

Part III

Transparency Masters

TABLE A Areas of a Standard Normal Distribution
(Alternate Version of Appendix II Table 5)

The table entries represent the area under the standard normal curve from 0 to the specified value of z .

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 |
| 3.6 | .4998 | .4998 | .4998 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 |

For values of z greater than or equal to 3.70, use 0.4999 to approximate the shaded area under the standard normal curve.

TABLE 1 Random Numbers

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 92630 | 78240 | 19267 | 95457 | 53497 | 23894 | 37708 | 79862 | 76471 | 66418 |
| 79445 | 78735 | 71549 | 44843 | 26404 | 67318 | 00701 | 34986 | 66751 | 99723 |
| 59654 | 71966 | 27386 | 50004 | 05358 | 94031 | 29281 | 18544 | 52429 | 06080 |
| 31524 | 49587 | 76612 | 39789 | 13537 | 48086 | 59483 | 60680 | 84675 | 53014 |
| 06348 | 76938 | 90379 | 51392 | 55887 | 71015 | 09209 | 79157 | 24440 | 30244 |
| 28703 | 51709 | 94456 | 48396 | 73780 | 06436 | 86641 | 69239 | 57662 | 80181 |
| 68108 | 89266 | 94730 | 95761 | 75023 | 48464 | 65544 | 96583 | 18911 | 16391 |
| 99938 | 90704 | 93621 | 66330 | 33393 | 95261 | 95349 | 51769 | 91616 | 33238 |
| 91543 | 73196 | 34449 | 63513 | 83834 | 99411 | 58826 | 40456 | 69268 | 48562 |
| 42103 | 02781 | 73920 | 56297 | 72678 | 12249 | 25270 | 36678 | 21313 | 75767 |
| 17138 | 27584 | 25296 | 28387 | 51350 | 61664 | 37893 | 05363 | 44143 | 42677 |
| 28297 | 14280 | 54524 | 21618 | 95320 | 38174 | 60579 | 08089 | 94999 | 78460 |
| 09331 | 56712 | 51333 | 06289 | 75345 | 08811 | 82711 | 57392 | 25252 | 30333 |
| 31295 | 04204 | 93712 | 51287 | 05754 | 79396 | 87399 | 51773 | 33075 | 97061 |
| 36146 | 15560 | 27592 | 42089 | 99281 | 59640 | 15221 | 96079 | 09961 | 05371 |
| 29553 | 18432 | 13630 | 05529 | 02791 | 81017 | 49027 | 79031 | 50912 | 09399 |
| 23501 | 22642 | 63081 | 08191 | 89420 | 67800 | 55137 | 54707 | 32945 | 64522 |
| 57888 | 85846 | 67967 | 07835 | 11314 | 01545 | 48535 | 17142 | 08552 | 67457 |
| 55336 | 71264 | 88472 | 04334 | 63919 | 36394 | 11196 | 92470 | 70543 | 29776 |
| 10087 | 10072 | 55980 | 64688 | 68239 | 20461 | 89381 | 93809 | 00796 | 95945 |
| 34101 | 81277 | 66090 | 88872 | 37818 | 72142 | 67140 | 50785 | 21380 | 16703 |
| 53362 | 44940 | 60430 | 22834 | 14130 | 96593 | 23298 | 56203 | 92671 | 15925 |
| 82975 | 66158 | 84731 | 19436 | 55790 | 69229 | 28661 | 13675 | 99318 | 76873 |
| 54827 | 84673 | 22898 | 08094 | 14326 | 87038 | 42892 | 21127 | 30712 | 48489 |
| 25464 | 59098 | 27436 | 89421 | 80754 | 89924 | 19097 | 67737 | 80368 | 08795 |

TABLE 1 *continued*

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 67609 | 60214 | 41475 | 84950 | 40133 | 02546 | 09570 | 45682 | 50165 | 15609 |
| 44921 | 70924 | 61295 | 51137 | 47596 | 86735 | 35561 | 76649 | 18217 | 63446 |
| 33170 | 30972 | 98130 | 95828 | 49786 | 13301 | 36081 | 80761 | 33985 | 68621 |
| 84687 | 85445 | 06208 | 17654 | 51333 | 02878 | 35010 | 67578 | 61574 | 20749 |
| 71886 | 56450 | 36567 | 09395 | 96951 | 35507 | 17555 | 35212 | 69106 | 01679 |
| 00475 | 02224 | 74722 | 14721 | 40215 | 21351 | 08596 | 45625 | 83981 | 63748 |
| 25993 | 38881 | 68361 | 59560 | 41274 | 69742 | 40703 | 37993 | 03435 | 18873 |
| 92882 | 53178 | 99195 | 93803 | 56985 | 53089 | 15305 | 50522 | 55900 | 43026 |
| 25138 | 26810 | 07093 | 15677 | 60688 | 04410 | 24505 | 37890 | 67186 | 62829 |
| 84631 | 71882 | 12991 | 83028 | 82484 | 90339 | 91950 | 74579 | 03539 | 90122 |
| 34003 | 92326 | 12793 | 61453 | 48121 | 74271 | 28363 | 66561 | 75220 | 35908 |
| 53775 | 45749 | 05734 | 86169 | 42762 | 70175 | 97310 | 73894 | 88606 | 19994 |
| 59316 | 97885 | 72807 | 54966 | 60859 | 11932 | 35265 | 71601 | 55577 | 67715 |
| 20479 | 66557 | 50705 | 26999 | 09854 | 52591 | 14063 | 30214 | 19890 | 19292 |
| 86180 | 84931 | 25455 | 26044 | 02227 | 52015 | 21820 | 50599 | 51671 | 65411 |
| 21451 | 68001 | 72710 | 40261 | 61281 | 13172 | 63819 | 48970 | 51732 | 54113 |
| 98062 | 68375 | 80089 | 24135 | 72355 | 95428 | 11808 | 29740 | 81644 | 86610 |
| 01788 | 64429 | 14430 | 94575 | 75153 | 94576 | 61393 | 96192 | 03227 | 32258 |
| 62465 | 04841 | 43272 | 68702 | 01274 | 05437 | 22953 | 18946 | 99053 | 41690 |
| 94324 | 31089 | 84159 | 92933 | 99989 | 89500 | 91586 | 02802 | 69471 | 68274 |
| 05797 | 43984 | 21575 | 09908 | 70221 | 19791 | 51578 | 36432 | 33494 | 79888 |
| 10395 | 14289 | 52185 | 09721 | 25789 | 38562 | 54794 | 04897 | 59012 | 89251 |
| 35177 | 56986 | 25549 | 59730 | 64718 | 52630 | 31100 | 62384 | 49483 | 11409 |
| 25633 | 89619 | 75882 | 98256 | 02126 | 72099 | 57183 | 55887 | 09320 | 72363 |
| 16464 | 48280 | 94254 | 45777 | 45150 | 68865 | 11382 | 11782 | 22695 | 41988 |

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TABLE 2 Binomial Coefficients $C_{n,r}$

| $n \setminus r$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------------|---|----|-----|-------|-------|--------|--------|--------|---------|---------|---------|
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | | |
| 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 | |
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |
| 11 | 1 | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 |
| 12 | 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 |
| 13 | 1 | 13 | 78 | 286 | 715 | 1,287 | 1,716 | 1,716 | 1,287 | 715 | 286 |
| 14 | 1 | 14 | 91 | 364 | 1,001 | 2,002 | 3,003 | 3,432 | 3,003 | 2,002 | 1,001 |
| 15 | 1 | 15 | 105 | 455 | 1,365 | 3,003 | 5,005 | 6,435 | 6,435 | 5,005 | 3,003 |
| 16 | 1 | 16 | 120 | 560 | 1,820 | 4,368 | 8,008 | 11,440 | 12,870 | 11,440 | 8,008 |
| 17 | 1 | 17 | 136 | 680 | 2,380 | 6,188 | 12,376 | 19,448 | 24,310 | 24,310 | 19,448 |
| 18 | 1 | 18 | 153 | 816 | 3,060 | 8,568 | 18,564 | 31,824 | 43,758 | 48,620 | 43,758 |
| 19 | 1 | 19 | 171 | 969 | 3,876 | 11,628 | 27,132 | 50,388 | 75,582 | 92,378 | 92,378 |
| 20 | 1 | 20 | 190 | 1,140 | 3,845 | 15,504 | 38,760 | 77,520 | 125,970 | 167,960 | 184,756 |

TABLE 3 Binomial Probability Distribution $C_{n,r}p^r q^{n-r}$

This table shows the probability of r successes in n independent trials, each with probability of success p .

| | | p | | | | | | | | | | | | | | | | | | | |
|-----|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| n | r | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 | .55 | .60 | .65 | .70 | .75 | .80 | .85 | .90 | .95 |
| 2 | 0 | .980 | .902 | .810 | .723 | .640 | .563 | .490 | .423 | .360 | .303 | .250 | .203 | .160 | .123 | .090 | .063 | .040 | .023 | .010 | .002 |
| | 1 | .020 | .095 | .180 | .255 | .320 | .375 | .420 | .455 | .480 | .495 | .500 | .495 | .480 | .455 | .420 | .375 | .320 | .255 | .180 | .095 |
| | 2 | .000 | .002 | .010 | .023 | .040 | .063 | .090 | .123 | .160 | .203 | .250 | .303 | .360 | .423 | .490 | .563 | .640 | .723 | .810 | .902 |
| 3 | 0 | .970 | .857 | .729 | .614 | .512 | .422 | .343 | .275 | .216 | .166 | .125 | .091 | .064 | .043 | .027 | .016 | .008 | .003 | .001 | .000 |
| | 1 | .029 | .135 | .243 | .325 | .384 | .422 | .441 | .444 | .432 | .408 | .375 | .334 | .288 | .239 | .189 | .141 | .096 | .057 | .027 | .007 |
| | 2 | .000 | .007 | .028 | .057 | .096 | .141 | .189 | .239 | .288 | .334 | .375 | .408 | .432 | .444 | .441 | .422 | .384 | .325 | .243 | .135 |
| | 3 | .000 | .000 | .001 | .003 | .008 | .016 | .027 | .043 | .064 | .091 | .125 | .166 | .216 | .275 | .343 | .422 | .512 | .614 | .729 | .857 |
| 4 | 0 | .961 | .815 | .656 | .522 | .410 | .316 | .240 | .179 | .130 | .092 | .062 | .041 | .026 | .015 | .008 | .004 | .002 | .001 | .000 | .000 |
| | 1 | .039 | .171 | .292 | .368 | .410 | .422 | .412 | .384 | .346 | .300 | .250 | .200 | .154 | .112 | .076 | .047 | .026 | .011 | .004 | .000 |
| | 2 | .001 | .014 | .049 | .098 | .154 | .211 | .265 | .311 | .346 | .368 | .375 | .368 | .346 | .311 | .265 | .211 | .154 | .098 | .049 | .014 |
| | 3 | .000 | .000 | .004 | .011 | .026 | .047 | .076 | .112 | .154 | .200 | .250 | .300 | .346 | .384 | .412 | .422 | .410 | .368 | .292 | .171 |
| | 4 | .000 | .000 | .000 | .001 | .002 | .004 | .008 | .015 | .026 | .041 | .062 | .092 | .130 | .179 | .240 | .316 | .410 | .522 | .656 | .815 |
| 5 | 0 | .951 | .774 | .590 | .444 | .328 | .237 | .168 | .116 | .078 | .050 | .031 | .019 | .010 | .005 | .002 | .001 | .000 | .000 | .000 | .000 |
| | 1 | .048 | .204 | .328 | .392 | .410 | .396 | .360 | .312 | .259 | .206 | .156 | .113 | .077 | .049 | .028 | .015 | .006 | .002 | .000 | .000 |
| | 2 | .001 | .021 | .073 | .138 | .205 | .264 | .309 | .336 | .346 | .337 | .312 | .276 | .230 | .181 | .132 | .088 | .051 | .024 | .008 | .001 |
| | 3 | .000 | .001 | .008 | .024 | .051 | .088 | .132 | .181 | .230 | .276 | .312 | .337 | .346 | .336 | .309 | .264 | .205 | .138 | .073 | .021 |
| | 4 | .000 | .000 | .000 | .002 | .006 | .015 | .028 | .049 | .077 | .113 | .156 | .206 | .259 | .312 | .360 | .396 | .410 | .392 | .328 | .204 |
| | 5 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .005 | .010 | .019 | .031 | .050 | .078 | .116 | .168 | .237 | .328 | .444 | .590 | .774 |
| 6 | 0 | .941 | .735 | .531 | .377 | .262 | .178 | .118 | .075 | .047 | .028 | .016 | .008 | .004 | .002 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .057 | .232 | .354 | .399 | .393 | .356 | .303 | .244 | .187 | .136 | .094 | .061 | .037 | .020 | .010 | .004 | .002 | .000 | .000 | .000 |
| | 2 | .001 | .031 | .098 | .176 | .246 | .297 | .324 | .328 | .311 | .278 | .234 | .186 | .138 | .095 | .060 | .033 | .015 | .006 | .001 | .000 |
| | 3 | .000 | .002 | .015 | .042 | .082 | .132 | .185 | .236 | .276 | .303 | .312 | .303 | .276 | .236 | .185 | .132 | .082 | .042 | .015 | .002 |
| | 4 | .000 | .000 | .001 | .006 | .015 | .033 | .060 | .095 | .138 | .186 | .234 | .278 | .311 | .328 | .324 | .297 | .246 | .176 | .098 | .031 |
| | 5 | .000 | .000 | .000 | .000 | .002 | .004 | .010 | .020 | .037 | .061 | .094 | .136 | .187 | .244 | .303 | .356 | .393 | .399 | .354 | .232 |
| | 6 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .004 | .008 | .016 | .028 | .047 | .075 | .118 | .178 | .262 | .377 | .531 | .735 |
| 7 | 0 | .932 | .698 | .478 | .321 | .210 | .133 | .082 | .049 | .028 | .015 | .008 | .004 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .066 | .257 | .372 | .396 | .367 | .311 | .247 | .185 | .131 | .087 | .055 | .032 | .017 | .008 | .004 | .001 | .000 | .000 | .000 | .000 |
| | 2 | .002 | .041 | .124 | .210 | .275 | .311 | .318 | .299 | .261 | .214 | .164 | .117 | .077 | .047 | .025 | .012 | .004 | .001 | .000 | .000 |
| | 3 | .000 | .004 | .023 | .062 | .115 | .173 | .227 | .268 | .290 | .292 | .273 | .239 | .194 | .144 | .097 | .058 | .029 | .011 | .003 | .000 |
| | 4 | .000 | .000 | .003 | .011 | .029 | .058 | .097 | .144 | .194 | .239 | .273 | .292 | .290 | .268 | .227 | .173 | .115 | .062 | .023 | .004 |
| | 5 | .000 | .000 | .000 | .001 | .004 | .012 | .025 | .047 | .077 | .117 | .164 | .214 | .261 | .299 | .318 | .311 | .275 | .210 | .124 | .041 |
| | 6 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .008 | .017 | .032 | .055 | .087 | .131 | .185 | .247 | .311 | .367 | .396 | .372 | .257 |
| | 7 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .004 | .008 | .015 | .028 | .049 | .082 | .133 | .210 | .321 | .478 | .698 | .932 |

