Section 3-10

3-117. a) E(X) = 300(0.4) = 120, V(X) = 300(0.4)(0.6) = 72 and $\sigma_X = \sqrt{72}$.

Then, $P(X \le 90) \cong P\left(Z \le \frac{90-120}{\sqrt{72}}\right) = P(Z \le -3.54) = 0.0002$

b) $P(70 < X \le 90) \cong P\left(\frac{70-120}{\sqrt{72}} < Z \le \frac{90-120}{\sqrt{72}}\right) = P(-5.89 < Z \le -3.54)$

$= 0.0002 - 0 = 0.0002$

3-118. a) $P(X < 8) = \binom{100}{0}0.1^0(0.9)^{100} + \binom{100}{1}0.1^1(0.9)^{99} + \ldots + \binom{100}{7}0.1^7 0.9^{93}$

$= 0.2061$

b) E(X) = 10, V(X) = 100(0.1)(0.9) = 9 and $\sigma_X = 3$.

Then, $P(X < 8) \cong P(Z < \frac{8-10}{3}) = P(Z < -0.667) = 0.2524$

c) $P(8 < X < 12) \cong P\left(\frac{8-10}{3} < Z < \frac{12-10}{3}\right) = P(-0.67 < Z < 0.67) = 0.4971$

3-119. Let X denote the number of defective chips in the lot.
Then, E(X) = 1000(0.02) = 20, V(X) = 1000(0.02)(0.98) = 19.6.

a) $P(X > 25) \cong P\left(Z > \frac{25-20}{\sqrt{19.6}}\right) = P(Z > 1.13) = 1 - P(Z \le 1.13) = 0.1294$

b) $P(20 < X < 30) \cong P(0 < Z < \frac{10}{\sqrt{19.6}}) = P(0 < Z < 2.26)$

$= P(Z \le 2.26) - P(Z < 0) = 0.98809 - 0.5 = 0.4881$

3-120. Let X = number of defective inspected parts
E(X) = 100(0.08) = 8
V(X) = 100(0.08)(0.92) = 7.36

a) $P(X < 8) = P(X \le 7) = \sum_{i=0}^{7} \binom{100}{i}(0.08)^i (0.92)^{100-i} = 0.4471$

b) $P(X < 8) \cong P\left(Z < \frac{8-8}{\sqrt{7.36}}\right) = P(Z < 0) = 0.5$

3-121. Let X denote the number of original components that fail during the useful life of the product. Then, X is a binomial random variable with p = 0.005 and n = 2000. Also, E(X) = 2000(0.005) = 10 and V(X) = 2000(0.005)(0.995) = 9.95.

$P(X \ge 5) \cong P\left(Z \ge \frac{5-10}{\sqrt{9.95}}\right) = P(Z \ge -1.59) = 1 - P(Z < -1.59) = 1 - 0.055917 = 0.9441$.

3-122. Let X denote the number of particles in 10 cm^2 of dust. Then, X is a Poisson random variable with $\lambda = 10(1000) = 10{,}000$. Also, $E(X) = \lambda = 10{,}000$ and $V(X) = \lambda^2 = 10^8$.

$P(X > 10{,}000) \cong P\left(Z > \frac{10{,}000 - 10{,}000}{\sqrt{10^8}}\right) = P(Z > 0) = 0.5$

3-123. E(X) = 50(0.1) = 5 and V(X) = 50(0.1)(0.9) = 4.5

a) $P(X \le 2) = P(X \le 2.5) \cong P\left(Z \le \frac{2.5-5}{\sqrt{4.5}}\right) = P(Z \le -1.18)$

$= 1 - P(Z \le 1.18) = 1 - 0.881 = 0.119$

b) $P(X \le 2) \cong P\left(Z \le \frac{2-5}{\sqrt{4.5}}\right) = P(Z \le -1.41) = 0.0793$

c) $P(X \le 2) = \binom{50}{0}0.1^0 0.9^{50} + \binom{50}{1}0.1^1 0.9^{49} + \binom{50}{2}0.1^2 0.9^{48} = 0.118$

The probability computed using the continuity correction is closer.

