \text{C}) = 63.1\%$, $P(\text{M},\ \textit{given}\ \text{C}) = 36.9\%$, and $P(\text{E}\ \textit{and}\ \text{C}) = 0$.

(a) $P(\text{W}\ \textit{and}\ \text{E}) = P(\text{E}) \cdot P(\text{W},\ \textit{given}\ \text{E})$

using a conditional probability rule

$= (0.56)(0.554) \approx 0.310$

(b) $P(\text{M}\ \textit{and}\ \text{E}) = P(\text{E}) \cdot P(\text{M},\ \textit{given}\ \text{E})$
$= (0.56)(0.446) \approx 0.250$

(c) $P(\text{W}\ \textit{and}\ \text{C}) = P(\text{C}) \cdot P(\text{W},\ \textit{given}\ \text{C})$
$= (0.036)(0.631) \approx 0.023$

(d) $P(\text{M}\ \textit{and}\ \text{C}) = P(\text{C}) \cdot P(\text{M},\ \textit{given}\ \text{C})$
$= (0.036)(0.369) \approx 0.013$

(e) $P(\text{none of the above}) = 1 - [P(\text{W}\ \textit{and}\ \text{E}) + P(\text{M}\ \textit{and}\ \text{E}) + P(\text{W}\ \textit{and}\ \text{C}) + P(\text{M}\ \textit{and}\ \text{C})]$
$= 1 - [0.310 + 0.250 + 0.023 + 0.013]$
$= 1 - 0.596 = 0.404$

18. $P(\$0) = 0.275$, $P(\text{less than } \$200) = 0.096$, $P(\$200 - \$599) = 0.135$, $P(\$600 - \$999) = 0.021$,
$P(\$1000 \text{ or more}) = 0.262$, $P(\text{don't know}) = 0.211$

$P(\$600 \text{ or more}) = P[(\$600 - \$999)\ \textit{or}\ (\$1000 \text{ or more})]$
$= P(\$600 - \$999) + P(\$1000 \text{ or more})$

since mutually exclusive price ranger

$= 0.021 + 0.262 = 0.283$

$P(\text{no more than } \$199) = P[(\$0)\ \textit{or}\ (\text{less than } \$200)]$
$= 0.275 + 0.096$

assuming "less than \$200" means \$1 to \$199 so that this category does not overlap with those who said \$0

$= 0.371$

19. We have $P(A) = \dfrac{580}{1160}$, $P(Pa) = \dfrac{580}{1160} = P(\textit{not}\ A)$, $P(S) = \dfrac{686}{1160}$, $P(N) = \dfrac{474}{1160} = P(\textit{not}\ S)$

(a) $P(S) = \dfrac{686}{1160}$

$P(S,\ \textit{given}\ A) = \dfrac{270}{580}$ (*given A* means stay in the *A*, aggressive row)

$P(S,\ \textit{given}\ Pa) = \dfrac{416}{580}$ (staying in row *Pa*)

(b) $P(S) = \dfrac{686}{1160} = \dfrac{343}{580}$

$P(S,\ \textit{given}\ Pa) = \dfrac{416}{580}$

They are not independent since the probabilities are not the same.

(c) $P(A \text{ and } S) = P(A) \cdot P(S, \text{ given } A)$

$$= \left(\frac{580}{1160}\right)\left(\frac{270}{580}\right) = \frac{270}{1160}$$

$P(Pa \text{ and } S) = P(Pa) \cdot P(S, \text{ given } Pa)$

$$= \left(\frac{580}{1160}\right)\left(\frac{416}{580}\right) = \frac{416}{1160}$$

(d) $P(N) = \dfrac{474}{1160}$

$P(N, \text{ given } A) = \dfrac{310}{580}$ (stay in the A row)

(e) $P(N) = \dfrac{474}{1160} = \dfrac{237}{580}$

$P(N, \text{ given } A) = \dfrac{310}{580}$

Since the probabilities are not the same, N and A are not independent.

(f) $P(A \text{ or } S) = P(A) + P(S) - P(A \text{ and } S)$

$$= \frac{580}{1160} + \frac{686}{1160} - \frac{270}{1160} = \frac{996}{1160}$$

20. (a) $P(+, \text{ given}$ condition present$) = \dfrac{110}{130}$ (stay in "condition present" row)

(b) $P(-, \text{ given}$ condition present$) = \dfrac{20}{130}$ (stay in "condition present" row)

[(a) and (b) are complementary events]

(c) $P(-, \text{ given}$ condition absent$) = \dfrac{50}{70}$ (stay in the row or column of the "*given*")

(d) $P(+, \text{ given}$ condition absent$) = \dfrac{20}{70}$

(e) $P(\text{condition present } and\ +) = P(\text{condition present}) \cdot P(+, \text{ given condition present})$

$$= \left(\frac{130}{200}\right)\left(\frac{110}{130}\right) = \frac{110}{200}$$

(f) $P(\text{condition present } and\ -) = P(\text{condition present}) \cdot P(-, \text{ given condition present})$

$$= \left(\frac{130}{200}\right)\left(\frac{20}{130}\right) = \frac{20}{200}$$

21. Let C denote the condition is present, and *not* C denote the condition is absent.

(a) $P(+, \text{ given C}) = \dfrac{72}{154}$ (stay in C column)

(b) $P(-, \text{ given C}) = \dfrac{82}{154}$ (stay in C column)

(c) $P(-, \text{ given not C}) = \dfrac{79}{116}$ (stay in *not* C column)

(d) $P(+, \text{ given not C}) = \dfrac{37}{116}$ (stay in *not* C column)

(e) $P(\text{C and } +) = P(\text{C}) \cdot P(+, \text{ given C}) = \left(\dfrac{154}{270}\right)\left(\dfrac{72}{154}\right) = \dfrac{72}{270}$

(f) $P(\text{C and } -) = P(\text{C}) \cdot P(-, \text{ given C}) = \left(\dfrac{154}{270}\right)\left(\dfrac{82}{154}\right) = \dfrac{82}{270}$

22. First determine the denominator. If it is a row or column total, the numerator will be in the body (inside) of the table in that same row or column. If the denominator is the grand total the numerator can be one or more row totals, one or more column totals, or a body-of-the-table cell entry. A cell entry is usually indicated when the problem mentions both a row category and a column category, in which case the desired cell is the one where the row and column intersect.

(a) customer at random, denominator is grand total, 2008; loyal 10–14 years, numerator is column total, 291; $\frac{291}{2008}$

(b) given: customer is from the East, so denominator is row total, 452; loyal 10–14 years: the cell entry in that row, 77; $\frac{77}{452}$

(c) no qualifiers on the customers, so denominator is grand total, 2008; at least 10 years: need entry for 10–14 years and 15+ years, so numerator is the sum of these 2 column totals, 291 + 535 = 826; $\frac{826}{2008}$

(d) given: from the West means the denominator is the West row total, 373; loyal at least 10 years means we sum the numbers in the West row for 10–14 and 15+ years, 45 + 86 = 131; $\frac{131}{373}$

(e) given: loyal less than 1 year means the denominator is column total, 157; from the West means the numerator is the cell entry for West in that column, 41; $\frac{41}{157}$

(f) given: loyal < 1 year, so denominator is 157; from South, so numerator is at the intersection of the < 1 year column and the South row, 53; $\frac{53}{157}$

(g) given: East, so denominator is 452; loyal 1+ years: either add up all the entries except < 1 year in the East row, or use the complementary event rule (less work!):

$$P(\text{loyal 1+ years, } \textit{given} \text{ East}) = 1 - P(\text{loyal} < 1 \text{ year, } \textit{given} \text{ East})$$
$$= 1 - \frac{32}{452} = \frac{420}{452}$$

(h) given: West, so denominator is West row total, 373; loyal 1+ years is either the sum of all the West row entries except < 1 year, or apply the complementary event rule in the probability calculation;

$$P(\text{loyal 1+ years, } \textit{given} \text{ West}) = 1 - P(\text{loyal} < 1 \text{ year, } \textit{given} \text{ West})$$
$$= 1 - \frac{41}{373} = \frac{332}{373}$$

(i) $P(\text{East}) = \frac{452}{2008} \approx 0.2251$

$P(\text{loyal 15+ years}) = \frac{535}{2008} \approx 0.2664$

$P(\text{East, } \textit{given} \text{ 15+ years}) = \frac{118}{535} \approx 0.2206$

$P(\text{loyal 15+ years, } \textit{given} \text{ East}) = \frac{118}{452} \approx 0.2611$

If they are independent, $P(\text{East}) = P(\text{East, } \textit{given} \text{ 15+ years})$ but $0.2251 \neq 0.2206$, and if they are independent, $P(\text{loyal 15+ years}) = P(\text{loyal 15+ years, } \textit{given} \text{ East})$ but $0.2664 \neq 0.2611$, so they aren't independent. (If you use decimal approximations, and the 2 probabilities are quite close, it's time to reduce fractions or use the least common denominator to get an accurate comparison.)

Note: independence is symmetric, i.e., if A is independent of B, then B is independent of A; this means you don't have to do *both* independence checks; one is sufficient.

23. First determine the denominator. If it is a row or column total, the numerator will be in the body (inside) of the table in that same row or column. If the denominator is the grand total the numerator can be one or more row totals, one or more column totals, or a body-of-the-table cell entry. A cell entry is usually indicated when the problem mentions both a row category and a column category, in which case the desired cell is the one where the row and column intersect.

(a) $P(2+) = 1 - P(<2) = 1 - P(1 \text{ visit}) = 1 - \frac{962}{1894} = \frac{932}{1894}$

There is no additional information about the customer, so the denominator is the grand total.

(b) $P(2+, \textit{given } 25\text{–}39 \text{ years old}) = 1 - P(<2, \textit{given } 25\text{–}39) = 1 - \frac{386}{739} = \frac{353}{739}$

The customer's age is given, so the denominator is the 25–39 row total, and the numerator is determined by the numbers in that row, using the complimentary rule for probabilities.

(c) $$\begin{aligned} P(>3 \text{ visits}) &= P(4 \textit{ or } 5 \textit{ or } 6 \text{ or more visits}) \\ &= P(4) + P(5) + P(6+) \\ &= \frac{66}{1894} + \frac{44}{1894} + \frac{32}{1894} = \frac{142}{1894} \end{aligned}$$

There is no qualifier on the customer, so we use the grand total, 1894, as the denominator.

(d) $$\begin{aligned} P(>3, \textit{given} \text{ age } 65+) &= 1 - P(\le 3, \textit{given } 65+) \\ &= 1 - P[(1, 2, \textit{or } 3 \text{ visits}), \textit{given } 65+] \\ &= 1 - [P(1 \text{ visit}, \textit{given } 65+) + P(2 \text{ visits}, \textit{given } 65+) + P(3 \text{ visits}, \textit{given } 65+)] \\ &= 1 - \left(\frac{115}{224} + \frac{69}{224} + \frac{18}{224}\right) = 1 - \frac{202}{224} = \frac{22}{224} \end{aligned}$$

Alternately, this can be solved without the complementary event rule:

$$\begin{aligned} P(>3 \text{ visits}, \textit{given } 65 \text{ or older}) &= P(4 \textit{ or } 5 \textit{ or } 6 \text{ or more visits}, \textit{given } 65+) \\ &= P(4, \textit{given } 65+) + P(5, \textit{given } 65+) + P(6 \text{ or more}, \textit{given } 65+) \\ &= \frac{12}{224} + \frac{7}{224} + \frac{3}{224} = \frac{22}{224} \end{aligned}$$

Here, the complementary event rule has no advantage, work-wise, over the regular method.

(e) $$\begin{aligned} P(40 \text{ or older}) &= P(40\text{–}49 \textit{ or } 50\text{–}64 \textit{ or } 65+) \\ &= P(40\text{–}49) + P(50\text{–}64) + P(65+) \\ &= \frac{434}{1894} + \frac{349}{1894} + \frac{224}{1894} = \frac{1007}{1894} \end{aligned}$$

(f) $$\begin{aligned} P(40 \text{ or older}, \textit{given } 4 \text{ visits}) &= P(40\text{–}49 \textit{ or } 50\text{–}64 \textit{ or } 65+ \text{or over}, \textit{given } 4 \text{ visits}) \\ &= P(40\text{–}49, \textit{given } 4) + P(50\text{–}64, \textit{given } 4) + P(65+, \textit{given } 4) \\ &= \frac{13}{66} + \frac{14}{66} + \frac{12}{66} = \frac{39}{66} \end{aligned}$$

The given 4 visits means the denominator is the 4 visit column total, and the numerator is composed from numbers in that column.

(g) $P(25\text{–}39 \text{ years old}) = \frac{739}{1894} \approx 0.3902$

$P(\text{visits more than once a week}) = 1 - P(\text{visits once}) = 1 - \frac{962}{1894} = \frac{932}{1894} \approx 0.4921$

$P(\text{visits} > 1, \textit{given } 25\text{–}39) = 1 - P(1 \text{ visit}, \textit{given } 25\text{–}39) = 1 - \frac{386}{739} = \frac{353}{739} \approx 0.4777$

25–39 years old is independent of more than 1 visit if $P(>1 \text{ visit}) = P(>1 \text{ visit}, \textit{given} \text{ age } 25\text{–}39)$ but $0.4921 \ne 0.4777$ so they are not independent.

24. P(female) = 85% so P(male) = 15%
P(BSN. *given* female) = 70%
P(BSN, *given* male) = 90%
70% <u>of females</u>. 90% <u>of males</u> are conditions further defining a randomly selected student. so gender is the "given..."

(a) P(BSN. *given* F) = 70% = 0.70

(b) P(BSN *and* F) = P(F) · P(BSN. *given* F) = (0.85)(0.70) = 0.595 conditional probability rule

(c) P(BSN. *given* M) = 90% = 0.90

(d) P(BSN *and* M) = P(M) · P(BSN, *given* M) = (0.15)(0.90) = 0.135

(e) Of the graduates, some are female and some are male, so if we add the numbers of male graduates and female graduates. we will have all the graduates.
P(BSN) = P[BSN and (M *or* F)]
= P[(BSN and M) *or* (BSN and F)]
= P(BSN and M) + P(BSN and F)
We can use the mutually exclusive addition rule here because no graduating student is both male and female: these are disjoint conditions.
= 0.135 + 0.595 from (b) and (d)
= 0.730
(Refer to the hint in 16(a) above which uses the same method.)
A and (B *or* C) = (A and B) *or* (A and C) is the distribution law of "and" over "or." Recall the distributive law for multiplication over addition gives us, for example,
5(6 + 7) = 5(6) + 5(7) = 30 + 35 = 65
The word "and" in mathematics is translated as "multiply" in algebra. and as "intersection" in set theory. "Or" is translated as "add" in algebra, and as "union" in set theory.

(f) If all incoming freshmen nursing students from the overall samples space, the phrase "(given) female" describes only part of the sample space. The "given" restricts the sample space *for this problem*.
(a) P(BSN, given F). to females only.

Consider the following 4 graphs. Figure A shows 100 squares partitioned into one group of 85 squares and one group of 15 squares. 85% women and 15% men, overall.

FIGURE A

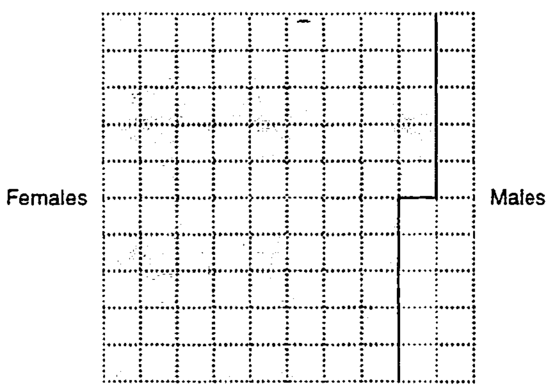
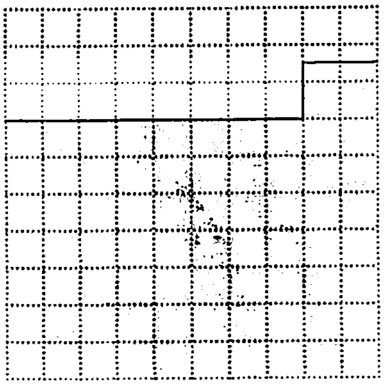


Figure B shows the same 100 squares with 73 squares total shaded, i.e., 73% of students overall will graduate [see part (e)].

FIGURE B



Graduates

Figure C shows the graduation squares overlaying the female squares, with their 59.5 squares in common shaded, showing those who are graduates and females among the 100 squares overall [part (b)].

FIGURE C

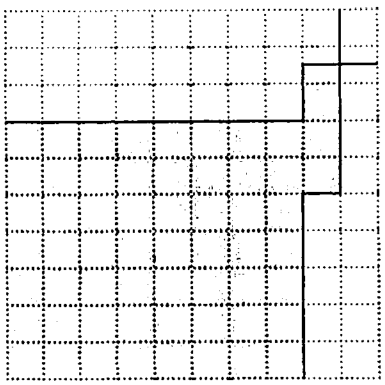
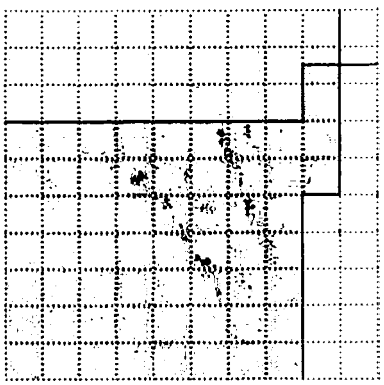


Figure D shows the same 59.5 squares shaded as a part of the 85 shaded female squares, showing $\frac{59.5}{85} = 70\%$ of the females graduating.

FIGURE D



In figures C and D we are looking at the same 59.5 squares, describing the same students, but in C we compare them to all 100 squares, and in D, we compare them only to the 85 female squares.

Formula (2) on page 151 shows the probability for equally likely outcomes:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

The phrases "will graduate and is female" and "will graduate, given female" *do* describe the same students represented by the 59.5 squares. This is the number of favorable outcomes (squares), i.e., the numerator of the probability calculation in parts (a) and (b).