TABLE 3 *continued*

| <i>n</i> | <i>r</i> | <i>p</i> | | | | | | | | | | | | | | | | | | | |
|----------|----------|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 | .55 | .60 | .65 | .70 | .75 | .80 | .85 | .90 | .95 |
| 8 | 0 | .923 | .663 | .430 | .272 | .168 | .100 | .058 | .032 | .017 | .008 | .004 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .075 | .279 | .383 | .385 | .336 | .267 | .198 | .137 | .090 | .055 | .031 | .016 | .008 | .003 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .003 | .051 | .149 | .238 | .294 | .311 | .296 | .259 | .209 | .157 | .109 | .070 | .041 | .022 | .010 | .004 | .001 | .000 | .000 | .000 |
| | 3 | .000 | .005 | .033 | .084 | .147 | .208 | .254 | .279 | .279 | .257 | .219 | .172 | .124 | .081 | .047 | .023 | .009 | .003 | .000 | .000 |
| | 4 | .000 | .000 | .005 | .018 | .046 | .087 | .136 | .188 | .232 | .263 | .273 | .263 | .232 | .188 | .136 | .087 | .046 | .018 | .005 | .000 |
| | 5 | .000 | .000 | .000 | .003 | .009 | .023 | .047 | .081 | .124 | .172 | .219 | .257 | .279 | .279 | .254 | .208 | .147 | .084 | .033 | .005 |
| | 6 | .000 | .000 | .000 | .000 | .001 | .004 | .010 | .022 | .041 | .070 | .109 | .157 | .209 | .259 | .296 | .311 | .294 | .238 | .149 | .051 |
| | 7 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .008 | .016 | .031 | .055 | .090 | .137 | .198 | .267 | .336 | .385 | .383 | .279 |
| | 8 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .004 | .008 | .017 | .032 | .058 | .100 | .168 | .272 | .430 | .663 |
| 9 | 0 | .914 | .630 | .387 | .232 | .134 | .075 | .040 | .021 | .010 | .005 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .083 | .299 | .387 | .368 | .302 | .225 | .156 | .100 | .060 | .034 | .018 | .008 | .004 | .001 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .003 | .063 | .172 | .260 | .302 | .300 | .267 | .216 | .161 | .111 | .070 | .041 | .021 | .010 | .004 | .001 | .000 | .000 | .000 | .000 |
| | 3 | .000 | .008 | .045 | .107 | .176 | .234 | .267 | .272 | .251 | .212 | .164 | .116 | .074 | .042 | .021 | .009 | .003 | .001 | .000 | .000 |
| | 4 | .000 | .001 | .007 | .028 | .066 | .117 | .172 | .219 | .251 | .260 | .246 | .213 | .167 | .118 | .074 | .039 | .017 | .005 | .001 | .000 |
| | 5 | .000 | .000 | .001 | .005 | .017 | .039 | .074 | .118 | .167 | .213 | .246 | .260 | .251 | .219 | .172 | .117 | .066 | .028 | .007 | .001 |
| | 6 | .000 | .000 | .000 | .001 | .003 | .009 | .021 | .042 | .074 | .116 | .164 | .212 | .251 | .272 | .267 | .234 | .176 | .107 | .045 | .008 |
| | 7 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .010 | .021 | .041 | .070 | .111 | .161 | .216 | .267 | .300 | .302 | .260 | .172 | .063 |
| | 8 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .008 | .018 | .034 | .060 | .100 | .156 | .225 | .302 | .368 | .387 | .299 |
| | 9 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .005 | .010 | .021 | .040 | .075 | .134 | .232 | .387 | .630 |
| 10 | 0 | .904 | .599 | .349 | .197 | .107 | .056 | .028 | .014 | .006 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .091 | .315 | .387 | .347 | .268 | .188 | .121 | .072 | .040 | .021 | .010 | .004 | .002 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .004 | .075 | .194 | .276 | .302 | .282 | .233 | .176 | .121 | .076 | .044 | .023 | .011 | .004 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 3 | .000 | .010 | .057 | .130 | .201 | .250 | .267 | .252 | .215 | .166 | .117 | .075 | .042 | .021 | .009 | .003 | .001 | .000 | .000 | .000 |
| | 4 | .000 | .001 | .011 | .040 | .088 | .146 | .200 | .238 | .251 | .238 | .205 | .160 | .111 | .069 | .037 | .016 | .006 | .001 | .000 | .000 |
| | 5 | .000 | .000 | .001 | .008 | .026 | .058 | .103 | .154 | .201 | .234 | .246 | .234 | .201 | .154 | .103 | .058 | .026 | .008 | .001 | .000 |
| | 6 | .000 | .000 | .000 | .001 | .006 | .016 | .037 | .069 | .111 | .160 | .205 | .238 | .251 | .238 | .200 | .146 | .088 | .040 | .011 | .001 |
| | 7 | .000 | .000 | .000 | .000 | .001 | .003 | .009 | .021 | .042 | .075 | .117 | .166 | .215 | .252 | .267 | .250 | .201 | .130 | .057 | .010 |
| | 8 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .011 | .023 | .044 | .076 | .121 | .176 | .233 | .282 | .302 | .276 | .194 | .075 |
| | 9 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .002 | .004 | .010 | .021 | .040 | .072 | .121 | .188 | .268 | .347 | .387 | .315 |
| | 10 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .006 | .014 | .028 | .056 | .107 | .197 | .349 | .599 |

TABLE 3 *continued*

| n | r | p | | | | | | | | | | | | | | | | | | | |
|----|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 | .55 | .60 | .65 | .70 | .75 | .80 | .85 | .90 | .95 |
| 11 | 0 | .895 | .569 | .314 | .167 | .086 | .042 | .020 | .009 | .004 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .099 | .329 | .384 | .325 | .236 | .155 | .093 | .052 | .027 | .013 | .005 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .005 | .087 | .213 | .287 | .295 | .258 | .200 | .140 | .089 | .051 | .027 | .013 | .005 | .002 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 3 | .000 | .014 | .071 | .152 | .221 | .258 | .257 | .225 | .177 | .126 | .081 | .046 | .023 | .010 | .004 | .001 | .000 | .000 | .000 | .000 |
| | 4 | .000 | .001 | .016 | .054 | .111 | .172 | .220 | .243 | .236 | .206 | .161 | .113 | .070 | .038 | .017 | .006 | .002 | .000 | .000 | .000 |
| | 5 | .000 | .000 | .002 | .013 | .039 | .080 | .132 | .183 | .221 | .236 | .226 | .193 | .147 | .099 | .057 | .027 | .010 | .002 | .000 | .000 |
| | 6 | .000 | .000 | .000 | .002 | .010 | .027 | .057 | .099 | .147 | .193 | .226 | .236 | .221 | .183 | .132 | .080 | .039 | .013 | .002 | .000 |
| | 7 | .000 | .000 | .000 | .000 | .002 | .006 | .017 | .038 | .070 | .113 | .161 | .206 | .236 | .243 | .220 | .172 | .111 | .054 | .016 | .001 |
| | 8 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .010 | .023 | .046 | .081 | .126 | .177 | .225 | .257 | .258 | .221 | .152 | .071 | .014 |
| | 9 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .005 | .013 | .027 | .051 | .089 | .140 | .200 | .258 | .295 | .287 | .213 | .087 |
| | 10 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .005 | .013 | .027 | .052 | .093 | .155 | .236 | .325 | .684 | .329 | |
| | 11 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .009 | .020 | .042 | .086 | .167 | .314 | .569 | | |
| 12 | 0 | .886 | .540 | .282 | .142 | .069 | .032 | .014 | .006 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .107 | .341 | .377 | .301 | .206 | .127 | .071 | .037 | .017 | .008 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .006 | .099 | .230 | .292 | .283 | .232 | .168 | .109 | .064 | .034 | .016 | .007 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 3 | .000 | .017 | .085 | .172 | .236 | .258 | .240 | .195 | .142 | .092 | .054 | .028 | .012 | .005 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 4 | .000 | .002 | .021 | .068 | .133 | .194 | .231 | .237 | .213 | .170 | .121 | .076 | .042 | .020 | .008 | .002 | .001 | .000 | .000 | .000 |
| | 5 | .000 | .000 | .004 | .019 | .053 | .103 | .158 | .204 | .227 | .223 | .193 | .149 | .101 | .059 | .029 | .011 | .003 | .001 | .000 | .000 |
| | 6 | .000 | .000 | .000 | .004 | .016 | .040 | .079 | .128 | .177 | .212 | .226 | .212 | .177 | .128 | .079 | .040 | .016 | .004 | .000 | .000 |
| | 7 | .000 | .000 | .000 | .001 | .003 | .011 | .029 | .059 | .101 | .149 | .193 | .223 | .227 | .204 | .158 | .103 | .053 | .019 | .004 | .000 |
| | 8 | .000 | .000 | .000 | .000 | .001 | .002 | .008 | .020 | .042 | .076 | .121 | .170 | .216 | .237 | .231 | .194 | .133 | .068 | .021 | .002 |
| | 9 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .005 | .012 | .028 | .054 | .092 | .142 | .195 | .240 | .258 | .236 | .172 | .085 | .017 |
| | 10 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .007 | .016 | .034 | .064 | .109 | .168 | .232 | .283 | .292 | .230 | .099 | |
| | 11 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .008 | .017 | .037 | .071 | .127 | .206 | .301 | .377 | .341 | |
| | 12 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .006 | .014 | .032 | .069 | .142 | .282 | .540 | | |
| 15 | 0 | .860 | .463 | .206 | .087 | .035 | .013 | .005 | .002 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .130 | .366 | .343 | .231 | .132 | .067 | .031 | .013 | .005 | .002 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .009 | .135 | .267 | .286 | .231 | .156 | .092 | .048 | .022 | .009 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 3 | .000 | .031 | .129 | .218 | .250 | .225 | .170 | .111 | .063 | .032 | .014 | .005 | .002 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 4 | .005 | .005 | .043 | .116 | .188 | .225 | .219 | .179 | .127 | .078 | .042 | .019 | .007 | .002 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 5 | .000 | .001 | .010 | .045 | .103 | .165 | .206 | .212 | .186 | .140 | .092 | .051 | .024 | .010 | .003 | .001 | .000 | .000 | .000 | .000 |
| | 6 | .000 | .000 | .002 | .013 | .043 | .092 | .147 | .191 | .207 | .191 | .153 | .105 | .061 | .030 | .012 | .003 | .001 | .000 | .000 | .000 |
| | 7 | .000 | .000 | .000 | .003 | .014 | .039 | .081 | .132 | .177 | .201 | .196 | .165 | .118 | .071 | .035 | .013 | .003 | .001 | .000 | .000 |
| | 8 | .000 | .000 | .000 | .001 | .003 | .013 | .035 | .071 | .118 | .165 | .196 | .201 | .177 | .132 | .081 | .039 | .014 | .003 | .000 | .000 |
| | 9 | .000 | .000 | .000 | .000 | .001 | .003 | .012 | .030 | .061 | .105 | .153 | .191 | .207 | .191 | .147 | .092 | .043 | .013 | .002 | .000 |
| | 10 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .010 | .024 | .051 | .092 | .140 | .186 | .212 | .206 | .165 | .103 | .045 | .010 | .001 |
| | 11 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .007 | .019 | .042 | .078 | .127 | .179 | .219 | .225 | .188 | .116 | .043 | .005 |
| | 12 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .002 | .005 | .014 | .032 | .063 | .111 | .170 | .225 | .250 | .218 | .129 | .031 |
| | 13 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .009 | .022 | .048 | .092 | .156 | .231 | .586 | .267 | .135 | |
| | 14 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .002 | .005 | .013 | .031 | .067 | .132 | .231 | .343 | .366 | |
| | 15 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .002 | .005 | .013 | .035 | .087 | .206 | .463 | |