d) $P(X \le 10) = P(X \le 10.5) \cong P\left(Z \le \frac{10.5-5}{\sqrt{4.5}}\right) = P(Z \le 2.59) = 0.995$

e) $P(X < 10) = P(X \le 9.5) \cong P\left(Z \le \frac{9.5-5}{\sqrt{4.5}}\right) = P(Z \le 2.12) = 0.983$

3-124. $E(X) = 50(0.1) = 5$ and $V(X) = 50(0.1)(0.9) = 4.5$

a) $P(X \ge 2) = P(X \ge 1.5) \cong P\left(Z \le \frac{1.5-5}{\sqrt{4.5}}\right) = P(Z \ge -1.65) = 0.951$

b) $P(X \ge 2) \cong P\left(Z \ge \frac{2-5}{\sqrt{4.5}}\right) = P(Z \ge -1.414) = 0.921$

c) $P(X \ge 2) = 1 - P(X < 2) = 1 - \binom{50}{0}0.1^0 0.9^{50} - \binom{50}{1}0.1^1 0.9^{49} = 0.966$

The probability computed using the continuity correction is closer.

d) $P(X \ge 6) = P(X \ge 5.5) \cong P(Z \ge 0.24) = 0.4052$

e) $P(X > 6) = P(X \ge 7) = P(X \ge 6.5) \cong P(Z \ge 0.707) = 0.24$

Section 3-11

3-125. a) $P(X < 9, Y < 2.5) = P(X < 9)\, P(Y < 2.5)$

$= P\left(Z < \frac{9-10}{1.5}\right) P\left(Z < \frac{2.5-2}{.25}\right)$

$= P(Z < -0.67)P(Z < 2)$

$= (0.25143)(0.97725)$

$= 0.2457$

b) $P(X > 8, Y < 2.25) = P(X > 8)P(Y < 2.25)$

$= P\left(Z > \frac{8-10}{1.5}\right) P\left(Z < \frac{2.25-2}{.25}\right)$

$= P(Z > -1.33)P(Z < 1)$

$= (1 - P(Z < -1.33))P(Z < 1)$

$= (0.90824)(0.84134)$

$= 0.7641$

c) $P(8.5 \le X \le 11.5, Y > 1.75) = P(8.5 \le X \le 11.5)P(Y > 1.75)$

$$= P\left(\frac{8.5-10}{1.5} \le Z \le \frac{11.5-10}{1.5}\right)P\left(Z > \frac{1.75-2}{.25}\right)$$
$$= P(-1 \le Z \le 1)P(Z > -1)$$
$$= P(-1 \le Z \le 1)(1 - P(Z < -1))$$
$$= (0.68268)(0.84134)$$
$$= 0.5743$$

d) $P(X < 13, 1.5 \le Y \le 1.8) = P(X < 13)P(1.5 \le Y \le 1.8)$

$$= P\left(Z < \frac{13-10}{1.5}\right)P\left(\frac{1.5-2}{.25} \le Z \le \frac{1.8-2}{.25}\right)$$
$$= P(Z < 2)P(-2 \le Z \le -0.8)$$
$$= (0.97725)(0.18911)$$
$$= 0.1848$$

3-126. a) $P(X < 12, Y < 19) = P(X < 12)P(Y < 19)$

$$= P\left(Z < \frac{12-15}{3}\right)P\left(Z < \frac{19-20}{1}\right)$$
$$= P(Z < -1)P(Z < -1)$$
$$= (0.15866)(0.15866)$$
$$= 0.0252$$

b) $P(X > 16, Y < 18) = P(X > 16)P(Y < 18)$

$$= P\left(Z > \frac{16-15}{3}\right)P\left(Z < \frac{18-20}{1}\right)$$
$$= P(Z > 0.33)P(Z < -2)$$
$$= (1 - P(Z < 0.33))P(Z < -2)$$
$$= (0.3707)(0.02275)$$
$$= 0.0084$$