However, the denominator changes depending on the context; in part (b), it is the 100 students (squares) overall, but in part (a), you are given 85 females (squares) as the total number of outcomes.

Thus, although part (a) and part (b) refer to the same students, the probabilities differ because the total number of outcomes being considered in part (a) is different from the total number of outcomes being considered in part (b).

(In part (a), each of the 85 squares is equally likely to be chosen, and in part (b), each of the 100 squares is equally likely to be chosen as the incoming freshman nursing student selected at random.)

25. Given: Let A be the event that a new store grosses > \$940,000 in year 1; then not A is the event the new store grosses ≤ \$940,000 the first year.
Let B be the event that the store grosses > \$940,000 in the second year; then not B is the event the store grosses ≤ \$940,000 in the second year of operation.

2 year results	Translations
A and B	profitable both years
A and *not* B	profitable first but not second year
not A and B	profitable second but not first year
not A and *not* B	not profitable either year

P(A) = 65% (show profit in first year)
P(*not* A) = 35%
P(B) = 71% (show profit in second year)
P(*not* B) = 29%
P(close) = P(*not* A and *not* B)
P(B, *given* A) = 87%

(a) P(A) = 65% = 0.65 (from the given)

(b) P(B) = 71% = 0.71 (from the given)

(c) P(B, *given* A) = 87% = 0.87 (from the given)

(d) P(A *and* B) = P(A) · P(B, *given* A) = (0.65)(0.87) = 0.5655 ≈ 0.57 (conditional probability rule)

(e) P(A *or* B) = P(A) + P(B) − P(A and B) = 0.65 + 0.71 − 0.57 = 0.79 (addition rule)

(f) P(not closed) = P(show a profit in year 1 or year 2 or both) (same question as (e))
P(closed) = 1 − P(not closed) = 1 − 0.79 = 0.21 (complimentary event rule)

26. Known: Let A be the event the client relapses in phase I.
Let B be the event the client relapses in phase II.
Let C be the event that the client has no relapse in phase I, i.e., C = *not* A.
Let D be the event that the client has no relapse in phase II, i.e., D = *not* B.
P(A) = 0.27 so P(*not* A) = P(C) = 1 − 0.27 = 0.73
P(B) = 0.23 so P(*not* B) = P(D) = 1 − 0.23 = 0.77
P(*not* B, *given not* A) = 0.95 = P(D, *given* C) = 0.95
P(B, *given* A) = 0.70

Possible outcomes	Translation
A, B	relapse in I, relapse in II
not A, B (= C, B)	no relapse in I, relapse in II
A, *not* B (= A, D)	relapse in I, no relapse in II
not A, *not* B (= C, D)	no relapse in I no relapse in II

(a) $P(A) = 0.27$, $P(B) = 0.23$, $P(C) = 0.73$, $P(D) = 0.77$ (from the given)

(b) $P(B, \textit{given } A) = 0.70$, $P(D, \textit{given } C) = 0.95$ (from the given)

(c) $P(A \textit{ and } B) = P(A)\cdot P(B, \textit{given } A)$
$= (0.27)(0.70) = 0.189$ (conditional probability rule)
$P(C \textit{ and } D) = P(C)\cdot P(D, \textit{given } C)$
$= (0.73)(0.95) = 0.6935 \approx 0.69$

(d) $P(A \textit{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (general addition rule)
$= 0.27 + 0.23 - 0.189 = 0.311 \approx 0.31$

(e) $P(C \textit{ and } D) = 0.69$ [from (c)]

(f) $P(A \textit{ and } B) = 0.189$ [from (c)]

(g) translate as "inclusive or," i.e., as "or both"
$P(A \textit{ or } B) = 0.31$ [from (d)]

27. Let TB denote that the person has tuberculosis, so *not* TB denotes that the person does not have tuberculosis.
Let + indicate the test for tuberculosis indicates the presence of the disease, so – indicates that the test for tuberculosis shows no disease.
Given: $P(+, \textit{given } TB) = 0.82$ (sensitivity of the test)
$P(+, \textit{given not } TB) = 0.09$ (false-positive rate)
$P(TB) = 0.04$

(a) $P(TB \textit{ and } +) = P(TB)\cdot P(+, \textit{given } TB)$ (by the conditional probability rule)
$= (0.04)(0.82) = 0.0328 \approx 0.033$ (predictive value of the test)

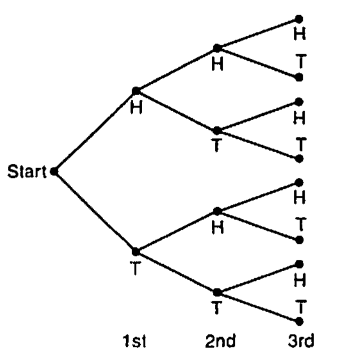
(b) $P(\textit{not } TB) = 1 - P(TB)$ (complementary events)
$= 1 - 0.04 = 0.96$

(c) $P(\textit{not } TB \textit{ and } +) = P(\textit{not } TB)\cdot P(+, \textit{given not } TB)$ (conditional probability rule and (b))
$= (0.96)(0.09) = 0.0864 \approx 0.086$

(Note: refer to #20 above to see terminology.)

Section 4.3

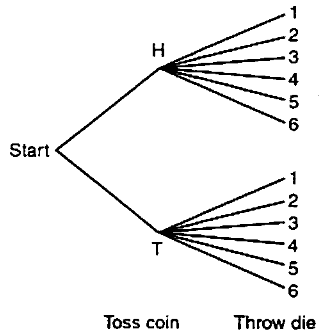
1. (a) Outcomes for Tossing a Coin Three Times



(b) HHT. HTH, THH: 3

(c) 8 possible outcomes, 3 with exactly 2 Hs: $\frac{3}{8}$

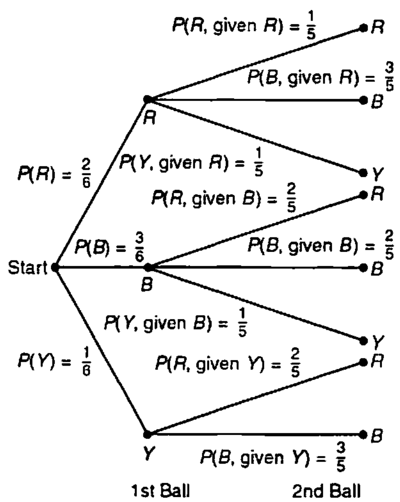
2. (a) Outcomes of Tossing a Coin and Throwing a Die



(b) outcomes with H and > 4
H5. H6:2

(c) 12 outcomes, two with H and > 4: $\frac{2}{12} = \frac{1}{6}$

3. (a) Outcomes for Drawing Two Balls (without replacement)



Because we drew without replacement the number of available balls drops to 5 and one of the colors drops by 1. Note that if the yellow ball is drawn first, there are only two possibilities for the second draw: red and blue; the yellow balls are exhausted.

(b) $P(R, R) = \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{2}{30} = \frac{1}{15}$

$P(R, B) = \left(\frac{2}{6}\right)\left(\frac{3}{5}\right) = \frac{6}{30} = \frac{1}{5}$

$P(R, Y) = \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{2}{30} = \frac{1}{15}$

$P(B, R) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{6}{30} = \frac{1}{5}$

$P(B, B) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{6}{30} = \frac{1}{5}$

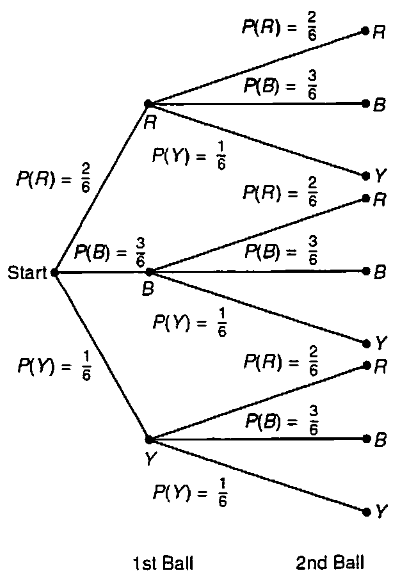
$P(B, Y) = \left(\frac{3}{6}\right)\left(\frac{1}{5}\right) = \frac{3}{30} = \frac{1}{10}$

$P(Y, R) = \left(\frac{1}{6}\right)\left(\frac{2}{5}\right) = \frac{2}{30} = \frac{1}{15}$

$P(Y, B) = \left(\frac{1}{6}\right)\left(\frac{3}{5}\right) = \frac{3}{30} = \frac{1}{10}$

where $P(x, y)$ is the probability the first ball is color x, and the second ball is color y. Multiply the branch probability values along each branch from start to finish. Observe the sum of the probabilities is 1.

4. (a) Outcomes for Drawing Two Balls with Replacement



Because the draws are with replacement, $P(R)$ stays at $\frac{2}{6}$, $P(B) = \frac{3}{6}$, and $P(Y) = \frac{1}{6}$ for each draw.

(b) Using $P(x, y)$ to denote the probability of x on the first draw and y on the second:

$$P(R, R) = \left(\frac{2}{6}\right)\left(\frac{2}{6}\right) = \frac{4}{36} = \frac{1}{9}$$

$$P(R, B) = \left(\frac{2}{6}\right)\left(\frac{3}{6}\right) = \frac{6}{36} = \frac{1}{6}$$

$$P(R, Y) = \left(\frac{2}{6}\right)\left(\frac{1}{6}\right) = \frac{2}{36} = \frac{1}{18}$$

$$P(B, R) = \left(\frac{3}{6}\right)\left(\frac{2}{6}\right) = \frac{6}{36} = \frac{1}{6}$$

$$P(B, B) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = \frac{9}{36} = \frac{1}{4}$$

$$P(B, Y) = \left(\frac{3}{6}\right)\left(\frac{1}{6}\right) = \frac{3}{36} = \frac{1}{12}$$

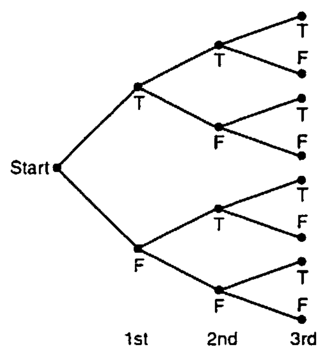
$$P(Y, R) = \left(\frac{1}{6}\right)\left(\frac{2}{6}\right) = \frac{2}{36} = \frac{1}{18}$$

$$P(Y, B) = \left(\frac{1}{6}\right)\left(\frac{3}{6}\right) = \frac{3}{36} = \frac{1}{12}$$

$$P(Y, Y) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$$

Multiplying the probabilities along each branch.

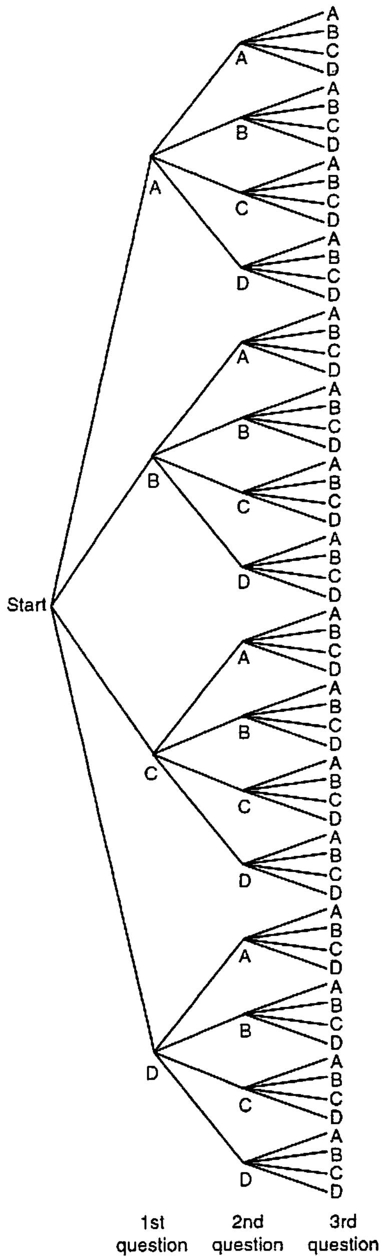
5. (a) Choices for Three True/False Questions



The tree diagram looks exactly like that of problem 1, because each event has 2 outcomes, and the events are independent, so possible outcomes for the second event are the same as for the first event.

(b) $P(\text{3 correct responses}) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

6. (a) Outcomes of Three Multiple-Choice Questions



This is a gaudier version of problems 1 and 5 where there are 3 questions, but now there are 4 responses (A, B, C, D) for each question at each step.

(b) If the outcomes are equally likely, then $P(\text{all 3 correct}) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{64}$.

7. 4 wire choices for the first leaves 3 wire choices for the second, 2 for the third, and only 1 wire choice for the fourth wire connection: $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$.

8. 4 choices for his first stop, 3 for the second, 2 for the third, and only 1 city for his (last) fourth stop: $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$. This problem is identical to problem 7 except wires were changed to cities.

9. **(a)** Choose 1 card from each deck. The number of pairs (one card from the first deck and one card from the second) is $52 \cdot 52 = 52^2 = 2704$.

(b) There are 4 kings in the first deck and four in the second, so $4 \cdot 4 = 16$.

(c) There are 16 ways to draw a king from each deck, and 2704 ways to draw a card from each deck, so $\frac{16}{2704} = \frac{1}{169} \approx 0.006$.

10. **(a)** The die rolls are independent, so multiply the 6 ways the first die can land by the 6 ways the second die can land: $6 \cdot 6 = 36$.

(b) Even numbers are 2, 4, and 6, three possibilities per die, so $3 \cdot 3 = 9$.

(c) $P(\text{even, even}) = \frac{9}{36} = \frac{1}{4} = 0.25$

using $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

11. There are 4 fertilizers to choose from, and 3 temperature zones to choose from for each fertilizer, and 3 possible water treatments for every fertilizer-temperature zone combination: $4 \cdot 3 \cdot 3 = 36$.

12. There are 3 possible sandwiches to choose from, and for each, 4 possible salads, and for each sandwich-salad combination, there are also 5 desserts to choose from: $3 \cdot 4 \cdot 5 = 60$.

Problems 13, 14, 15, and 16 deal with permutations, $P_{n,r} = \frac{n!}{(n-r)!}$. This counts the number of ways r objects can be selected from n when the order of the result is important. For example, if we choose two people from a group, the first of which is to be the group's chair, and the second, the assistant chair, then (John, Mary) is distinct from (Mary, John).

13. $P_{5,2}: n = 5, r = 2$

$$P_{5,2} = \frac{5!}{(5-2)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3!} = 20$$

14. $P_{8,3}: n = 8, r = 3$

$$P_{8,3} = \frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 336$$

15. $P_{7,7}: n = r = 7$

$$P_{7,7} = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 7! = 5040 \text{ (recall } 0! = 1)$$

In general, $P_{n,n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$.

16. $P_{9,9}: n = r = 9$

$$P_{9,9} = \frac{9!}{(9-9)!} = \frac{9!}{0!} = \frac{9!}{1} = 362{,}880$$

In general, $P_{n,n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$.

Problems 17, 18, 19, and 20 deal with combination, $C_{n,r} = \frac{n!}{r!(n-r)!}$. This counts the number of ways r items can be selected from among n items when the order of the result doesn't matter. For example, when choosing two people from an office to pick up coffee and doughnuts, (John, Mary) is the same as (Mary, John) – both get to carry the goodies back to the office.

17. $C_{5,2}$: $n = 5$, $r = 2$

$$C_{5,2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1} = \frac{20}{2} = 10$$

18. $C_{8,3}$: $n = 8$, $r = 3$

$$C_{8,3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8\cdot7\cdot6\cdot5!}{3\cdot2\cdot1\cdot5!} = 56$$

19. $C_{7,7}$: $n = r = 7$

$$C_{7,7} = \frac{7!}{7!(7-7)!} = \frac{7!}{7!0!} = \frac{7!}{7!(1)} = 1 \text{ (recall } 0! = 1)$$

In general, $C_{n,n} = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!(1)} = 1$. There is only 1 way to choose all n objects without regard to order.

20. $C_{8,8}$: $n = r = 8$

$$P_{9,9} = \frac{8!}{8!(8-8)!} = \frac{8!}{8!0!} = \frac{8!}{8!(1)} = 1 \text{ (recall } 0! = 1)$$

In general, $C_{n,n} = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!(1)} = 1$. There is only 1 way to choose all n objects without regard to order.

21. Since the order matters (first is day supervisor, second is night supervisor, and third is coordinator), this is a permutation of 15 nurse candidates to fill 3 positions.

$$P_{15,3} = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{15\cdot14\cdot13\cdot12!}{12!} = 2730$$

22. Order matters here, since the order of the finalists selected determines who gets what amount of money.

$$P_{10,3} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10\cdot9\cdot8\cdot7!}{7!} = 720$$

23. **(a)** Think of this as 2 urns, the first having 8 red balls numbered 1 to 8, indicating the word to be defined, and the second urn containing 8 blue balls numbered 1 to 8, representing the possible definitions. Draw 1 ball from each urn without replacement. The (red number, blue number) pair indicates which word is "matched" with which definition–maybe correctly matched, maybe not. Here, (red 4, blue 6) is different from (red 6, blue 4)

$n = r = 8$

$$P_{8,8} = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 40,320$$

(b) This is the same as part (a), except we throw 3 definitions out; 3 words will be left with no definition.

$$P_{8,5} = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{40,320}{6} = 6720$$

24. Order of the books matter: history, art, fiction, ... is distinct from fiction, art, history, ...

$$P_{6,6} = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6!}{1} = 720$$

25. Order matters because the resulting sequence determines who wins first place, who wins second place, and who wins third place.

$$P_{5,3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$$

26. The order of the software packages selected doesn't matter, since all three are going home with the customer. (Assume the software packages are of equal interest to the customer.)

$$C_{10,3} = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3!7!} = \frac{720}{6} = 120$$

27. The order of trainee selection doesn't matter, since they are all going to be trained the same.

$$C_{15,5} = \frac{15!}{5!(15-5)!} = \frac{15!}{5!10!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{5!10!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 3003$$

28. It doesn't matter in which order the professor grades the problems, the 5 selected problems all get graded.

(a) $C_{12,5} = \frac{12!}{5!(12-5)!} = \frac{12!}{5!7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{5!7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 792$

(b) Jerry must have the very same 5 problems as the professor selected to grade, so

$P(\text{Jerry chose the right problems}) = \frac{1}{792} \approx 0.001$. (Jerry is pushing his luck.)