TABLE 3 *continued*

| <i>n</i> | <i>r</i> | <i>p</i> | | | | | | | | | | | | | | | | | | | |
|----------|----------|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| | | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 | .55 | .60 | .65 | .70 | .75 | .80 | .85 | .90 | .95 |
| 16 | 0 | .851 | .440 | .185 | .074 | .028 | .010 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .138 | .371 | .329 | .210 | .113 | .053 | .023 | .009 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .010 | .146 | .275 | .277 | .211 | .134 | .073 | .035 | .015 | .006 | .002 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 3 | .000 | .036 | .142 | .229 | .246 | .208 | .146 | .089 | .047 | .022 | .009 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 4 | .000 | .006 | .051 | .131 | .200 | .225 | .204 | .155 | .101 | .057 | .028 | .011 | .004 | .001 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 5 | .000 | .001 | .014 | .056 | .120 | .180 | .210 | .201 | .162 | .112 | .067 | .034 | .014 | .005 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 6 | .000 | .000 | .003 | .018 | .055 | .110 | .165 | .198 | .198 | .168 | .122 | .075 | .039 | .017 | .006 | .001 | .000 | .000 | .000 | .000 |
| | 7 | .000 | .000 | .000 | .005 | .020 | .052 | .101 | .152 | .189 | .197 | .175 | .132 | .084 | .044 | .019 | .006 | .001 | .000 | .000 | .000 |
| | 8 | .000 | .000 | .000 | .001 | .006 | .020 | .049 | .092 | .142 | .181 | .196 | .181 | .142 | .092 | .049 | .020 | .006 | .001 | .000 | .000 |
| | 9 | .000 | .000 | .000 | .000 | .001 | .006 | .019 | .044 | .084 | .132 | .175 | .197 | .189 | .152 | .101 | .052 | .020 | .005 | .000 | .000 |
| | 10 | .000 | .000 | .000 | .000 | .000 | .001 | .006 | .017 | .039 | .075 | .122 | .168 | .198 | .198 | .165 | .110 | .055 | .018 | .003 | .000 |
| | 11 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .005 | .014 | .034 | .067 | .112 | .162 | .201 | .210 | .180 | .120 | .056 | .014 | .001 |
| | 12 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .011 | .028 | .057 | .101 | .155 | .204 | .225 | .200 | .131 | .051 | .006 |
| | 13 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .009 | .022 | .047 | .089 | .146 | .208 | .246 | .229 | .142 | .036 |
| | 14 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .002 | .006 | .015 | .035 | .073 | .134 | .211 | .277 | .275 | .146 |
| | 15 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .009 | .023 | .053 | .113 | .210 | .329 | .371 |
| | 16 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .010 | .028 | .074 | .185 | .440 |
| 20 | 0 | .818 | .358 | .122 | .039 | .012 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 1 | .165 | .377 | .270 | .137 | .058 | .021 | .007 | .002 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 2 | .016 | .189 | .285 | .229 | .137 | .067 | .028 | .010 | .003 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 3 | .001 | .060 | .190 | .243 | .205 | .134 | .072 | .032 | .012 | .004 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 4 | .000 | .013 | .090 | .182 | .218 | .190 | .130 | .074 | .035 | .014 | .005 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 5 | .000 | .002 | .032 | .103 | .175 | .202 | .179 | .127 | .075 | .036 | .015 | .005 | .001 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 6 | .000 | .000 | .009 | .045 | .109 | .169 | .192 | .171 | .124 | .075 | .036 | .015 | .005 | .001 | .000 | .000 | .000 | .000 | .000 | .000 |
| | 7 | .000 | .000 | .002 | .016 | .055 | .112 | .164 | .184 | .166 | .122 | .074 | .037 | .015 | .005 | .001 | .000 | .000 | .000 | .000 | .000 |
| | 8 | .000 | .000 | .000 | .005 | .022 | .061 | .114 | .161 | .180 | .162 | .120 | .073 | .035 | .014 | .004 | .001 | .000 | .000 | .000 | .000 |
| | 9 | .000 | .000 | .000 | .001 | .007 | .027 | .065 | .116 | .160 | .177 | .160 | .119 | .071 | .034 | .012 | .003 | .000 | .000 | .000 | .000 |
| | 10 | .000 | .000 | .000 | .000 | .002 | .010 | .031 | .069 | .117 | .159 | .176 | .159 | .117 | .069 | .031 | .010 | .002 | .000 | .000 | .000 |
| | 11 | .000 | .000 | .000 | .000 | .000 | .003 | .012 | .034 | .071 | .119 | .160 | .177 | .160 | .116 | .065 | .027 | .007 | .001 | .000 | .000 |
| | 12 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .014 | .035 | .073 | .120 | .162 | .180 | .161 | .114 | .061 | .022 | .005 | .000 | .000 |
| | 13 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .005 | .015 | .037 | .074 | .122 | .166 | .184 | .164 | .112 | .055 | .016 | .002 | .000 |
| | 14 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .005 | .015 | .037 | .075 | .124 | .171 | .192 | .169 | .109 | .045 | .009 | .000 |
| | 15 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .005 | .015 | .036 | .075 | .127 | .179 | .202 | .175 | .103 | .032 | .002 |
| | 16 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .005 | .014 | .035 | .074 | .130 | .190 | .218 | .182 | .090 | .013 |
| | 17 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .004 | .012 | .032 | .072 | .134 | .205 | .243 | .190 | .060 |
| | 18 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .010 | .028 | .067 | .137 | .229 | .285 | .189 |
| | 19 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .002 | .007 | .021 | .058 | .137 | .270 | .377 |
| | 20 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | .003 | .012 | .039 | .122 | .358 |

TABLE 4 Poisson Probability Distribution

For a given value of λ , entry indicates the probability of obtaining a specified value of r .

| | | λ | | | | | | | | | |
|-----|-------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| r | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | 1.0 | |
| 0 | .9048 | .8187 | .7408 | .6703 | .6065 | .5488 | .4966 | .4493 | .4066 | .3679 | |
| 1 | .0905 | .1637 | .2222 | .2681 | .3033 | .3293 | .3476 | .3595 | .3659 | .3679 | |
| 2 | .0045 | .0164 | .0333 | .0536 | .0758 | .0988 | .1217 | .1438 | .1647 | .1839 | |
| 3 | .0002 | .0011 | .0033 | .0072 | .0126 | .0198 | .0284 | .0383 | .0494 | .0613 | |
| 4 | .0000 | .0001 | .0003 | .0007 | .0016 | .0030 | .0050 | .0077 | .0111 | .0153 | |
| 5 | .0000 | .0000 | .0000 | .0001 | .0002 | .0004 | .0007 | .0012 | .0020 | .0031 | |
| 6 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0002 | .0003 | .0005 | |
| 7 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | |

| | | λ | | | | | | | | | |
|-----|-------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| r | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | |
| 0 | .3329 | .3012 | .2725 | .2466 | .2231 | .2019 | .1827 | .1653 | .1496 | .1353 | |
| 1 | .3662 | .3614 | .3543 | .3452 | .3347 | .3230 | .3106 | .2975 | .2842 | .2707 | |
| 2 | .2014 | .2169 | .2303 | .2417 | .2510 | .2584 | .2640 | .2678 | .2700 | .2707 | |
| 3 | .0738 | .0867 | .0998 | .1128 | .1255 | .1378 | .1496 | .1607 | .1710 | .1804 | |
| 4 | .0203 | .0260 | .0324 | .0395 | .0471 | .0551 | .0636 | .0723 | .0812 | .0902 | |
| 5 | .0045 | .0062 | .0084 | .0111 | .0141 | .0176 | .0216 | .0260 | .0309 | .0361 | |
| 6 | .0008 | .0012 | .0018 | .0026 | .0035 | .0047 | .0061 | .0078 | .0098 | .0120 | |
| 7 | .0001 | .0002 | .0003 | .0005 | .0008 | .0011 | .0015 | .0020 | .0027 | .0034 | |
| 8 | .0000 | .0000 | .0001 | .0001 | .0001 | .0002 | .0003 | .0005 | .0006 | .0009 | |
| 9 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 | .0002 | |

| | | λ | | | | | | | | | |
|-----|-------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| r | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 | |
| 0 | .1225 | .1108 | .1003 | .0907 | .0821 | .0743 | .0672 | .0608 | .0550 | .0498 | |
| 1 | .2572 | .2438 | .2306 | .2177 | .2052 | .1931 | .1815 | .1703 | .1596 | .1494 | |
| 2 | .2700 | .2681 | .2652 | .2613 | .2565 | .2510 | .2450 | .2384 | .2314 | .2240 | |
| 3 | .1890 | .1966 | .2033 | .2090 | .2138 | .2176 | .2205 | .2225 | .2237 | .2240 | |
| 4 | .0992 | .1082 | .1169 | .1254 | .1336 | .1414 | .1488 | .1557 | .1622 | .1680 | |
| 5 | .0417 | .0476 | .0538 | .0602 | .0668 | .0735 | .0804 | .0872 | .0940 | .1008 | |
| 6 | .0146 | .0174 | .0206 | .0241 | .0278 | .0319 | .0362 | .0407 | .0455 | .0504 | |
| 7 | .0044 | .0055 | .0068 | .0083 | .0099 | .0118 | .0139 | .0163 | .0188 | .0216 | |
| 8 | .0011 | .0015 | .0019 | .0025 | .0031 | .0038 | .0047 | .0057 | .0068 | .0081 | |
| 9 | .0003 | .0004 | .0005 | .0007 | .0009 | .0011 | .0014 | .0018 | .0022 | .0027 | |
| 10 | .0001 | .0001 | .0001 | .0002 | .0002 | .0003 | .0004 | .0005 | .0006 | .0008 | |
| 11 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 | .0002 | .0002 | |
| 12 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | |

