Chapter 3

Section 3-2

3-1. Continuous
3-2. Discrete
3-3. Continuous
3-4. Discrete
3-5. Discrete
3-6. Continuous
3-7. Discrete

Section 3-3

3-8. a) Engineers with at least 36 months of full-time employment.
b) Samples of cement blocks with compressive strength of at least 6000 kg per square centimeter.
c) Measurements of the diameter of forged pistons that conform to engineering specifications.
d) Cholesterol levels at most 180 or at least 220.

3-9. The intersection of A and B is empty, therefore $P(X \in A \cap B) = 0$.
a) Yes, since $P(X \in A \cap B) = 0$.
b) $P(X \in A') = 1 - P(X \in A) = 1 – 0.4 = 0.6$
c) $P(X \in B') = 1 - P(X \in B) = 1 – 0.6 = 0.4$
d) $P(X \in A \cup B) = P(X \in A) + P(X \in B) - P(X \in A \cap B) = 0.4 + 0.6 – 0 = 1$

3-10. a) $P(X \in A') = 1 - P(X \in A) = 1 – 0.3 = 0.7$
b) $P(X \in B') = 1 - P(X \in B) = 1 – 0.25 = 0.75$
c) $P(X \in C') = 1 - P(X \in C) = 1 – 0.6 = 0.4$
d) A and B are mutually exclusive if $P(X \in A \cap B) = 0$. To determine if A and B are mutually exclusive, solve the following for $P(X \in A \cap B)$:
$P(X \in A \cup B) = P(X \in A) + P(X \in B) - P(X \in A \cap B)$
$0.55 = 0.3 + 0.25 - P(X \in A \cap B)$
$0.55 = 0.55 - P(X \in A \cap B)$ and $P(X \in A \cap B) = 0$.
Therefore, A and B are mutually exclusive.
e) B and C are mutually exclusive if $P(X \in B \cap C) = 0$. To determine if B and C are mutually exclusive, solve the following for $P(X \in B \cap C)$:
$P(X \in B \cup C) = P(X \in B) + P(X \in C) - P(X \in B \cap C)$
$0.70 = 0.25 + 0.60 - P(X \in B \cap C)$
$0.70 = 0.85 - P(X \in B \cap C)$ and $P(X \in B \cap C) = 0.15$.
Therefore, B and C are not mutually exclusive.

3-11. a) $P(X > 15) = 1 – P(X \le 15) = 1 – 0.3 = 0.7$
b) $P(X \le 24) = P(X \le 15) + P(15 < X \le 24) = 0.3 + 0.6 = 0.9$
c) $P(15 < X \le 20) = P(X \le 20) – P(X \le 15) = 0.5 – 0.3 = 0.2$
d) $P(X \le 18) = P(15 < X \le 18) + P(X \le 15)$
where $P(15 < X \le 18) = P(15 < X \le 24) – P(18 < X \le 24) = 0.6 – 0.4 = 0.2$
Therefore, $P(X \le 18) = P(15 < X \le 18) + P(X \le 15) = 0.2 + 0.3 = 0.5$
Alternatively, $P(X \le 18) = P(X \le 24) - P(18 < X \le 24) = 0.9 – 0.4 = 0.5$.

3-12. A - Overfilled, B - Medium filled, C - Underfilled
a) $P(X \in C') = 1 - P(X \in C) = 1 – 0.15 = 0.85$
b) $P(X \in A \cup C) = P(X \in A) + P(X \in C) – P(X \in A \cap C) = 0.40 + 0.15 – 0 = 0.55$ ($P(X \in A \cap C)$ = 0 since A and C are mutually exclusive)

3-13. a) $P(X \le 7000) = 1 – P(X > 7000) = 1 – 0.45 = 0.55$
b) $P(X > 5000) = 1 – P(X \le 5000) = 1 – 0.05 = 0.95$
c) $P(5000 < X \le 7000) = P(X \le 7000) – P(X \le 5000) = 0.55 – 0.05 = 0.50$

3-14. a) Probability that a component does not fail: $P(E_1') = 1 – P(E_1) = 1 – 0.15 = 0.85$
b) $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = 0.15 + 0.30 = 0.45$

c) $P(E_1 \text{ or } E_2)' = 1 - 0.45 = 0.55$

Section 3-4

3-15. a) $\int_0^4 kx^2 dx = k\frac{x^3}{3}\Big|_0^4 = k\frac{64}{3}$. Therefore, k = 3/64.

$$E(X) = \int_0^4 \frac{3}{64}x^3 dx = \frac{3}{64}\frac{x^4}{4}\Big|_0^4 = 3$$

$$V(X) = \int_0^4 \frac{3}{64}x^2(x-3)^2 dx = \frac{3}{64}\int_0^4 (x^4 - 6x^3 + 9x^2)dx$$

$$= \frac{3}{64}\left(\frac{x^5}{5} - \frac{6x^4}{4} + \frac{9x^3}{3}\right)\Big|_0^4 = 0.6$$

b) $\int_0^2 k(1+2x)dx = k(x+x^2)\Big|_0^2 = k6$. Therefore, k = 1/6.

$$E(X) = \int_0^2 \frac{1}{6}x(1+2x)dx = \frac{1}{6}\int_0^2 (x+2x^2)dx = \frac{1}{6}\left(\frac{x^2}{2} + \frac{2x^3}{3}\right)\Big|_0^2 = 11/9$$

$$V(X) = \int_0^2 \tfrac{1}{6}(1+2x)(x - \tfrac{11}{9})^2 dx$$

$$= \tfrac{1}{6}\int_0^2 (x^2 - \tfrac{22}{9}x + \tfrac{121}{81} + 2x^3 - \tfrac{44}{9}x^2 + \tfrac{242}{81}x)dx$$

$$= \tfrac{1}{6}\left(\tfrac{x^3}{3} - \tfrac{22}{9}\tfrac{x^2}{2} + \tfrac{121}{81}x + +2\tfrac{x^4}{4} - \tfrac{44}{9}\tfrac{x^3}{3} + \tfrac{242}{81}\tfrac{x^2}{2}\right)\Big|_0^2 = \frac{1.704}{6} = 0.284$$

c) $\int_0^\infty ke^{-x}dx = k(-e^{-x})\Big|_0^\infty = k$. Therefore, k = 1.

$E(X) = \int_0^\infty xe^{-x}dx$, using integration by parts $E(X) = -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}dx = -e^{-x}\Big|_0^\infty = 1$

$V(X) = \int_0^\infty (x-1)^2 e^{-x}dx = \int_0^\infty (x^2e^{-x} - 2xe^{-x} + e^{-x})dx$. Now, using integration by parts

$\int_0^\infty x^2e^{-x}dx = 2\int_0^\infty xe^{-x}$. Therefore, $V(X) = \int_0^\infty e^{-x}dx = 1$, because e^{-x} is a probability density function.

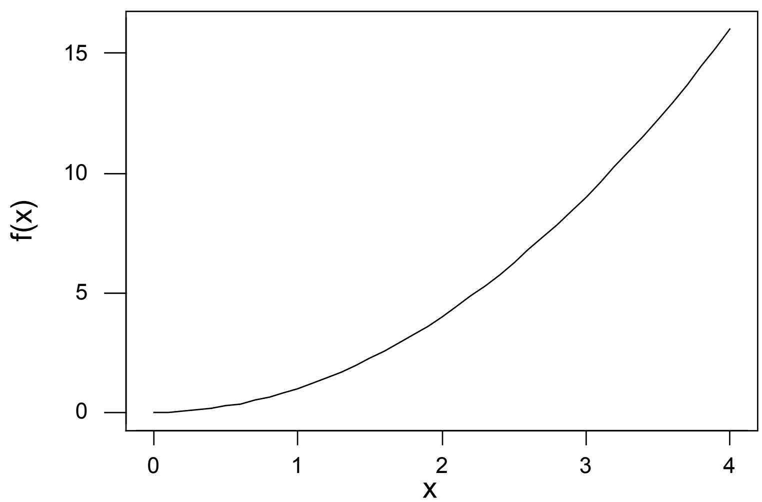
d) $\int_{100}^{100+k} kdx = kx\Big|_{100}^{100+k} = k^2$, $k^2 = 1$, so k =1 (where k > 0)

$$E(X) = \int_{100}^{101} 1xdx = 100.5$$

$$V(X) = \int_{100}^{101} (x - 100.5)^2 1dx = 0.08333$$

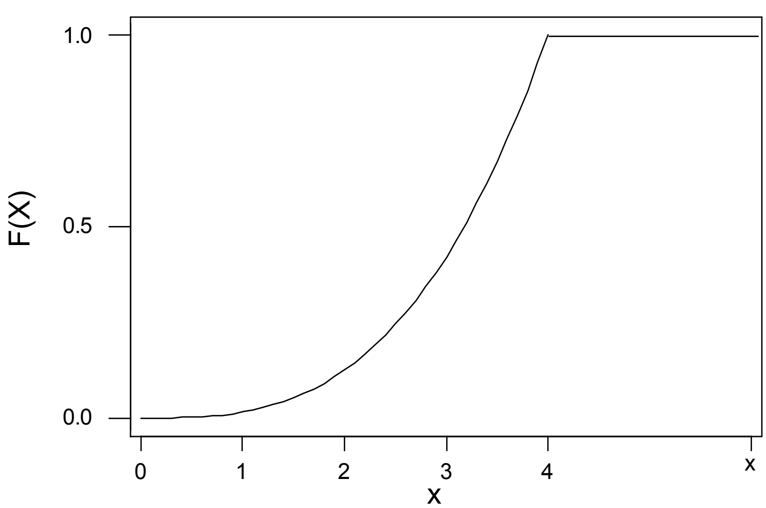
3-16. For 3-15a:

a) $f(x) = \dfrac{3x^2}{64}, \quad 0 < x < 4$



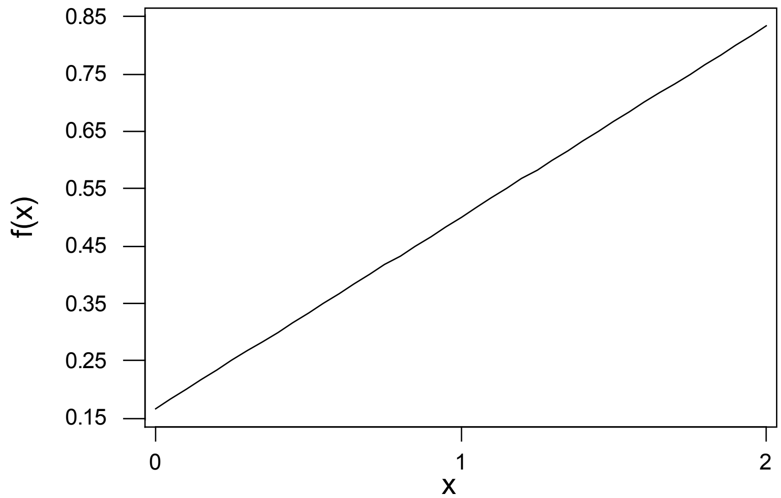
b) F(X) = $\int_0^x f(t)dt = \int_0^x \frac{3t^2}{64}dt = \frac{x^3}{64}$, 0 < x < 4

c)