(c) Silvia did seven problems, which have $C_{7,5}$ subsets of 5 problems which would be among 792 subsets of 5 the professor selected from.

$$C_{7,5} = \frac{7!}{5!(7-5)!} = \frac{7!}{5!2!} = \frac{7 \cdot 6 \cdot 5!}{5!2!} = \frac{7 \cdot 6}{2 \cdot 1} = \frac{42}{2} = 21$$

$P(\text{Silvia lucked out}) = \frac{21}{792} \approx 0.027$

(Silvia is pushing her luck, too, but she increased her chances by a factor of 21, compared to Jerry, just by doing two more problems. Now, if these two had just done all the problems, or even split them half and half, ...)

29. (a) Six applicants are selected from among 12 without regard to order.

$$C_{12,6} = \frac{12!}{6!6!} = \frac{479{,}001{,}600}{(720)^2} = 924$$

(b) There are 7 women and 5 men. This problem really asks, in how many ways can 6 women be selected from among 7, and zero men be selected from 5?

$$(C_{7,6})(C_{5,0}) = \left(\frac{7!}{6!(7-6)!}\right)\left(\frac{5!}{0!(5-0)!}\right) = \frac{7!}{6!1!} \cdot \frac{5!}{0!5!} = 7$$

Since the zero men are "selected" be default, all positions being filled. This problem reduces to, in how many ways can 6 applicants be selected from 7 women?

(c) $P(\text{event A}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

$P(\text{all hired are women}) = \frac{7}{924} = \frac{1}{132} \approx 0.008$

30. It doesn't matter in which order you or the state select the 6 numbers each. It only matters that you and the state pick the *same* six numbers. While you spend your zillion dollars, you can always reorder your numbers if you want to.

(a) $C_{42,6} = \frac{42!}{6!(42-6)!} = \frac{42!}{6!36!} = \frac{42 \cdot 41 \cdot 40 \cdot 39 \cdot 38 \cdot 37 \cdot 36!}{6!36!} = \frac{3{,}776{,}965{,}920}{720} = 5{,}245{,}786$

(Most calculators will handle numbers through 69! But if you hate to see numbers like 1.771×10^{98}, cancel out the common factorial factors, such as 36! here.)

(b) This problem asks, what is the chance you choose the very same 6 numbers the state chose.

$P(\text{winning ticket}) = \frac{1}{5{,}245{,}786} \approx 0.000000191$

(c) What is the chance one of your 10 tickets is the winning ticket? (We'll assume each ticket is different from the other 9 you have, but, it really doesn't matter much ...)

$P(\text{win}) = \frac{10}{5{,}245{,}786} = \frac{5}{2{,}622{,}893} \approx 0.0000019$

Chapter 4 Review

1. $P(\text{asked}) = 24\% = 0.24$
$P(\text{received}, \textit{given} \text{ asked}) = 45\% = 0.45$
$P(\text{asked } \textit{and} \text{ received}) = P(\text{asked}) \cdot P(\text{received}, \textit{given} \text{ asked}) = (0.24)(0.45) = 0.108 = 10.8\%$

2. $P(\text{asked}) = 20\% = 0.20$
$P(\text{received}, \textit{given} \text{ asked}) = 59\% = 0.59$
$P(\text{asked } \textit{and} \text{ received}) = P(\text{asked}) \cdot P(\text{received}, \textit{given} \text{ asked}) = (0.20)(0.59) = 0.118 = 11.8\%$

3. (a) If the first card is replaced before the second is chosen (sampling with replacement), they are independent. If the sampling is without replacements they are dependent.

(b) $P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$

with replacement, independent

$P(\text{H on both}) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{16} = 0.0625 \approx 0.063$

(c) without replacement, dependent

$P(\text{H on first } \textit{and} \text{ H on second}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} \approx 0.059$

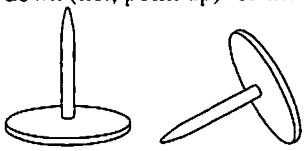
4. (a) There are 11 other outcomes besides 3H. The sample space is the 12 outcomes shown.

1H	1T
2H	2T
(3H)	3T
4H	4T
5H	5T
6H	6T

(b) Yes; the die and the coin are independent. Each outcome has probability $\left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$.

(c) $P(\text{H } \textit{and} \text{ number} < 3) = P[\text{H } \textit{and} \text{ (1 } \textit{or} \text{ 2)}] = P(\text{1H } \textit{or} \text{ 2H}) = P(\text{1H}) + P(\text{2H}) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6} \approx 0.167$

5. (a) Throw a large number of similar thumbtacks, or one thumbtack a large number of times, and record the frequency of occurrence of the various outcomes. Assume the thumbtack falls either flat side down (i.e., point up). or tilted (with the point down, resting on the edge of the flat side). (We will



assume these are the only two ways the tack can land.) To estimate the probability the tack lands on its flat side with the point up. find the relative frequency of this occurrence, dividing the number of times this occurred by the total number of thumbtack tosses.

(b) The sample space is the two outcomes flat side down (point up) and tilted (point down).

(c) $P(\text{flat side down, point up}) = \frac{340}{500} = 0.68$

$P(\text{tilted, point down}) = 1 - 0.68 = 0.32$

6. (a) $P(N) = \frac{470}{1000} = 0.470$

$P(M) = \frac{390}{1000} = 0.390$

$P(S) = \frac{140}{1000} = 0.140$

(b) $P(N, \textit{given } W) = \frac{420}{500} = 0.840$

$P(S, \textit{given } W) = \frac{20}{500} = 0.040$

(c) $P(N, \textit{given } A) = \frac{50}{500} = 0.100$

$P(S, \textit{given } A) = \frac{120}{500} = 0.240$

(d) $P(N \textit{ and } W) = P(W) \cdot P(N, \textit{ given } W)$

$= \left(\frac{500}{1000}\right)(0.840) = 0.420$

$P(M \textit{ and } W) = P(W) \cdot P(M, \textit{ given } W)$

$= \left(\frac{500}{1000}\right)\left(\frac{60}{500}\right) = 0.060$

(e) $P(N \textit{ or } M) = P(N) + P(M)$ if mutually exclusive

$= \left(\frac{470}{1000}\right) + \left(\frac{390}{1000}\right) = \frac{860}{1000} = 0.860$

They are mutually exclusive because the reactions are defined into 3 distinct, mutually exclusive categories, a reaction can't be both mild and non-existent.

(f) If N and W were independent, $P(N \text{ and } W) = P(N) \cdot P(W) = (0.470)(0.500) = 0.235$. However, from (d). we have $P(N \text{ and } W) = 0.420$. They are not independent.

7. (a) possible values for x, the sum of the two dice faces, is 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12

(b) 2:1 and 1
3:1 and 2. or 2 and 1
4:1 and 3. 2 and 2. 3 and 1
5:1 and 4. 2 and 3, 3 and 2. 4 and 1
6:1 and 5. 2 and 4, 3 and 3, 4 and 2, 5 and 1
7:1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, 6 and 1
8:2 and 6, 3 and 5, 4 and 4, 5 and 3, 6 and 2
9:3 and 6. 4 and 5. 5 and 4, 6 and 3
10:4 and 6. 5 and 5, 6 and 4
11:5 and 6, 6 and 5
12:6 and 6

x	$P(x)$
2	$\frac{1}{36} \approx 0.028$
3	$\frac{2}{36} \approx 0.056$
4	$\frac{3}{36} \approx 0.083$
5	$\frac{4}{36} \approx 0.111$
6	$\frac{5}{36} \approx 0.139$
7	$\frac{6}{36} \approx 0.167$
8	$\frac{5}{36} \approx 0.139$
9	$\frac{4}{36} \approx 0.111$
10	$\frac{3}{36} \approx 0.083$
11	$\frac{2}{36} \approx 0.056$
12	$\frac{1}{36} \approx 0.028$

Where there are (6)(6) = 36 possible. equally likely outcomes (the sums. however. are not equally likely).

8. $P(\text{pass } 101) = 0.77$
$P(\text{pass 102}, \textit{given} \text{ pass } 101) = 0.90$
$P(\text{pass 101 } \textit{and} \text{ pass } 102) = P(\text{pass } 101) \cdot P(\text{pass 102}, \textit{given} \text{ pass } 101) = 0.77(0.90) = 0.693$

9. $C_{8,2} = \frac{8!}{2!6!} = \frac{8 \cdot 7 \cdot 6!}{(2 \cdot 1)6!} = \frac{56}{2} = 28$

10. (a) $P_{7,2} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 7(6) = 42$

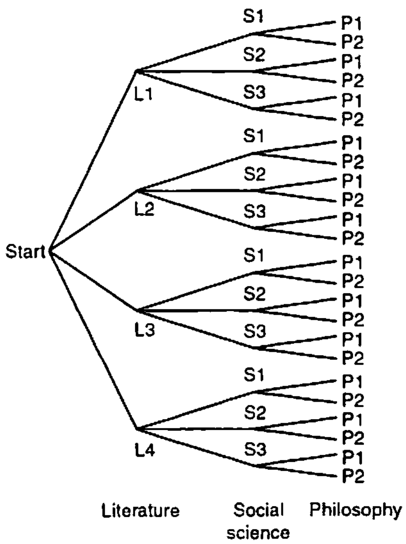
(b) $C_{7,2} = \frac{7!}{2!5!} = \frac{7 \cdot 6}{2} = 21$

(c) $P_{3,3} = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 6$

(d) $C_{4,4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4!0!} = 1$

11. $3 \cdot 2 \cdot 1 = 6$

12. Ways to Satisfy Literature, Social Science, and Philosophy Requirements



Let Li, $i = 1, \cdots, 4$ denote the 4 literature courses.
Let Si, $i = 1, 2, 3$ denote the 3 social science courses.
Let Pi, $i = 1, 2$ denote the 2 philosophy courses.
There are $4 \cdot 3 \cdot 2 = 24$ possible course combinations.

13. 5 multiple choice questions, each with 4 possible answers (A, B, C, or D), so 4 answers for first question; and for each of those, 4 answers for the second question; and for each of those, 4 answers for the third question; and for each of those, 4 answers for the fifth question. There are $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5 = 1024$ possible sequences, such as A, D, B, B, C or C, B, A, D, D, etc.

$P(\text{getting the correct sequence}) = \frac{1}{1024} \approx 0.00098$

14. Two possible outcomes per coin toss; 6 tosses to get a sequence such as THTHHT
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$ possible sequences.

15. 10 possible numbers per turn of dial; 3 dial turns
$10 \cdot 10 \cdot 10 = 1000$ possible combinations

16. The combination uses the three numbers 2, 9, and 5, in some sequence.

The number of sequences is $P_{3,3} = \frac{3!}{(3-3)!} = \frac{3 \cdot 2 \cdot 1}{0!} = 6.$

They are 2, 9, 5 9, 2, 5 5, 2, 9 2, 5, 9 9, 5, 2 5, 9, 2
Since all three numbers must be used, think of it as drawing without replacement from an urn containing the numbers 2, 9, and 5.

Chapter 5 The Binomial Probability Distribution and Related Topics

Section 5.1

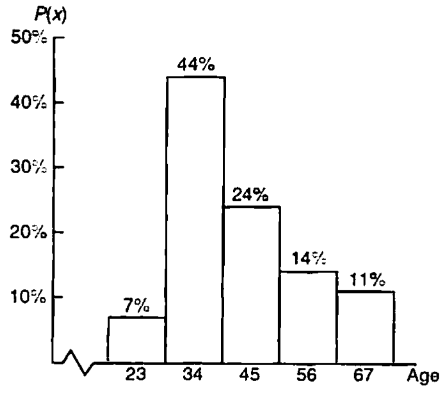
1. (a) The number of traffic fatalities can be only a whole number. This is a discrete random variable.
   (b) Distance can assume any value, so this is a continuous random variable.
   (c) Time can take on any value, so this is a continuous random variable.
   (d) The number of ships can be only a whole number. This is a discrete random variable.
   (e) Weight can assume any value, so this is a continuous random variable.

2. (a) Speed can assume any value, so this is a continuous random variable.
   (b) Age can take on any value, so this is a continuous random variable.
   (c) Number of books can be only a whole number. This is a discrete random variable.
   (d) Weight can assume any value, so this is a continuous random variable.
   (e) Number of lightning strikes can be only a whole number. This is a discrete random variable.

3. (a) $\Sigma P(x) = 0.25 + 0.60 + 0.15 = 1.00$
   Yes, this is a valid probability distribution because a probability is assigned to each distinct value of the random variable and the sum of these probabilities is 1.
   (b) $\Sigma P(x) = 0.25 + 0.60 + 0.20 = 1.05$
   No, this is not a probability distribution because the probabilities total to more than 1.

4. (a) $\Sigma P(x) = 0.07 + 0.44 + 0.24 + 0.4 + 0.11 = 1.00$
   Yes, this is a valid probability distribution because the events are distinct and the probabilities total to 1.

   (b) Age of Promotion Sensitive Shoppers



   (c) $\mu = \Sigma xP(x)$
   $= 23(0.07) + 34(0.44) + 45(0.24) + 56(0.14) + 67(0.11)$
   $= 42.58$

(d) $\sigma = \sqrt{\Sigma(x-\mu)^2 P(x)}$

$= \sqrt{(-19.58)^2(0.07)+(-8.58)^2(0.44)+(2.42)^2(0.24)+(13.42)^2(0.14)+(24.42)^2(0.11)}$

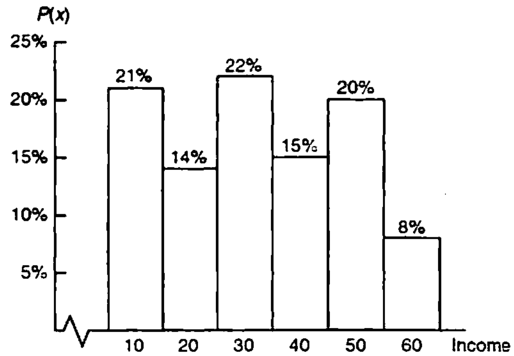
$= \sqrt{151.44}$

≈ 12.31

5. (a) $\Sigma P(x) = 0.21+0.14+0.22+0.15+0.20+0.08 = 1.00$

Yes, this is a valid probability distribution because the events are distinct and the probabilities total to 1.

(b) Income Distribution ($1000)



(c) $\mu = \Sigma xP(x)$

$= 10(0.21)+20(0.14)+30(0.22)+40(0.15)+50(0.20)+60(0.08)$

$= 32.3$

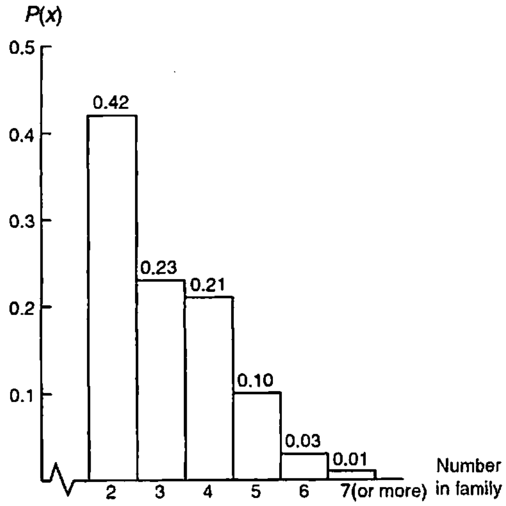
(d) $\sigma = \sqrt{\Sigma(x-\mu)^2 P(x)}$

$= \sqrt{(-22.3)^2(0.21)+(-12.3)^2(0.14)+(-2.3)^2(0.22)+(7.7)^2(0.15)+(17.7)^2(0.20)+(27.7)^2(0.08)}$

$= \sqrt{259.71}$

≈ 16.12

6. (a) Sizes of Families



(b) $P(2) = 0.42$

(c)
$$\begin{aligned} P(\text{more than } 3) &= P(4, 5, 6, 7 \text{ or more}) \\ &= P(4) + P(5) + P(6) + P(7 \text{ or more}) \\ &= 0.21 + 0.10 + 0.03 + 0.01 \\ &= 0.35 \end{aligned}$$

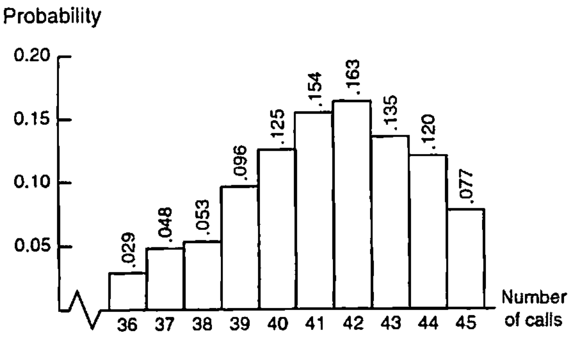
(d)
$$\begin{aligned} \mu &= \Sigma x P(x) \\ &= 2(0.42) + 3(0.23) + 4(0.21) + 5(0.10) + 6(0.03) + 7(0.01) \\ &= 3.12 \end{aligned}$$

(e)
$$\begin{aligned} \sigma &= \sqrt{\Sigma (x-\mu)^2 P(x)} \\ &= \sqrt{(-1.12)^2 (0.42) + (-0.12)^2 (0.23) + (0.88)^2 (0.21) + (1.88)^2 (0.10) + (2.88)^2 (0.03) + (3.88)^2 (0.01)} \\ &= \sqrt{1.4456} \\ &\approx 1.20 \end{aligned}$$

7. (a)

x	f	Relative Frequency	$P(x)$
36	6	6/208	0.029
37	10	10/208	0.048
38	11	11/208	0.053
39	20	20/208	0.096
40	26	26/208	0.125
41	32	32/208	0.154
42	34	34/208	0.163
43	28	28/208	0.135
44	25	25/208	0.120
45	16	16/208	0.077

(b)



(c) $P(39,40,41,42,43)=P(39)+P(40)+P(41)+P(42)+P(43)$
$=0.096+0.125+0.154+0.163+0.135$
$=0.673$

(d) $P(36,37.38,39,40)=P(36)+P(37)+P(38)+P(39)+P(40)$
$=0.029+0.048+0.053+0.096+0.125$
$=0.351$

(e) $\mu=\Sigma xP(x)$
$=36(0.029)+37(0.048)+38(0.053)+39(0.096)+40(0.125)+41(0.154)$
$+42(0.163)+43(0.135)+44(0.120)+45(0.077)$
$=41.288$

(f) $\sigma=\sqrt{\Sigma(x-\mu)^2 P(x)}$

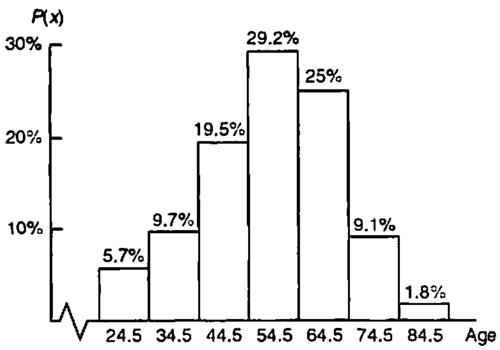
$$=\sqrt{\begin{array}{l}(-5.288)^2(0.029)+(-4.288)^2(0.048)+(-3.288)^2(0.053)+(-2.288)^2(0.096)+(-1.288)^2(0.125)\\+(-0.288)^2(0.154)+(0.712)^2(0.163)+(1.712)^2(0.135)+(2.712)^2(0.120)+(3.712)^2(0.077)\end{array}}$$

$=\sqrt{5.411}$
≈ 2.326

8. (a) $\Sigma P(x)=0.057+0.097+0.195+0.292+0.250+0.091+0.018$
$=1.000$

Yes, this is a valid probability distribution because the outcomes are distinct and the probabilities total to 1.