TABLE 4 *continued*

| <i>r</i> | λ | | | | | | | | | |
|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 | 3.8 | 3.9 | 4.0 |
| 0 | .0450 | .0408 | .0369 | .0334 | .0302 | .0273 | .0247 | .0224 | .0202 | .0183 |
| 1 | .1397 | .1304 | .1217 | .1135 | .1057 | .0984 | .0915 | .0850 | .0789 | .0733 |
| 2 | .2165 | .2087 | .2008 | .1929 | .1850 | .1771 | .1692 | .1615 | .1539 | .1465 |
| 3 | .2237 | .2226 | .2209 | .2186 | .2158 | .2125 | .2087 | .2046 | .2001 | .1954 |
| 4 | .1734 | .1781 | .1823 | .1858 | .1888 | .1912 | .1931 | .1944 | .1951 | .1954 |
| 5 | .1075 | .1140 | .1203 | .1264 | .1322 | .1377 | .1429 | .1477 | .1522 | .1563 |
| 6 | .0555 | .0608 | .0662 | .0716 | .0771 | .0826 | .0881 | .0936 | .0989 | .1042 |
| 7 | .0246 | .0278 | .0312 | .0348 | .0385 | .0425 | .0466 | .0508 | .0551 | .0595 |
| 8 | .0095 | .0111 | .0129 | .0148 | .0169 | .0191 | .0215 | .0241 | .0269 | .0298 |
| 9 | .0033 | .0040 | .0047 | .0056 | .0066 | .0076 | .0089 | .0102 | .0116 | .0132 |
| 10 | .0010 | .0013 | .0016 | .0019 | .0023 | .0028 | .0033 | .0039 | .0045 | .0053 |
| 11 | .0003 | .0004 | .0005 | .0006 | .0007 | .0009 | .0011 | .0013 | .0016 | .0019 |
| 12 | .0001 | .0001 | .0001 | .0002 | .0002 | .0003 | .0003 | .0004 | .0005 | .0006 |
| 13 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 |
| 14 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 |

| <i>r</i> | λ | | | | | | | | | |
|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 | 5.0 |
| 0 | .0166 | .0150 | .0136 | .0123 | .0111 | .0101 | .0091 | .0082 | .0074 | .0067 |
| 1 | .0679 | .0630 | .0583 | .0540 | .0500 | .0462 | .0427 | .0395 | .0365 | .0337 |
| 2 | .1393 | .1323 | .1254 | .1188 | .1125 | .1063 | .1005 | .0948 | .0894 | .0842 |
| 3 | .1904 | .1852 | .1798 | .1743 | .1687 | .1631 | .1574 | .1517 | .1460 | .1404 |
| 4 | .1951 | .1944 | .1933 | .1917 | .1898 | .1875 | .1849 | .1820 | .1789 | .1755 |
| 5 | .1600 | .1633 | .1662 | .1687 | .1708 | .1725 | .1738 | .1747 | .1753 | .1755 |
| 6 | .1093 | .1143 | .1191 | .1237 | .1281 | .1323 | .1362 | .1398 | .1432 | .1462 |
| 7 | .0640 | .1686 | .0732 | .0778 | .0824 | .0869 | .0914 | .0959 | .1002 | .1044 |
| 8 | .0328 | .0360 | .0393 | .0428 | .0463 | .0500 | .0537 | .0575 | .0614 | .0653 |
| 9 | .0150 | .0168 | .0188 | .0209 | .0232 | .0255 | .0280 | .0307 | .0334 | .0363 |
| 10 | .0061 | .0071 | .0081 | .0092 | .0104 | .0118 | .0132 | .0147 | .0164 | .0181 |
| 11 | .0026 | .0027 | .0032 | .0037 | .0043 | .0049 | .0056 | .0064 | .0073 | .0082 |
| 12 | .0008 | .0009 | .0011 | .0014 | .0016 | .0019 | .0022 | .0026 | .0030 | .0034 |
| 13 | .0002 | .0003 | .0004 | .0005 | .0006 | .0007 | .0008 | .0009 | .0011 | .0013 |
| 14 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0003 | .0003 | .0004 | .0005 |
| 15 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 |

TABLE 4 *continued*

| <i>r</i> | λ | | | | | | | | | |
|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 5.1 | 5.2 | 5.3 | 5.4 | 5.5 | 5.6 | 5.7 | 5.8 | 5.9 | 6.0 |
| 0 | .0061 | .0055 | .0050 | .0045 | .0041 | .0037 | .0033 | .0030 | .0027 | .0025 |
| 1 | .0311 | .0287 | .0265 | .0244 | .0225 | .0207 | .0191 | .0176 | .0162 | .0149 |
| 2 | .0793 | .0746 | .0701 | .0659 | .0618 | .0580 | .0544 | .0509 | .0477 | .0446 |
| 3 | .1348 | .1293 | .1239 | .1185 | .1133 | .1082 | .1033 | .0985 | .0938 | .0892 |
| 4 | .1719 | .1681 | .1641 | .1600 | .1558 | .1515 | .1472 | .1428 | .1383 | .1339 |
| 5 | .1753 | .1748 | .1740 | .1728 | .1714 | .1697 | .1678 | .1656 | .1632 | .1606 |
| 6 | .1490 | .1515 | .1537 | .1555 | .1571 | .1584 | .1594 | .1601 | .1605 | .1606 |
| 7 | .1086 | .1125 | .1163 | .1200 | .1234 | .1267 | .1298 | .1326 | .1353 | .1377 |
| 8 | .0692 | .0731 | .0771 | .0810 | .0849 | .0887 | .0925 | .0962 | .0998 | .1033 |
| 9 | .0392 | .0423 | .0454 | .0486 | .0519 | .0552 | .0586 | .0620 | .0654 | .0688 |
| 10 | .0200 | .0220 | .0241 | .0262 | .0285 | .0309 | .0334 | .0359 | .0386 | .0413 |
| 11 | .0093 | .0104 | .0116 | .0129 | .0143 | .0157 | .0173 | .0190 | .0207 | .0225 |
| 12 | .0039 | .0045 | .0051 | .0058 | .0065 | .0073 | .0082 | .0092 | .0102 | .0113 |
| 13 | .0015 | .0018 | .0021 | .0024 | .0028 | .0032 | .0036 | .0041 | .0046 | .0052 |
| 14 | .0006 | .0007 | .0008 | .0009 | .0011 | .0013 | .0015 | .0017 | .0019 | .0022 |
| 15 | .0002 | .0002 | .0003 | .0003 | .0004 | .0005 | .0006 | .0007 | .0008 | .0009 |
| 16 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 |
| 17 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 | .0001 |

| <i>r</i> | λ | | | | | | | | | |
|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 6.1 | 6.2 | 6.3 | 6.4 | 6.5 | 6.6 | 6.7 | 6.8 | 6.9 | 7.0 |
| 0 | .0022 | .0020 | .0018 | .0017 | .0015 | .0014 | .0012 | .0011 | .0010 | .0009 |
| 1 | .0137 | .0126 | .0116 | .0106 | .0098 | .0090 | .0082 | .0076 | .0070 | .0064 |
| 2 | .0417 | .0390 | .0364 | .0340 | .0318 | .0296 | .0276 | .0258 | .0240 | .0223 |
| 3 | .0848 | .0806 | .0765 | .0726 | .0688 | .0652 | .0617 | .0584 | .0552 | .0521 |
| 4 | .1294 | .1249 | .1205 | .1162 | .1118 | .1076 | .1034 | .0992 | .0952 | .0912 |
| 5 | .1579 | .1549 | .1519 | .1487 | .1454 | .1420 | .1385 | .1349 | .1314 | .1277 |
| 6 | .1605 | .0601 | .1595 | .1586 | .1575 | .1562 | .1546 | .1529 | .1511 | .1490 |
| 7 | .1399 | .1418 | .1435 | .1450 | .1462 | .1472 | .1480 | .1486 | .1489 | .1490 |
| 8 | .1066 | .1099 | .1130 | .1160 | .1188 | .1215 | .1240 | .1263 | .1284 | .1304 |
| 9 | .0723 | .0757 | .0791 | .0825 | .0858 | .0891 | .0923 | .0954 | .0985 | .1014 |
| 10 | .0441 | .0469 | .0498 | .0528 | .0558 | .0588 | .0618 | .0649 | .0679 | .0710 |
| 11 | .0245 | .0265 | .0285 | .0307 | .0330 | .0353 | .0377 | .0401 | .0426 | .0452 |
| 12 | .0124 | .0137 | .0150 | .0164 | .0179 | .0194 | .0210 | .0227 | .0245 | .0264 |
| 13 | .0058 | .0065 | .0073 | .0081 | .0089 | .0098 | .0108 | .0119 | .0130 | .0142 |
| 14 | .0025 | .0029 | .0033 | .0037 | .0041 | .0046 | .0052 | .0058 | .0064 | .0071 |
| 15 | .0010 | .0012 | .0014 | .0016 | .0018 | .0020 | .0023 | .0026 | .0029 | .0033 |
| 16 | .0004 | .0005 | .0005 | .0006 | .0007 | .0008 | .0010 | .0011 | .0013 | .0014 |
| 17 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 | .0004 | .0004 | .0005 | .0006 |
| 18 | .0000 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 |
| 19 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 |

TABLE 4 *continued*

| <i>r</i> | λ | | | | | | | | | |
|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 7.1 | 7.2 | 7.3 | 7.4 | 7.5 | 7.6 | 7.7 | 7.8 | 7.9 | 8.0 |
| 0 | .0008 | .0007 | .0007 | .0006 | .0006 | .0005 | .0005 | .0004 | .0004 | .0003 |
| 1 | .0059 | .0054 | .0049 | .0045 | .0041 | .0038 | .0035 | .0032 | .0029 | .0027 |
| 2 | .0208 | .0194 | .0180 | .0167 | .0156 | .0145 | .0134 | .0125 | .0116 | .0107 |
| 3 | .0492 | .0464 | .0438 | .0413 | .0389 | .0366 | .0345 | .0324 | .0305 | .0286 |
| 4 | .0874 | .0836 | .0799 | .0764 | .0729 | .0696 | .0663 | .0632 | .0602 | .0573 |
| 5 | .1241 | .1204 | .1167 | .1130 | .1094 | .1057 | .1021 | .0986 | .0951 | .0916 |
| 6 | .1468 | .1445 | .1420 | .1394 | .1367 | .1339 | .1311 | .1282 | .1252 | .1221 |
| 7 | .1489 | .1486 | .1481 | .1474 | .1465 | .1454 | .1442 | .1428 | .1413 | .1396 |
| 8 | .1321 | .1337 | .1351 | .1363 | .1373 | .1382 | .1388 | .1392 | .1395 | .1396 |
| 9 | .1042 | .1070 | .1096 | .1121 | .1144 | .1167 | .1187 | .1207 | .1224 | .1241 |
| 10 | .0740 | .0770 | .0800 | .0829 | .0858 | .0887 | .0914 | .0941 | .0967 | .0993 |
| 11 | .0478 | .0504 | .0531 | .0558 | .0585 | .0613 | .0640 | .0667 | .0695 | .0722 |
| 12 | .0283 | .0303 | .0323 | .0344 | .0366 | .0388 | .0411 | .0434 | .0457 | .0481 |
| 13 | .0154 | .0168 | .0181 | .0196 | .0211 | .0227 | .0243 | .0260 | .0278 | .0296 |
| 14 | .0078 | .0086 | .0095 | .0104 | .0113 | .0123 | .0134 | .0145 | .0157 | .0169 |
| 15 | .0037 | .0041 | .0046 | .0051 | .0057 | .0062 | .0069 | .0075 | .0083 | .0090 |
| 16 | .0016 | .0019 | .0021 | .0024 | .0026 | .0030 | .0033 | .0037 | .0041 | .0045 |
| 17 | .0007 | .0008 | .0009 | .0010 | .0012 | .0013 | .0015 | .0017 | .0019 | .0021 |
| 18 | .0003 | .0003 | .0004 | .0004 | .0005 | .0006 | .0006 | .0007 | .0008 | .0009 |
| 19 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0003 | .0003 | .0003 | .0004 |
| 20 | .0000 | .0000 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 |
| 21 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 |

TABLE 4 *continued*

| <i>r</i> | λ | | | | | | | | | |
|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 8.1 | 8.2 | 8.3 | 8.4 | 8.5 | 8.6 | 8.7 | 8.8 | 8.9 | 9.0 |
| 0 | .0003 | .0003 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0001 | .0001 |
| 1 | .0025 | .0023 | .0021 | .0019 | .0017 | .0016 | .0014 | .0013 | .0012 | .0011 |
| 2 | .0100 | .0092 | .0086 | .0079 | .0074 | .0068 | .0063 | .0058 | .0054 | .0050 |
| 3 | .0269 | .0252 | .0237 | .0222 | .0208 | .0195 | .0183 | .0171 | .0160 | .0150 |
| 4 | .0544 | .0517 | .0491 | .0466 | .0443 | .0420 | .0398 | .0377 | .0357 | .0337 |
| 5 | .0882 | .0849 | .0816 | .0784 | .0752 | .0722 | .0692 | .0663 | .0635 | .0607 |
| 6 | .1191 | .1160 | .1128 | .1097 | .1066 | .1034 | .1003 | .0972 | .0941 | .0911 |
| 7 | .1378 | .1358 | .1338 | .1317 | .1294 | .1271 | .1247 | .1222 | .1197 | .1171 |
| 8 | .1395 | .1392 | .1388 | .1382 | .1375 | .1366 | .1356 | .1344 | .1332 | .1318 |
| 9 | .1256 | .1269 | .1280 | .1290 | .1299 | .1306 | .1311 | .1315 | .1317 | .1318 |
| 10 | .1017 | .1040 | .1063 | .1084 | .1104 | .1123 | .1140 | .1157 | .1172 | .1186 |
| 11 | .0749 | .0776 | .0802 | .0828 | .0853 | .0878 | .0902 | .0925 | .0948 | .0970 |
| 12 | .0505 | .0530 | .0555 | .0579 | .0604 | .0629 | .0654 | .0679 | .0703 | .0728 |
| 13 | .0315 | .0334 | .0354 | .0374 | .0395 | .0416 | .0438 | .0459 | .0481 | .0504 |
| 14 | .0182 | .0196 | .0210 | .0225 | .0240 | .0256 | .0272 | .0289 | .0306 | .0324 |
| 15 | .0098 | .0107 | .0116 | .0126 | .0136 | .0147 | .0158 | .0169 | .0182 | .0194 |
| 16 | .0050 | .0055 | .0060 | .0066 | .0072 | .0079 | .0086 | .0093 | .0101 | .0109 |
| 17 | .0024 | .0026 | .0029 | .0033 | .0036 | .0040 | .0044 | .0048 | .0053 | .0058 |
| 18 | .0011 | .0012 | .0014 | .0015 | .0017 | .0019 | .0021 | .0024 | .0026 | .0029 |
| 19 | .0005 | .0005 | .0006 | .0007 | .0008 | .0009 | .0010 | .0011 | .0012 | .0014 |
| 20 | .0002 | .0002 | .0002 | .0003 | .0003 | .0004 | .0004 | .0005 | .0005 | .0006 |
| 21 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 | .0002 | .0002 | .0003 |
| 22 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 |