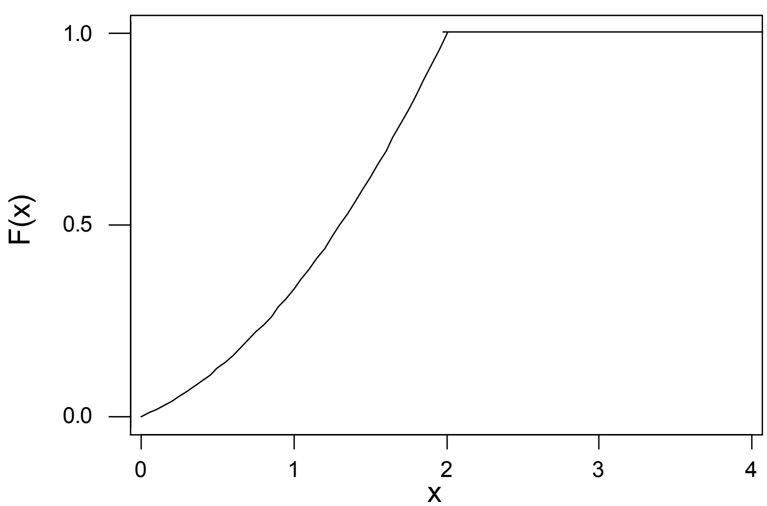
3-15b:

a) $f(x) = \frac{(1+2x)}{6}, \quad 0 < x < 2$



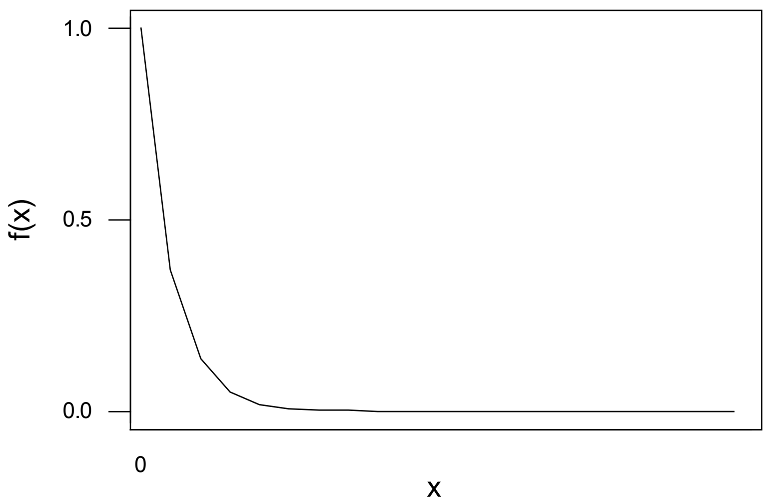
b) F(X) $= \int_0^x \frac{(1+2t)}{6} dt = \frac{1}{6}\left(x + x^2\right), \quad 0 < x < 2$

c)

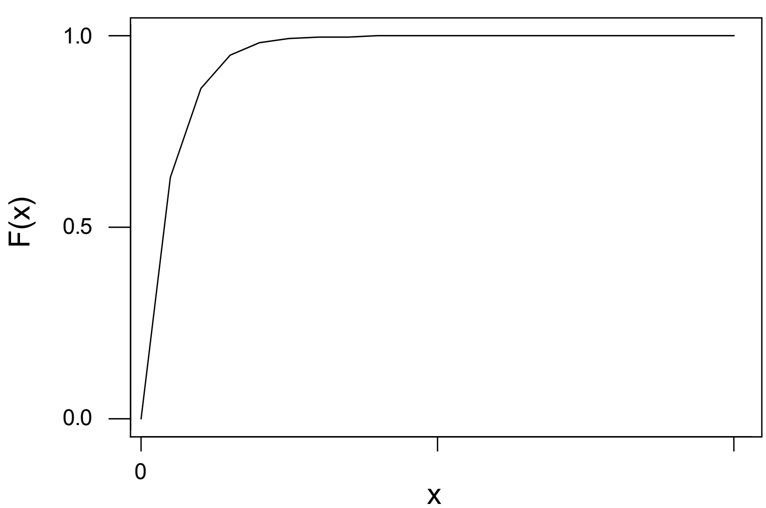


3-15c: $f(x) = e^{-x}$, x >0

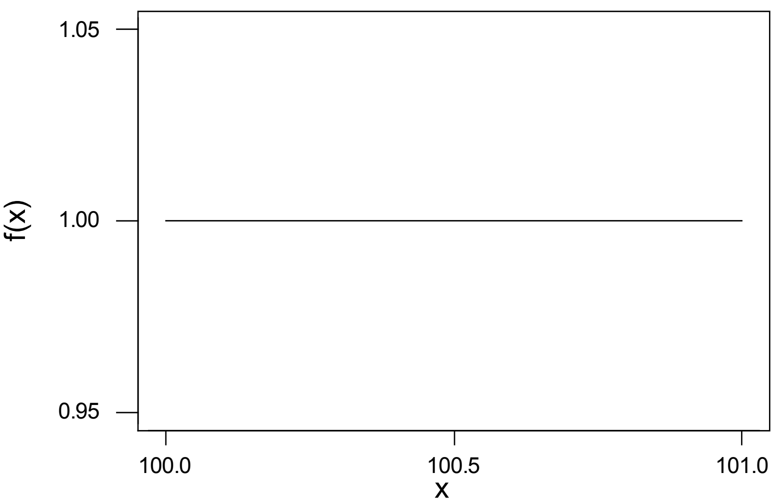
a)



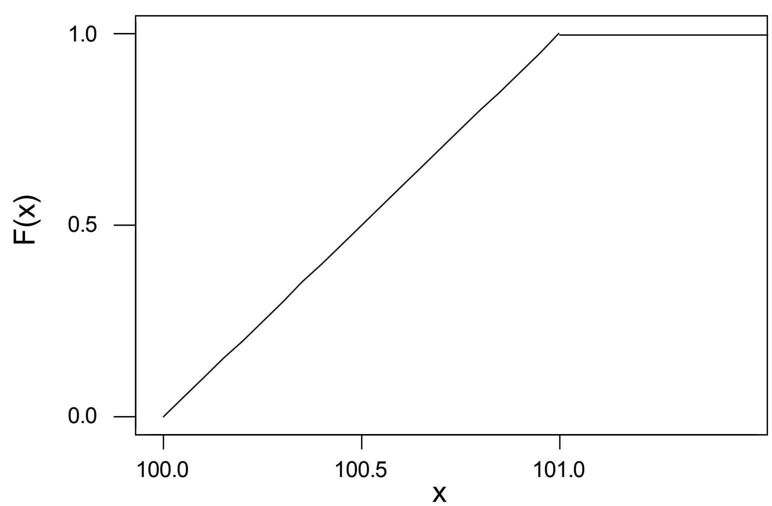
b) $F(x) = 1 - e^{-x}$, $x > 0$
c)



3-15 d: $f(x) = 1$, $100 < x < 101$
a)



b) $F(X) = x - 100,\ \ 100 < x < 101$
c)



3-17. a) $P(X > 6) = \int_{6}^{\infty} e^{-(x-6)} dx = 1$

b) $P(6 \le X < 8) = \int_{6}^{8} e^{-(x-6)} dx = e^{0} - 0.1353 = 0.8647$

c) $P(X < 8) = \int_6^8 e^{-(x-6)}dx = 0.8647$

d) $P(X > 8) = 1 - P(X \le 8) = 1 - P(X < 8) = 1 - 0.8647 = 0.1353$

e) $P(X < x) = \int_6^x e^{-(x-6)}dx = 0.95$

$$\int_6^x e^{-(x-6)}dx = 1 - e^{-(x-6)}$$

$$1 - e^{-(x-6)} = 0.95$$

$$x = -\ln(0.05) + 6$$

$$x = 9$$

3-18. a) $P(0 < X) = 0.5$, by symmetry.

b) $P(0.5 < X) = \int_{0.5}^{1} 1.5x^2 dx = 0.5x^3 \Big|_{0.5}^{1} = 0.5 - 0.0625 = 0.4375$

c) $P(-0.5 \le X \le 0.5) = \int_{-0.5}^{0.5} 1.5x^2 dx = 0.5x^3 \Big|_{-0.5}^{0.5} = 0.125$

d) $P(X < -2) = 0$

e) $P(X < 0 \text{ or } X > -0.5) = 1$

f) $P(x < X) = \int_x^1 1.5x^2 dx = 0.5x^3 \Big|_x^1 = 0.5 - 0.5x^3 = 0.05$

Then, $x = 0.9655$

3-19. a) $P(X > 1000) = \int_{1000}^{\infty} \frac{e^{-\frac{x}{3000}}}{3000} dx = 0.7165$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-\frac{x}{3000}}}{3000} dx = 0.2031$

c) $P(X < 3000) = \int_{0}^{3000} \frac{e^{-\frac{x}{3000}}}{3000} dx = 0.6321$

d) $P(X < x) = \int_{0}^{x} \frac{e^{-\frac{x}{3000}}}{3000} dx = 0.10$

$$\int_0^x \frac{e^{-\frac{x}{3000}}}{3000}dx = -e^{-\frac{x}{3000}}\Big|_0^x = 1-e^{-\frac{x}{3000}} = 0.1$$

$$e^{-\frac{x}{3000}} = 0.9$$

$$x = 316.2$$

e) $E(X) = \int_0^\infty x\frac{e^{-\frac{x}{3000}}}{3000}dx = 3000$

$V(X) = (3000)^2 = 9{,}000{,}000$

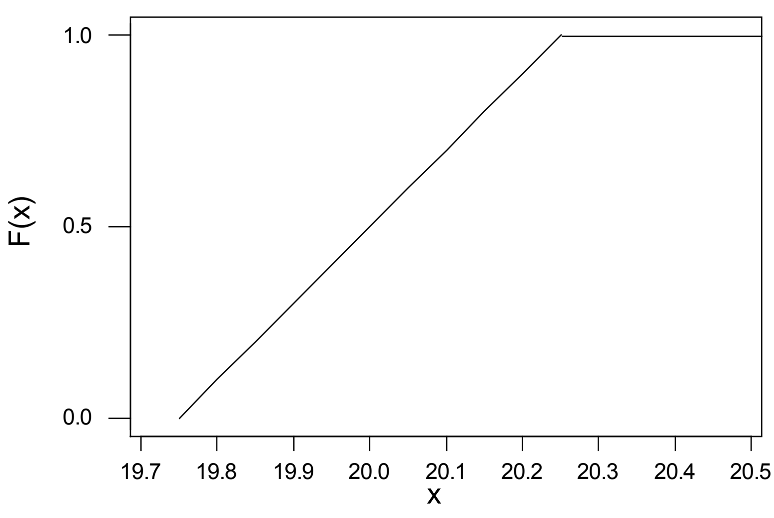
3-20. a) $P(19.75 < X < 20) = \int_{19.75}^{20} 2.0dx = 0.5$

b) $P(19.9 < X < 20.1) = \int_{19.9}^{20.1} 2.0dx = 0.4$

c) $E(X) = \int_{19.75}^{20.25} 2.0xdx = 20$

$V(X) = \int_{19.75}^{20.25} 2.0x^2dx - E\{X\}^2 = 400.02-(20)^2 = 0.02$

d) $F(x) = \int_{19.75}^{x} 2.0dx = 2x-39.5$, $19.75 < x < 20.25$



3-21. a) $P(X < 805) = F(805) = 0.1(805) - 80 = 0.5$

b) $P(800 < X < 805) = F(805) - F(800) = 0.5 - 0 = 0.5$

c) $P(X > 808) = 1 - P(X \le 808) = 1 - [0.1(808) - 80] = 0.2$

d) In specification = $P(802 < X < 808) = F(808) - F(802) = 0.8 - 0.2 = 0.6$.

Therefore, out of specification is 1 – 0.6 = 0.4.

3-22. a) P(X ≤ 2.0080) = $\int_{2.0050}^{2.0080} 200dx = 0.6$

b) P(X > 2.0055) = $\int_{2.0055}^{2.0100} 200dx = 0.9$

c) P(2.0080 < X < 2.0090) = $\int_{2.0080}^{2.0090} 200dx = 0.2$

3-23.a) Show that $\int_{1}^{\infty} 2x^{-3}dx = 1$:

$$\int_{1}^{\infty} 2x^{-3}dx = -x^{-2}\Big|_{1}^{\infty} = 0-(-1^{-2}) = 1$$

b) F(X) = $\int_{1}^{x} 2x^{-3}dx = -x^{-2}\Big|_{1}^{x} = 1-x^{-2}$

c) E(X) = $\int_{1}^{\infty} 2x^{-2}dx = 2.0$

d) P(X < 5) = $\int_{1}^{5} 2x^{-3}dx = 0.96$

e) P(X > 7) = $\int_{7}^{\infty} 2x^{-3}dx = 0.0204$

3-24. a) E(X) = $\int_{5}^{\infty} x10e^{-10(x-5)}dx$.

Using integration by parts with u = x and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -xe^{-10(x-5)}\Big|_{5}^{\infty} + \int_{5}^{\infty} e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Bigg|_{5}^{\infty} = 5.1$$

Now, $V(X) = \int_{5}^{\infty} (x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2 e^{-10(x-5)}\Big|_{5}^{\infty} + 2\int_{5}^{\infty} (x-5.1)e^{-10(x-5)}dx$.