c) $P(14 \le X < 16, Y > 22) = P(14 \le X < 16)P(Y > 22)$

$$= P\left(\frac{14-15}{3} \le Z < \frac{16-15}{3}\right)P\left(Z > \frac{22-20}{1}\right)$$
$$= P(-0.33 \le Z < 0.33)P(Z > 2)$$
$$= P(-0.33 \le Z < 0.33)(1 - P(Z < 2))$$
$$= (0.2586)(0.02275)$$
$$= 0.0059$$

d) $P(11 \le X \le 20, 17.5 \le Y \le 21) = P(11 \le X \le 20)P(17.5 \le Y \le 21)$

$$= P\left(\frac{11-15}{3} \le Z \le \frac{20-15}{3}\right)P\left(\frac{17.5-20}{1} \le Z \le \frac{21-20}{1}\right)$$
$$= P(-1.33 \le Z \le 1.67)P(-2.5 \le Z \le 1)$$
$$= (0.86018)(0.83513)$$
$$= 0.7184$$

3-127. a) $P(X < 4, Y < 4) = P(X < 4)P(Y < 4)$

$$= P(X \le 3)P(Y \le 3)$$
$$= (0.857)(0.4335)$$
$$= 0.372$$

b) $P(X > 2, Y < 4) = P(X > 2)P(Y < 4)$

$$= (1 - P(X \le 2))P(Y \le 3)$$
$$= (1 - 0.6767)(0.4335)$$

$= 0.1402$

c) $P(2 \le X < 4, Y \ge 3) = P(2 \le X < 4)P(Y \ge 3)$
$= P(2 \le X \le 3)(1 - P(Y < 3))$
$= (P(X \le 3) - P(X \le 1))(1 - P(Y \le 2))$
$= (0.4511)(0.7619)$
$= 0.3437$

d) $P(X < 5, 1 \le Y \le 4) = P(X < 5)P(1 \le Y \le 4)$
$= P(X \le 4)(P(Y \le 4) - P(Y = 0))$
$= (0.9473)(0.6105)$
$= 0.5783$

3-128. a) $P(X \le 5, Y \le 8) = P(X \le 5)P(Y \le 8)$
$= (0.6321)(0.6321)$
$= 0.3996$

b) $P(X > 5, Y \le 6) = P(X > 5)P(Y \le 6)$
$= (1 - P(X \le 5))P(Y \le 6)$
$= (0.3679)(0.5276)$
$= 0.1941$

c) $P(3 < X \le 7, Y > 7) = P(3 < X \le 7)P(Y > 7)$
$= (P(Y \le 7) - P(Y \le 3))(1 - P(Y \le 7))$
$= (0.3022)(0.4169)$
$= 0.1260$

d) $P(X > 7, 5 < Y \le 7) = P(X > 7)P(5 < Y \le 7)$
$= (1 - P(X \le 7))(P(Y \le 7) - P(Y \le 5))$
$= (0.2466)(0.1184)$
$= 0.0292$

3-129. The two vendors are assumed independent of one another.
a) $P(X_1 < 6100, X_2 < 6100) = P(X_1 < 6100)P(X_2 < 6100)$

$$= P\left(Z < \frac{6100 - 6000}{100}\right)P\left(Z < \frac{6100 - 5825}{90}\right)$$
$$= P(Z < 1)P(Z < 3.05)$$
$$= (0.84134)(0.9989)$$
$$= 0.8404$$

b) $P(5800 < X_1 < 6050, 5800 < X_2 < 6050) = P(5800 < X_1 < 6050)P(5800 < X_2 < 6050)$

$$= P\left(\frac{5800 - 6100}{100} < Z < \frac{6050 - 6000}{100}\right) - P\left(\frac{5800 - 5825}{90} < Z < \frac{6050 - 5825}{90}\right)$$
$$= P(-2 < Z < 0.5)P(-0.28 < Z < 2.5)$$
$$= (0.66871)(0.60405)$$
$$= 0.4033$$

c) $P(X_1 > 6200, X_2 > 6200) = P(X_1 > 6200)P(X_2 > 6200)$

$$= P\left(Z > \frac{6200 - 6000}{100}\right)P\left(Z > \frac{6200 - 5825}{90}\right)$$
$$= P(Z > 2)P(Z > 4.16)$$
$$= (1 - P(Z < 2))(1 - P(Z < 4.16))$$
$$= 0$$