(b) Age of Nurses



(c) $P(60 \text{ years of age or older}) = P(64.5) + P(74.5) + P(84.5)$

$$= 0.250 + 0.091 + 0.018$$
$$= 0.359$$

The probability is 35.9%.

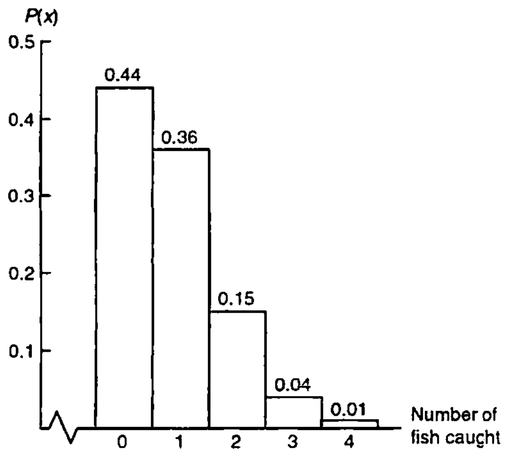
(d) $\mu = \Sigma xP(x)$

$$= 24.5(0.057) + 34.5(0.097) + 44.5(0.195) + 54.5(0.292) + 64.5(0.250) + 74.5(0.091) + 84.5(0.018)$$
$$= 53.76$$

(e) $\sigma = \sqrt{\Sigma(x-\mu)^2 P(x)}$

$$= \sqrt{(-29.26)^2(0.057) + (-19.26)^2(0.097) + (-9.26)^2(0.195) + (0.74)^2(0.292) + (10.74)^2(0.250) + (20.74)^2(0.091) + (30.74)^2(0.018)}$$
$$= \sqrt{186.65}$$
$$\approx 13.66$$

9. (a) Number of Fish Caught in a 6-Hour Period at Pyramid Lake, Nevada



(b) $P(1 \text{ or more}) = 1 - P(0)$

$$= 1 - 0.44$$
$$= 0.56$$

(c) $P(2 \text{ or more}) = P(2) + P(3) + P(4 \text{ or more})$

$$= 0.15 + 0.04 + 0.01$$
$$= 0.20$$

(d) $\mu = \Sigma xP(x)$

$$= 0(0.44) + 1(0.36) + 2(0.15) + 3(0.04) + 4(0.01)$$
$$= 0.82$$

(e) $\sigma = \sqrt{\Sigma(x-\mu)^2 P(x)}$

$= \sqrt{(-0.82)^2(0.44)+(0.18)^2(0.36)+(1.18)^2(0.15)+(2.18)^2(0.04)+(3.18)^2(0.01)}$

$= \sqrt{0.8076}$

≈ 0.899

10. $\Sigma P(x) \neq 1.000$ due to rounding

(a) $P(1 \text{ or more}) = 1 - P(0)$

$= 1 - 0.237$

$= 0.763$

This is the complement of the probability that none of the parolees will be repeat offenders.

(b) $P(2 \text{ or more}) = P(2)+P(3)+P(4)+P(5)$

$= 0.264+0.088+0.015+0.001$

$= 0.368$

(c) $P(4 \text{ or more}) = P(4)+P(5)$

$= 0.015+0.001$

$= 0.016$

(d) $\mu = \Sigma xP(x)$

$= 0(0.237)+1(0.396)+2(0.264)+3(0.088)+4(0.015)+5(0.001)$

$= 1.253$

(e) $\sigma = \sqrt{\Sigma(x-\mu)^2 P(x)}$

$= \sqrt{\begin{aligned}&(-1.253)^2(0.237)+(-0.253)^2(0.396)+(0.747)^2(0.264)+(1.747)^2(0.088)\\&+(2.747)^2(0.015)+(3.747)^2(0.001)\end{aligned}}$

$= \sqrt{0.941}$

≈ 0.97

11. (a) $P(\text{win}) = \dfrac{15}{719} \approx 0.021$

$P(\text{not win}) = \dfrac{719-15}{719} = \dfrac{704}{719} \approx 0.979$

(b) Expected earnings $=$ (value of dinner)(probability of winning)

$= \$35\left(\dfrac{15}{719}\right)$

$\approx \$0.73$

Lisa's expected earnings are \$0.73.

contribution $= \$15 - \$0.73 = \$14.27$

Lisa effectively contributed \$14.27 to the hiking club.

12. (a) $P(\text{win}) = \dfrac{6}{2852} \approx 0.0021$

$$P(\text{not win}) = \frac{2852-6}{2852} = \frac{2846}{2852} \approx 0.9979$$

(b) Expected earnings = (value of cruise)(probability of winning)

$$\approx \$2000(0.0021)$$
$$\approx \$4.20$$

Kevin spent 6($5) = $30 for the tickets. His expected earnings are less than the amount he paid.

contribution = $30 − $4.20 = $25.80

Kevin effectively contributed $25.80 to the homeless center.

13. (a) $P(60 \text{ years}) = 0.01191$

Expected loss = $50,000(0.01191) = $595.50

The expected loss for Big Rock Insurance is $595.50.

(b)

Probability	Expected Loss
$P(61) = 0.01292$	$50.000(0.01292) = $646
$P(62) = 0.01396$	$50.000(0.01396) = $698
$P(63) = 0.01503$	$50.000(0.01503) = $751.50
$P(64) = 0.01613$	$50.000(0.01613) = $806.50

Expected loss = $595.50 + $646 + $698 + $751.50 + $806.50
= $3497.50

The total expected loss is $3497.50.

(c) $3497.50 + $700 = $4197.50

They should charge $4197.50.

(d) $5000 − $3497.50 = $1502.50

They can expect to make $1502.50.

Comment: losses are usually denoted by negative numbers such as −$50,000.

14. (a) $P(60 \text{ years}) = 0.00756$

Expected loss = $50,000(0.00756) = $378

The expected loss for Big Rock Insurance is $378.

(b)

Probability	Expected Loss
$P(61) = 0.00825$	$50,000(0.00825) = $412.50
$P(62) = 0.00896$	$50.000(0.00896) = $448
$P(63) = 0.00965$	$50.000(0.00965) = $482.50
$P(64) = 0.01035$	$50.000(0.01035) = $517.50

expected loss = $378 + $412.50 + $448 + $482.50 + $517.50
= $2238.50

The total expected loss is $2238.50.

(c) $2238.50 + $700 = $2938.50
They should charge $2938.50.

(d) $5000 − $2238.50 = $2761.50
They can expect to make $2761.50.

15. (a) $W = x_1 - x_2$; $a = 1$, $b = -1$

$\mu_W = \mu_1 - \mu_2 = 115 - 100 = 15$

$\sigma_W^2 = 1^2\sigma_1^2 + (-1)^2\sigma_2^2 = 12^2 + 8^2 = 208$

$\sigma_W = \sqrt{\sigma_W^2} = \sqrt{208} \approx 14.4$

(b) $W = 0.5x_1 + 0.5x_2$; $a = 0.5$, $b = 0.5$

$\mu_W = 0.5\mu_1 + 0.5\mu_2 = 0.5(115) + 0.5(100) = 107.5$

$\sigma_W^2 = (0.5)^2\sigma_1^2 + (0.5)^2\sigma_2^2 = 0.25(12)^2 + 0.25(8)^2 = 52$

$\sigma_W = \sqrt{\sigma_W^2} = \sqrt{52} \approx 7.2$

(c) $L = 0.8x_1 - 2$; $a = -2$, $b = 0.8$

$\mu_L = -2 + 0.8\mu_1 = -2 + 0.8(115) = 90$

$\sigma_L^2 = (0.8)^2\sigma_1^2 = 0.64(12)^2 = 92.16$

$\sigma_L = \sqrt{\sigma_L^2} = \sqrt{92.16} = 9.6$

(d) $L = 0.95x_2 - 5$; $a = -5$, $b = 0.95$

$\mu_L = -5 + 0.95\mu_2 = -5 + 0.95(100) = 90$

$\sigma_L^2 = (0.95)^2\sigma_2^2 = 0.9025(8)^2 = 57.76$

$\sigma_L = \sqrt{\sigma_L^2} = \sqrt{57.76} = 7.6$

16. (a) $W = x_1 + x_2$; $a = 1$, $b = 1$

$\mu_W = \mu_1 + \mu_2 = 28.1 + 90.5 = 118.6$ minutes

$\sigma_W^2 = \sigma_1^2 + \sigma_2^2 = (8.2)^2 + (15.2)^2 = 298.28$

$\sigma_W = \sqrt{\sigma_W^2} = \sqrt{298.28} \approx 17.27$ minutes

(b) $W = 1.50x_1 + 2.75x_2$; $a = 1.50$, $b = 2.75$

$\mu_W = 1.50\mu_1 + 2.75\mu_2 = 1.50(28.1) + 2.75(90.5) \approx \$291.03$

$\sigma_W^2 = (1.50)^2\sigma_1^2 + (2.75)^2\sigma_2^2 = 2.25(8.2)^2 + 7.5625(15.2)^2 = 1898.53$

$\sigma_W = \sqrt{\sigma_W^2} = \sqrt{1898.53} \approx \$43.57$

(c) $L = 1.5x_1 + 50$; $a = 50$, $b = 1.5$

$\mu_L = 50 + 1.5\mu_1 = 50 + 1.5(28.1) = \$92.15$

$\sigma_L^2 = (1.5)^2\sigma_1^2 = 2.25(8.2)^2 = 151.29$

$\sigma_L = \sqrt{\sigma_L^2} = \sqrt{151.29} = \$12.30$

17. (a) $W = 0.5x_1 + 0.5x_2$; $a = 0.5$, $b = 0.5$

$\mu_W = 0.5\mu_1 + 0.5\mu_2 = 0.5(50.2) + 0.5(50.2) = 50.2$

$\sigma_W^2 = 0.5^2\sigma_1^2 + 0.5^2\sigma_2^2 = 0.5^2(11.5)^2 + 0.5^2(11.5)^2 = 66.125$

$\sigma_W = \sqrt{\sigma_W^2} = \sqrt{66.125} \approx 8.13$

(b) Single policy (x_1): $\mu_1 = 50.2$
Two policies (W): $\mu_W \approx 50.2$
The means are the same.

(c) Single policy (x_1): $\sigma_1 = 11.5$
Two policies (W): $\sigma_W \approx 8.13$
The standard deviation for two policies is smaller.

(d) Yes, the risk decreases by a factor of $\frac{1}{\sqrt{n}}$ because $\sigma_W = \frac{1}{\sqrt{n}}\sigma$.

Section 5.2

1. A trial is one flip of a fair quarter. Success = head. Failure = tail.
$n = 3$, $p = 0.5$, $q = 1 - 0.5 = 0.5$

(a) $P(3) = C_{3,3}(0.5)^3(0.5)^{3-3}$
$= 1(0.5)^3(0.5)^0$
$= 0.125$

To find this value in Table 3 of Appendix II, use the group in which $n = 3$, the column headed by $p = 0.5$, and the row headed by $r = 3$.

(b) $P(2) = C_{3,2}(0.5)^2(0.5)^{3-2}$
$= 3(0.5)^2(0.5)^1$
$= 0.375$

To find this value in Table 3 of Appendix II, use the group in which $n = 3$, the column headed by $p = 0.5$, and the row headed by $r = 2$.

(c) $P(r \geq 2) = P(2) + P(3)$
$= 0.125 + 0.375$
$= 0.5$

(d) The probability of getting exactly three tails is the same as getting exactly zero heads.

$P(0) = C_{3,0}(0.5)^0(0.5)^{3-0}$
$= 1(0.5)^0(0.5)^3$
$= 0.125$

To find this value in Table 3 of Appendix II, use the group in which $n = 3$, the column headed by $p = 0.5$, and the row headed by $r = 0$.

The results from Table 3 of Appendix II are the same.

In the problems that follow, there are often other ways the solve the problems than those shown. As long as you get the same answer, your method is probably correct.

2. A trial is answering a question on the quiz. Success = correct answer. Failure = incorrect answer.

$n = 10,\ p = \frac{1}{5} = 0.2,\ q = 1 - 0.2 = 0.8$

(a) $P(10) = C_{10,10}(0.2)^{10}(0.8)^{10-10}$
$= 1(0.2)^{10}(0.8)^{0}$
$= 0.000$ (to three digits)

(b) 10 incorrect is the same as 0 correct.
$P(0) = C_{10,0}(0.2)^{0}(0.8)^{10-0}$
$= 1(0.2)^{0}(0.8)^{10}$
$= 0.107$

(c) First method:
$P(r \ge 1) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$
$= 0.268 + 0.302 + 0.201 + 0.088 + 0.026 + 0.006 + 0.001 + 0.000 + 0.000 + 0.000$
$= 0.892$

Second method:
$P(r \ge 1) = 1 - P(0)$
$= 1 - 0.107$
$= 0.893$

The two results should be equal, but because of rounding error, they differ slightly.

(d) $P(r \ge 5) = P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$
$= 0.026 + 0.006 + 0.001 + 0.000 + 0.000 + 0.000$
$= 0.033$

3. (a) A trial is a man's response to the question, "Would you marry the same woman again?"
Success = a positive response. Failure = a negative response.
$n = 10,\ p = 0.80,\ q = 1 - 0.80 = 0.20$
Using values in Table 3 of Appendix II:

$P(r \ge 7) = P(7) + P(8) + P(9) + P(10)$
$= 0.201 + 0.302 + 0.268 + 0.107$
$= 0.878$

$P(r \text{ is less than half of } 10) = P(r < 5)$
$= P(0) + P(1) + P(2) + P(3) + P(4)$
$= 0.000 + 0.000 + 0.000 + 0.001 + 0.006$
$= 0.007$

(b) A trial is a woman's response to the question. "Would you marry the same man again?"
Success = a positive response. Failure = a negative response.
$n = 10,\ p = 0.5,\ q = 1 - 0.5 = 0.5$
Using values in Table 3 of Appendix II:

$$\begin{aligned}P(r \geq 7) &= P(7) + P(8) + P(9) + P(10)\\ &= 0.117 + 0.044 + 0.010 + 0.001\\ &= 0.172\end{aligned}$$

$$\begin{aligned}P(r < 5) &= P(0) + P(1) + P(2) + P(3) + P(4)\\ &= 0.001 + 0.010 + 0.044 + 0.117 + 0.205\\ &= 0.377\end{aligned}$$

4. A trial is a one-time fling. Success = has done a one-time fling. Failure = has not done a one-time fling.
$n = 7,\ p = 0.10,\ q = 1 - 0.10 = 0.90$

(a) $$\begin{aligned}P(0) &= C_{7,0}(0.10)^0(0.90)^{7-0}\\ &= 1(0.10)^0(0.90)^7\\ &= 0.478\end{aligned}$$

(b) $$\begin{aligned}P(r \geq 1) &= 1 - P(0)\\ &= 1 - 0.478\\ &= 0.522\end{aligned}$$

(c) $$\begin{aligned}P(r \leq 2) &= P(0) + P(1) + P(2)\\ &= 0.478 + 0.372 + 0.124\\ &= 0.974\end{aligned}$$

5. A trial consists of a woman's response regarding her mother-in-law. Success = dislike. Failure = like.
$n = 6,\ p = 0.90,\ q = 1 - 0.90 = 0.10$

(a) $$\begin{aligned}P(6) &= C_{6,6}(0.90)^6(0.10)^{6-6}\\ &= 1(0.90)^6(0.10)^0\\ &= 0.531\end{aligned}$$

(b) $$\begin{aligned}P(0) &= C_{6,0}(0.90)^0(0.10)^{6-0}\\ &= 1(0.90)^0(0.10)^6\\ &\approx 0.000 \text{ (to 3 digits)}\end{aligned}$$

(c) $$\begin{aligned}P(r \geq 4) &= P(4) + P(5) + P(6)\\ &= 0.098 + 0.354 + 0.531\\ &= 0.983\end{aligned}$$

(d) $P(r \le 3) = 1 - P(r \ge 4)$
$\approx 1 - 0.983$
$= 0.017$

From the table:
$P(r \le 3) = P(0) + P(1) + P(2) + P(3)$
$= 0.000 + 0.000 + 0.001 + 0.015$
$= 0.016$

6. A trial is how a businessman wears a tie. Success = too tight. Failure = not too tight.
$n = 20,\ p = 0.10,\ q = 1 - 0.10 = 0.90$

(a) $P(r \ge 1) = 1 - P(r = 0)$
$= 1 - 0.122$
$= 0.878$

(b) $P(r > 2) = 1 - P(r \le 2)$
$= 1 - [P(r = 0) + P(r = 1) + P(r = 2)]$
$= 1 - P(r = 0) - P(r = 1) - P(r = 2)$
$= [1 - P(r = 0)] - P(r = 1) - P(r = 2)$
$= P(r \ge 1) - P(r = 1) - P(r = 2)$
$= 0.878 - 0.270 - 0.285$ using (a)
$= 0.323$

(c) $P(r = 0) = 0.122$

(d) At least 18 are not too tight is the same as at most 2 are too tight. (To see this, note that at least 18 failures is the same as 18 or 19 or 20 failures, which is 2, 1, or 0 successes. i.e., at most 2 successes.)
$P(r \le 2) = 1 - P(r > 2)$
$= 1 - 0.323$ using (b)
$= 0.677$

7. A trial consists of taking a polygraph examination. Success = pass. Failure = fail.
$n = 9,\ p = 0.85,\ q = 1 - 0.85 = 0.15$

(a) $P(9) = 0.232$

(b) $P(r \ge 5) = P(5) + P(6) + P(7) + P(8) + P(9)$
$= 0.028 + 0.107 + 0.260 + 0.368 + 0.232$
$= 0.995$

(c) $P(r \le 4) = 1 - P(r \ge 5)$
$= 1 - 0.995$
$= 0.005$

From the table:
$P(r \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$
$= 0.000 + 0.000 + 0.000 + 0.001 + 0.005$
$= 0.006$

The two results should be equal, but because of rounding error, they differ slightly.