From the definition of E(X), the integral above is recognized to equal 0. Therefore, $V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$

3-25. a) P(X < 1208) = F(1208) = 0.1(1208) – 120 = 0.8

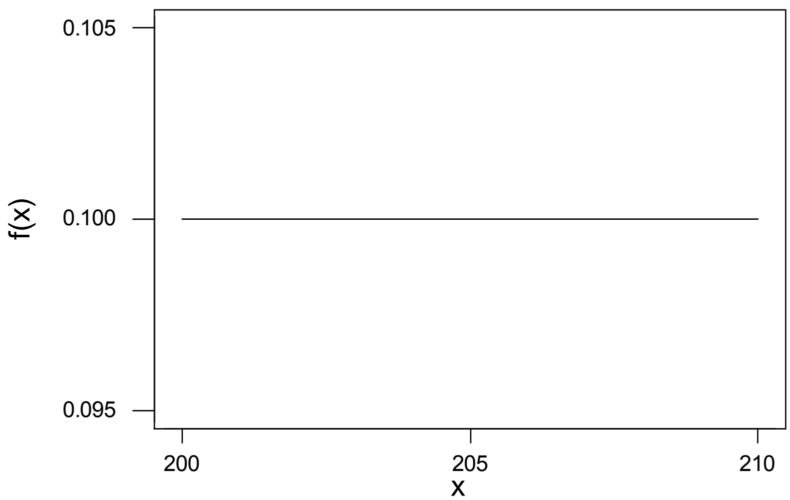
b) P(1195 < X < 1205) = F(1205) – F(1195) = [0.1(1205)-120] – [0.1(1195)-120] = 0.5

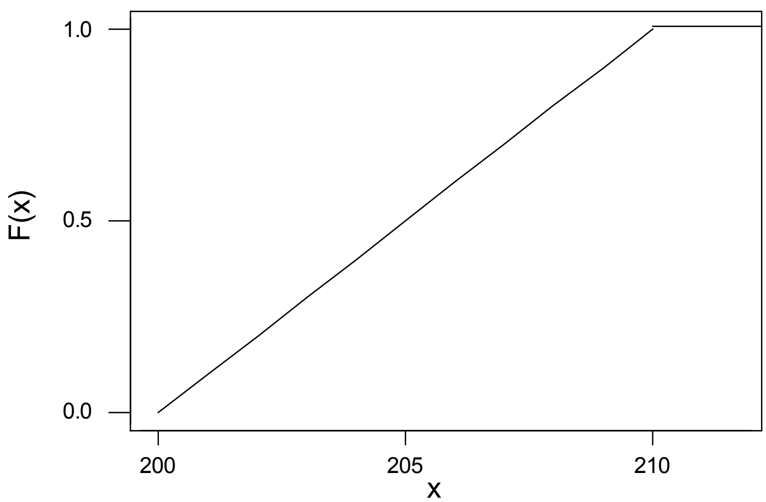
3-26. a) E(X) = $\int_{100}^{120} x\frac{600}{x^2}dx = 600\ln x \big|_{100}^{120} = 600(\ln 120 - \ln 100) = 109.39\mu m$

$$V(X) = \int_{100}^{120} x^2 \frac{600}{x^2} dx - E(x)^2 = 600x \Big|_{100}^{120} - 109.39^2 = 33.19\mu m^2$$

b) Average Cost = Coating Cost × Mean of the Coating Thickness
= 0.50 × 109.39 = \$54.70

3-27. a) $P(X < 209) = 0.1(209) - 20 = 0.9$
b) $P(200 < X < 208) = F(208) - F(200) = 0.8 - 0 = 0.8$
c) $P(X > 209) = 1 - F(209) = 1 - 0.9 = 0.1$
d) $f(x) = 0.1$ for $200 \le x \le 210$
e)





f) E(X) = 205, V(X) = 8.3333

Section 3-5

3-28. a) P(-1 < Z < 1) = P(Z < 1) – P(Z < -1) = 0.841345 – 0.158655 = 0.68269
b) P(-2 < Z < 2) = P(Z < 2) – P(Z < -2) = 0.97725 – 0.02275 = 0.9545
c) P(-3 < Z < 3) = P(Z < 3) – P(Z < -3) = 0.998999 – 0.00135 = 0.997649
d) P(Z < -3) = 0.00135
e) P(0 < Z ≤ 3) = P(Z < 3) – P(Z < 0) = 0.998999 – 0. 5 = 0.498999

3-29. a) P(Z < z) = 0.500000
z = 0
b) P(Z < z) = 0.001001
z = -3.09
c) P(Z > z) = 0.881000
Can be rewritten as: 1 – P(Z < z) = 0.881000
P(Z < z) = 0.119000
z = -1.18
d) P(Z > z) = 0.866500
Can be rewritten as: 1 – P(Z < z) = 0.866500
P(Z < z) = 0.133500
z = -1.11
e) P(-1.3 < Z < z) = 0.863140
P(Z < z) – P(Z < -1.3) = 0.863140
P(Z < z) – 0.096801 = 0.863140
P(Z < z) = 0.959941, Z = 1.75

3-30. a) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.95
So P(Z<-z) = 0.5(1 – 0.95) = 0.025
z = 1.96
b) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.99
So P(Z<-z) = 0.5(1 – 0.99) = 0.005
z = 2.58
c) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.68

So $P(Z<-z) = 0.5(1 – 0.68) = 0.16$
$z = 1$
d) $P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.9973$
So $P(Z<-z) = 0.5(1 – 0.9973) = 0.00135$
$z = 3$

3-31. a) $P(X < 24) = P(Z < 2) = 0.97725$
b) $P(X > 18) = P(Z > -1) = 0.84134$
c) $P(18 < X < 22) = P(-1 < Z < 1) = P(Z < 1) – P(Z < -1) = 0.84134 – 0.15866 = 0.68268$
d) $P(14 < X < 26) = P(-3 < Z < 3) = 0.99865 – 0.00135 = 0.9973$
e) $P(16 < X < 20) = P(-2 < Z < 0) = P(Z < 0) – P(Z < -2) = 0.5 – 0.02275 = 0.47725$
f) $P(20 < X < 26) = P(0 < Z < 3) = P(Z < 3) – P(Z < 0) = 0.99865 – 0.50 = 0.49865$

3-32. a) $P(X > x) = P(Z > z) = 0.5$ where $z = \frac{x-20}{2} = 0$. Thus $x = 20$
b) $P(X > x) = P(Z > z) = 0.95$ where $z = \frac{x-20}{2} = -2.58$. Thus $x = 14.84$
c) $P(x < X < 20) = P(Z < 0) – P(Z < z)= 0.5 – P(Z < z) = 0.2$
So $P(Z < z) = 0.3$ where $z = \frac{x-20}{2} = -0.53$. Thus $x = 18.94$

3-33. a) $P(X < 31) = P(Z < 3) = 0.99865$
b) $P(X > 30) = P(Z > 3.5) = 0.00023$
c) $P(33 < X < 37) = P(-2 < Z < 0) = P(Z < 0) – P(Z < -2) = 0.5 – 0.02275 = 0.47725$
d) $P(32 < X < 39) = P(-2.5 < Z < 1) = 0.84134 – 0.00621 = 0.83513$
e) $P(30 < X < 38) = P(-3.5 < Z < 0.5) = 0.69146 – 0.00023 = 0.69123$

3-34. a) $P(X > x) = 1 – P(X < x)$
$= 1 - P\left(Z < \frac{x-6}{3}\right) = 0.50.$

And, $P\left(Z < \frac{x-6}{3}\right) = 0.50$

Therefore, $\frac{x-6}{3} = 0$ and $x = 6$.

b) $P(X > x) = 1 – P(X < x)$
$= 1 - P\left(Z < \frac{x-6}{3}\right) = 0.95.$

And, $P\left(Z < \frac{x-6}{3}\right) = 0.05$

Therefore, $\frac{x-6}{3} = -1.645$ and $x = 1.065$.

c) $P(x < X < 9) = 0.20$

$$P\left(\frac{x-6}{3} < Z < \frac{9-6}{3}\right) = P\left(\frac{x-6}{3} < Z < 1\right)$$

$$P(Z < 1) - P\left(Z < \frac{x-6}{3}\right) = 0.20$$

$$0.841345 - P\left(Z < \frac{x-6}{3}\right) = 0.20$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.841345 - 0.20$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.641345$$

Therefore, $\frac{x-6}{3}$ =0.36, and x = 7.08.

d) P(3 < X < x) = 0.80

$$P\left(\frac{3-6}{3} < Z < \frac{x-6}{3}\right) = P\left(-1 < Z < \frac{x-6}{3}\right)$$

$$P\left(Z < \frac{x-6}{3}\right) - P(Z < -1) = 0.80$$

$$P\left(Z < \frac{x-6}{3}\right) - 0.158655 = 0.80$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.958655$$

Therefore, $\frac{x-6}{3} \cong 1.74$, and x = 11.22.

3-35. a) $P(X < 6250) = P\left(Z < \frac{6250-6000}{100}\right)$
$= P(Z < 2.5)$
$= 0.9938$

b) $P(5800 < X < 5900) = P(-2 < Z < -1)$
$= P(Z < -1) - P(Z < -2)$
$= 0.1359$

c) $P(X > x) = P\left(Z > \frac{x-6000}{100}\right) = 0.95.$

Therefore, $\frac{x-6000}{100} = -1.64$ and x = 5835.51.

3-36. a) $P(X < 39) = P(Z < 2) = 0.97725$
b) $P(X < 29) = P(Z < -3) = 0.00135$

3-37. a) $P(X > 0.62) = P\left(Z > \frac{0.62-0.5}{0.05}\right)$
$= P(Z > 2.4)$
$= 1 - P(Z < 2.4)$
$= 0.0082$

b) $P(0.47 < X < 0.63) = P(-0.6 < Z < 2.6)$
$= P(Z < 2.6) - P(Z < -0.6)$
$= 0.99534 - 0.27425$
$= 0.7211$

c) $P(X < x) = P\left(Z < \frac{x-0.5}{0.05}\right) = 0.90.$

Therefore, $\frac{x-0.5}{0.05} = 1.28$ and x = 0.5641.

3-38. a) $P(X < 12) = P(Z < \frac{12-12.4}{0.1}) = P(Z < -4) \cong 0$

b) $P(X < 12.1) = P\left(Z < \frac{12.1-12.4}{0.1}\right)$
$= P(Z < -3)$
$= 0.0014$

and

$$P(X > 12.6) = P\left(Z > \frac{12.6 - 12.4}{0.1}\right)$$
$$= P(Z > 2)$$
$$= 0.0228.$$

Therefore, the proportion of cans scrapped is 0.00135 + 0.02275 = 0.0241.

c) $P(12.4 - x < X < 12.4 + x) = 0.99.$

Therefore, $P\left(-\frac{x}{0.1} < Z < \frac{x}{0.1}\right) = 0.99$

Consequently, $P\left(Z < \frac{x}{0.1}\right) = 0.995$ and $x = 0.1(2.58) = 0.258$.

The specifications are (12.142, 12.658).

3-39. a) If $P(X > 12) = 0.999$, then $P\left(Z > \frac{12 - \mu}{0.1}\right) = 0.999.$

Therefore, $\frac{12-\mu}{0.1} = -3.09$ and $\mu = 12.309$.

b) If $P(X > 12) = 0.999$, then $P\left(Z > \frac{12 - \mu}{0.05}\right) = 0.999.$

Therefore, $\frac{12-\mu}{0.05} = -3.09$ and $\mu = 12.155$.

3-40. a) $P(X > 0.5) = P\left(Z > \frac{0.5 - 0.4}{0.05}\right)$
$$= P(Z > 2)$$
$$= 1 - 0.97725$$
$$= 0.0228$$

b) $P(0.4 < X < 0.5) = P(0 < Z < 2)$
$$= P(Z < 2) - P(Z < 0)$$
$$= 0.4773$$

c) $P(X > x) = 0.90$, then $P\left(Z > \frac{x - 0.4}{0.05}\right) = 0.90.$

Therefore, $\frac{x-0.4}{0.05} = -1.28$ and $x = 0.336$.