TABLE 4 *continued*

| r | λ | | | | | | | | | |
|-----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 9.1 | 9.2 | 9.3 | 9.4 | 9.5 | 9.6 | 9.7 | 9.8 | 9.9 | 10 |
| 0 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0000 |
| 1 | .0010 | .0009 | .0009 | .0008 | .0007 | .0007 | .0006 | .0005 | .0005 | .0005 |
| 2 | .0046 | .0043 | .0040 | .0037 | .0034 | .0031 | .0029 | .0027 | .0025 | .0023 |
| 3 | .0140 | .0131 | .0123 | .0115 | .0107 | .0100 | .0093 | .0087 | .0081 | .0076 |
| 4 | .0319 | .0302 | .0285 | .0269 | .0254 | .0240 | .0226 | .0213 | .0201 | .0189 |
| 5 | .0581 | .0555 | .0530 | .0506 | .0483 | .0460 | .0439 | .0418 | .0398 | .0378 |
| 6 | .0881 | .0851 | .0822 | .0793 | .0764 | .0736 | .0709 | .0682 | .0656 | .0631 |
| 7 | .1145 | .1118 | .1091 | .1064 | .1037 | .1010 | .0982 | .0955 | .0928 | .0901 |
| 8 | .1302 | .1286 | .1269 | .1251 | .1232 | .1212 | .1191 | .1170 | .1148 | .1126 |
| 9 | .1317 | .1315 | .1311 | .1306 | .1300 | .1293 | .1284 | .1274 | .1263 | .1251 |
| 10 | .1198 | .1210 | .1219 | .1228 | .1235 | .1241 | .1245 | .1249 | .1250 | .1251 |
| 11 | .0991 | .1012 | .1031 | .1049 | .1067 | .1083 | .1098 | .1112 | .1125 | .1137 |
| 12 | .0752 | .0776 | .0799 | .0822 | .0844 | .0866 | .0888 | .0908 | .0928 | .0948 |
| 13 | .0526 | .0549 | .0572 | .0594 | .0617 | .0640 | .0662 | .0685 | .0707 | .0729 |
| 14 | .0342 | .0361 | .0380 | .0399 | .0419 | .0439 | .0459 | .0479 | .0500 | .0521 |
| 15 | .0208 | .0221 | .0235 | .0250 | .0265 | .0281 | .0297 | .0313 | .0330 | .0347 |
| 16 | .0118 | .0127 | .0137 | .0147 | .0157 | .0168 | .0180 | .0192 | .0204 | .0217 |
| 17 | .0063 | .0069 | .0075 | .0081 | .0088 | .0095 | .0103 | .0111 | .0119 | .0128 |
| 18 | .0032 | .0035 | .0039 | .0042 | .0046 | .0051 | .0055 | .0060 | .0065 | .0071 |
| 19 | .0015 | .0017 | .0019 | .0021 | .0023 | .0026 | .0028 | .0031 | .0034 | .0037 |
| 20 | .0007 | .0008 | .0009 | .0010 | .0011 | .0012 | .0014 | .0015 | .0017 | .0019 |
| 21 | .0003 | .0003 | .0004 | .0004 | .0005 | .0006 | .0006 | .0007 | .0008 | .0009 |
| 22 | .0001 | .0001 | .0002 | .0002 | .0002 | .0002 | .0003 | .0003 | .0004 | .0004 |
| 23 | .0000 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0002 | .0002 |
| 24 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0001 |

TABLE 4 *continued*

| r | λ | | | | | | | | | |
|-----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 0 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| 1 | .0002 | .0001 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| 2 | .0010 | .0004 | .0002 | .0001 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |
| 3 | .0037 | .0018 | .0008 | .0004 | .0002 | .0001 | .0000 | .0000 | .0000 | .0000 |
| 4 | .0102 | .0053 | .0027 | .0013 | .0006 | .0003 | .0001 | .0001 | .0000 | .0000 |
| 5 | .0224 | .0127 | .0070 | .0037 | .0019 | .0010 | .0005 | .0002 | .0001 | .0001 |
| 6 | .0411 | .0255 | .0152 | .0087 | .0048 | .0026 | .0014 | .0007 | .0004 | .0002 |
| 7 | .0646 | .0437 | .0281 | .0174 | .0104 | .0060 | .0034 | .0018 | .0010 | .0005 |
| 8 | .0888 | .0655 | .0457 | .0304 | .0194 | .0120 | .0072 | .0042 | .0024 | .0013 |
| 9 | .1085 | .0874 | .0661 | .0473 | .0324 | .0213 | .0135 | .0083 | .0050 | .0029 |
| 10 | .1194 | .1048 | .0859 | .0663 | .0486 | .0341 | .0230 | .0150 | .0095 | .0058 |
| 11 | .1194 | .1144 | .1015 | .0844 | .0663 | .0496 | .0355 | .0245 | .0164 | .0106 |
| 12 | .1094 | .1144 | .1099 | .0984 | .0829 | .0661 | .0504 | .0368 | .0259 | .0176 |
| 13 | .0926 | .1056 | .1099 | .1060 | .0956 | .0814 | .0658 | .0509 | .0378 | .0271 |
| 14 | .0728 | .0905 | .1021 | .1060 | .1024 | .0930 | .0800 | .0655 | .0514 | .0387 |
| 15 | .0534 | .0724 | .0885 | .0989 | .1024 | .0992 | .0906 | .0786 | .0650 | .0516 |
| 16 | .0367 | .0543 | .0719 | .0866 | .0960 | .0992 | .0963 | .0884 | .0772 | .0646 |
| 17 | .0237 | .0383 | .0550 | .0713 | .0847 | .0934 | .0963 | .0936 | .0863 | .0760 |
| 18 | .0145 | .0256 | .0397 | .0554 | .0706 | .0830 | .0909 | .0936 | .0911 | .0844 |
| 19 | .0084 | .0161 | .0272 | .0409 | .0557 | .0699 | .0814 | .0887 | .0911 | .0888 |
| 20 | .0046 | .0097 | .0177 | .0286 | .0418 | .0559 | .0692 | .0798 | .0866 | .0888 |
| 21 | .0024 | .0055 | .0109 | .0191 | .0299 | .0426 | .0560 | .0684 | .0783 | .0846 |
| 22 | .0012 | .0030 | .0065 | .0121 | .0204 | .0310 | .0433 | .0560 | .0676 | .0769 |
| 23 | .0006 | .0016 | .0037 | .0074 | .0133 | .0216 | .0320 | .0438 | .0559 | .0669 |
| 24 | .0003 | .0008 | .0020 | .0043 | .0083 | .0144 | .0226 | .0328 | .0442 | .0557 |
| 25 | .0001 | .0004 | .0010 | .0024 | .0050 | .0092 | .0154 | .0237 | .0336 | .0446 |
| 26 | .0000 | .0002 | .0005 | .0013 | .0029 | .0057 | .0101 | .0164 | .0246 | .0343 |
| 27 | .0000 | .0001 | .0002 | .0007 | .0016 | .0034 | .0063 | .0109 | .0173 | .0254 |
| 28 | .0000 | .0000 | .0001 | .0003 | .0009 | .0019 | .0038 | .0070 | .0117 | .0181 |
| 29 | .0000 | .0000 | .0001 | .0002 | .0004 | .0011 | .0023 | .0044 | .0077 | .0125 |
| 30 | .0000 | .0000 | .0000 | .0001 | .0002 | .0006 | .0013 | .0026 | .0049 | .0083 |
| 31 | .0000 | .0000 | .0000 | .0000 | .0001 | .0003 | .0007 | .0015 | .0030 | .0054 |
| 32 | .0000 | .0000 | .0000 | .0000 | .0001 | .0001 | .0004 | .0009 | .0018 | .0034 |
| 33 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0002 | .0005 | .0010 | .0020 |
| 34 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0002 | .0006 | .0012 |
| 35 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0003 | .0007 |
| 36 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0002 | .0004 |
| 37 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0002 |
| 38 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 |
| 39 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 |

Source: Extracted from William H. Beyer (ed.), *CRC Basic Statistical Tables* (Cleveland, Ohio: The Chemical Reber Co. 1971).

TABLE 5 Areas of a Standard Normal Distribution(a) Table of Areas to the Left of z

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

For values of z less than -3.49 , use 0.000 to approximate the area.

TABLE 5(a) *continued*

| <i>z</i> | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

For *z* values greater than 3.49, use 1.000 to approximate the area.

TABLE 5 *continued*

| (b) Confidence Interval, Critical Values z_c | |
|--|----------------------|
| Level of Confidence c | Critical Value z_c |
| 0.75, or 75% | 1.15 |
| 0.80, or 80% | 1.28 |
| 0.85, or 85% | 1.44 |
| 0.90, or 95% | 1.645 |
| 0.95, or 95% | 1.96 |
| 0.98, or 98% | 2.33 |
| 0.99, or 99% | 2.58 |

TABLE 5 *continued*

| (c) Hypothesis Testing, Critical Values z_0 | | |
|---|-----------------|-----------------|
| Level of Significance | $\alpha = 0.05$ | $\alpha = 0.01$ |
| Critical value z_0 for a left-tailed test | -1.645 | -2.33 |
| Critical value z_0 for a right-tailed test | 1.645 | 2.33 |
| Critical value $\pm z_0$ for a two-tailed test | ± 1.96 | ± 2.58 |

TABLE 6 Student's *t* Distribution

| Student's <i>t</i> values generated by Minitab Version 9.2 | | | | | | | | |
|--|------------|-------|-------|-------|-------|--------|--------|--------|
| <i>d.f.</i> | <i>c</i> | 0.750 | 0.800 | 0.850 | 0.900 | 0.950 | 0.980 | 0.990 |
| | α' | 0.125 | 0.100 | 0.075 | 0.050 | 0.025 | 0.010 | 0.005 |
| | α'' | 0.250 | 0.200 | 0.150 | 0.100 | 0.050 | 0.020 | 0.010 |
| 1 | | 2.414 | 3.078 | 4.165 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | | 1.604 | 1.886 | 2.282 | 2.290 | 4.303 | 6.965 | 9.925 |
| 3 | | 1.423 | 1.638 | 1.924 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | | 1.344 | 1.533 | 1.778 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | | 1.301 | 1.476 | 1.699 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | | 1.273 | 1.440 | 1.650 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | | 1.254 | 1.415 | 1.617 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | | 1.240 | 1.397 | 1.592 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | | 1.230 | 1.383 | 1.574 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | | 1.221 | 1.372 | 1.559 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | | 1.214 | 1.363 | 1.548 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | | 1.209 | 1.356 | 1.538 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | | 1.204 | 1.350 | 1.530 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | | 1.200 | 1.345 | 1.523 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | | 1.197 | 1.341 | 1.517 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | | 1.194 | 1.337 | 1.512 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | | 1.191 | 1.333 | 1.508 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | | 1.189 | 1.330 | 1.504 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | | 1.187 | 1.328 | 1.500 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | | 1.185 | 1.325 | 1.497 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | | 1.183 | 1.323 | 1.494 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | | 1.182 | 1.321 | 1.492 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | | 1.180 | 1.319 | 1.489 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | | 1.179 | 1.318 | 1.487 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | | 1.178 | 1.316 | 1.485 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | | 1.177 | 1.315 | 1.483 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | | 1.176 | 1.314 | 1.482 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | | 1.175 | 1.313 | 1.480 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | | 1.174 | 1.311 | 1.479 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | | 1.173 | 1.310 | 1.477 | 1.697 | 2.042 | 2.457 | 2.750 |
| 35 | | 1.170 | 1.306 | 1.472 | 1.690 | 2.030 | 2.438 | 2.724 |
| 40 | | 1.167 | 1.303 | 1.468 | 1.684 | 2.021 | 2.423 | 2.704 |
| 45 | | 1.165 | 1.301 | 1.465 | 1.679 | 2.014 | 2.412 | 2.690 |
| 50 | | 1.164 | 1.299 | 1.462 | 1.676 | 2.009 | 2.403 | 2.678 |
| 55 | | 1.163 | 1.297 | 1.460 | 1.673 | 2.004 | 2.396 | 2.668 |
| 60 | | 1.162 | 1.296 | 1.458 | 1.671 | 2.000 | 2.390 | 2.660 |
| 90 | | 1.158 | 1.291 | 1.452 | 1.662 | 1.987 | 2.369 | 2.632 |
| 120 | | 1.156 | 1.289 | 1.449 | 1.658 | 1.980 | 2.358 | 2.617 |
| ∞ | | 1.150 | 1.282 | 1.440 | 1.645 | 1.960 | 2.326 | 2.576 |