(d) All students fail is the same as no students pass.
$P(0) = 0.000$ (to 3 digits)

8. A trial is asking a person if he/she has a cellular phone. Success = has a cellular phone. Failure = does not have a cellular phone.
$n = 11,\ p = 0.35,\ q = 1 - p = 0.65$

(a) $P(11) = 0.000$ (to 3 digits)

(b) $P(r > 4) = P(r \ge 5)$
$= P(5) + P(6) + P(7) + P(8) + P(9) + P(10) + P(11)$
$= 0.183 + 0.099 + 0.038 + 0.010 + 0.002 + 0.000 + 0.000$
$= 0.332$

(c) Fewer than 5 do not have a cellular phone is the same as more than 6 have a cellular phone.
$P(r > 6) = P(7) + P(8) + P(9) + P(10) + P(11)$
$= 0.038 + 0.010 + 0.002 + 0.000 + 0.000$
$= 0.050$

(d) More than 7 do not have a cellular phone is the same as fewer than 4 have a cellular phone.
$P(r < 4) = P(0) + P(1) + P(2) + P(3)$
$= 0.009 + 0.052 + 0.140 + 0.225$
$= 0.426$

9. A trial consists of checking the gross receipts of the Green Parrot Italian Restaurant for one business day. Success = gross is over \$2200. Failure = gross is at or below \$2200.
$p = 0.85,\ q = 1 - 0.85 = 0.15$

(a) $n = 7$
$P(r \ge 5) = P(5) + P(6) + P(7)$
$= 0.210 + 0.396 + 0.321$
$= 0.927$

(b) $n = 10$
$P(r \ge 5) = P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$
$= 0.008 + 0.040 + 0.130 + 0.276 + 0.347 + 0.197$
$= 0.998$

(c) $n = 5$

$$P(r < 3) = P(0) + P(1) + P(2)$$
$$= 0.000 + 0.002 + 0.024$$
$$= 0.026$$

(d) $n = 10$

$$P(r < 7) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$
$$= 0.000 + 0.000 + 0.000 + 0.000 + 0.001 + 0.008 + 0.040$$
$$= 0.049$$

(e) $n = 7$

$$P(r < 3) = P(0) + P(1) + P(2)$$
$$= 0.000 + 0.000 + 0.001$$
$$= 0.001$$

Yes. If p were really 0.85, then the event of a 7-day period with gross income exceeding \$2200 fewer than 3 days would be very rare. If it happened again, we would suspect that $p = 0.85$ is too high.

10. A trial consists of checking the gross receipts of the store for one business day. Success = gross over \$850. Failure = gross is at or below \$850. $p = 0.6$, $q = 1 - p = 0.4$

(a) $n = 5$

$$P(r \geq 3) = P(3) + P(4) + P(5)$$
$$= 0.346 + 0.259 + 0.078$$
$$= 0.683$$

(b) $n = 10$

$$P(r \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10)$$
$$= 0.251 + 0.215 + 0.121 + 0.040 + 0.006$$
$$= 0.633$$

(c) $n = 10$

$$P(r < 5) = P(0) + P(1) + P(2) + P(3) + P(4)$$
$$= 0.000 + 0.002 + 0.011 + 0.042 + 0.111$$
$$= 0.166$$

(d) $n = 20$

$$P(r < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$
$$= 0.000 + 0.000 + 0.000 + 0.000 + 0.000 + 0.001$$
$$= 0.001$$

Yes. If p were really 0.60, then the event of a 20-day period with gross income exceeding \$850 fewer than 6 days would be very rare. If it happened again, we would suspect that $p = 0.60$ is too high.

(e) $n = 20$

$$P(r > 17) = P(18) + P(19) + P(20)$$
$$= 0.003 + 0.000 + 0.000$$
$$= 0.003$$

Yes. If p were really 0.60, then the event of a 20-day period with gross income exceeding \$850 more than 17 days would be very rare. If it happened again, we would suspect that $p = 0.60$ is too low.

11. A trial is catching and releasing a pike. Success = pike dies. Failure = pike lives.
$n = 16,\ p = 0.05,\ q = 1 - 0.05 = 0.95$

(a) $P(0) = 0.440$

(b) $P(r < 3) = P(0) + P(1) + P(2)$
$= 0.440 + 0.371 + 0.146$
$= 0.957$

(c) All of the fish lived is the same as none of the fish died.
$P(0) = 0.440$

(d) More than 14 fish lived is the same as less than 2 fish died.
$P(r < 2) = P(0) + P(1)$
$= 0.440 + 0.371$
$= 0.811$

12. A trial is tasting coffee. Success = choose Tasty Bean. Failure = do not choose Tasty Bean.
$n = 4,\ p = \frac{1}{5} = 0.2,\ q = 1 - 0.2 = 0.8$

(a) $P(4) = 0.002$

(b) $P(0) = 0.410$

(c) $P(r \ge 3) = P(3) + P(4)$
$= 0.026 + 0.002$
$= 0.028$

13. (a) A trial consists of using the Meyers-Briggs instrument to determine if a person in marketing is an extrovert. Success = extrovert. Failure = not extrovert.
$n = 15,\ p = 0.75,\ q = 1 - 0.75 = 0.25$

$P(r \ge 10) = P(10) + P(11) + P(12) + P(13) + P(14) + P(15)$
$= 0.165 + 0.225 + 0.225 + 0.156 + 0.067 + 0.013$
$= 0.851$

$P(r \ge 5) = P(5) + P(6) + P(7) + P(8) + P(9) + P(r \ge 10)$
$= 0.001 + 0.003 + 0.013 + 0.039 + 0.092 + 0.851$
$= 0.999$

$P(15) = 0.013$

(b) A trial consists of using the Meyers-Briggs instrument to determine if a computer programmer is an introvert. Success = introvert. Failure = not introvert.

$n = 5$, $p = 0.60$, $q = 1 - 0.60 = 0.40$

$$P(0) = 0.010$$

$$\begin{aligned} P(r \geq 3) &= P(3) + P(4) + P(5) \\ &= 0.346 + 0.259 + 0.078 \\ &= 0.683 \end{aligned}$$

$$P(5) = 0.078$$

14. A trial consists of a man's response regarding welcoming a woman taking the initiative in asking for a date. Success = yes. Failure = no.

$n = 20$, $p = 0.70$, $q = 1 - 0.70 = 0.30$

(a) $$\begin{aligned} P(r \geq 18) &= P(18) + P(19) + P(20) \\ &= 0.028 + 0.007 + 0.001 \\ &= 0.036 \end{aligned}$$

(b) $$\begin{aligned} P(r < 3) &= P(0) + P(1) + P(2) \\ &= 0.000 + 0.000 + 0.000 \\ &= 0.000 \text{ (to 3 digits)} \end{aligned}$$

(c) $P(0) = 0.000$ (to 3 digits)

(d) At least 5 say no is the same as at most 15 say yes.

$$\begin{aligned} P(r \leq 15) &= 1 - P(r \geq 16) \\ &= 1 - [P(16) + P(17) + P(18) + P(19) + P(20)] \\ &= 1 - (0.130 + 0.072 + 0.028 + 0.007 + 0.001) \\ &= 1 - 0.238 \\ &= 0.762 \end{aligned}$$

15. A trial is checking the development of hypertension in patients with diabetes. Success = yes. Failure = no.

$n = 10$, $p = 0.40$, $q = 1 - 0.40 = 0.60$

(a) $P(0) = 0.006$

(b) $$\begin{aligned} P(r < 5) &= P(0) + P(1) + P(2) + P(3) + P(4) \\ &= 0.006 + 0.040 + 0.121 + 0.215 + 0.251 \\ &= 0.633 \end{aligned}$$

A trial is checking the development of an eye disease in patients with diabetes. Success = yes. Failure = no.

$n = 10$, $p = 0.30$, $q = 1 - 0.30 = 0.70$

(c) $$\begin{aligned} P(r \leq 2) &= P(0) + P(1) + P(2) \\ &= 0.028 + 0.121 + 0.233 \\ &= 0.382 \end{aligned}$$

(d) At least 6 will never develop a related eye disease is the same as at most 4 will never develop a related eye disease.

$$P(r \le 4) = P(0) + P(1) + P(2) + P(3) + P(4)$$
$$= 0.028 + 0.121 + 0.233 + 0.267 + 0.200$$
$$= 0.849$$

16. A trial consists of the response of adults regarding their concern that employers are monitoring phone calls. Success = yes. Failure = no.

$p = 0.37$, $q = 1 - 0.37 = 0.63$

(a) $n = 5$

$$P(0) = C_{5,0}(0.37)^0(0.63)^{5-0}$$
$$= 1(0.37)^0(0.63)^5$$
$$\approx 0.099$$

(b) $n = 5$

$$P(5) = C_{5,5}(0.37)^5(0.63)^{5-5}$$
$$= 1(0.37)^5(0.63)^0$$
$$\approx 0.007$$

(c) $n = 5$

$$P(3) = C_{5,3}(0.37)^3(0.63)^{5-3}$$
$$= 10(0.37)^3(0.63)^2$$
$$\approx 0.201$$

17. A trial consists of the response of adults regarding their concern that Social Security numbers are used for general identification. Success = concerned that SS numbers are being used for identification. Failure = not concerned that SS numbers are being used for identification.

$n = 8$, $p = 0.53$, $q = 1 - 0.53 = 0.47$

(a) $P(r \le 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$$= 0.002381 + 0.021481 + 0.084781 + 0.191208 + 0.269521 + 0.243143$$
$$= 0.812515$$

$P(r \le 5) = 0.81251$ from the cumulative probability is the same, truncated to 5 digits.

(b) $P(r > 5) = P(6) + P(7) + P(8)$

$$= 0.137091 + 0.044169 + 0.006726$$
$$= 0.187486$$

$$P(r > 5) = 1 - P(r \le 5)$$
$$= 1 - 0.81251$$
$$= 0.18749$$

Yes, this is the same result rounded to 5 digits.

18. A trial consists of determining the sex of a wolf. Success = male. Failure = female.

(a) $n = 12$. $p = 0.55$, $q = 0.45$

$$\begin{aligned} P(r \geq 6) &= P(6)+P(7)+P(8)+P(9)+P(10)+P(11)+P(12) \\ &= 0.212+0.223+0.170+0.092+0.034+0.008+0.001 \\ &= 0.740 \end{aligned}$$

Six or more female is the same as six or fewer male.

$$\begin{aligned} P(r \leq 6) &= P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6) \\ &= 0.000+0.001+0.007+0.028+0.076+0.149+0.212 \\ &= 0.473 \end{aligned}$$

Fewer than 4 female is the same as more than 8 male.

$$\begin{aligned} P(r > 8) &= P(9)+P(10)+P(11)+P(12) \\ &= 0.092+0.034+0.008+0.001 \\ &= 0.135 \end{aligned}$$

(b) $n = 12$, $p = 0.70$, $q = 0.30$

$$\begin{aligned} P(r \geq 6) &= P(6)+P(7)+P(8)+P(9)+P(10)+P(11)+P(12) \\ &= 0.079+0.158+0.231+0.240+0.168+0.071+0.014 \\ &= 0.961 \end{aligned}$$

$$\begin{aligned} P(r \leq 6) &= P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6) \\ &= 0.000+0.000+0.000+0.001+0.008+0.029+0.079 \\ &= 0.117 \end{aligned}$$

$$\begin{aligned} P(r > 8) &= P(9)+P(10)+P(11)+P(12) \\ &= 0.240+0.168+0.071+0.014 \\ &= 0.493 \end{aligned}$$

19. A trial consists of determining the kind of stone in a chipped stone tool.

(a) $n = 11$; Success = obsidian. Failure = not obsidian.

$p = 0.15$, $q = 1-0.15 = 0.85$

$$\begin{aligned} P(r \geq 3) &= 1-P(r \leq 2) \\ &= 1-[P(0)+P(1)+P(2)] \\ &= 1-(0.167+0.325+0.287) \\ &= 1-0.779 \\ &= 0.221 \end{aligned}$$

(b) $n = 5$; Success = basalt. Failure = not basalt.

$p = 0.55$, $q = 1-0.55 = 0.45$

$$\begin{aligned} P(r \geq 2) &= P(2)+P(3)+P(4)+P(5) \\ &= 0.276+0.337+0.206+0.050 \\ &= 0.869 \end{aligned}$$

(c) $n = 10$: Success = neither obsidian nor basalt. Failure = either obsidian or basalt. The two outcomes, tool is obsidian or tool is basalt, are mutually exclusive. Therefore, P(obsidian *or* basalt) = 0.55 + 0.15 = 0.70. P(neither obsidian nor basalt) = 1 − 0.70 = 0.30. Therefore, $p = 0.30$. $q = 1 - 0.30 = 0.70$.

$$\begin{aligned}P(r \geq 4) &= P(4)+P(5)+P(6)+P(7)+P(8)+P(9)+P(10)\\ &= 0.200+0.103+0.037+0.009+0.001+0.000+0.000\\ &= 0.350\end{aligned}$$

Note that the phrase "neither obsidian nor basalt" is the English translation of the math phrase "not (obsidian or basalt)" and therefore, it describes the complement of the event "obsidian or basalt." Using $P(\bar{A}) = 1 - P(A)$, we get $p = P(\text{Success}) = 0.30$.

20. A trial consists of an office visit.

(a) Success = visitor age is under 15 years old.
Failure = visitor age is 15 years old or older.

$n = 8,\ p = 0.20,\ q = 1 - 0.20 = 0.80$

$$\begin{aligned}P(r \geq 4) &= P(4)+P(5)+P(6)+P(7)+P(8)\\ &= 0.046+0.009+0.001+0.000+0.000\\ &= 0.056\end{aligned}$$

(b) Success = visitor age is 65 years old or older.
Failure = visitor age is under 65 years old.

$n = 8,\ p = 0.25,\ q = 1 - 0.25 = 0.75$

$$\begin{aligned}P(2 \leq r \leq 5) &= P(2)+P(3)+P(4)+P(5)\\ &= 0.311+0.208+0.087+0.023\\ &= 0.629\end{aligned}$$

(c) Success = visitor age is 45 years old or older.
Failure = visitor age is less than 65 years old.

$n = 8,\ p = 0.20+0.25 = 0.45,\ q = 1 - 0.45 = 0.55$

$$\begin{aligned}P(2 \leq r \leq 5) &= P(2)+P(3)+P(4)+P(5)\\ &= 0.157+0.257+0.263+0.172\\ &= 0.849\end{aligned}$$

(d) Success = visitor age is under 25 years old.
Failure = visitor age is 25 years old or older.

$n = 8,\ p = 0.20+0.10 = 0.30,\ q = 1 - 0.30 = 0.70$

$P(8) = 0.000$ (to 3 digits)

(e) Success = visitor age is 15 years old or older.
Failure = visitor age is under 15 years old.

$n = 8,\ p = 0.10+0.25+0.20+0.25 = 0.80,\ q = 0.20$

$P(8) = 0.168$

21. (a) $p = 0.30,\ P(3) = 0.132$
$p = 0.70,\ P(2) = 0.132$
They are the same.

(b) $p = 0.30.\ P(r \geq 3) = 0.132 + 0.028 + 0.002 = 0.162$
$p = 0.70.\ P(r \leq 2) = 0.002 + 0.028 + 0.132 = 0.162$
They are the same.

(c) $p = 0.30.\ P(4) = 0.028$
$p = 0.70,\ P(1) = 0.028$
$r = 1$

(d) The column headed by $p = 0.80$ is symmetrical with the one headed by $p = 0.20$.