3-41. a) $P(90.3 < X) = P\left(\frac{90.3 - 90.2}{0.1} < Z\right)$
$$= P(1 < Z)$$
$$= P(Z > 1)$$
$$= 1 - P(Z < 1)$$
$$=1 - 0.841345$$
$$= 0.1587.$$

$$P(X < 89.7) = P\left(Z < \frac{89.7 - 90.2}{0.1}\right)$$
$$= P(Z < -5)$$
$$\cong 0 .$$

Therefore, the answer is 0.1587.

b) The process mean should be set at the center of the specifications; that is, at $\mu = 90.0$.

c) $P(89.7 < X < 90.3) = P\left(\frac{89.7-90}{0.1} < Z < \frac{90.3-90}{0.1}\right)$

$= P(-3 < Z < 3) = 0.9973.$

3-42. a) $P(X > 1140) = P(Z > \frac{1140-1000}{60}) = P(Z > 2.33) = P(Z < -2.33) = 0.009903$

b) $P(X < 900) = P(Z > \frac{900-1000}{60}) = P(Z > -0.17) = 0.432505$

3-43. a) $P(X > 9) = P(Z > 1.34) = 0.09012$

b) $P(5.5 < X < 8.5) = P(-1.69 < Z < -0.242) = 0.40439 - 0.04551 = 0.35888$

c) Threshold = $\mu + 3.75\sigma = 6.23 + 3.75(2.064) = 13.97$

3-44. a) $P(X < 5000) = P\left(Z < \frac{5000-7000}{600}\right)$

$= P(Z < -3.33)$

$= 0.0004.$

b) $P(X > x) = 0.95$. Therefore, $P\left(Z > \frac{x-7000}{600}\right) = 0.95$ and $\frac{x-7000}{600} = -1.64$. Consequently, $x = 6016$.

3-45. a) $P(X > 0.0026) = P\left(Z > \frac{0.0026-0.002}{0.0004}\right)$

$= P(Z > 1.5)$

$= 0.0668.$

b) $P(0.0014 < X < 0.0026) = P(-1.5 < Z < 1.5)$

$= 0.8664.$

c) $P(0.0014 < X < 0.0026) = P\left(\frac{0.0014-0.002}{\sigma} < Z < \frac{0.0026-0.002}{\sigma}\right)$

$= P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right) = 0.995$

Therefore, $P\left(Z < \frac{0.0006}{\sigma}\right) = 0.9975$. Consequently, $\frac{0.0006}{\sigma} = 2.81$ and $\sigma = 0.000214$.

3-46. a) $P(X > 2.10) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.977250 = 0.02275$

b) $P(X < 2.10) = 0.999$

$P\left(Z < \frac{2.10-2}{\hat{\sigma}}\right) = 0.999$

$\frac{2.10-2}{\hat{\sigma}} = 3.1$

$\hat{\sigma} = 0.03226$

c) $P(X < 2.10) = P(Z < z) = 0.999$ where $z = \frac{2.10-\hat{\mu}}{0.05} = 3.1$. Thus $\hat{\mu} = 1.945$

3-47. X is a lognormal distribution with $\theta=5$ and $\omega^2=9$

a)

$$P(X < 13300) = P(e^W < 13300) = P(W < \ln(13300)) = \Phi\left(\frac{\ln(13330)-5}{3}\right)$$

$= \Phi(1.50) = 0.9332$

b) Find the value for which $P(X \le x) = 0.95$

$$P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-5}{3}\right) = 0.95$$

$$\frac{\ln(x)-5}{3} = 1.65 \quad x = e^{1.65(3)+5} = 20952.2$$

c) $\mu = E(X) = e^{\theta+\omega^2/2} = e^{5+9/2} = e^{9.5} = 13359.7$

$$V(X) = e^{2\theta+\omega^2}(e^{\omega^2}-1) = e^{10+9}(e^9-1) = e^{19}(e^9-1) = 1.45x10^{12}$$

3-48. a) X is a lognormal distribution with θ=2 and $\omega^2=4$

$$P(X < 500) = P(e^W < 500) = P(W < \ln(500)) = \Phi\left(\frac{\ln(500)-2}{2}\right)$$

$$= \Phi(2.11) = 0.9826$$

b)

$$P(500 < X < 1000) = P(500 < e^W < 1000) = P(\ln(500) < W < \ln(1000)) = \Phi\left(\frac{\ln(1000)-2}{2}\right) - \Phi\left(\frac{\ln(500)-2}{2}\right)$$

$$= \Phi(2.45) - \Phi(2.11) = 0.99286 - 0.9826$$

$$= 0.01026$$

c)

$$P(1500 < X < 2000) = P(1500 < e^W < 2000) = P(\ln(1500) < W < \ln(2000)) = \Phi\left(\frac{\ln(2000)-2}{2}\right) - \Phi\left(\frac{\ln(1500)-2}{2}\right)$$

$$= \Phi(2.80) - \Phi(2.66) = 0.99744 - 0.99609$$

$$= 0.00135$$

d) The product has significantly degraded over the first 500 hours. The degradation is less significant after 500 hours.

3-49. X is a lognormal distribution with θ=0.5 and $\omega^2=1$

a)

$$P(X > 10) = P(e^W > 10) = P(W > \ln(10)) = 1 - \Phi\left(\frac{\ln(10)-0.5}{1}\right)$$

$$= 1 - \Phi(1.80) = 1 - 0.96407 = 0.03593$$

b) $$P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-0.5}{1}\right) = 0.50$$

$$\frac{\ln(x)-0.5}{1} = 0 \quad x = e^{0(1)+0.5} = 1.65 \text{ seconds}$$

c) $\mu = E(X) = e^{\theta+\omega^2/2} = e^{0.5+1/2} = e^1 = 2.7183$

$$V(X) = e^{2\theta+\omega^2}(e^{\omega^2}-1) = e^{1+1}(e^1-1) = e^2(e^1-1) = 12.6965$$

3-50. a) Find the values of θand ω^2 given that E(X) = 10000 and σ= 20,000

$$10000 = e^{\theta+\omega^2/2} \qquad 20000^2 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$$

Let $x = e^{\theta}$ and $y = e^{\omega^2}$ then (1) $10000 = x\sqrt{y}$ and (2) $20000^2 = x^2y(y-1) = x^2y^2 - x^2y$

Square (1): $10000^2 = x^2 y$ and substitute into (2)

$$20000^2 = 10000^2(y-1)$$

$$y = 5$$

Substitute y into (1) and solve for x: $x = \frac{10000}{\sqrt{5}} = 4472.1360$

$\theta = \ln(4472.1360) = 8.4056$ and $\omega^2 = \ln(5) = 1.6094$

b)

$$P(X > 10000) = P(e^W > 10000) = P(W > \ln(10000)) = 1 - \Phi\left(\frac{\ln(10000)-8.4056}{1.2686}\right)$$

$$= 1 - \Phi(-1.18) = 1 - 0.1190 = 0.8810$$

c) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\frac{\ln(x) - 8.4056}{1.2686}\right) = 0.1$

$\frac{\ln(x) - 8.4056}{1.2686} = -1.28 \quad x = e^{-1.280(1.2686)+8.4056} = 881.65$ hours

3-51. $\beta = 0.2$ and $\delta = 100$ hours

$E(X) = 100\Gamma(1 + \frac{1}{0.2}) = 100 \times 5! = 12,000$

$V(X) = 100^2\Gamma(1 + \frac{2}{0.2}) - 100^2[\Gamma(1 + \frac{1}{0.2})]^2 = 3.61 \times 10^{10}$

3-52. a) $P(X < 10000) = F_X(10000) = 1 - e^{-100^{0.2}} = 1 - e^{-2.512} = 0.9189$

b) $P(X > 5000) = 1 - F_X(5000) = e^{-50^{0.2}} = 0.1123$

3-53. Let X denote lifetime of a bearing. $\beta=2$ and $\delta=10000$ hours

a) $P(X > 8000) = 1 - F_X(8000) = e^{-\left(\frac{8000}{10000}\right)^2} = e^{-0.8^2} = 0.5273$

b)

$E(X) = 10000\Gamma(1 + \frac{1}{2}) = 10000\Gamma(1.5)$

$= 10000(0.5)\Gamma(0.5) = 5000\sqrt{\pi} = 8862.3$

= 8862.3 hours

c) Let Y denote the number of bearings out of 10 that last at least 8000 hours. Then, Y is a binomial random variable with n = 10 and p = 0.5273.

$P(Y = 10) = \binom{10}{10}0.5273^{10}(1 - 0.5273)^0 = 0.00166$.

3-54. a) $E(X) = \delta\Gamma(1 + \frac{1}{\beta}) = 900\Gamma(1 + 1/3) = 900\Gamma(4/3) = 900(0.89298). = 803.68$ hours

b)

$V(X) = \delta^2\Gamma(1 + \frac{2}{\beta}) - \delta^2\left[\Gamma(1 + \frac{2}{\beta})\right]^2 = 900^2\Gamma(1 + \frac{2}{3}) - 900^2\left[\Gamma(1 + \frac{1}{3})\right]^2$

$= 900^2(0.90274)\text{-}900^2(0.89298)^2 = 85314.64 \text{ hours}^2$

c) $P(X < 500) = F_X(500) = 1 - e^{-\left(\frac{500}{900}\right)^3} = 0.1576$

3-55. a) $\Gamma(6) = 5! = 120$

b) $\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{3}{4}\pi^{1/2} = 1.32934$

c) $\Gamma\left(\frac{9}{2}\right) = \frac{7}{2}\Gamma\left(\frac{7}{2}\right) = \frac{7}{2}\frac{5}{2}\frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{105}{16}\pi^{1/2} = 11.6317$

3-56. $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$\Gamma(r) = -x^{r-1}e^{-x}\Big|_0^\infty + (r-1)\int_0^\infty x^{r-2}e^{-x}dx = (r-1)\Gamma(r-1)$.

3-57. $\int_0^\infty f(x;\lambda,r)dx = \int_0^\infty \frac{\lambda^r x^{r-1}e^{-\lambda x}}{\Gamma(r)}dx$. Let y = λx, then the integral is $\int_0^\infty \frac{\lambda\ y^{r-1}e^{-y}}{\Gamma(r)}\frac{dy}{\lambda}$. From the definition of $\Gamma(r)$, this integral is recognized to equal 1.

3-58. E(X) = 6/3 = 2; V(X) = $6/3^2$ = 0.6667

3-59. E(X) = 3.2/2.5 = 1.28; V(X) = $3.2/(2.5)^2$ = 0.512

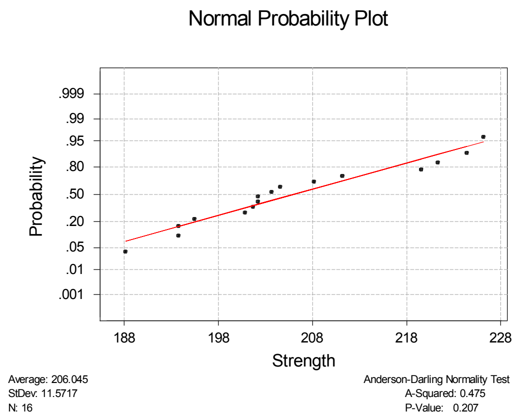
3-60. $r/\lambda = 3$, so $r = 3\lambda$; Also, $r/\lambda^2 = 1.5$; Using substitution, $3\lambda/\lambda^2 = 1.5$, and $\lambda = 2$;
Giving r = 6.

3-61. $r/\lambda = 4.5$, so $r = 4.5\lambda$; Also, $r/\lambda^2 = 6.25$; Using substitution, $4.5\lambda/\lambda^2 = 6.25$, and $\lambda = 0.72$;
Giving r = 3.24.

3-62. $r/\lambda = 4$, so $r = 4\lambda$; Also, $r/\lambda^2 = 2$; Using substitution, $4\lambda/\lambda^2 = 2$, and $\lambda = 2$; Giving $r = 8$.