3-130. Let X, Y, and Z denote the time until a problem on line 1, 2, and 3, respectively.
The random variables are independent with the same distribution.

a) $P(X > 40, Y > 40, Z > 40) = P(X > 40)P(Y > 40)P(Z > 40)$
$= [P(X > 40)]^3$

Now,

$$P(X > 40) = \int_{40}^{\infty} \frac{1}{40} e^{-x/40} dx$$
$$= -e^{-x/40} \Big|_{40}^{\infty}$$
$$= e^{-1}$$

Therefore, $P(X > 40, Y > 40, Z > 40) = [e^{-1}]^3 = 0.0498$

b) $P(20 < X < 40, 20 < Y < 40, 20 < Z < 40) = P(20 < X < 40)P(20 < Y < 40)P(20 < Z < 40)$
$= [P(20 < X < 40)]^3$

and

$$P(20 < X < 40) = \int_{20}^{40} \frac{1}{40} e^{-x/40} dx$$
$$= -e^{-x/40} \Big|_{20}^{40}$$
$$= e^{-0.5} - e^{-1}$$
$$= 0.2387$$

Therefore, $P(20 < X < 40, 20 < Y < 40, 20 < Z < 40) = (0.2387)^3 = 0.0136$.

3-131. a) $P(T_1) = 1 - (0.4 + 0.25) = 0.35$, $P(T_2) = 1 - (0.25 + 0.30) = 0.45$,
$P(T_3) = 1 - (0.35 + 0.40) = 0.25$, $P(T_4) = 1 - (0.50 + 0.40) = 0.10$
b) $P(F_1F_2F_3F_4) = P(F_1)P(F_2)P(F_3)P(F_4) = (0.40)(0.25)(0.35)(0.50) = 0.0175$
c) P(Four caps at first quality or Four caps at second quality)
= P(Four caps at first quality) + P(Four caps at second quality)
= $P(F_1F_2F_3F_4) + P(S_1S_2S_3S_4) = 0.0175 + (0.25)(0.30)(0.40)(0.40) = 0.0295$

3-132. Let X denote the production yield on a day. Then,

$$P(X > 1400) = P\left(Z > \frac{1400 - 1500}{\sqrt{10000}}\right) = P(Z > -1) = 1 - P(Z < -1) = 0.8413$$

a) Let Y denote the number of days out of five such that the yield exceeds 1400. Then, by independence, Y has a binomial distribution with n = 5 and p = 0.8413. The answer is then

$$P(Y = 5) = \binom{5}{5}(0.8413)^5(0.1587)^0 = 0.4215$$

b) $$P(Y = 0) = \binom{5}{0}(0.8413)^0(0.1587)^5 = 0.0001$$

3-133. a) $P(C_1)P(C_2) = (0.95)(0.92) = 0.8740$
b) $1 - 0.8740 = 0.1260$

3-134. a) $P(C_1)P(C_2)P(C_3) = (0.90)(0.99)(0.95) = 0.8465$
b) $1 - 0.8465 = 0.1535$

3-135. a) $P(C_1') = 0.15$
b) $P(C_2') = 0.08$
c) $P(C_1 \text{ or } C_2) = 1 - P(C_1')\,P(C_2') = 1 - (0.15)(0.08) = 0.988$
d) $1 - P(C_1 \text{ or } C_2) = 1 - 0.988 = 0.012$

3-136. a) $P(C_1 \text{ or } C_2 \text{ or } C_3) = 1 - P(C_1')\,P(C_2')\,P(C_3') = 1 - (0.10)(0.01)(0.05) = 0.99995$

Section 3-12

3-137. a) $3E(X_1) + 5E(X_2) = 3(2) + 5(5) = 31$
b) $9V(X_1) + 25V(X_2) = 9(2) + 25(10) = 268$