TABLE 7 The χ^2 Distribution

| <i>d.f.</i> \alpha | .995 | .990 | .975 | .950 | .900 | .100 | .050 | .025 | .010 | .005 |
|--------------------|----------------------|----------------------|----------------------|----------------------|--------|-------|-------|-------|-------|-------|
| 1 | 0.0 ⁴ 393 | 0.0 ³ 157 | 0.0 ³ 982 | 0.0 ² 393 | 0.0158 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 |
| 2 | 0.0100 | 0.0201 | 0.0506 | 0.103 | 0.211 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 |
| 6 | 0.676 | 0.872 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 |
| 7 | 0.989 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 17.28 | 19.68 | 21.92 | 24.72 | 26.76 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 20 | 7.43 | 8.26 | 8.59 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 21 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 23 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 |
| 27 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 |
| 29 | 13.21 | 14.26 | 16.05 | 17.71 | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 51.80 | 55.76 | 59.34 | 63.69 | 66.77 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 |

Source: From H. L. Herter, *Biometrika*, June 1964. Printed by permission of Biometrika Trustees.

TABLE 8 The F Distribution

| Degrees of Freedom for Denominator | α | Degrees of Freedom for Numerator | | | | | | | | |
|------------------------------------|----------|----------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | .050 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| 1 | .025 | 647.79 | 799.50 | 864.16 | 899.58 | 921.85 | 937.11 | 948.22 | 956.66 | 963.28 |
| 1 | .010 | 4052.2 | 4999.5 | 5403.4 | 5624.6 | 5763.6 | 5859.0 | 5928.4 | 5981.1 | 6022.5 |
| 2 | .050 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 |
| 2 | .025 | 38.51 | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 |
| 2 | .010 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 |
| 3 | .050 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| 3 | .025 | 17.44 | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 |
| 3 | .010 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 |
| 4 | .050 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 |
| 4 | .025 | 12.22 | 10.65 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.90 |
| 4 | .010 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 |
| 5 | .050 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 |
| 5 | .025 | 10.01 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.68 |
| 5 | .010 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 |
| 6 | .050 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 |
| 6 | .025 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.52 |
| 6 | .010 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 |
| 7 | .050 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 |
| 7 | .025 | 8.07 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.82 |
| 7 | .010 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 |
| 8 | .050 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| 8 | .025 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 |
| 8 | .010 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| 9 | .050 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| 9 | .025 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 |
| 9 | .010 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| 10 | .050 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| 10 | .025 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 |
| 10 | .010 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| 11 | .050 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 |
| 11 | .025 | 6.72 | 5.26 | 4.63 | 4.28 | 4.04 | 3.88 | 3.76 | 3.66 | 3.59 |
| 11 | .010 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 |
| 12 | .050 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| 12 | .025 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 |
| 12 | .010 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |

TABLE 8 *continued*

| Degrees of Freedom for Numerator | | | | | | | | | | |
|----------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 1000 |
| 241.88 | 243.91 | 245.95 | 248.01 | 249.26 | 250.10 | 251.14 | 251.77 | 252.20 | 253.25 | 254.19 |
| 968.63 | 976.71 | 984.87 | 993.10 | 998.08 | 1001.4 | 1005.6 | 1008.1 | 1009.8 | 1014.0 | 1017.7 |
| 6055.8 | 6106.3 | 6157.3 | 6208.7 | 6239.8 | 6260.6 | 6286.8 | 6302.5 | 6313.0 | 6339.4 | 6362.7 |
| 19.40 | 19.41 | 19.43 | 19.45 | 19.46 | 19.46 | 19.47 | 19.48 | 19.48 | 19.49 | 19.49 |
| 39.40 | 39.41 | 39.43 | 39.45 | 39.46 | 39.46 | 39.47 | 39.48 | 39.48 | 39.49 | 39.50 |
| 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.48 | 99.49 | 99.50 |
| 8.79 | 8.74 | 8.70 | 8.66 | 8.63 | 8.62 | 8.59 | 8.58 | 8.57 | 8.55 | 8.53 |
| 14.42 | 14.34 | 14.25 | 14.17 | 14.12 | 14.08 | 14.04 | 14.01 | 13.99 | 13.95 | 13.91 |
| 27.23 | 27.05 | 26.87 | 26.69 | 26.58 | 26.50 | 26.41 | 26.35 | 26.32 | 26.22 | 26.14 |
| 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.70 | 5.69 | 5.66 | 5.63 |
| 8.84 | 8.75 | 8.66 | 8.56 | 8.580 | 8.46 | 8.41 | 8.38 | 8.36 | 8.31 | 8.26 |
| 14.55 | 14.37 | 14.20 | 14.02 | 13.91 | 13.84 | 13.75 | 13.69 | 13.65 | 13.56 | 13.47 |
| 4.74 | 4.68 | 4.62 | 4.56 | 4.52 | 4.50 | 4.46 | 4.44 | 4.43 | 4.40 | 4.37 |
| 6.62 | 6.52 | 6.43 | 6.33 | 6.27 | 6.23 | 6.18 | 6.14 | 6.12 | 6.07 | 6.02 |
| 10.05 | 9.89 | 9.72 | 9.55 | 9.45 | 9.38 | 9.29 | 9.24 | 9.20 | 9.11 | 9.03 |
| 4.06 | 4.00 | 3.94 | 3.87 | 3.83 | 3.81 | 3.77 | 3.75 | 3.74 | 3.70 | 3.67 |
| 5.46 | 5.37 | 5.27 | 5.17 | 5.11 | 5.07 | 5.01 | 4.98 | 4.96 | 4.90 | 4.86 |
| 7.87 | 7.72 | 7.56 | 7.40 | 7.30 | 7.23 | 7.14 | 7.09 | 7.06 | 6.97 | 6.89 |
| 3.64 | 3.57 | 3.51 | 3.44 | 3.40 | 3.38 | 3.34 | 3.32 | 3.30 | 3.27 | 3.23 |
| 4.76 | 4.67 | 4.57 | 4.47 | 4.40 | 4.36 | 4.31 | 4.28 | 4.25 | 4.20 | 4.15 |
| 6.62 | 6.47 | 6.31 | 6.16 | 6.06 | 5.99 | 5.91 | 5.86 | 5.82 | 5.74 | 5.66 |
| 3.35 | 3.28 | 3.22 | 3.15 | 3.11 | 3.08 | 3.04 | 3.02 | 3.01 | 2.97 | 2.93 |
| 4.30 | 4.20 | 4.10 | 4.00 | 3.94 | 3.89 | 3.84 | 3.81 | 3.78 | 3.73 | 3.68 |
| 5.81 | 5.67 | 5.52 | 5.36 | 5.26 | 5.20 | 5.12 | 5.07 | 5.03 | 4.95 | 4.87 |
| 3.14 | 3.07 | 3.01 | 2.94 | 2.89 | 2.86 | 2.83 | 2.80 | 2.79 | 2.75 | 2.71 |
| 3.96 | 3.87 | 3.77 | 3.67 | 3.60 | 3.56 | 3.51 | 3.47 | 3.45 | 3.39 | 3.34 |
| 5.26 | 5.11 | 4.96 | 4.81 | 4.71 | 4.65 | 4.57 | 4.52 | 4.48 | 4.40 | 4.32 |
| 2.98 | 2.91 | 2.85 | 2.77 | 2.73 | 2.70 | 2.66 | 2.64 | 2.62 | 2.58 | 2.54 |
| 3.72 | 3.62 | 3.52 | 3.42 | 3.35 | 3.31 | 3.26 | 3.22 | 3.20 | 3.14 | 3.09 |
| 4.85 | 4.71 | 4.56 | 4.41 | 4.31 | 4.25 | 4.17 | 4.12 | 4.08 | 4.00 | 3.92 |
| 2.85 | 2.79 | 2.72 | 2.65 | 2.60 | 2.57 | 2.53 | 2.51 | 2.49 | 2.45 | 2.41 |
| 3.53 | 3.43 | 3.33 | 3.23 | 3.16 | 3.12 | 3.06 | 3.03 | 3.00 | 2.94 | 2.89 |
| 4.54 | 4.40 | 4.25 | 4.10 | 4.01 | 3.94 | 3.86 | 3.81 | 3.78 | 3.69 | 3.61 |
| 2.75 | 2.69 | 2.62 | 2.54 | 2.50 | 2.47 | 2.43 | 2.40 | 2.38 | 2.34 | 2.30 |
| 3.37 | 3.28 | 3.18 | 3.07 | 3.01 | 2.96 | 2.91 | 2.87 | 2.85 | 2.79 | 2.73 |
| 4.30 | 4.16 | 4.01 | 3.86 | 3.76 | 3.70 | 3.62 | 3.57 | 3.54 | 3.45 | 3.37 |

TABLE 8 *continued*

| Degrees of Freedom for Denominator | α | Degrees of Freedom for Numerator | | | | | | | | |
|------------------------------------|----------|----------------------------------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 13 | .050 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 |
| 13 | .025 | 6.41 | 4.97 | 4.35 | 4.00 | 3.77 | 3.60 | 3.48 | 3.39 | 3.31 |
| 13 | .010 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 |
| 14 | .050 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 |
| 14 | .025 | 6.30 | 4.86 | 4.24 | 3.89 | 3.66 | 3.50 | 3.38 | 3.29 | 3.21 |
| 14 | .010 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 |
| 15 | .050 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| 15 | .025 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 |
| 15 | .010 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| 16 | .050 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 |
| 16 | .025 | 6.12 | 4.69 | 4.08 | 3.73 | 3.50 | 3.34 | 3.22 | 3.12 | 3.05 |
| 16 | .010 | 8.53 | 6.12 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 |
| 17 | .050 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 |
| 17 | .025 | 6.04 | 4.62 | 4.01 | 3.66 | 3.44 | 3.28 | 3.16 | 3.06 | 2.98 |
| 17 | .010 | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 |
| 18 | .050 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |
| 18 | .025 | 5.98 | 4.56 | 3.95 | 3.61 | 3.38 | 3.22 | 3.10 | 3.01 | 2.93 |
| 18 | .010 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |
| 19 | .050 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
| 19 | .025 | 5.92 | 4.51 | 3.90 | 3.56 | 3.33 | 3.17 | 3.05 | 2.96 | 2.88 |
| 19 | .010 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
| 20 | .050 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| 20 | .025 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 |
| 20 | .010 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| 21 | .050 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
| 21 | .025 | 5.83 | 4.42 | 3.82 | 3.48 | 3.25 | 3.09 | 2.97 | 2.87 | 2.80 |
| 21 | .010 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
| 22 | .050 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |
| 22 | .025 | 5.79 | 4.38 | 3.78 | 3.44 | 3.22 | 3.05 | 2.93 | 2.84 | 2.76 |
| 22 | .010 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |
| 23 | .050 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |
| 23 | .025 | 5.75 | 4.35 | 3.75 | 3.41 | 3.18 | 3.02 | 2.90 | 2.81 | 2.73 |
| 23 | .010 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |
| 24 | .050 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| 24 | .025 | 5.72 | 4.32 | 3.72 | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 |
| 24 | .010 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |