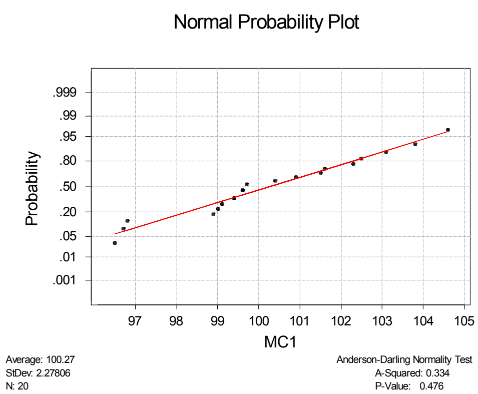
Section 3-6

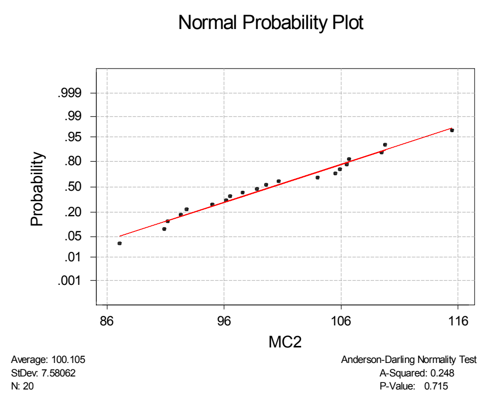
3-63.



Because the points in normal probability plot pass the fat pencil test, we conclude that the normal distribution is an appropriate model.

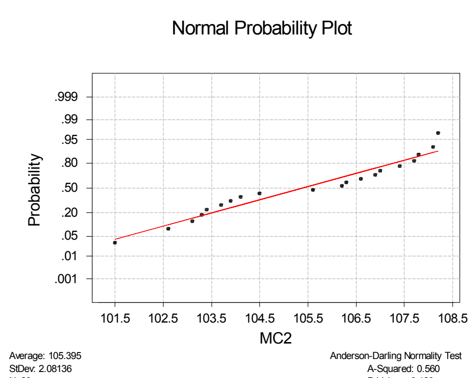
3-64.





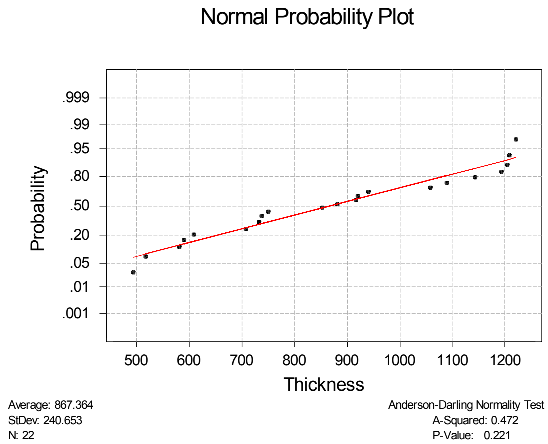
The dimension from both machine one and two seem to have a normal distribution with approximately the same mean. Dimension from machine two tends to have higher variability than those from machine one, note the difference in the horizontal scale.

3-65.



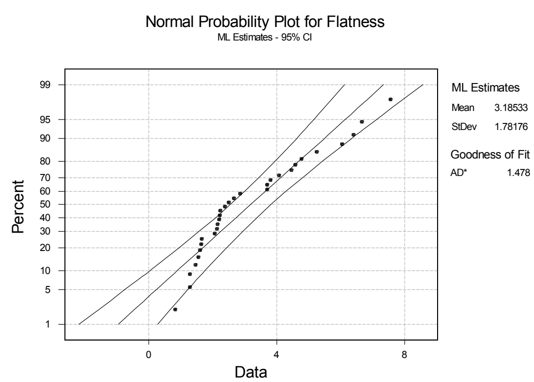
The normal distribution is a reasonable model for the data. It also appears that the variance has been reduced.

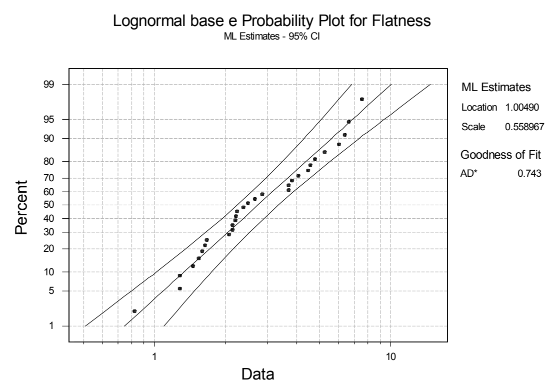
3-66.

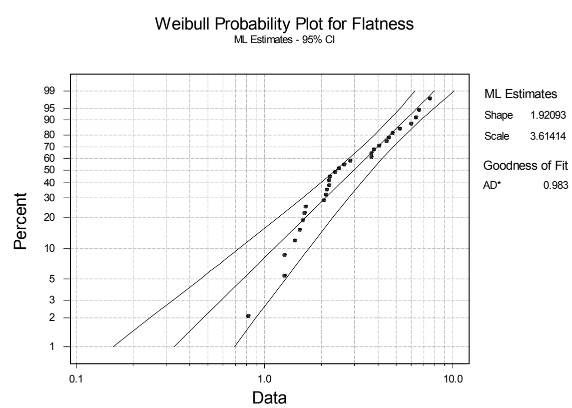


It is reasonable to model these data using a normal probability distribution.

3-67.

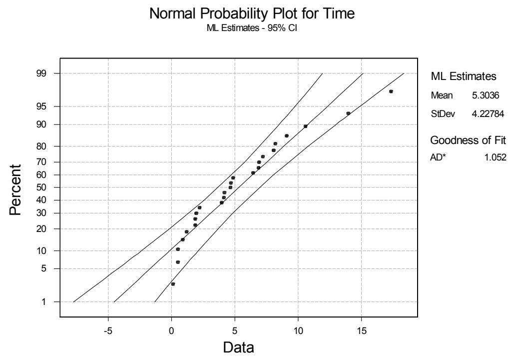


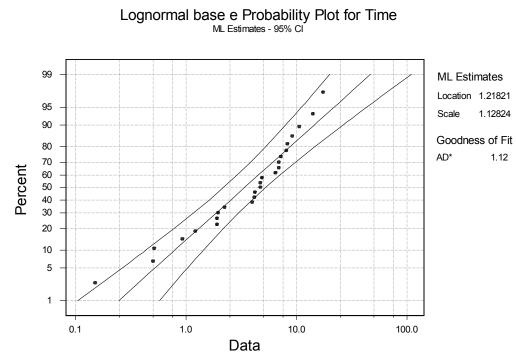


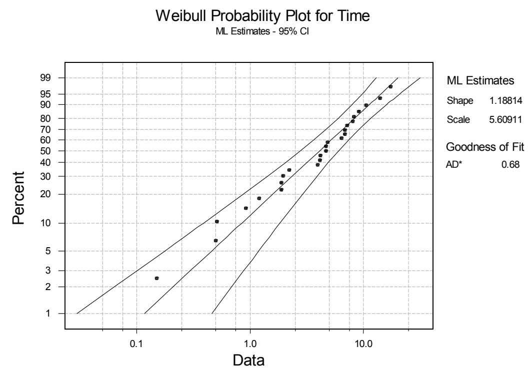


Based on three probability plots, the log normal (base e) appears to be the most appropriate choice as a model for the flatness data.

3-68.

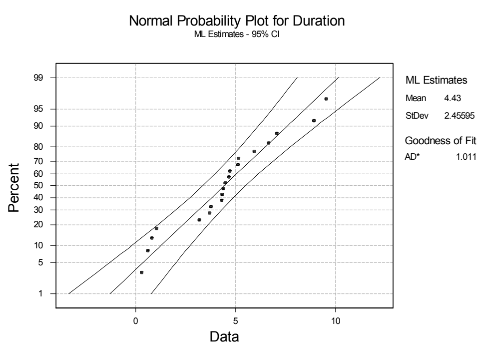


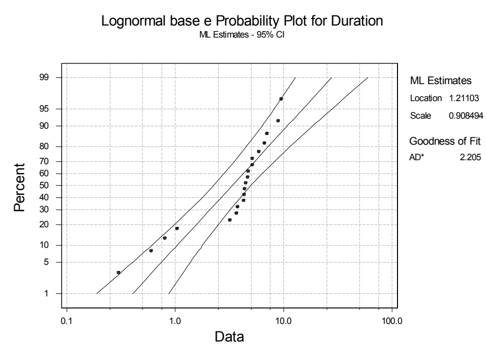


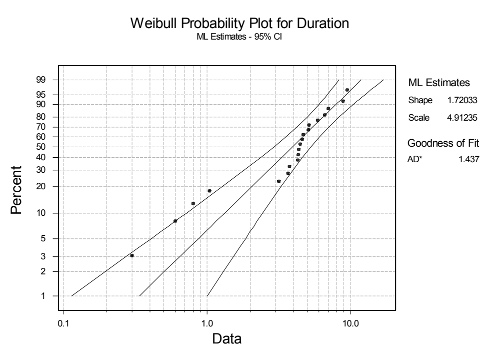


Weibull probability density model appears to provide the most suitable fit to the data.

3-69.

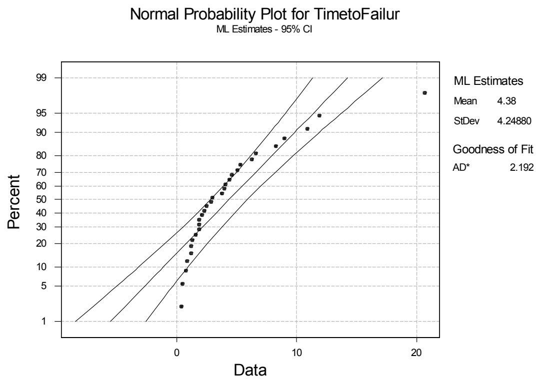


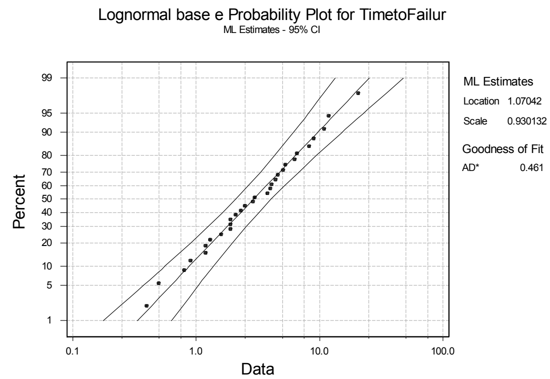


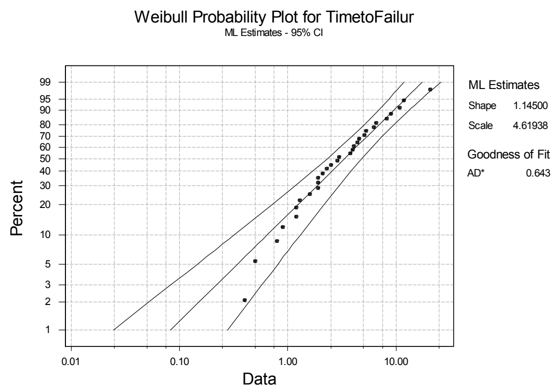


The normal distribution appears to be the most appropriate choice as a model for the data.

3-70.







Based on three probability plots, the log normal (base e) appears to be the most appropriate choice as a model for the time-to-failure data.

<u>Section 3-7</u>

3-71. The function is a probability mass function. All probabilities are nonnegative and sum to 1.

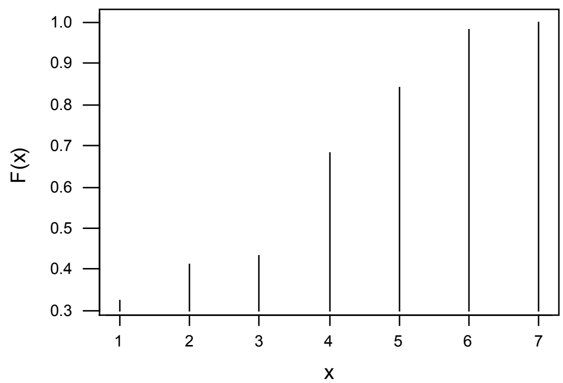
a) $P(X \le 3) = P(X = 1) + P(X = 2) + (PX = 3) = 0.326 + 0.088 + 0.019 = 0.433$

b) $P(3 < X < 5.1) = P(X = 4) + P(X = 5) = 0.251 + 0.158 = 0.409$

c) $P(X > 4.5) = P(X = 5) + P(X = 6) + P(X = 7) = 0.158 + 0.140 + 0.018 = 0.316$

d) $E(X) = 1(0.326) + 2(0.088) + 3(0.019) + 4(0.251) + 5(0.158) + 6(0.140) + 7(0.018)$
$= 3.319$
$V(X) = 1^2 (0.326) + 2^2 (0.088) + 3^2 (0.019) + 4^2 (0.251) + 5^2 (0.158) + 6^2 (0.140)$
$+ 7^2 (0.018) – (3.319)^2$
$= 3.7212$

f) Graph of F(x)



3-72. The function is a probability mass function. All probabilities are nonnegative and sum to 1.