22. $n = 3,\ p = 0.0228,\ q = 1 - p = 0.9772$

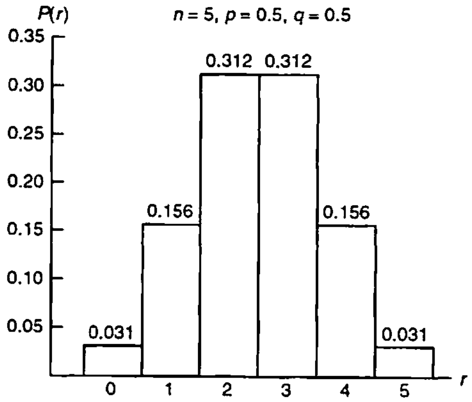
(a) $P(2) = C_{3,2}p^2q^{3-2} = 3(0.0228)^2(0.9772)^1 = 0.00152$

(b) $P(3) = C_{3,3}p^3q^{3-3} = 1(0.0228)^3(0.9772)^0 = 0.00001$

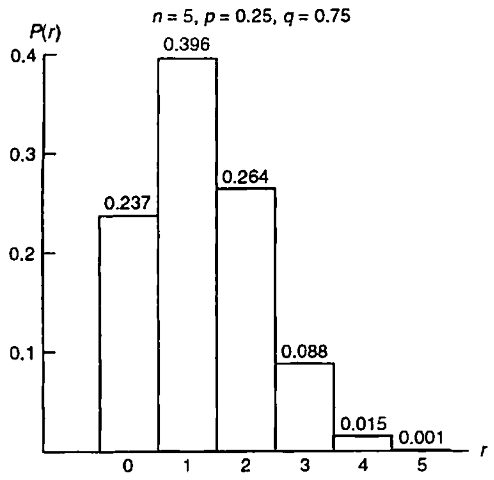
(c) $P(2 \text{ or } 3) = P(2) + P(3) = 0.00153$

Section 5.3

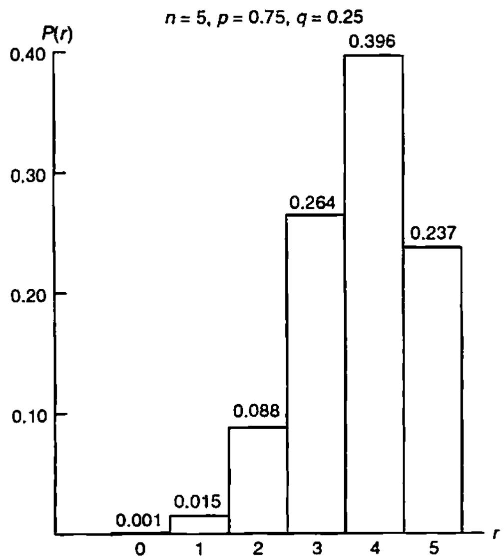
1. (a) Binomial Distribution
The distribution is symmetrical.



(b) Binomial Distribution
The distribution is skewed right.



(c) Binomial Distribution
The distribution is skewed left.



(d) The distributions are mirror images of one another.

(e) The distribution would be skewed left for $p = 0.73$ because the more likely number of successes are to the right of the middle.

2. **(a)** $p = 0.30$ goes with graph II since it is slightly skewed right.

(b) $p = 0.50$ goes with graph I since it is symmetrical.

(c) $p = 0.65$ goes with graph III since it is slightly skewed left.

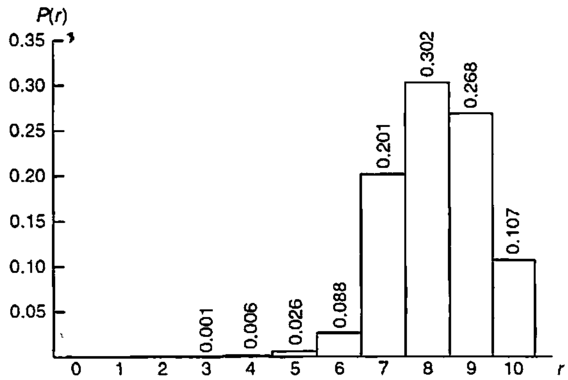
(d) $p = 0.90$ goes with graph IV since it is drastically skewed left.

(e) The graph is more symmetrical when p is close to 0.5. The graph is skewed left when p is close to 1 and skewed right when p is close to 0.

3. The probabilities can be taken directly from Table 3 in Appendix II.

(a) $n = 10,\ p = 0.80$

Households with Children Under 2 That Buy Film

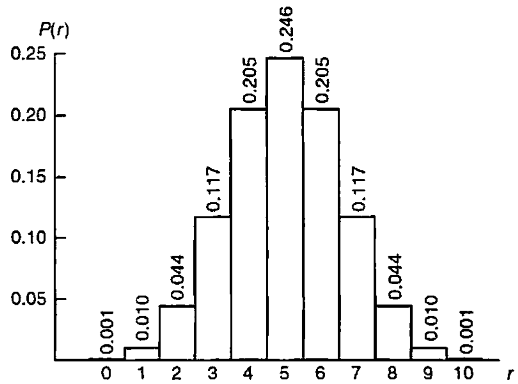


$\mu = np = 10(0.8) = 8$

$\sigma = \sqrt{npq} = \sqrt{10(0.8)(0.2)} \approx 1.26$

(b) $n = 10,\ p = 0.5$

Households with No Children Under 21 that Buy Film



$\mu = np = 10(0.5) = 5$

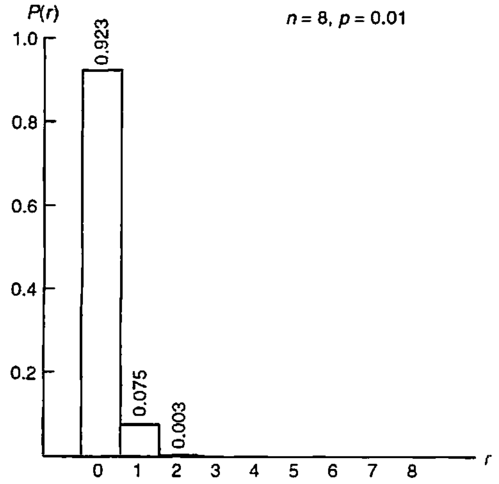
$\sigma = \sqrt{npq} = \sqrt{10(0.5)(0.5)} \approx 1.58$

(c) Yes; since the graph in part (a) is skewed left, it supports the claim that more households buy film that have children under 2 years than households that have no children under 21 years.

4. **(a)** $n = 8,\ p = 0.01$

The probabilities can be taken directly from Table 3 in Appendix II.

Binomial Distribution for Number of Defective Syringes



(b) $\mu = np = 8(0.01) = 0.08$

The expected number of defective syringes the inspector will find is 0.08.

(c) The batch will be accepted if less than 2 defectives are found.

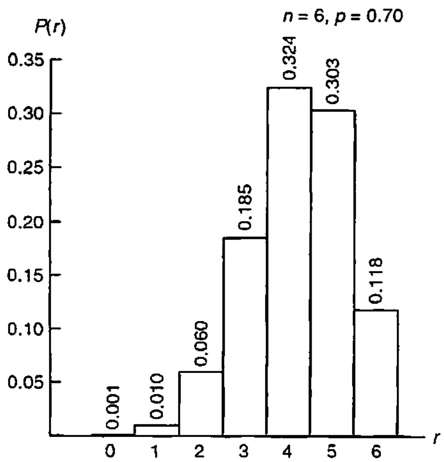
$$P(r<2) = P(0) + P(1)$$
$$= 0.923 + 0.075$$
$$= 0.998$$

(d) $\sigma = \sqrt{npq} = \sqrt{8(0.01)(0.99)} \approx 0.281$

5. **(a)** $n = 6,\ p = 0.70$

The probabilities can be taken directly from Table 3 in Appendix II.

Binomial Distribution for Number of Addresses Found



(b) $\mu = np = 6(0.70) = 4.2$

$\sigma = \sqrt{npq} = \sqrt{6(0.70)(0.30)} \approx 1.122$

The expected number of friends for whom addresses will be found is 4.2.

(c) Find n such that $P(r \geq 2) = 0.97$.

Try $n = 5$.

$$\begin{aligned} P(r \geq 2) &= P(2) + P(3) + P(4) + P(5) \\ &= 0.132 + 0.309 + 0.360 + 0.168 \\ &= 0.969 \\ &\approx 0.97 \end{aligned}$$

You would have to submit 5 names to be 97% sure that at least two addresses will be found.

If you solve this problem as

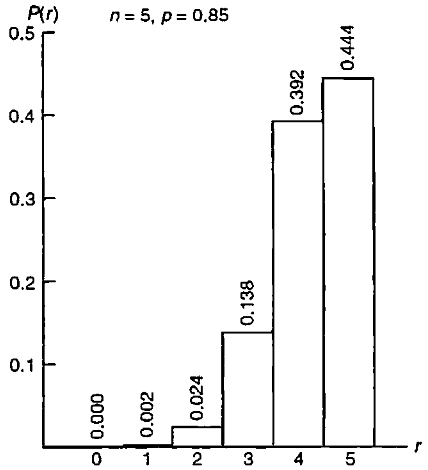
$$\begin{aligned} P(r \geq 2) &= 1 - P(r < 2) \\ &= 1 - [P(r = 0) + P(r = 1)] \\ &= 1 - 0.002 - 0.028 = 0.97, \end{aligned}$$

the answers differ due to rounding error in the table.

6. (a) $n = 5,\ p = 0.85$

The probabilities can be taken directly from Table 3 in Appendix II.

Binomial Distribution for Number of Automobile Damage Claims by People Under Age 25



(b) $\mu = np = 5(0.85) = 4.25$

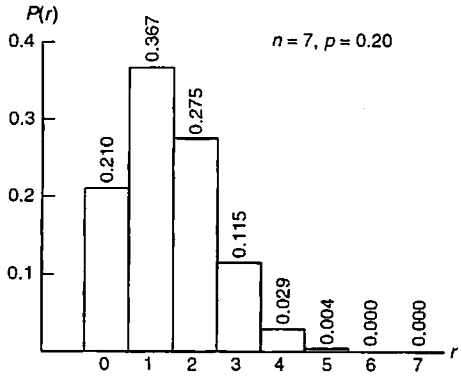
$\sigma = \sqrt{npq} = \sqrt{5(0.85)(0.15)} \approx 0.798$

For samples of size 5, the expected number of claims made by people under 25 years of age is about 4.

7. (a) $n = 7,\ p = 0.20$

The probabilities can be taken directly from Table 3 in Appendix II.

Binomial Distribution for Number of Illiterate People



(b) $\mu = np = 7(0.20) = 1.4$

$$\sigma = \sqrt{npq} = \sqrt{7(0.20)(0.80)} \approx 1.058$$

The expected number of people in this sample who are illiterate is 1.4.

(c) Let success = literate and $p = 0.80$.
Find n such that

$$P(r \geq 7) = 0.98.$$

Try $n = 12$.

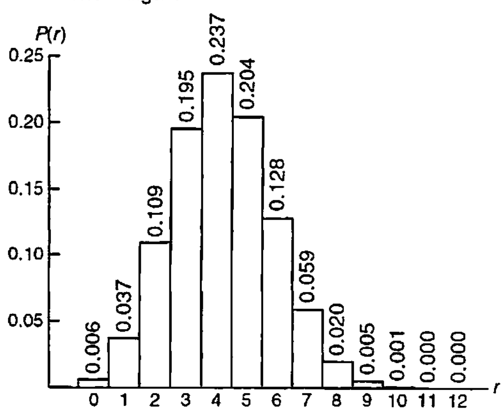
$$\begin{aligned} P(r \geq 7) &= P(7) + P(8) + P(9) + P(10) + P(11) + P(12) \\ &= 0.053 + 0.133 + 0.236 + 0.283 + 0.206 + 0.069 \\ &= 0.98 \end{aligned}$$

You would need to interview 12 people to be 98% sure that at least seven of these people are not illiterate.

8. (a) $n = 12,\ p = 0.35$

The probabilities can be taken directly from Table 3 in Appendix II.

Drivers Who Tailgate

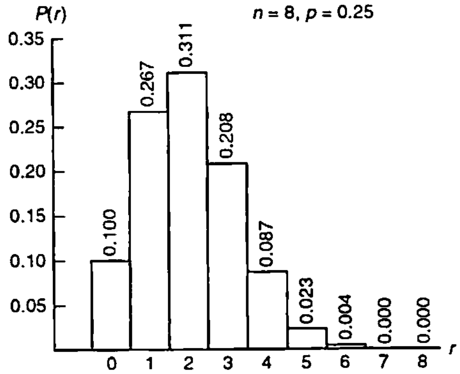


(b) $\mu = np = 12(0.35) = 4.2$
The expected number of vehicles out of 12 that will tailgate is 4.2.

(c) $\sigma = \sqrt{npq} = \sqrt{12(0.35)(0.65)} \approx 1.65$

9. (a) $n = 8,\ p = 0.25$
The probabilities can be taken directly from Table 3 in Appendix II.

Binomial Distribution for Number of
Gullible Customers



(b) $\mu = np = 8(0.25) = 2$

$$\sigma = \sqrt{npq} = \sqrt{8(0.25)(0.75)} \approx 1.225$$

The expected number of people in this sample who believe the product is improved is 1.4.

(c) Find n such that

$$P(r \geq 1) = 0.99.$$

Try n = 16.

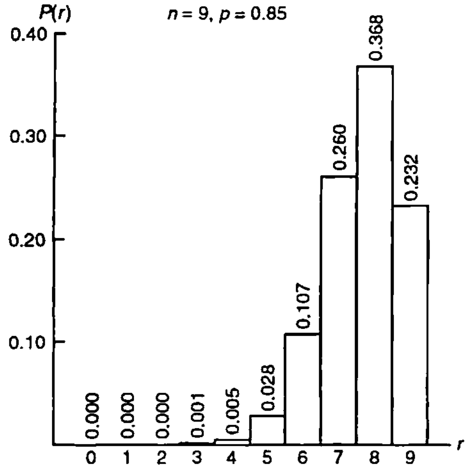
$$P(r \geq 1) = 1 - P(0)$$
$$= 1 - 0.01$$
$$= 0.99$$

Sixteen people are needed in the marketing study to be 99% sure that at least one person believes the product to be improved.

10. (a) From Table 3 in Appendix II and n = 9, p = 85.

r	0	1	2	3	4	5	6	7	8	9
$P(r)$	0.000	0.000	0.000	0.001	0.005	0.028	0.107	0.260	0.368	0.232

(b) Binomial Distribution for Number of Hot Spots that are Forest Fires



(c) $\mu = np = 9(0.85) = 7.65$

The expected number of real forest fires is 7.65.

(d) $\sigma = \sqrt{npq} = \sqrt{9(0.85)(0.15)} \approx 1.071$

(e) Find n such that

$$P(r \geq 1) = 0.999.$$

Try $n = 4$.

$$\begin{aligned} P(r \geq 1) &= 1 - P(r = 0) \\ &= 1 - 0.001 \\ &= 0.999 \end{aligned}$$

The satellite must report 4 hot spots to be 99.9% sure of at least one real forest fire.

11. $p = 0.40$

Find n such that

$$P(r \geq 5) = 0.95.$$

Try $n = 20$.

$$\begin{aligned} P(r \geq 5) &= 1 - [P(0) + P(1) + P(2) + P(3) + P(4)] \\ &= 1 - (0.000 + 0.000 + 0.003 + 0.012 + 0.035) \\ &= 1 - 0.05 \\ &= 0.95 \end{aligned}$$

He must make 20 sales calls to be 95% sure of meeting the quota.

12. $p = 0.55$

Find n such that

$$P(r \geq 4) = 0.964.$$

Try $n = 12$.

$$\begin{aligned} P(r \geq 4) &= 1 - [P(0) + P(1) + P(2) + P(3)] \\ &= 1 - (0.000 + 0.001 + 0.007 + 0.028) \\ &= 1 - 0.036 \\ &= 0.964 \end{aligned}$$

She must make 12 phone calls to be 96.4% sure of meeting the quota.

13. $p = 0.10$

Find n such that

$$P(r \geq 1) = 0.90$$

From a calculator or a computer, we determine $n = 22$ gives $P(r \geq 1) = 0.9015$.

14. **(a)** $p = 0.40, n = 7$

$$\begin{aligned} P(r \geq 1) &= 1 - P(0) \\ &= 1 - 0.028 \\ &= 0.972 \end{aligned}$$

(b) $\mu = np = 7(0.40) = 2.8$

The expected number of these seven drivers who will warn oncoming traffic is 2.8.

$$\sigma = \sqrt{npq} = \sqrt{7(0.40)(0.60)} \approx 1.30$$

(c) Find n such that

$$P(r \geq 1) = 0.998.$$

Try $n = 12$.

$$\begin{aligned} P(r \geq 1) &= 1 - P(0) \\ &= 1 - 0.002 \\ &= 0.998 \end{aligned}$$

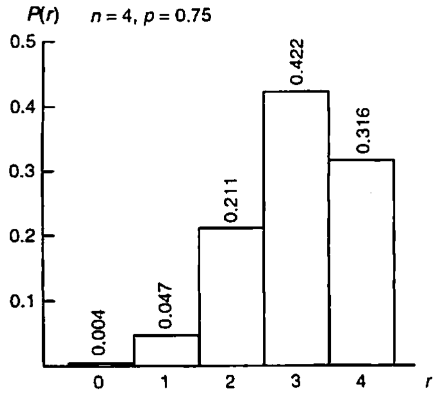
Twelve cars would need to go by the speed trap to be 99.8% sure that at least one driver will warn others.

15. **(a)** Since success = not a repeat offender, then $p = 0.75$.

From Table 3 in Appendix II and $n = 4, p = 0.75$.

r	0	1	2	3	4
$P(r)$	0.004	0.047	0.211	0.422	0.316

(b) Binomial Distribution for Number of Parolees Who Do Not Become Repeat Offenders



(c) $\mu = np = 4(0.75) = 3$

The expected number of parolees in Alice's group who will not be repeat offenders is 3.

$$\sigma = \sqrt{npq} = \sqrt{4(0.75)(0.25)} \approx 0.866$$

(d) Find n such that

$$P(r \geq 3) = 0.98.$$

Try $n = 7$.

$$P(r \geq 3) = P(3) + P(4) + P(5) + P(6) + P(7)$$
$$= 0.058 + 0.173 + 0.311 + 0.311 + 0.133$$
$$= 0.986$$

This is slightly higher than needed, but $n = 6$ yields $P(r \geq 3) = 0.963$.

Alice should have a group of 7 to be about 98% sure three or more will not become repeat offenders.

16. (a) $p = 0.65$

Find n such that

$$P(r \geq 1) = 0.98.$$

Try $n = 4$.

$$P(r \geq 1) = 1 - P(0)$$
$$= 1 - 0.015$$
$$= 0.985$$

Four stations are required to be 98% certain that an enemy plane flying over will be detected by at least one station.