3-138. a) $E(2X_1 + 0.5X_2 - 3X_3) = 2(4) + 0.5(3) - 3(2) = 3.5$
b) $V(2X_1 + 0.5X_2 - 3X_3) = 4(1) + 0.25(5) + 9(2) = 23.25$

3-139. a) $4E(X_1) - 2E(X_2) = 4(6) - 2(1) = 22$
b) $16V(X_1) + 4V(X_2) = 16(2) + 4(4) = 48$
c) $2E(Y) = 2(22) = 44$
d) $4V(Y) = 4(48) = 192$

3-140. a) $E(Y) = E(2.5X_1 - 0.5X_2 + 1.5X_3) = 2.5(1.2) - 0.5(0.8) + 1.5(0.5) = 3.35$
b) $V(Y) = V(2.5X_1 - 0.5X_2 + 1.5X_3) = 6.25(1)^2 + 0.25(0.5)^2 + 2.25(2.2)^2 = 17.2025$
c) $E(-3Y) = -3E(Y) = -3(3.35) = -10.05$
d) $V(-3Y) = 9V(Y) = 9(17.2025) = 154.8225$

3-141. a) $P(Z < 1.16) = 0.8770$
b) $P(-0.367 < Z < 0.367) = 0.6432 - 0.3568 = 0.2864$
c) $P(-1 < Z < 1) = 0.6826$

3-142. a) $P(Z > -0.311) = 1 - P(Z < -0.311) = 1 - 0.378281 = 0.621719$
b) $P(-0.455 < Z < 0.993) = 0.838913 - 0.322758 = 0.516155$

3-143. a) Let T denote the total thickness. Then, T = X + Y and E(T) = 3 mm,
$V(T) = 0.1^2 + 0.1^2 = 0.02\text{mm}^2$ and $\sigma_T = 0.141$ mm.
b) $P(T > 3.3) = P\left(Z > \frac{3.3 - 3}{0.1414}\right) = P(Z > 2.12) = 1 - P(Z < 2.12) = 1 - 0.982997 = 0.0169$

3-144. Let D denote the width of the casing minus the width of the door. Then, D is normally distributed.
a) $E(D) = 1/8$ $\quad V(D) = \left(\frac{1}{8}\right)^2 + \left(\frac{1}{16}\right)^2 = \frac{5}{256}$
b) $P(D > \frac{1}{4}) = P\left(Z > \frac{1/4 - 1/8}{\sqrt{5/256}}\right) = P(Z > 0.89) = 1 - P(Z < .89) = 1 - 0.8133 = 0.1867$
c) $P(D < 0) = P\left(Z < \frac{0 - 1/8}{\sqrt{5/256}}\right) = P(Z < -0.89) = 0.1867$

3-145. $D = A - B - C$
a) $E(D) = 10 - 2 - 2 = 6$ mm
$$V(D) = 0.1^2 + 0.05^2 + 0.05^2 = 0.015\text{mm}^2$$
$$\sigma_D = 0.1225\text{mm}$$
b) $P(D < 5.9) = P\left(Z < \frac{5.9 - 6}{0.1225}\right) = P(Z < -0.82) = 0.2072.$

3-146. $E(3X_1 + 5X_2) = 31$, $V(3X_1 + 5X_2) = 18 + 250 + 30(2) = 328$

3-147. $E(Y) = 22$, $V(Y) = 48 + 4(2)(5) = 88$

3-148. $E(R) = E(R_1 + R_2) = 20 + 50 = 70$ ohms, $V(R) = V(R_1 + R_2) = 0.5 + 1 = 1.5$ square ohms

3-149. $E(Y) \cong 2(20)^2 = 800$, $V(Y) \cong 57600$

3-150. $E(Y) \cong (100)^2 + 2(100) + 1 = 1201$, $V(Y) \cong [2(100) + 2]^2(25) = 1020100$

3-151. $E(P) \cong 40(100) = 4000$, $V(P) \cong [2(40)(100)]^2(0.5)^2 = 16{,}000{,}000$