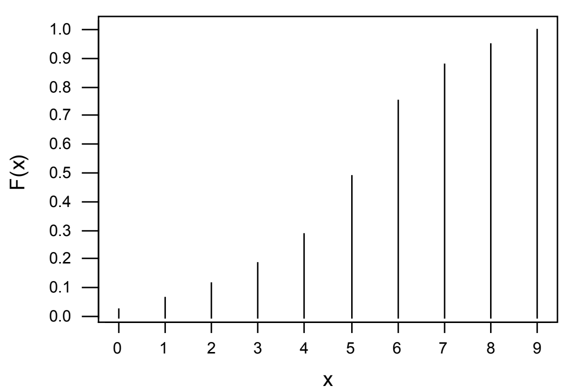
a) $P(X \le 1) = P(X=0) + P(X=1) = 0.025 + 0.041 = 0.066$

b) $P(2<X<7.2) = P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) = 0.762$

c) $P(X\ge 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9) = 0.508$

d) $E(X) = 0(0.025) + 1(0.041) + 2(0.049) + 3(0.074) + 4(0.098) + 5(0.205) + 6(0.262)$
$+ 7(0.123) + 8(0.074) + 9(0.049)$
$= 5.244$
$V(X) = 0^2 (0.025) + 1^2 (0.041) + 2^2 (0.049) + 3^2 (0.074) + 4^2 (0.098) + 5^2 (0.205)$
$+ 6^2 (0.262) + 7^2 (0.123) + 8^2\ 0.074) + 9^2 (0.049) - (5.244)^2$
$= 4.260$

e) Graph of F(x)



3-73. The function is a probability mass function. All probabilities are nonnegative and sum to 1.

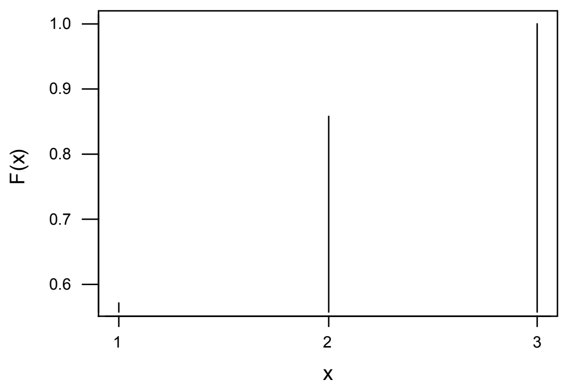
a) $P(X \le 1) = (8/7)(1/2) = 4/7$

b) $P(X > 1) = 1 - P(X \le 1) = 1 - 4/7 = 3/7$

c) $E(X) = 1\left(\frac{8}{7}\times\frac{1}{2}\right) + 2\left(\frac{8}{7}\times\frac{1}{4}\right) + 3\left(\frac{8}{7}\times\frac{1}{8}\right) = \frac{11}{7}$

$V(X) = 1^2\left(\frac{8}{7}\times\frac{1}{2}\right) + 2^2\left(\frac{8}{7}\times\frac{1}{4}\right) + 3^2\left(\frac{8}{7}\times\frac{1}{8}\right) - \left(\frac{11}{7}\right)^2 = \frac{26}{49}$

d) Graph of F(x)



3-74. The function is a probability mass function. All probabilities are nonnegative and sum to 1.

a) $P(X = 2) = (1/2)(2/5) = 1/5$

b) $P(X \le 3) = P(X=1) + P(X=2) + P(X=3) = 1/10 + 1/5 + 3/10 = 6/10 = 3/5$

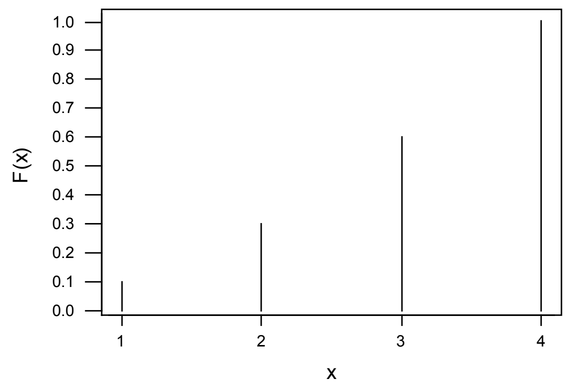
c) $P(X > 2.5) = 1 - P(X \le 2.5) = 1 - P(X \le 2) = 1 - (3/10) = 7/10$

d) $P(X \ge 1) = 1$

e) $E(X) = 1\left(\frac{1}{2}\times\frac{1}{5}\right) + 2\left(\frac{1}{2}\times\frac{2}{5}\right) + 3\left(\frac{1}{2}\times\frac{3}{5}\right) + 4\left(\frac{1}{2}\times\frac{4}{5}\right) = \frac{30}{10} = 3$

$V(X) = 1^2\left(\frac{1}{2}\times\frac{1}{5}\right) + 2^2\left(\frac{1}{2}\times\frac{2}{5}\right) + 3^2\left(\frac{1}{2}\times\frac{3}{5}\right) + 4^2\left(\frac{1}{2}\times\frac{4}{5}\right) - (3)^2 = 0.6$

f) Graph of F(x)



3-75. The function is a probability mass function. All probabilities are nonnegative and sum to 1.

a) $P(X < 9) = P(X = 7) + P(X = 8) = 0.170$

b) $P(X > 11) = P(X = 12) + P(X = 13) = 0.05 + 0.05 = 0.10$

c) $P(8 \le X \le 12) = P(X = 8) + P(X = 9) + P(X = 10) + P(X = 11) + P(X = 12) = 0.91$

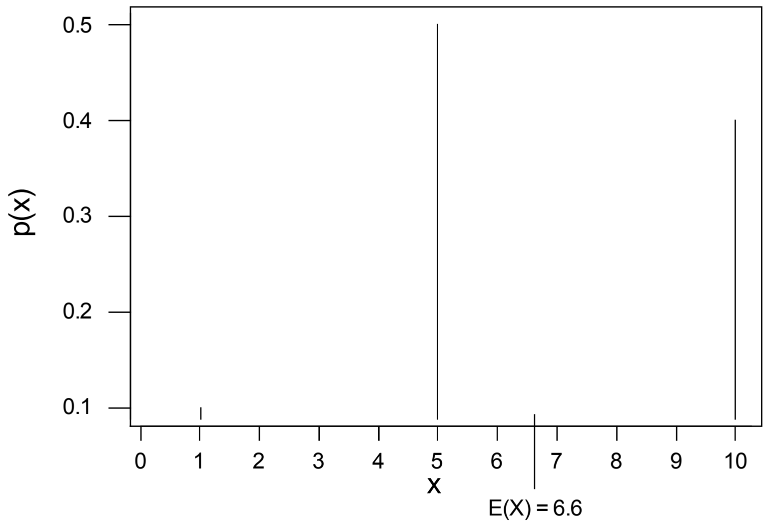
d) $E(X) = 7(0.040) + 8(0.130) + 9(0.190) + 10(0.240) + 11(0.300) + 12(0.050) + 13(0.050)$
$= 9.98$

$V(X) = 7^2(0.040) + 8^2(0.130) + 9^2(0.190) + 10^2(0.240) + 11^2(0.300) + 12^2(0.050)$
$+ 13^2(0.050) - (9.98)^2$
$= 2.02$

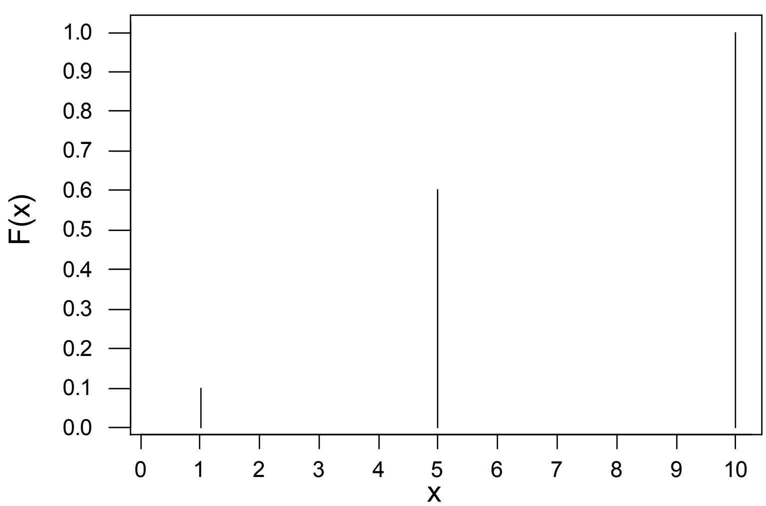
3-76. a) $P(X = 10 \text{ million}) = 0.4$, $P(X = 5 \text{ million}) = 0.5$, $P(X = 1 \text{ million}) = 0.1$

b) $E(X) = 10(0.4) + 5(0.5) + 1(0.1) = 6.6$ million
$V(X) = 10^2(0.4) + 5^2(0.5) + 1^2(0.1) - (6.6)^2 = 9.04 \text{ (million)}^2$
Standard Deviation is then $\sqrt{9.04} = 3.006$ million

c)



d)



Section 3-8

3-77. A binomial distribution is based on independent trials with two outcomes and a constant probability of success on each trial.

a) reasonable.

b) independence assumption not reasonable.

c) The probability that the second component fails depends on the failure time of the first component. The binomial distribution is not reasonable.

d) not independent trials with constant probability.

e) probability of a correct answer not constant.

f) reasonable.

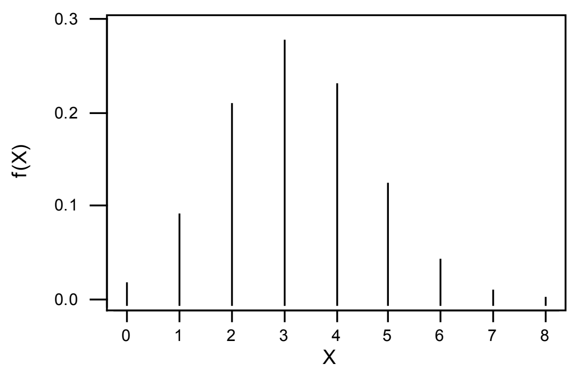
g) probability of finding a hole in the tube is not constant.

h) If the fills are independent with a constant probability of an underfill, then the binomial distribution for the number packages underfilled is reasonable.

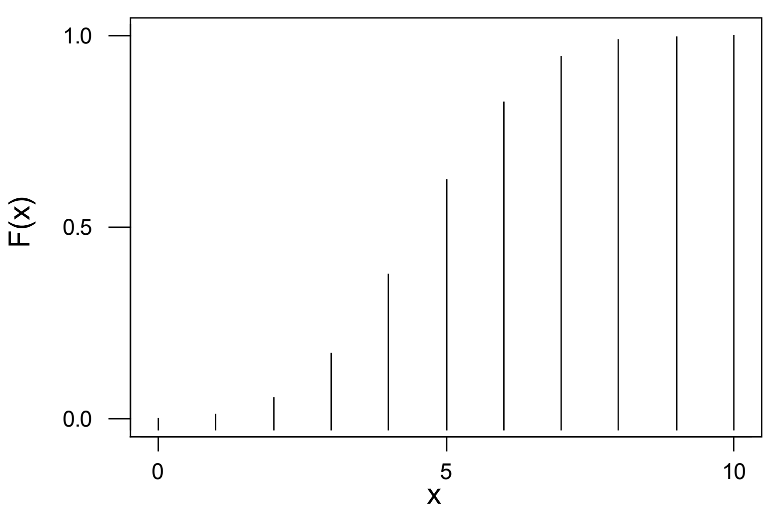
i) Because of the bursts, each trial (that consists of sending a bit) is not independent.

j) not independent trials with constant probability

3-78. a)



b)



c) The value X = 3 appears to be most likely.

d) The least likely values are X = 0, 7, and 8.