(b) $n = 4, p = 0.65$

$$\mu = np = 4(0.65) = 2.6$$

If four stations are in use, 2.6 is the expected number of stations that will detect an enemy plane.

17. (a) Let success = available, then $p = 0.75$, $n = 12$.

$$P(12) = 0.032$$

(b) Let success = not available, then $p = 0.25$, $n = 12$.

$$P(r \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12)$$
$$= 0.040 + 0.011 + 0.002 + 0.000 + 0.000 + 0.000 + 0.000$$
$$= 0.053$$

(c) $n = 12, p = 0.75$

$$\mu = np = 12(0.75) = 9$$

The expected number of those available to serve on the jury is 9.

$$\sigma = \sqrt{npq} = \sqrt{12(0.75)(0.25)} = 1.5$$

(d) $p = 0.75$

Find n such that

$$P(r \geq 12) = 0.959.$$

Try $n = 20$.

$$P(r \geq 12) = P(12) + P(13) + P(14) + P(15) + P(16) + P(17) + P(18) + P(19) + P(20)$$
$$= 0.061 + 0.112 + 0.169 + 0.202 + 0.190 + 0.134 + 0.067 + 0.021 + 0.003$$
$$= 0.959$$

The jury commissioner must contact 20 people to be 95.9% sure of finding at least 12 people who available to serve.

18. (a) Let success = emergency, then $p = 0.15$, $n = 4$.

$$P(4) = 0.001$$

(b) Let success = not emergency, then $p = 0.85$, $n = 4$.

$$P(r \geq 3) = P(3) + P(4)$$
$$= 0.368 + 0.522$$
$$= 0.890$$

(c) $p = 0.15$

Find n such that

$$P(r \geq 1) = 0.96.$$

Try $n = 20$.

$$P(r \geq 1) = 1 - P(0)$$
$$= 1 - 0.039$$
$$= 0.961$$

The operators need to answer 20 calls to be 96% (or more) sure that at least one call was in fact an emergency.

19. Let success = case solved, then $p = 0.2$, $n = 6$.

(a) $P(0) = 0.262$

(b) $P(r \geq 1) = 1 - P(0)$
$$= 1 - 0.262$$
$$= 0.738$$

(c) $\mu = np = 6(0.20) = 1.2$

The expected number of crimes that will be solved is 1.2.

$$\sigma = \sqrt{npq} = \sqrt{6(0.20)(0.80)} \approx 0.98$$

(d) Find n such that

$$P(r \geq 1) = 0.90.$$

Try $n = 11$.

$$\begin{aligned} P(r \geq 1) &= 1 - P(0) \\ &= 1 - 0.086 \\ &= 0.914 \end{aligned}$$

[Note: For $n = 10$, $P(r \geq 1) = 0.893$.]

The police must investigate 11 property crimes before they can be at least 90% sure of solving one or more cases.

20. (a) $p = 0.55$

Find n such that

$$P(r \geq 1) = 0.99.$$

Try $n = 6$.

$$\begin{aligned} P(r \geq 1) &= 1 - P(0) \\ &= 1 - 0.008 \\ &= 0.992 \end{aligned}$$

Six alarms should be used to be 99% certain that a burglar trying to enter is detected by at least one alarm.

(b) $n = 9$, $p = 0.55$

$$\mu = np = 9(0.55) = 4.95$$

The expected number of alarms that would detect a burglar is about 5.

21. (a) Japan: $n = 7, p = 0.95$

$$P(7) = 0.698$$

United States: $n = 7, p = 0.60$

$$P(7) = 0.028$$

(b) Japan: $n = 7, p = 0.95$

$$\mu = np = 7(0.95) = 6.65$$

$$\sigma = \sqrt{npq} = \sqrt{7(0.95)(0.05)} \approx 0.58$$

United States: $n = 7, p = 0.60$

$$\mu = np = 7(0.60) = 4.2$$

$$\sigma = \sqrt{npq} = \sqrt{7(0.60)(0.40)} \approx 1.30$$

The expected number of verdicts in Japan is 6.65 and in the United States is 4.2.

(c) United States: $p = 0.60$
Find n such that
$$P(r \geq 2) = 0.99.$$
Try $n = 8$.
$$\begin{aligned} P(r \geq 2) &= 1 - [P(0) + P(1)] \\ &= 1 - (0.001 + 0.008) \\ &= 0.991 \end{aligned}$$
Japan: $p = 0.95$
Find n such that
$$P(r \geq 2) = 0.99.$$
Try $n = 3$.
$$\begin{aligned} P(r \geq 2) &= P(2) + P(3) \\ &= 0.135 + 0.857 \\ &= 0.992 \end{aligned}$$
Cover 8 trials in the U.S. and 3 trials in Japan.

22. $n = 6$, $p = 0.45$

(a) $P(6) = 0.008$

(b) $P(0) = 0.028$

(c) $$\begin{aligned} P(r \geq 2) &= P(2) + P(3) + P(4) + P(5) + P(6) \\ &= 0.278 + 0.303 + 0.186 + 0.061 + 0.008 \\ &= 0.836 \end{aligned}$$

(d) $\mu = np = 6(0.45) = 2.7$
The expected number is 2.7.
$$\sigma = \sqrt{npq} = \sqrt{6(0.45)(0.55)} \approx 1.219$$

(e) Find n such that
$$P(r \geq 3) = 0.90.$$
Try $n = 10$.
$$\begin{aligned} P(r \geq 3) &= 1 - [P(0) + P(1) + P(2)] \\ &= 1 - (0.003 + 0.021 + 0.076) \\ &= 1 - 0.100 \\ &= 0.900 \end{aligned}$$
You need to interview 10 professors to be at least 90% sure of filling the quota.

23. (a) $p = 0.40$

Find n such that

$$P(r \geq 1) = 0.99.$$

Try $n = 9$.

$$\begin{aligned} P(r \geq 1) &= 1 - P(0) \\ &= 1 - 0.010 \\ &= 0.990 \end{aligned}$$

The owner must answer 9 inquiries to be 99% sure of renting at least one room.

(b) $n = 25, p = 0.40$

$\mu = np = 25(0.40) = 10$

The expected number is 10 room rentals.

Section 5.4

1. **(a)** Geometric probability distribution, $p = 0.77$.

$$P(n) = p(1-p)^{n-1}$$

$$P(n) = (0.77)(0.23)^{n-1}$$

(b) $P(1) = (0.77)(0.23)^{1-1}$

$$\begin{aligned} &= (0.77)(0.23)^0 \\ &= 0.77 \end{aligned}$$

(c) $P(2) = (0.77)(0.23)^{2-1}$

$$\begin{aligned} &= (0.77)(0.23)^1 \\ &= 0.1771 \end{aligned}$$

(d) $P(3 \text{ or more tries}) = 1 - P(1) - P(2)$

$$\begin{aligned} &= 1 - 0.77 - 0.1771 \\ &= 0.0529 \end{aligned}$$

(e) $\mu = \frac{1}{p} = \frac{1}{0.77} \approx 1.29$

The expected number is 1 attempt.

2. **(a)** Geometric probability distribution, $p = 0.57$.

$$P(n) = p(1-p)^{n-1}$$

$$P(n) = (0.57)(0.43)^{n-1}$$

(b) $P(2) = (0.57)(0.43)^{2-1}$

$$\begin{aligned} &= (0.57)(0.43)^1 \\ &= 0.2451 \end{aligned}$$

(c) $P(3)=(0.57)(0.43)^{3-1}$
$=(0.57)(0.43)^2$
≈ 0.1054

(d) $P(\text{more than 3 attempts})=1-P(1)-P(2)-P(3)$
$=1-0.57-0.2451-0.1054$
$=0.0795$

(e) $\mu=\frac{1}{p}=\frac{1}{0.57}\approx 1.75$
The expected number is 2 attempts.

3. (a) Geometric probability distribution. $p = 0.05$.
$P(n)=p(1-p)^{n-1}$
$P(n)=(0.05)(0.95)^{n-1}$

(b) $P(5)=(0.05)(0.95)^{5-1}$
$=(0.05)(0.95)^4$
≈ 0.0407

(c) $P(10)=(0.05)(0.95)^{10-1}$
$=(0.05)(0.95)^9$
≈ 0.0315

(d) $P(\text{more than 3})=1-P(1)-P(2)-P(3)$
$=1-0.05-(0.05)(0.95)-(0.05)(0.95)^2$
$=1-0.05-0.0475-0.0451$
$=0.8574$

(e) $\mu=\frac{1}{p}=\frac{1}{0.05}\approx 20$
The expected number is 20 pot shards.

4. (a) Geometric probability distribution, $p = 0.80$.
$P(n)=p(1-p)^{n-1}$
$P(n)=(0.80)(0.20)^{n-1}$

(b) $P(1)=(0.80)(0.20)^{1-1}=0.80$
$P(2)=(0.80)(0.20)^{2-1}=0.16$
$P(3)=(0.80)(0.20)^{3-1}=0.032$

(c) $P(n\geq 4)=1-P(1)-P(2)-P(3)$
$=1-0.80-0.16-0.032$
$=0.008$

(d) $P(n)=(0.04)(0.96)^{n-1}$

$$P(1)=(0.04)(0.96)^{1-1}=0.04$$

$$P(2)=(0.04)(0.96)^{2-1}=0.0384$$

$$P(3)=(0.04)(0.96)^{3-1}=0.0369$$

$$\begin{aligned}P(n\geq 4)&=1-P(1)-P(2)-P(3)\\&=1-0.04-0.0384-0.0369\\&=0.8847\end{aligned}$$

5. (a) Geometric probability distribution. $p=0.71$.

$$P(n)=p(1-p)^{n-1}$$

$$P(n)=(0.71)(0.29)^{n-1}$$

(b) $P(1)=(0.71)(0.29)^{1-1}=0.71$

$$P(2)=(0.71)(0.29)^{2-1}=0.2059$$

$$\begin{aligned}P(n\geq 3)&=1-P(1)-P(2)\\&=1-0.71-0.2059\\&=0.0841\end{aligned}$$

(c) $P(n)=(0.83)(0.17)^{n-1}$

$$P(1)=(0.83)(0.17)^{1-1}=0.83$$

$$P(2)=(0.83)(0.17)^{2-1}=0.1411$$

$$\begin{aligned}P(n\geq 3)&=1-P(1)-P(2)\\&=1-0.83-0.1411\\&=0.0289\end{aligned}$$

6. (a) Geometric probability distribution. $p=0.36$.

$$P(n)=p(1-p)^{n-1}$$

$$P(n)=(0.036)(0.964)^{n-1}$$

(b) $P(3)=(0.036)(0.964)^{3-1}\approx 0.03345$

$$P(5)=(0.036)(0.964)^{5-1}\approx 0.0311$$

$$P(12)=(0.036)(0.964)^{12-1}\approx 0.0241$$

(c) $P(n\geq 5)=1-P(1)-P(2)-P(3)-P(4)$

$$\begin{aligned}&=1-0.036-(0.036)(0.964)-(0.036)(0.964)^2-(0.036)(0.964)^3\\&=1-0.036-0.0347-0.03345-0.03225\\&=0.8636\end{aligned}$$

(d) $\mu = \frac{1}{p} = \frac{1}{0.036} \approx 27.8$
The expected number is 28 apples.

7. (a) Geometric probability distribution, $p = 0.30$.

$$P(n) = p(1-p)^{n-1}$$
$$P(n) = (0.30)(0.70)^{n-1}$$

(b) $P(3) = (0.30)(0.70)^{3-1} = 0.147$

(c)
$$\begin{aligned} P(n > 3) &= 1 - P(1) - P(2) - P(3) \\ &= 1 - 0.30 - (0.30)(0.70) - 0.147 \\ &= 1 - 0.30 - 0.21 - 0.147 \\ &= 0.343 \end{aligned}$$

(d) $\mu = \frac{1}{p} = \frac{1}{0.30} = 3.33$
The expected number is 3 trips.

8. (a) The Poisson distribution would be a good choice because finding prehistoric artifacts is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of artifacts found in a fixed amount of sediment.

$$\lambda = \frac{1.5}{10\text{ L}} \cdot \frac{5}{5} = \frac{7.5}{50\text{ L}}; \ \lambda = 7.5 \text{ per 50 liters}$$

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(r) = \frac{e^{-7.5}(7.5)^r}{r!}$$

(b) $P(2) = \frac{e^{-7.5}(7.5)^2}{2!} \approx 0.0156$

$$P(3) = \frac{e^{-7.5}(7.5)^3}{3!} \approx 0.0389$$

$$P(4) = \frac{e^{-7.5}(7.5)^4}{4!} \approx 0.0729$$

(c)
$$\begin{aligned} P(r \geq 3) &= 1 - P(0) - P(1) - P(2) \\ &= 1 - 0.0006 - 0.0041 - 0.0156 \\ &= 0.9797 \end{aligned}$$

(d) $P(r<3)=P(0)+P(1)+P(2)$
$=0.0006+0.0041+0.0156$
$=0.0203$

or

$P(r<3)=1-P(r\geq 3)$
$=1-0.9797$
$=0.0203$

9. **(a)** The Poisson distribution would be a good choice because frequency of grooming is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of times that one otter grooms another in a fixed time interval.

$$\lambda=\frac{1.7}{10\text{ min}}\cdot\frac{3}{3}=\frac{5.1}{30\text{ min}};\ \lambda=5.1 \text{ per } 30\text{ min interval}$$

$$P(r)=\frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(r)=\frac{e^{-5.1}(5.1)^r}{r!}$$

(b) $P(4)=\dfrac{e^{-5.1}(5.1)^4}{4!}\approx 0.1719$

$P(5)=\dfrac{e^{-5.1}(5.1)^5}{5!}\approx 0.1753$

$P(6)=\dfrac{e^{-5.1}(5.1)^6}{6!}\approx 0.1490$

(c) $P(r\geq 4)=1-P(0)-P(1)-P(2)-P(3)$
$=1-0.0061-0.0311-0.0793-0.1348$
$=0.7487$

(d) $P(r<4)=P(0)+P(1)+P(2)+P(3)$
$=0.0061-0.0311-0.0793+0.1348$
$=0.2513$

or

$P(r<4)=1-P(r\geq 4)$
$=1-0.7487$
$=0.2513$

10. **(a)** The Poisson distribution would be a good choice because frequency of shoplifting is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of incidents in a fixed time interval.

$$\lambda=\frac{1}{3\text{ hr}}\cdot\frac{\frac{11}{3}}{\frac{11}{3}}=\frac{\frac{11}{3}}{11\text{ hr}};\ \lambda=\frac{11}{3}\approx 3.7 \text{ per } 11 \text{ hours (rounded to nearest tenth)}$$

(b) $P(r \geq 1) = 1 - P(0)$
$= 1 - 0.0247$
$= 0.9753$

(c) $P(r \geq 3) = 1 - P(0) - P(1) - P(2)$
$= 1 - 0.0247 - 0.0915 - 0.1692$
$= 0.7146$

(d) $P(0) = 0.0247$

11. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because frequency of births is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of births (or deaths) for a community of a given population size.

(b) For 1000 people, $\lambda = 16$ births; $\lambda = 8$ deaths

By Table 4 in Appendix II:

$P(10 \text{ births}) = 0.0341$
$P(10 \text{ deaths}) = 0.0993$
$P(16 \text{ births}) = 0.0992$
$P(16 \text{ deaths}) = 0.0045$

(c) For 1500 people,

$$\lambda = \frac{16}{1000} \cdot \frac{1.5}{1.5} = \frac{24}{1500}; \ \lambda = 24 \text{ births per 1500 people}$$
$$\lambda = \frac{8}{1000} \cdot \frac{1.5}{1.5} = \frac{12}{1500}; \ \lambda = 12 \text{ deaths per 1500 people}$$

By Table 4 or a calculator:

$P(10 \text{ births}) = 0.00066$
$P(10 \text{ deaths}) = 0.1048$
$P(16 \text{ births}) = 0.02186$
$P(16 \text{ deaths}) = 0.0543$

(d) For 750 people,

$$\lambda = \frac{16}{1000} \cdot \frac{0.75}{0.75} = \frac{12}{750}; \ \lambda = 12 \text{ births per 750 people}$$
$$\lambda = \frac{8}{1000} \cdot \frac{0.75}{0.75} = \frac{6}{750}; \ \lambda = 6 \text{ deaths per 750 people}$$

$P(10 \text{ births}) = 0.1048$
$P(10 \text{ deaths}) = 0.0413$
$P(16 \text{ births}) = 0.0543$
$P(16 \text{ deaths}) = 0.0003$

12. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because frequency of hairline cracks is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of hairline cracks for a given length of retaining wall.

(b) $\lambda = \frac{4.2}{30 \text{ ft}} \cdot \frac{\frac{5}{3}}{\frac{5}{3}} = \frac{7}{50 \text{ ft}};$

$\lambda = 7$ per 50 ft

From Table 4 in Appendix II:

$$P(3) = 0.0521$$

$$\begin{aligned} P(r \geq 3) &= 1 - P(0) - P(1) - P(2) \\ &= 1 - 0.0009 - 0.0064 - 0.0223 \\ &= 0.9704 \end{aligned}$$

(c) $\lambda = \frac{4.2}{30 \text{ ft}} \cdot \frac{\frac{2}{3}}{\frac{2}{3}} = \frac{2.8}{20 \text{ ft}};$

$\lambda = 2.8$ per 20 ft

$$P(3) = 0.2225$$

$$\begin{aligned} P(r \geq 3) &= 1 - P(0) - P(1) - P(2) \\ &= 1 - 0.0608 - 0.1703 - 0.2384 \\ &= 0.5305 \end{aligned}$$

(d) $\lambda = \frac{4.2}{30 \text{ ft}} \cdot \frac{\frac{1}{15}}{\frac{1}{15}} = \frac{0.28}{2 \text{ ft}};$

$\lambda = 0.3$ per 2 ft

$$P(3) = 0.0033$$

$$\begin{aligned} P(r \geq 3) &= 1 - P(0) - P(1) - P(2) \\ &= 1 - 0.7408 - 0.2222 - 0.0333 \\ &= 0.0037 \end{aligned}$$

(e) Discussion

13. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because frequency of gale-force winds is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of gale-force winds in a given time interval.