TABLE 8 *continued*

| Degrees of Freedom for Numerator | | | | | | | | | | |
|----------------------------------|------|------|------|------|------|------|------|------|------|------|
| 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 1000 |
| 2.67 | 2.60 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 | 2.31 | 2.30 | 2.25 | 2.21 |
| 3.25 | 3.15 | 3.05 | 2.95 | 2.88 | 2.84 | 2.78 | 2.74 | 2.72 | 2.66 | 2.60 |
| 4.10 | 3.96 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 | 3.38 | 3.34 | 3.25 | 3.18 |
| 2.60 | 2.53 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 | 2.24 | 2.22 | 2.18 | 2.14 |
| 3.15 | 3.05 | 2.95 | 2.84 | 2.78 | 2.73 | 2.67 | 2.64 | 2.61 | 2.55 | 2.50 |
| 3.94 | 3.80 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 | 3.22 | 3.18 | 3.09 | 3.02 |
| 2.54 | 2.48 | 2.40 | 2.33 | 2.28 | 2.25 | 2.20 | 2.18 | 2.16 | 2.11 | 2.07 |
| 3.06 | 2.96 | 2.86 | 2.76 | 2.69 | 2.64 | 2.59 | 2.55 | 2.52 | 2.46 | 2.40 |
| 3.80 | 3.67 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 | 3.08 | 3.05 | 2.96 | 2.88 |
| 2.49 | 2.42 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 | 2.12 | 2.11 | 2.06 | 2.02 |
| 2.99 | 2.89 | 2.79 | 2.68 | 2.61 | 2.57 | 2.51 | 2.47 | 2.45 | 2.38 | 2.32 |
| 3.69 | 3.55 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 | 2.97 | 2.93 | 2.84 | 2.76 |
| 2.45 | 2.38 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 | 2.08 | 2.06 | 2.01 | 1.97 |
| 2.92 | 2.82 | 2.72 | 2.62 | 2.55 | 2.50 | 2.44 | 2.41 | 2.38 | 2.32 | 2.26 |
| 3.59 | 3.46 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 | 2.87 | 2.83 | 2.75 | 2.66 |
| 2.41 | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 | 2.04 | 2.02 | 1.97 | 1.92 |
| 2.87 | 2.77 | 2.67 | 2.56 | 2.49 | 2.44 | 2.38 | 2.35 | 2.32 | 2.26 | 2.20 |
| 3.51 | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 | 2.78 | 2.75 | 2.66 | 2.58 |
| 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 2.00 | 1.98 | 1.93 | 1.88 |
| 2.82 | 2.72 | 2.62 | 2.51 | 2.44 | 2.39 | 2.33 | 2.30 | 2.27 | 2.20 | 2.14 |
| 3.43 | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 | 2.71 | 2.67 | 2.58 | 2.50 |
| 2.35 | 2.28 | 2.20 | 2.12 | 2.07 | 2.04 | 1.99 | 1.97 | 1.95 | 1.90 | 1.85 |
| 2.77 | 2.68 | 2.57 | 2.46 | 2.40 | 2.35 | 2.29 | 2.25 | 2.22 | 2.16 | 2.09 |
| 3.37 | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 | 2.64 | 2.61 | 2.52 | 2.43 |
| 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.94 | 1.92 | 1.87 | 1.82 |
| 2.73 | 2.64 | 2.53 | 2.42 | 2.36 | 2.31 | 2.25 | 2.24 | 2.18 | 2.11 | 2.05 |
| 3.31 | 3.17 | 3.03 | 2.88 | 2.79 | 2.72 | 2.64 | 2.58 | 2.55 | 2.46 | 2.37 |
| 2.30 | 2.23 | 2.15 | 2.07 | 2.02 | 1.98 | 1.94 | 1.91 | 1.89 | 1.84 | 1.79 |
| 2.70 | 2.60 | 2.50 | 2.39 | 2.32 | 2.27 | 2.24 | 2.17 | 2.14 | 2.08 | 2.01 |
| 3.26 | 3.12 | 2.98 | 2.83 | 2.73 | 2.67 | 2.58 | 2.53 | 2.50 | 2.40 | 2.32 |
| 2.27 | 2.20 | 2.13 | 2.05 | 2.00 | 1.96 | 1.91 | 1.88 | 1.86 | 1.81 | 1.76 |
| 2.67 | 2.57 | 2.47 | 2.36 | 2.29 | 2.24 | 2.18 | 2.14 | 2.11 | 2.04 | 1.98 |
| 3.21 | 3.07 | 2.93 | 2.78 | 2.69 | 2.62 | 2.54 | 2.48 | 2.45 | 2.35 | 2.27 |
| 2.25 | 2.18 | 2.11 | 2.03 | 1.97 | 1.94 | 1.89 | 1.86 | 1.84 | 1.79 | 1.74 |
| 2.64 | 2.54 | 2.44 | 2.33 | 2.26 | 2.21 | 2.15 | 2.11 | 2.08 | 2.01 | 1.94 |
| 3.17 | 3.03 | 2.89 | 2.74 | 2.64 | 2.58 | 2.49 | 2.44 | 2.40 | 2.31 | 2.22 |

TABLE 8 *continued*

| Degrees of Freedom for Denominator | α | Degrees of Freedom for Numerator | | | | | | | | |
|------------------------------------|----------|----------------------------------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 25 | .050 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |
| 25 | .025 | 5.69 | 4.29 | 3.69 | 3.35 | 3.13 | 2.97 | 2.85 | 2.75 | 2.68 |
| 25 | .010 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |
| 26 | .050 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |
| 26 | .025 | 5.66 | 4.27 | 3.67 | 3.33 | 3.10 | 2.94 | 2.82 | 2.73 | 2.65 |
| 26 | .010 | 7.72 | 2.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |
| 27 | .050 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |
| 27 | .025 | 5.63 | 4.24 | 3.65 | 3.31 | 3.08 | 2.92 | 2.80 | 2.71 | 2.63 |
| 27 | .010 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |
| 28 | .050 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 |
| 28 | .025 | 5.61 | 4.22 | 3.63 | 3.29 | 3.06 | 2.90 | 2.78 | 2.69 | 2.61 |
| 28 | .010 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 |
| 29 | .050 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 |
| 29 | .025 | 5.59 | 4.20 | 3.61 | 3.27 | 3.04 | 2.88 | 2.76 | 2.67 | 2.59 |
| 29 | .010 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 |
| 30 | .050 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| 30 | .025 | 5.57 | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 |
| 30 | .010 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| 40 | .050 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 |
| 40 | .025 | 5.42 | 4.05 | 3.46 | 3.13 | 2.90 | 2.74 | 2.62 | 2.53 | 2.45 |
| 40 | .010 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 3.99 | 2.89 |
| 50 | .050 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 |
| 50 | .025 | 5.34 | 3.97 | 3.39 | 3.05 | 2.83 | 2.67 | 2.55 | 2.46 | 2.38 |
| 50 | .010 | 7.17 | 5.16 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 |
| 60 | .050 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| 60 | .025 | 5.29 | 3.93 | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 |
| 60 | .010 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| 100 | .050 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 |
| 100 | .025 | 5.18 | 3.83 | 3.25 | 2.92 | 2.70 | 2.54 | 2.42 | 2.32 | 2.24 |
| 100 | .010 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 |
| 200 | .050 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 |
| 200 | .025 | 5.10 | 3.76 | 3.18 | 2.85 | 2.63 | 2.47 | 2.35 | 2.26 | 2.18 |
| 200 | .010 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 |
| 1000 | .050 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 |
| 1000 | .025 | 5.04 | 3.70 | 3.13 | 2.80 | 2.58 | 2.42 | 2.30 | 2.20 | 2.13 |
| 1000 | .010 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 |

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TABLE 8 *continued*

| Degrees of Freedom for Numerator | | | | | | | | | | |
|----------------------------------|------|------|------|------|------|------|------|------|------|------|
| 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 1000 |
| 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.84 | 1.82 | 1.77 | 1.72 |
| 2.61 | 2.51 | 2.41 | 2.30 | 2.23 | 2.18 | 2.12 | 2.08 | 2.05 | 1.98 | 1.91 |
| 3.13 | 2.99 | 2.85 | 2.70 | 2.60 | 2.54 | 2.45 | 2.40 | 2.36 | 2.27 | 2.18 |
| 2.22 | 2.15 | 2.07 | 1.99 | 1.94 | 1.90 | 1.85 | 1.82 | 1.80 | 1.75 | 1.70 |
| 2.59 | 2.49 | 2.39 | 2.28 | 2.21 | 2.16 | 2.09 | 2.05 | 2.03 | 1.95 | 1.89 |
| 3.09 | 2.96 | 2.81 | 2.66 | 2.57 | 2.50 | 2.42 | 2.36 | 2.33 | 2.23 | 2.14 |
| 2.20 | 2.13 | 2.06 | 1.97 | 1.92 | 1.88 | 1.84 | 1.81 | 1.79 | 1.73 | 1.68 |
| 2.57 | 2.47 | 2.36 | 2.25 | 2.18 | 2.13 | 2.07 | 2.03 | 2.00 | 1.93 | 1.86 |
| 3.06 | 2.93 | 2.78 | 2.63 | 5.54 | 2.47 | 2.38 | 2.33 | 2.29 | 2.20 | 2.11 |
| 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.79 | 1.44 | 1.71 | 1.66 |
| 2.55 | 2.45 | 2.34 | 2.23 | 2.16 | 2.11 | 2.05 | 2.01 | 1.98 | 1.91 | 1.84 |
| 3.03 | 2.90 | 2.75 | 2.60 | 2.51 | 2.44 | 2.35 | 2.30 | 2.26 | 2.17 | 2.08 |
| 2.18 | 2.10 | 2.03 | 1.94 | 1.89 | 1.85 | 1.81 | 1.77 | 1.75 | 1.70 | 1.65 |
| 2.53 | 2.43 | 2.32 | 2.21 | 2.14 | 2.09 | 2.03 | 1.99 | 1.96 | 1.89 | 1.82 |
| 3.00 | 2.87 | 2.73 | 2.57 | 2.48 | 2.41 | 2.33 | 2.27 | 2.23 | 2.14 | 2.05 |
| 2.16 | 2.09 | 2.01 | 1.93 | 1.88 | 1.84 | 1.79 | 1.76 | 1.74 | 1.68 | 1.63 |
| 2.51 | 2.41 | 2.31 | 2.20 | 2.12 | 2.07 | 2.01 | 1.97 | 1.94 | 1.87 | 1.80 |
| 2.98 | 2.84 | 2.70 | 2.55 | 2.45 | 2.39 | 2.30 | 2.25 | 2.21 | 2.11 | 2.02 |
| 2.08 | 2.00 | 1.92 | 1.84 | 1.78 | 1.74 | 1.69 | 1.66 | 1.64 | 1.58 | 1.52 |
| 2.39 | 2.29 | 2.18 | 2.07 | 1.99 | 1.94 | 1.88 | 1.83 | 1.80 | 1.72 | 1.65 |
| 2.80 | 2.66 | 2.52 | 2.37 | 2.27 | 2.20 | 2.11 | 2.06 | 2.02 | 1.92 | 1.82 |
| 2.03 | 1.95 | 1.87 | 1.78 | 1.73 | 1.69 | 1.63 | 1.60 | 1.58 | 1.51 | 1.45 |
| 2.32 | 2.22 | 2.11 | 1.99 | 1.92 | 1.87 | 1.80 | 1.75 | 1.72 | 1.64 | 1.56 |
| 2.70 | 2.56 | 2.42 | 2.27 | 2.17 | 2.10 | 2.01 | 1.95 | 1.91 | 1.80 | 1.70 |
| 1.99 | 1.92 | 1.84 | 1.75 | 1.69 | 1.65 | 1.59 | 1.56 | 1.53 | 1.47 | 1.40 |
| 2.27 | 2.17 | 2.06 | 1.94 | 1.87 | 1.82 | 1.74 | 1.70 | 1.67 | 1.58 | 1.49 |
| 2.63 | 2.50 | 2.35 | 2.20 | 2.10 | 2.03 | 1.94 | 1.88 | 1.84 | 1.73 | 1.62 |
| 1.93 | 1.85 | 1.77 | 1.68 | 1.62 | 1.57 | 1.52 | 1.48 | 1.45 | 1.38 | 1.30 |
| 2.18 | 2.08 | 1.97 | 1.85 | 1.77 | 1.71 | 1.64 | 1.59 | 1.56 | 1.46 | 1.36 |
| 2.50 | 2.37 | 2.22 | 2.07 | 1.97 | 1.89 | 1.80 | 1.74 | 1.69 | 1.57 | 1.45 |
| 1.88 | 1.80 | 1.72 | 1.62 | 1.56 | 1.52 | 1.46 | 1.41 | 1.39 | 1.30 | 1.21 |
| 2.11 | 2.01 | 1.90 | 1.78 | 1.70 | 1.64 | 1.56 | 1.51 | 1.47 | 1.37 | 1.25 |
| 2.41 | 2.27 | 2.46 | 1.97 | 1.87 | 1.79 | 1.69 | 1.63 | 1.58 | 1.45 | 1.30 |
| 1.84 | 1.76 | 1.68 | 1.58 | 1.52 | 1.47 | 1.41 | 1.36 | 1.33 | 1.24 | 1.11 |
| 2.06 | 1.96 | 1.85 | 1.72 | 1.64 | 1.58 | 1.50 | 1.45 | 1.41 | 1.29 | 1.13 |
| 2.34 | 2.20 | 2.06 | 1.90 | 1.79 | 1.72 | 1.61 | 1.54 | 1.50 | 1.35 | 1.16 |