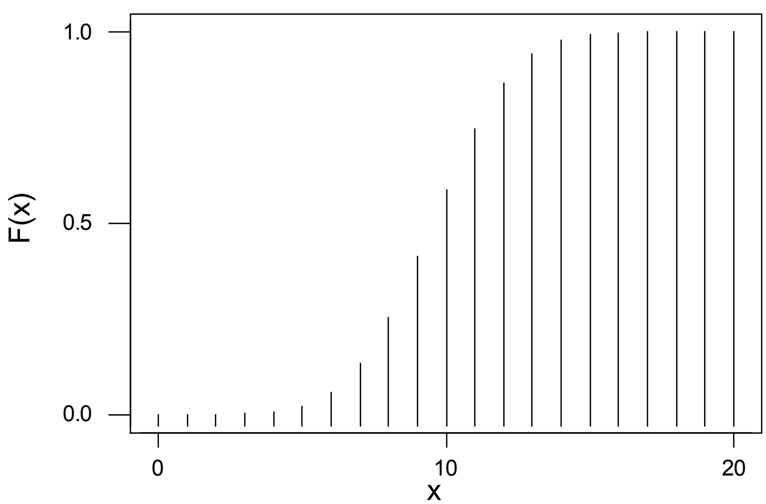
3-79. a) $P(X=15)=\binom{20}{15}0.5^{15}(0.5)^{5}=0.0148$

b) $P(X\le 12)=\binom{20}{0}0.5^{0}0.5^{20}+\binom{20}{1}0.5^{1}0.5^{19}+\ldots+\binom{20}{12}0.5^{12}0.5^{8}=0.8684$

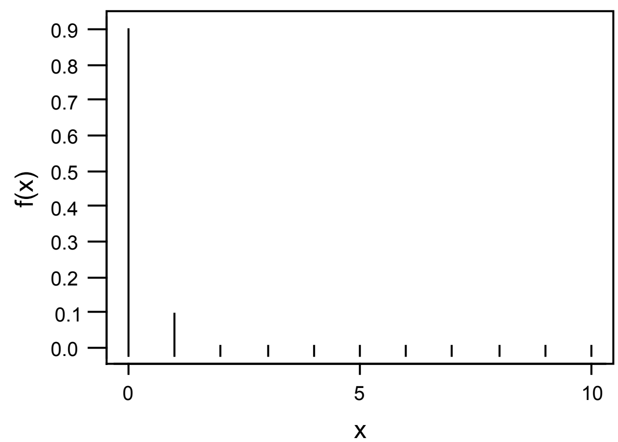
c) $P(X\ge 19)=\binom{20}{19}0.5^{19}(0.5)^{1}+\binom{20}{20}0.5^{20}(0.5)^{0}=0$

d) $P(13\le X<15)=\binom{20}{13}0.5^{13}0.5^{7}+\binom{20}{14}0.5^{14}0.5^{6}=0.1109$

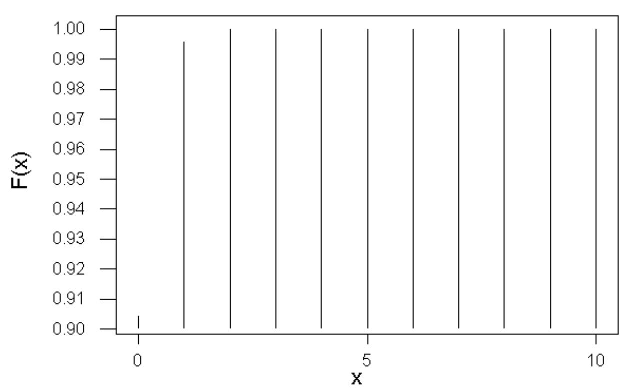
e)



3-80. a) The probability mass function is



b) The cumulative distribution function is



c) The value of X that appears to be most likely is 0.

d) The value of X that appears to be least likely is 10, although the probabilities for values of x greater than 1 are very small.

3-81. a) $P(X = 5) = \binom{10}{5} 0.1^5 (0.90)^5 = 0.0015$

b) $P(X \le 2) = \binom{10}{0} 0.10^0 0.90^{10} + \binom{10}{1} 0.10^1 0.90^9 + \binom{10}{2} 0.10^2 0.90^8$
$= 0.9298$

c) $P(X \ge 9) = \binom{10}{9} 0.10^9 (0.90)^1 + \binom{10}{10} 0.10^{10} (0.90)^0 = 0$

d) $P(3 \le X < 5) = \binom{10}{3} 0.10^3 0.90^7 + \binom{10}{4} 0.10^4 0.90^6 = 0.0686$

3-82. Let X denote the number of defective circuits. Then, X has a binomial distribution with n = 40 and p = 0.01. Then, $P(X = 0) = \binom{40}{0} 0.01^0 0.99^{40} = 0.6690$.

3-83. Let X denote the number of parts that will successfully pass the test. Then X has a binomial distribution with n = 7 and p = 0.80. $P(X=2) = \binom{7}{2} 0.8^2 0.2^5 = 0.0043$

3-84. Let X denote the number of times the line is occupied. Then, X has a binomial distribution with n = 8 and p = 0.45

a) $P(X = 2) = \binom{8}{2} 0.45^2 (0.55)^6 = 0.1569$

b) $P(X \ge 1) = 1 - P(X = 0) = 1 - \binom{8}{0} 0.45^0 (0.55)^8 = 0.9916$

c) E(X) = 8(0.45) = 3.6

3-85. a) n = 50, p = 5/50 = 0.1, since E(X) = 5 = np.

b) $P(X \le 2) = \binom{50}{0} 0.1^0 (0.9)^{50} + \binom{50}{1} 0.1^1 (0.9)^{49} + \binom{50}{2} 0.1^2 (0.9)^{48} = 0.1117$

c) $P(X \ge 49) = \binom{50}{49} 0.1^{49} (0.9)^1 + \binom{50}{50} 0.1^{50} (0.9)^0 = 0$

3-86. E(X) = 20 (0.01) = 0.2

V(X) = 20 (0.01) (0.99) = 0.198

$\mu_X + 3\sigma_X = 0.2 + 3\sqrt{0.198} = 1.53$

a) $P(X > 1.53) = P(X \ge 2) = 1 - P(X \le 1)$

$= 1 - \left[\binom{20}{0} 0.01^0 0.99^{20} + \binom{20}{1} 0.01^1 0.99^{19} \right]$

$= 0.0169$

b) X is binomial with n = 20 and p = 0.04

$P(X > 1) = 1 - P(X \le 1)$

$= 1 - \left[\binom{20}{0} 0.04^0 (0.96)^{20} + \binom{20}{1} 0.04^1 (0.96)^{19} \right]$

$= 0.1897$

c) Let Y denote the number of times X exceeds 1 in the next five samples. Then, Y is binomial with n = 5 and p = 0.1897 from part b.

$P(Y \ge 1) = 1 - P(Y = 0) = 1 - \left[\binom{5}{0} 0.1897^0 (0.8103)^5 \right] = 0.6507$

The probability is 0.6507 that at least one sample from the next five will contain more than one defective.

3-87. Let X denote the passengers with tickets that do not show up for the flight. Then, X is

Binomial with n = 125 and p = 0.1.

$$a)\, P(X \ge 5) = 1 - P(X \le 4)$$

$$= 1 - \left[\binom{125}{0} 0.1^0 (0.9)^{125} + \binom{125}{1} 0.1^1 (0.9)^{124} + \binom{125}{2} 0.1^2 (0.9)^{123} + \binom{125}{3} 0.1^3 (0.9)^{122} + \binom{125}{4} 0.1^4 (0.9)^{121} \right]$$

$$= 0.9961$$

$$b)\, P(X < 5) = 1 - P(X \ge 5) = 0.0039$$

c) $E(X) = np = 125(0.9) = 112.5$

$$\sigma_X = \sqrt{np(1-p)} = \sqrt{125(0.9)(0.1)} = 3.354$$

3-88. Let X denote the number of defective components among those stocked.

$$a)\, P(X = 0) = \binom{100}{0} 0.02^0 (0.98)^{100} = 0.133$$

$$b)\, P(X \le 2) = \binom{102}{0} 0.02^0 0.98^{102} + \binom{102}{1} 0.02^1 0.98^{101} + \binom{102}{2} 0.02^2 0.98^{100} = 0.666$$

$$c)\, P(X \le 5) = 0.981$$

3-89. Let X denote the number of students who successfully land the plane.

a) $P(X = 9) = \left[\binom{9}{9} (0.80)^9 (0.20)^0 \right] = 0.13422$

b) $P(X = 0) = \left[\binom{9}{0} (0.80)^0 (0.20)^9 \right] = 0.000001$

c) $P(X = 8) = \left[\binom{9}{8} (0.80)^8 (0.20)^1 \right] = 0.301990$

3-90. Let X denote the number of bulbs that fail.

a) $P(X<2) = P(X = 0) + P(X = 1) = \left[\binom{10}{0} (0.25)^0 (0.75)^{10} + \binom{10}{1} (0.25)^1 (0.75)^9 \right] = 0.2440$

b) $P(X = 0) = 0.0563$

c) $P(X>4) = 1 - P(X \le 4) = 1 - 0.921873 = 0.07813$

<u>Section 3-9.1</u>

3-91. a) $P(X = 0) = e^{-0.3} = 0.7408$

b) $P(X \le 3) = e^{-0.3} + \frac{e^{-0.3}(0.3)}{1!} + \frac{e^{-0.3}(0.3)^2}{2!} + \frac{e^{-0.3}(0.3)^3}{3!} = 0.9997$

c) $P(X = 6) = \frac{e^{-0.3}(0.3)^6}{6!} = 0$

d) $P(X = 2) = \frac{e^{-0.3}(0.3)^2}{2!} = 0.0333$

3-92. a) $P(X = 0) = \frac{e^{-5}5^0}{0!} = e^{-5} = 0.00673$

b) $P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= e^{-5} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!}$$
$$= 0.2650$$

c) $P(X = 6) = \frac{e^{-5}5^6}{6!} = 0.146$

d) $P(X = 9) = \frac{e^{-5}5^9}{9!} = 0.0363$

3-93. $P(X = 0) = \exp(-\lambda)$. Therefore, $\lambda = -\ln(0.02) = 3.912$.
Consequently, $E(X) = V(X) = 3.912$.

3-94. $P(X = 0) = \exp(-\lambda)$. Therefore, $\lambda = -\ln(0.04) = 3.219$.
Consequently, $E(X) = V(X) = 3.219$.

3-95. a) Let X denote the number of calls in one hour. Then, X is a Poisson random variable with $\lambda = 20$. $P(X = 18) = \frac{e^{-20}20^{18}}{18!} = 0.0844$.

b) $\lambda = 10$ for a thirty minute period. $P(X \le 3) = e^{-20} + \frac{e^{-10}10^1}{1!} + \frac{e^{-10}10^2}{2!} + \frac{e^{-10}10^3}{3!} = 0.0103$

c) Let Y denote the number of calls in two hours. Then, Y is a Poisson random variable with $\lambda= 40$. $P(Y = 30) = \frac{e^{-40}40^{30}}{30!} = 0.0185$

d) Let W denote the number of calls in 30 minutes. Then W is a Poisson random variable with $\lambda= 10$. $P(W = 10) = \frac{e^{-10}10^{10}}{10!} = 0.1251$

3-96. a) Let X denote the number of tremors in a 12 month period. Then, X is a Poisson random variable with $\lambda = 6$. $P(W = 10) = \frac{e^{-6}6^{10}}{10!} = 0.0413$

b) $\lambda =2(6) = 12$ for a two year period. Let Y denote the number of tremors in a two year period. $P(Y = 18) = \frac{e^{-12}12^{18}}{18!} = 0.0255$

c) $\lambda = (1/12)6 = 0.5$ for a one month period. Let W denote the number of tremors in a one-month period. $P(W = 0) = \frac{e^{-0.5}0.5^0}{0!} = 0.6065$

d) $\lambda = (1/2)6 = 3$ for a six month period. Let V denote the number of tremors in a six-month period.

$$P(V > 5) = 1 - P(V \le 5)$$
$$= 1 - \left[\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} + \frac{e^{-3}3^4}{4!} + \frac{e^{-3}3^5}{5!}\right]$$
$$= 1 - 0.9161$$
$$= 0.0839$$

3-97. a) Let X denote the number of cracks in 5 miles of highway. Then, X is a Poisson random variable with $\lambda = 10$. $P(X = 0) = e^{-10} = 4.54 \times 10^{-5}$

b) Let Y denote the number of cracks in a half mile of highway. Then, Y is a Poisson random variable with $\lambda = 1$. $P(Y \ge 1) = 1 - P(Y = 0) = 1 - e^{-1} = 0.6321$.

c) The assumptions of a Poisson process require that the probability of a count is constant for all intervals. If the probability of a count depends on traffic load and the load varies, then the assumptions of a Poisson process are not valid. Separate Poisson random variables might be appropriate for the heavy and light load sections of the highway.