(b) $\lambda = \frac{1}{60 \text{ hr}} \cdot \frac{1.8}{1.8} = \frac{1.8}{108 \text{ hours}}$;

$\lambda = 1.8$ per 108 hours

From Table 4 in Appendix II:

$P(2) = 0.2678$

$P(3) = 0.1607$

$P(4) = 0.0723$

$$P(r < 2) = P(0) + P(1)$$
$$= 0.1653 + 0.2975$$
$$= 0.4628$$

(c) $\lambda = \frac{1}{60 \text{ hr}} \cdot \frac{3}{3} = \frac{3}{180 \text{ hours}}$;

$\lambda = 3$ per 180 hours

$P(3) = 0.2240$

$P(4) = 0.1680$

$P(5) = 0.1008$

$$P(r < 2) = P(0) + P(1) + P(2)$$
$$= 0.0498 + 0.1494 + 0.2240$$
$$= 0.4232$$

14. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because frequency of earthquakes is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of earthquakes in a given time interval.

(b) $\lambda = 1.00$ per 22 years

$P(r \geq 1) = 0.6321$

(c) $\lambda = 1.00$ per 22 years

$P(0) = 0.3679$

(d) $\lambda = \frac{1}{22 \text{ years}} \cdot \frac{\frac{25}{11}}{\frac{25}{11}} \approx \frac{2.27}{50 \text{ years}}$;

$\lambda = 2.27$ per 50 years

$P(r \geq 1) = 0.8967$

(e) $\lambda = 2.27$ per 50 years

$P(0) = 0.1033$

15. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because frequency of commercial building sales is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of buildings sold in a given time interval.

(b) $\lambda = \frac{8}{275 \text{ days}} \cdot \frac{\frac{12}{55}}{\frac{12}{55}} \approx \frac{\frac{96}{55}}{60 \text{ days}};$

$\lambda = \frac{96}{55} \approx 1.7$ per 60 days

From Table 4 in Appendix II:

$$\begin{aligned} P(0) &= 0.1827 \\ P(1) &= 0.3106 \\ P(r \geq 2) &= 1 - P(0) - P(1) \\ &= 1 - 0.1827 - 0.3106 \\ &= 0.5067 \end{aligned}$$

(c) $\lambda = \frac{8}{275 \text{ days}} \cdot \frac{\frac{18}{55}}{\frac{18}{55}} \approx \frac{2.6}{90 \text{ days}};$

$\lambda \approx 2.6$ per 90 days

$$\begin{aligned} P(0) &= 0.0743 \\ P(2) &= 0.2510 \\ P(r \geq 3) &= 1 - P(0) - P(1) - P(2) \\ &= 1 - 0.0743 - 0.1931 - 0.2510 \\ &= 0.4816 \end{aligned}$$

16. (a) Essay. Answer could include:

The problem satisfies the conditions for a binomial experiment with

n large, $n = 316$, and p small. $p = \frac{661}{100{,}000} = 0.00661$.

$np = 316(0.00661) \approx 2.1 < 10$.

The Poisson distribution would be a good approximation to the binomial.

$n = 316,\ p = 0.00661,\ \lambda = np \approx 2.1$

(b) From Table 4 in Appendix II,

$P(0) = 0.1225$

(c) $$\begin{aligned} P(r \leq 1) &= P(0) + P(1) \\ &= 0.1225 + 0.2572 \\ &= 0.3797 \end{aligned}$$

(d) $$\begin{aligned} P(r \geq 2) &= 1 - P(r \leq 1) \\ &= 1 - 0.3797 \\ &= 0.6203 \end{aligned}$$

17. (a) Essay. Answer could include:

The problem satisfies the conditions for a binomial experiment with

n large, $n = 1000$, and p small, $p = \frac{1}{569} \approx 0.0018$.
$np \approx 1000(0.0018) = 1.8 < 10$.

The Poisson distribution would be a good approximation to the binomial.

$$\lambda = np \approx 1.8$$

(b) From Table 4 in Appendix II.

$$P(0) = 0.1653$$

(c) $P(r > 1) = 1 - P(0) - P(1)$
$= 1 - 0.1653 - 0.2975$
$= 0.5372$

(d) $P(r > 2) = P(r > 1) - P(2)$
$= 0.5372 - 0.2678$
$= 0.2694$

(e) $P(r > 3) = P(r > 2) - P(3)$
$= 0.2694 - 0.1607$
$= 0.1087$

18. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because frequency of lost bags is a relatively rare occurrence. It is reasonable to assume that the events are independent and the variable is the number of bags lost per 1000 passengers.

$\lambda = 6.02$ or 6.0 per 1000 passengers

(b) From Table 4 in Appendix II:

$P(0) = 0.0025$
$P(r \geq 3) = 1 - P(0) - P(1) - P(2)$
$= 1 - 0.0025 - 0.0149 - 0.0446$
$= 0.9380$
$P(r \geq 6) = P(r \geq 3) - P(3) - P(4) - P(5)$
$= 0.9380 - 0.0892 - 0.1339 - 0.1606$
$= 0.5543$

(c) $\lambda = 13.0$ per 1000 passengers

$P(0) = 0.000$ (to 3 digits)
$P(r \geq 6) = 1 - P(r \leq 5)$
$= 1 - 0.0107$
$= 0.9893$
$P(r \geq 12) = 1 - P(r \leq 11)$
$= 1 - 0.3532$
$= 0.6468$

19. (a) Essay. Answer could include:

The problem satisfies the conditions for a binomial experiment with n large, $n = 175$, and p small, $p = 0.005$. $np = (175)(0.005) = 0.875 < 10$. The Poisson distribution would be a good approximation to the binomial. $n = 175$, $p = 0.005$, $\lambda = np = 0.9$.

(b) From Table 4 in Appendix II,

$$P(0) = 0.4066$$

(c)
$$\begin{aligned} P(r \geq 1) &= 1 - P(0) \\ &= 1 - 0.4066 \\ &= 0.5934 \end{aligned}$$

(d)
$$\begin{aligned} P(r \geq 2) &= P(r \geq 1) - P(1) \\ &= 0.5934 - 0.3659 \\ &= 0.2275 \end{aligned}$$

20. (a) Essay. Answer could include:

The problem satisfies the conditions for a binomial experiment with n large, $n = 137$, and p small, $p = 0.02$. $np = (137)(0.02) = 2.74 < 10$. The Poisson distribution would be a good approximation to the binomial. $n = 175$, $p = 0.02$, $\lambda = np = 2.74 \approx 2.7$.

(b) From Table 4 in Appendix II,

$$P(0) = 0.0672$$

(c)
$$\begin{aligned} P(r \geq 2) &= 1 - P(0) - P(1) \\ &= 1 - 0.0672 - 0.1815 \\ &= 0.7513 \end{aligned}$$

(d)
$$\begin{aligned} P(r \geq 4) &= P(r \geq 2) - P(2) - P(3) \\ &= 0.7513 - 0.2450 - 0.2205 \\ &= 0.2858 \end{aligned}$$

21. (a) $n = 100$, $p = 0.02$, $r = 2$

$$P(r) = C_{n,r}p^r(1-p)^{n-r}$$

$$\begin{aligned} P(2) &= C_{100,2}(0.02)^2(0.98)^{100-2} \\ &= 4950(0.0004)(0.1381) \\ &= 0.2734 \end{aligned}$$

(b) $\lambda = np = 100(0.02) = 2$

From Table 4 in Appendix II,

$$P(2) = 0.2707$$

(c) The approximation is correct to two decimal places.

(d) $n = 100; p = 0.02; r = 3$

By the formula for the binomial distribution,

$$\begin{aligned}P(3) &= C_{100,3}(0.02)^3(0.98)^{100-3}\\ &= 161{,}700(0.000008)(0.1409)\\ &= 0.1823\end{aligned}$$

By the Poisson approximation, $\lambda = 3$, $P(3) = 0.1804$. The approximation is correct to two decimal places.

Chapter 5 Review

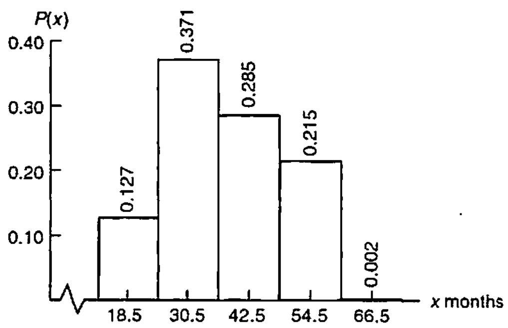
1. (a) $\mu = \Sigma xP(x)$

$$\begin{aligned}&= 18.5(0.127)+30.5(0.371)+42.5(0.285)+54.5(0.215)+66.5(0.002)\\ &= 37.628\\ &\approx 37.63\end{aligned}$$

The expected lease term is about 38 months.

$$\begin{aligned}\sigma &= \sqrt{\Sigma(x-\mu)^2 P(x)}\\ &= \sqrt{(-19.13)^2(0.127)+(-7.13)^2(0.371)+(4.87)^2(0.285)+(16.87)^2(0.215)+(28.87)^2(0.002)}\\ &\approx \sqrt{134.95}\\ &\approx 11.6 \text{ (using } \mu = 37.63 \text{ in the calculations)}\end{aligned}$$

(b) Leases in Months



2. (a)

Number killed by Wolves	Relative Frequency	$P(x)$
112	112/296	0.378
53	53/296	0.179
73	73/296	0.247
56	56/296	0.189
2	2/296	0.007

(b) $\mu = \sum xP(x)$

$= 0.5(0.378) + 3(0.179) + 8(0.247) + 13(0.189) + 18(0.007)$

≈ 5.28 yr

$\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$

$= \sqrt{(-4.78)^2 (0.378) + (-2.28)^2 (0.179) + (2.72)^2 (0.247) + (7.72)^2 (0.189) + (12.72)^2 (0.007)}$

$= \sqrt{23.8}$

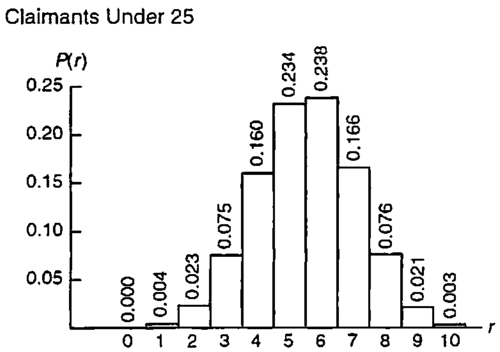
≈ 4.88 yr

3. This is a binomial experiment with 10 trials. A trial consists of a claim.

Success = submitted by a male under 25 years of age.

Failure = not submitted by a male under 25 years of age.

(a) The probabilities can be taken directly from Table 3 in Appendix II, $n = 10$, $p = 0.55$.



(b) $P(r \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10)$

$= 0.238 + 0.166 + 0.076 + 0.021 + 0.003$

$= 0.504$

(c) $\mu = np = 10(0.55) = 5.5$

The expected number of claims made by males under age 25 is 5.5.

$\sigma = \sqrt{npq} = \sqrt{10(0.55)(0.45)} \approx 1.57$

4. (a) $n = 20, p = 0.05$

$P(r \leq 2) = P(0) + P(1) + P(2)$

$= 0.358 + 0.377 + 0.189$

$= 0.924$

(b) $n = 20, p = 0.15$

Probability accepted:

$$P(r \le 2) = P(0) + P(1) + P(2)$$
$$= 0.039 + 0.137 + 0.229$$
$$= 0.405$$

Probability not accepted:

$$1 - 0.405 = 0.595$$

5. $n = 16, p = 0.50$

(a) $P(r \ge 12) = P(12) + P(13) + P(14) + P(15) + P(16)$
$$= 0.028 + 0.009 + 0.002 + 0.000 + 0.000$$
$$= 0.039$$

(b) $P(r \le 7) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)$
$$= 0.000 + 0.000 + 0.002 + 0.009 + 0.028 + 0.067 + 0.122 + 0.175$$
$$= 0.403$$

(c) $\mu = np = 16(0.50) = 8$

The expected number of inmates serving time for drug dealing is 8.

6. $n = 200, p = 0.80$

$$\mu = np = 200(0.80) = 160$$

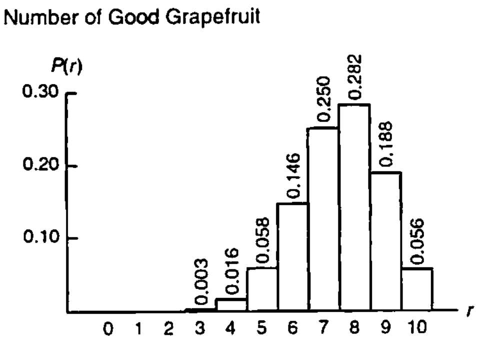
The expected number that will arrive on time is 160 flights.

$$\sigma = \sqrt{npq} = \sqrt{200(0.80)(0.20)} \approx 5.66$$

The standard deviation is 5.66 flights.

7. $n = 10, p = 0.75$

(a) The probabilities can be obtained directed from Table 3 in Appendix II.



(b) No more than one bad is the same as at least nine good.

$$P(r \geq 9) = P(9) + P(10)$$
$$= 0.188 + 0.056$$
$$= 0.244$$

$$P(r \geq 1) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$$
$$= 0.000 + 0.000 + 0.003 + 0.016 + 0.058 + 0.146 + 0.250 + 0.282 + 0.188 + 0.056$$
$$= 0.999$$

(c) $\mu = np = 10(0.75) = 7.5$

The expected number of good grapefruit in a sack is 7.5.

(d) $\sigma = \sqrt{npq} = \sqrt{10(0.75)(0.25)} \approx 1.37$

8. Let success = show up, then $p = 0.95$, $n = 82$.

$$\mu = np = 82(0.95) = 77.9$$

If 82 party reservations have been made. 77.9 or about 78 can be expected to show up.

$$\sigma = \sqrt{npq} = \sqrt{82(0.95)(0.05)} \approx 1.97$$

9. $p = 0.85$, $n = 12$

$$P(r \leq 2) = P(0) + P(1) + P(2)$$
$$= 0.000 + 0.000 + 0.000$$
$$= 0.000 \quad \text{(to 3 digits)}$$

The data seem to indicate that the percent favoring the increase in fees is less than 85%.

10. Let success = do not default, then $p = 0.50$.

Find n such that

$$P(r \geq 5) = 0.941.$$

Try $n = 15$.

$$P(r \geq 5) = 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$$
$$= 1 - (0.000 + 0.000 + 0.003 + 0.014 + 0.042)$$
$$= 1 - 0.059$$
$$= 0.941$$

You should buy 15 bonds if you want to be 94.1% sure that five or more will not default.

11. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because coughs are a relatively rare occurrence. It is reasonable to assume that they are independent events, and the variable is the number of coughs in a fixed time interval.

(b) $\lambda = 11$ per 1 minute

From Table 4 in Appendix II,

$$P(r \le 3) = P(0) + P(1) + P(2) + P(3)$$
$$= 0.0000 + 0.0002 + 0.0010 + 0.0037$$
$$= 0.0049$$

(c) $\lambda = \frac{11}{60 \text{ sec}} \cdot \frac{0.5}{0.5} = \frac{5.5}{30 \text{ sec}}$; $\lambda = 5.5$ per 30 sec.

$$P(r \ge 3) = 1 - P(0) - P(1) - P(2)$$
$$= 1 - 0.0041 - 0.0225 - 0.0618$$
$$= 0.9116$$

12. (a) Essay. Answer could include:

The Poisson distribution would be a good choice because number of accidents is a relatively rare occurrence. It is reasonable to assume that they are independent events, and the variable is the number of accidents for a given number of operations.

(b) $\lambda = 2.4$ per 100,000 flight operations

From Table 4 in Appendix II,

$$P(0) = 0.0907$$

(c) $\lambda = \frac{2.4}{100{,}000} \cdot \frac{2}{2} = \frac{4.8}{200{,}000}$;

$\lambda = 4.8$ per 200,000 flight operations.

$$P(r \ge 4) = 1 - P(0) - P(1) - P(2) - P(3)$$
$$= 1 - 0.0082 - 0.0395 - 0.0948 - 0.1517$$
$$= 0.7058$$

13. The loan-default problem satisfies the conditions for a binomial experiment. Moreover, p is small, n is large, and $np < 10$. Use of the Poisson approximation to the binomial distribution is appropriate.

$$n = 300,\ p = \frac{1}{350} = 0.0029,\ \lambda = np = 300(0.0029) \approx 0.86 \approx 0.9$$

From Table 4 in Appendix II,

$$P(r \ge 2) = 1 - P(0) - P(1)$$
$$= 1 - 0.4066 - 0.3659$$
$$= 0.2275$$

14. This problem satisfies the conditions for a binomial experiment. Moreover, p is small, n is large, and $np < 10$. Use of the Poisson approximation to the binomial distribution is appropriate.

$$n = 500{,}000,\ p = \frac{1}{5{,}000{,}000},\ \lambda = np = \frac{500{,}000}{5{,}000{,}000} = 0.1$$

From Table 4 in Appendix II,

$$P(0) = 0.9048$$
$$P(1) = 0.0905$$

15. (a) Use the geometric distribution with $p = 0.5$.

$$P(n=2) = (0.5)(0.5) = (0.5)^2 = 0.25$$

As long as you toss the coin at least twice, it does not matter how many more times you toss it. To get the first head on the second toss, you must get a tail on the first and a head on the second.

(b)
$$P(4) = (0.5)(0.5)^3 = (0.5)^4 = 0.0625$$
$$\begin{aligned} P(n>4) &= 1 - P(1) - P(2) - P(3) - P(4) \\ &= 1 - 0.5 - 0.5^2 - 0.5^3 - 0.5^4 \\ &= 0.0625 \end{aligned}$$

16. (a) Use the geometric distribution with $p = 0.83$.

$$P(1) = (0.83)(0.17)^{1-1} = 0.83$$

(b)
$$P(2) = (0.83)(0.17)^{2-1} = 0.1411$$
$$P(3) = (0.83)(0.17)^{3-1} = 0.0240$$
$$P(2 \text{ or } 3) = 0.1411 + 0.0240 \approx 0.165$$