TABLE 9 Critical Values for Spearman Rank Correlation, r_s

For a right- (left-) tailed test, use the positive (negative) critical value found in the table under significance level for a one-tailed test. For a two-tailed test, use both the positive and negative of the critical value found in the table under significance level for a two-tailed test, n = number of pairs.

| n | Significance level for a one-tailed test at | | | |
|-----|---|-------|-------|-------|
| | 0.05 | 0.025 | 0.005 | 0.001 |
| | Significance level for a two-tailed test at | | | |
| | 0.10 | 0.05 | 0.01 | 0.002 |
| 5 | 0.900 | 1.000 | | |
| 6 | 0.829 | 0.886 | 1.000 | |
| 7 | 0.715 | 0.786 | 0.929 | 1.000 |
| 8 | 0.620 | 0.715 | 0.881 | 0.953 |
| 9 | 0.600 | 0.700 | 0.834 | 0.917 |
| 10 | 0.564 | 0.649 | 0.794 | 0.879 |
| 11 | 0.537 | 0.619 | 0.764 | 0.855 |
| 12 | 0.504 | 0.588 | 0.735 | 0.826 |
| 13 | 0.484 | 0.561 | 0.704 | 0.797 |
| 14 | 0.464 | 0.539 | 0.680 | 0.772 |
| 15 | 0.447 | 0.522 | 0.658 | 0.750 |
| 16 | 0.430 | 0.503 | 0.636 | 0.730 |
| 17 | 0.415 | 0.488 | 0.618 | 0.711 |
| 18 | 0.402 | 0.474 | 0.600 | 0.693 |
| 19 | 0.392 | 0.460 | 0.585 | 0.676 |
| 20 | 0.381 | 0.447 | 0.570 | 0.661 |
| 21 | 0.371 | 0.437 | 0.556 | 0.647 |
| 22 | 0.361 | 0.426 | 0.544 | 0.633 |
| 23 | 0.353 | 0.417 | 0.532 | 0.620 |
| 24 | 0.345 | 0.407 | 0.521 | 0.608 |
| 25 | 0.337 | 0.399 | 0.511 | 0.597 |
| 26 | 0.331 | 0.391 | 0.501 | 0.587 |
| 27 | 0.325 | 0.383 | 0.493 | 0.577 |
| 28 | 0.319 | 0.376 | 0.484 | 0.567 |
| 29 | 0.312 | 0.369 | 0.475 | 0.558 |
| 30 | 0.307 | 0.363 | 0.467 | 0.549 |

Source: From G. J. Glasser and R. F. Winter, "Critical Values of the Coefficient of Rank Correlation for Testing the Hypothesis of Independence," *Biometrika*, 48, 444 (1961). Printed by permission of Biometrika Trustees.

FREQUENTLY USED FORMULAS

n = sample size N = population size f = frequency

Chapter 2

$$\text{Class Width} = \frac{\text{high} - \text{low}}{\text{number classes}} \quad (\text{increase to next integer})$$

$$\text{Class Midpoint} = \frac{\text{upper limit} + \text{lower limit}}{2}$$

Lower boundary = lower boundary of previous class + class width

Chapter 3

$$\text{Sample mean} \quad \bar{x} = \frac{\sum x}{n}$$

$$\text{Population mean} \quad \mu = \frac{\sum x}{N}$$

$$\text{Weighted average} = \frac{\sum xw}{\sum w}$$

Range = largest data value – smallest data value

$$\text{Sample standard deviation} \quad s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$\text{Computation formula} \quad s = \sqrt{\frac{SS_x}{n - 1}} \quad \text{where } SS_x = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$\text{Population standard deviation} \quad \sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

Sample variance s^2

Population variance σ^2

$$\text{Sample Coefficient of Variation} \quad CV = \frac{s}{\bar{x}} \cdot 100$$

$$\text{Sample mean for grouped data} \quad \bar{x} = \frac{\sum xf}{n}$$

$$\text{Sample standard deviation for grouped data} \quad s = \sqrt{\frac{\sum (x - \bar{x})^2 f}{n - 1}} = \sqrt{\frac{\sum x^2 f - (\sum xf)^2 / n}{n - 1}}$$

Chapter 4

Probability of the complement of event A $P(\text{not } A) = 1 - P(A)$

Multiplication rule for independent events $P(A \text{ and } B) = P(A) \cdot P(B)$

General multiplication rules $P(A \text{ and } B) = P(A) \cdot P(B, \text{ given } A)$
 $P(A \text{ and } B) = P(B) \cdot P(A, \text{ given } B)$

Addition rule for mutually exclusive events $P(A \text{ or } B) = P(A) + P(B)$

General addition rule $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Permutation rule $P_{n,r} = \frac{n!}{(n-r)!}$

Combination rule $C_{n,r} = \frac{n!}{r!(n-r)!}$

Chapter 5

Mean of a discrete probability distribution $\mu = \sum xP(x)$

Standard deviation of a discrete probability distribution $\sigma = \sqrt{\sum (x - \mu)^2 P(x)}$

Given $L = a + bx$ $\mu_L = a + b\mu$
 $\sigma_L = |b|\sigma$

Given $W = ax_1 + bx_2$ (x_1 and x_2 independent) $\mu_W = a\mu_1 + b\mu_2$
 $\sigma_W = \sqrt{a^2\sigma_1^2 + b^2\sigma_2^2}$

For Binomial Distributions

r = number of successes; p = probability of success; $q = 1 - p$

Binomial probability distribution $P(r) = \frac{n!}{r!(n-r)!} p^r q^{n-r}$

Mean $\mu = np$

Standard deviation $\sigma = \sqrt{npq}$

Geometric Probability Distribution

n = number of trial on which first success occurs $P(n) = p(1-p)^{n-1}$

Poisson Probability Distribution

λ = mean number of successes over given interval $P(\lambda) = \frac{e^{-\lambda} \lambda^r}{r!}$

Chapter 6

Raw score $x = z\sigma + \mu$

Standard score $z = \frac{x - \mu}{\sigma}$

Chapter 7

Mean of \bar{x} distribution $\mu_{\bar{x}} = \mu$

Standard deviation of \bar{x} distribution $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Mean of \hat{p} distribution $\mu_{\hat{p}} = p$

Standard deviation of \hat{p} distribution $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$; $q = 1 - p$

Standard score for \bar{x} $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

Chapter 8

Confidence Interval

for μ ($n \geq 30$) $\bar{x} - z_c \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_c \frac{\sigma}{\sqrt{n}}$

for μ ($n < 30$); $d.f. = n - 1$ $\bar{x} - t_c \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_c \frac{s}{\sqrt{n}}$

for p ($np > 5$ and $nq > 5$) $\hat{p} - z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_c \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where $\hat{p} = r/n$

for difference of means ($n_1 \geq 30$ and $n_2 \geq 30$)

$$(\bar{x}_1 - \bar{x}_2) - z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

for difference of means ($n_1 < 30$ and/or $n_2 < 30$ and $\sigma_1 = \sigma_2$); $d.f. = n_1 + n_2 - 2$

$$(\bar{x}_1 - \bar{x}_2) - t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \text{ where } s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

for difference of proportions where $\hat{p}_1 = r_1/n_1$; $\hat{p}_2 = r_2/n_2$; $\hat{q}_1 = 1 - \hat{p}_1$; $\hat{q}_2 = 1 - \hat{p}_2$

$$(\hat{p}_1 - \hat{p}_2) - z_c \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_c \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

Sample Size for Estimating

$$\text{means } n = \left(\frac{z_c \sigma}{E} \right)^2$$

$$\text{proportions } n = p(1-p) \left(\frac{z_c}{E} \right)^2 \text{ with preliminary estimate for } p$$

$$n = \frac{1}{4} \left(\frac{z_c}{E} \right)^2 \text{ without preliminary estimate for } p$$

Chapter 9

Sample Test Statistics for Tests of Hypotheses

$$\text{for } \mu (n \geq 30) \quad z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$\text{for } \mu (n < 30); d.f. = n - 1 \quad t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$\text{for } p \quad z = \frac{\hat{p} - p}{\sqrt{pq/n}} \text{ where } q = 1 - p$$

$$\text{for paired differences } d \quad t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \text{ with } d.f. = n - 1$$

$$\text{difference of means large sample} \quad z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

difference of means small sample with $\sigma_1 = \sigma_2$; $d.f. = n_1 + n_2 - 2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{ where } s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$\text{difference of proportions} \quad z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}} \text{ where } \hat{p} = \frac{r_1 + r_2}{n_1 + n_2}; \hat{q} = 1 - \hat{p}; \hat{p}_1 = r_1 / n_1; \hat{p}_2 = r_2 / n_2$$

Chapter 10

Regression and Correlation

$$\text{In all these formulas } SS_x = \Sigma x^2 - \frac{(\Sigma x)^2}{n}, SS_y = \Sigma y^2 - \frac{(\Sigma y)^2}{n}, SS_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$\text{Least squares line } y = a + bx \text{ where } b = \frac{SS_{xy}}{SS_x} \text{ and } a = \bar{y} - b\bar{x}$$

$$\text{Standard error of estimate} \quad S_e = \sqrt{\frac{SS_y - bSS_{xy}}{n - 2}} \text{ where } b = \frac{SS_{xy}}{SS_x}$$

$$\text{Pearson product moment correlation coefficient} \quad r = \frac{SS_{xy}}{\sqrt{SS_x SS_y}}$$

Coefficient of determination $= r^2$

Confidence interval for y $y_p - E < y < y_p + E$ where y_p is the predicted y value for x

$$E = t_c S_e \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{SS_x}} \text{ with } d.f. = n - 2$$

$$\text{Sample test statistic for } r \quad t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \quad d.f. = n - 2$$

$$\text{Sample test statistic for } b \quad t = \frac{b - \beta}{S_e / \sqrt{SS_x}} \quad d.f. = n - 2$$

$$\text{Confidence interval for } \beta \quad b - t_c \frac{S_e}{\sqrt{SS_x}} < \beta < b + t_c \frac{S_e}{\sqrt{SS_x}} \quad d.f. = n - 2$$

Chapter 11

$$\chi^2 = \sum \frac{(O - E)^2}{E} \text{ where } E = \frac{(\text{row total})(\text{column total})}{\text{sample size}}$$

Tests of Independence $d.f. = (R - 1)(C - 1)$

Goodness of fit $d.f. = (\text{number of entries}) - 1$

$$\text{Confidence Interval for } \sigma^2; d.f. = n - 1 \quad \frac{(n-1)s^2}{\chi_U^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

$$\text{Sample test statistic for } H_0: \sigma^2 = k; d.f. = n - 1 \quad \chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Testing Two Variances

$$\text{Sample test statistic } F = \frac{s_1^2}{s_2^2} \text{ where } s_1^2 \geq s_2^2 \quad d.f._N = n_1 - 1; d.f._D = n_2 - 1$$

ANOVA

k = number of groups; N = total sample size

$$SS_{TOT} = \sum x_{TOT}^2 - \frac{(\sum x_{TOT})^2}{N}; SS_{BET} = \sum_{\text{all groups}} \left(\frac{(\sum x_i)^2}{n_i} \right) - \frac{(\sum x_{TOT})^2}{N}; SS_W = \sum_{\text{all groups}} \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n_i} \right)$$

$$SS_{TOT} = SS_{BET} + SS_W; MS_{BET} = \frac{SS_{BET}}{d.f._{BET}} \text{ where } d.f._{BET} = k - 1; MS_W = \frac{SS_W}{d.f._W} \text{ where } d.f._W = N - k;$$

$$F = \frac{MS_{BET}}{MS_W} \text{ where } d.f. \text{ numerator} = d.f._{BET} = k - 1; d.f. \text{ denominator} = d.f._W = N - k$$

Two-Way ANOVA

r = number of rows; c = number of columns

$$\text{Row factor } F: \frac{MS \text{ row factor}}{MS \text{ error}}; \text{ Column factor } F: \frac{MS \text{ column factor}}{MS \text{ error}}; \text{ Interaction } F: \frac{MS \text{ interaction}}{MS \text{ error}}$$

with degrees of freedom for

row factor = $r - 1$; interaction = $(r - 1)(c - 1)$; column factor = $c - 1$; error = $rc(n - 1)$

Chapter 12

$$\text{Sample test statistic for } x = \text{proportion of plus signs to all signs } (n \geq 12) \quad z = \frac{x - 0.5}{\sqrt{0.25/n}}$$

Sample test statistic for R = sum of ranks

$$z = \frac{R - \mu_R}{\sigma_R} \text{ where } \mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} \text{ and } \sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$$\text{Spearman rank correlation coefficient } r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} \text{ where } d = x - y$$