3-98. a) Let X denote the number of flaws in 10 square feet of plastic panel. Then, X is a Poisson random variable with $\lambda_x = 0.05/ft^2$. Let Y be a Poisson random variable with $\lambda_y = 0.5/10ft^2$. Then, the probability there are no surface flaws in the auto's interior is $P(Y = 0) = e^{-0.5} = 0.6065$.

b) Let Y denote the number of cars with no flaws, $P(Y = 0) = \left[e^{-0.5}\right]^{10} = e^{-5} = 0.00674$

c) Let W denote the number of cars with surface flaws. Because the number of flaws has a Poisson distribution, the occurrences of surface flaws in cars are independent events with constant probability. From part a), the probability a car contains surface flaws is 1–0.6065 = 0.3935. Consequently, W is binomial with n = 10 and p = 0.3935.

$$P(W \le 1) = P(W = 0) + P(W = 1)$$

$$= \binom{10}{0} 0.3935^0 (0.6065)^{10} + \binom{10}{1} 0.3935^1 (0.6065)^9$$

$$= 0.0504$$

3-99. a) Let X denote the failures in 8 hours. Then, X has a Poisson distribution with $\lambda = 0.32$.

$$P(X = 0) = e^{-0.32} = 0.7261$$

b) Let Y denote the number of failure in 24 hours. Then, Y has a Poisson distribution with $\lambda = 0.96$. $P(Y \ge 3) = 1 - P(Y \le 2) = 1 - \left[e^{-.96} + \frac{e^{-0.96}(0.96)^1}{1!} + \frac{e^{-0.96}(0.96)^2}{2!} \right] = 1 - 0.9269 = 0.0731$

3-100. a) $\lambda = 3.2$, $P(X = 5) = 0.1140$
b) $\lambda_W = 6.4$, $P(W = 8) = 0.1160$
c) $\lambda_Y = 9.6$, $P(Y = 0) = 0.0001$

3-101. Let X denote number of calls exceeding the maximum design constraint. X follows a Poisson distribution with $\lambda = 9$. $P(X > 10) = 1 - P(X \le 10) = 1 - 0.706 = 0.2941$.

3-102. a) Let X denote the number of flaws in 25 square yards. Then, X is a Poisson random variable with $\lambda = 25(0.01) = 0.25$. $P(X = 0) = e^{-0.25} = 0.7788$.

b) Let Y denote the number of flaws in one square yard, then $P(Y = 0) = e^{-0.01} = 0.99$

c) $P(Y = 0) = 0.99$. Let V denote the number of square yards out of 10 that contain no flaws. Then, V is a binomial random variable with n = 10 and p = 0.99.

$$P(V \ge 8) = P(V = 8) + P(V = 9) + P(V = 10) = 0.9957$$

3-103. For one hour $\lambda = 10$

a) $P(X = 3) = \frac{e^{-10}(10)^3}{3!} = 0.0076$

b) For a thirty minute period, $\lambda = 5$

$$P(X = 6) = \frac{e^{-5}(5)^6}{6!} = 0.1462$$

Section 3-9.2

3-104. a) $P(X \le 0) = 0$

b) $P(X \ge 3) = \int_3^\infty 3e^{-3x}dx = -e^{-3x}\Big|_3^\infty = e^{-9} = 0.0001234$

c) $P(X \le 2) = \int_0^2 3e^{-3x}dx = -e^{-3x}\Big|_0^2 = 1 - e^{-6} = 0.99752$

d) $P(2 < X < 3) = \int_2^3 3e^{-3x}dx = -e^{-3x}\Big|_2^3 = e^{-6} - e^{-9} = 0.002355$

e) $P(X \le x) = \int_0^x 3e^{-3t}dt = -e^{-3t}\Big|_0^x = 1 - e^{-3x} = 0.05$ and x = 0.0171

3-105. If E(X) = 5, then λ =0.20

a) $P(X > 5) = \int_5^\infty 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_5^\infty = e^{-1} = 0.3679$

b) $P(X > 15) = -e^{-0.2x}\Big|_{15}^\infty = e^{-3} = 0.0498$

c) $P(X > 20) = -e^{-0.2x}\Big|_{20}^\infty = e^{-4} = 0.0183$

d) $P(X < x) = \int_0^x 0.2e^{-0.2t}dt = -e^{-0.2t}\Big|_0^x = 1 - e^{-0.2x} = 0.95$ and x = 14.9787.

3-106. Let X denote the time until the first count. Then, X is an exponential random variable with λ = 3 counts per minute.

a) $P(X > 0.5) = \int_{0.5}^\infty 3e^{-3x}dx = -e^{-3x}\Big|_{0.5}^\infty = e^{-1.5} = 0.2231$

b) $P\left(X < \frac{10}{60}\right) = \int_0^{1/6} 3e^{-3x}dx = -e^{-3x}\Big|_0^{1/6} = 1 - e^{-1/2} = 0.3935$

c) $P(1 < X < 2) = -e^{-3x}\Big|_1^2 = e^{-3} - e^{-6} = 0.0473$

3-107. a) E(X) = 1/3 minutes.

b) V(X) = $1/3^2$ = 1/9 minutes^2 and σ_X =1/3 minutes.

c) P(X < x) = 0.95 and P(X < x) = $-e^{-3t}\Big|_0^x = 1 - e^{-3x} = 0.95$.

Therefore, x = 0.9986 minutes.

3-108. Let X denote the time until the first call. Then, X is exponential and

$\lambda = \frac{1}{E(X)} = \frac{1}{12}$ calls/minute.

a) $P(X > 30) = \int_{30}^\infty \frac{1}{12}e^{-\frac{x}{12}}dx = -e^{-\frac{x}{12}}\Big|_{30}^\infty = e^{-2.5} = 0.0821$

b) The probability of at least one call in a 10-minute interval equals one minus the probability of zero calls in a 10-minute interval and that is P(X > 10).

$$P(X > 10) = -e^{-\frac{x}{12}}\Big|_{10}^\infty = e^{-5/6} = 0.4346 .$$

Therefore, the answer is 1– 0.4346 = 0.5654.

c) $P(5 < X < 10) = -e^{-\frac{x}{12}}\Big|_{5}^{10} = e^{-5/12} - e^{-10/12} = 0.2246$

d) P(X < x) = 0.90 and $P(X < x) = -e^{-\frac{t}{12}}\Big|_{0}^{x} = 1 - e^{-x/12} = 0.90$. Therefore, x = 27.63 minutes.

3-109. X follows an exponential distribution with λ = 50.

a) $P(X < 2) = \int_{1/30}^{\infty} 50e^{-50x}dx = -e^{-50x}\Big|_{1/30}^{\infty} = e^{-5/3} = 0.1889$

b) $P(2.5 < X < 4.5) = \int_{1/24}^{3/40} 50e^{-50x}dx = -e^{-50x}\Big|_{1/24}^{3/40} = e^{-25/12} - e^{-15/4} = 0.0007$

3-110. E(X) = λ = 50 in 1 hour

a) In a twenty-minute interval, E(X) = 50/3 = 16.667.

b) P(X=2) = $\frac{e^{-16.667}16.667^2}{2!} = 0$

3-111. Let X denote the distance between major cracks. Then, X is an exponential random variable with λ = 1/E(X) = 0.2 cracks/mile.

a) P(X > 10) = $\int_{10}^{\infty} 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_{10}^{\infty} = e^{-2} = 0.1353$

b) Let Y denote the number of cracks in 10 miles of highway. Because the distance between cracks is exponential, Y is a Poisson random variable with λ 10(0.2) = 2 cracks per 10 miles.

$$P(Y = 2) = \frac{e^{-2}2^2}{2!} = 0.2707$$

c) $\sigma_X = 1/\lambda = 5$ miles

3-112. a) $P(12 < X < 15) = \int_{12}^{15} 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_{12}^{15} = e^{-2.4} - e^{-3} = 0.0409$

b) $P(X > 5) = -e^{-0.2x}\Big|_{5}^{\infty} = e^{-1} = 0.3679$. By independence of the intervals in a Poisson process, the answer is 0.3679^2 = 0.1353. Alternatively, the answer can also be found as P(X > 10) = e^{-2} = 0.1353. The probability does depend on whether or not the lengths of highway are consecutive.

c) By the memoryless property, the answer is P(X > 10) = 0.1353 from part b.

3-113. Let X denote the time until a message is received. Then, X is an exponential random variable and λ =1/2.

a) $P(X > 2) = \int_{2}^{\infty} \frac{1}{2}e^{-x/2}dx = -e^{-x/2}\Big|_{2}^{\infty} = e^{-1} = 0.3679$

b) The same as part a.

c) E(X) = 2 hours.

3-114. Let X denote the time until a failure occurs. Then, X is an exponential random variable and λ = 1/4.

a) $P(X<1)=\int_0^1 \frac{1}{4}e^{-x/4}dx = 1-e^{-x/4}\Big|_0^1 = 0.2212$

b) $P(X<2)=\int_0^2 \frac{1}{4}e^{-x/4}dx = 1-e^{-x/4}\Big|_0^2 = 0.3935$

c) $P(X<4)=\int_0^4 \frac{1}{4}e^{-x/4}dx = 1-e^{-x/4}\Big|_0^4 = 0.6321$

3-115. Let X denote the time until the component fails. Then, X is an exponential random variable with $\lambda = 1/4$.

a) $P(X < b) = 0.03$

$$\int_0^b \frac{1}{4}e^{-x/4}dx = -e^{-x/4}\Big|_0^b = 1-e^{-b/4} = 0.03$$

$$e^{-b/4} = 0.97$$

$$-\frac{b}{4} = \ln(0.97)$$

$$b = 0.122$$

Where b = 0.122 years is equivalent to 1.4 months.

b) The mean time to failure in this case is $E(X) = \lambda$. Solve the following for λ.

$$\int_0^1 \frac{1}{\lambda}e^{-x/\lambda}dx = -e^{-x/\lambda}\Big|_0^1 = 1-e^{-1/\lambda} = 0.03$$

$$e^{-1/\lambda} = 0.97$$

$$-\frac{1}{\lambda} = \ln(0.97)$$

$$\frac{1}{\lambda} = 0.03$$

$$\lambda = 33.3$$

3-116. Let X denote the number of calls in 30 minutes. Because the time between calls is an exponential random variable, X is a Poisson random variable with $\lambda = 1/E(X) = 0.1$ calls per minute $\Rightarrow$ 3 calls per 30 minutes.

a) $P(X>3) = 1-P(X\le 3) = 1-\left[\frac{e^{-3}3^0}{0!}+\frac{e^{-3}3^1}{1!}+\frac{e^{-3}3^2}{2!}+\frac{e^{-3}3^3}{3!}\right] = 0.3528$

b) $P(X=0) = \frac{e^{-3}3^0}{0!} = 0.04979$

c) Let Y denote the time between calls in minutes. Then, $P(Y \ge x) = 0.01$ and

$$P(Y\ge x) = \int_x^\infty 0.1e^{-0.1t}dt = -e^{-0.1t}\Big|_x^\infty = e^{-0.1x}.$$

Therefore, $e^{-0.1x} = 0.01$ and x = 46.05 minutes.

d) $P(Y\ge 120) = \int_{120}^\infty 0.1e^{-0.1t}dt = -e^{-0.1t}\Big|_{120}^\infty = e^{-12} = 6.14\times 10^{-6}$

e) The probability of no calls in one-half hour is (from part b) $e^{-3} = 0.04979$. Therefore, for four non-overlapping one-half hour intervals, the probability of no calls is $(e^{-3})^4 = (0.04979)^4 = 6.14 \times 10^{-6}$.