3-152. $E(T) \cong 10.954\Pi/g^{1/2}$; $V(T) \cong 0.00001333\Pi^2/g$

3-153. $E(G) \cong 0.07396d$, $V(G) \cong 3.23 \times 10^{-7}d$

3-154. $E(Y) = 10$, $V(Y) = 258$

3-155. $E(Y) \cong 4(3)(2) = 24$, $V(Y) \cong 1[3(2)]^2 + 5[4(2)]^2 + 2[4(3)]^2 = 644$

3-156. $E(V) = 8$, $V(V) \cong 0.48$

Section 3-13

3-157. a) Mean: 100, Variance: 81/16 = 5.0625
b) $P(\overline{X} \le 98) = 0.1870$
c) $P(\overline{X} \ge 103) = 0.0912$

d) $P(96 \le \overline{X} \le 102) = 0.8130 - 0.0377 = 0.7753$

3-158. a) Mean: 50, Variance: 16/25 = 0.64
b) $P(\overline{X} \le 49) = 0.10565$
c) $P(\overline{X} > 52) = 0.00621$
d) $P(49 \le \overline{X} \le 51.5) = 0.969946 - 0.10565 = 0.864296$

3-159. a) Mean: 20, Variance: 2/40 = 1/20 = 0.05
b) $P(\overline{X} \le 19) = 0$
c) $P(\overline{X} > 22) = 0$
d) $P(16 \le \overline{X} \le 21.5) = 1$

3-160. a) $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.08}{\sqrt{20}} = 0.018$

b) $P(\overline{X} < 5.95) = P\left(\frac{\overline{X}-\mu}{\sigma_{\overline{X}}} < \frac{5.95 - 6.16}{0.018}\right) = P(Z < -11.67) = 0$

c) $P\left(\frac{\overline{X}-\mu}{\sigma_{\overline{X}}} < \frac{6-\mu}{0.018}\right) = 0.001$

$P(Z < z_0) = 0.001$, From the standard normal table, $z_0 = -3.00$

$z_0 = \frac{6-\mu}{0.018} = -3.00$

$\mu = 6.054$

3-161. Let $\overline{X}$ denote the average thickness of 10 wafers. Then, $E(\overline{X}) = 10$ and $V(\overline{X}) = 0.1$.

a) $P(9 < \overline{X} < 11) = P\left(\frac{9-10}{\sqrt{0.1}} < Z < \frac{11-10}{\sqrt{0.1}}\right) = P(-3.16 < Z < 3.16) = 0.9984$.

The answer is 1 – 0.9984 = 0.0016

b) $P(\overline{X} > 11) = 0.01$ and $\sigma_{\overline{X}} = \frac{1}{\sqrt{n}}$.

Therefore, $P(\overline{X} > 11) = P\left(Z > \frac{11-10}{1/\sqrt{n}}\right) = 0.01$, $\frac{11-10}{1/\sqrt{n}} = 2.33$ and n = 5.43 which is rounded up to 6.

3-162. $P(\overline{X} < 0.465) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{0.465-0.5}{0.05/\sqrt{49}}\right) = P(Z < -4.9) = 0$

3-163. $\mu_{\overline{X}} = 75.5\text{psi}$

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{6}} = 1.429$

$P(\overline{X} \ge 75.75) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \ge \frac{75.75-75.5}{1.429}\right)$

$= P(Z \ge 0.175) = 1 - P(Z \le 0.175)$

$= 1 - 0.56945 = 0.4306$

3-164. Assuming a normal distribution,

$\mu_{\overline{X}} = 2500$

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{5}} = 22.361$

$$P(2499 \le \overline{X} \le 2510) = P\left(\frac{2499 - 2500}{22.361} \le \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \le \frac{2510 - 2500}{22.361}\right)$$

$$= P(-0.04 \le Z \le 0.45) = P(Z \le 0.45) - P(Z \le -0.45)$$

$$= 0.673645 - 0.484047 = 0.1896$$

3-165.

$\mu_X = 8.2$ minutes

$\sigma_X = 1.5$ minutes

$n = 49$

$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{1.5}{\sqrt{49}} = 0.2143$ (minutes)

$\mu_{\overline{X}} = \mu_X = 8.2$

Using the central limit theorem, $\overline{X}$ is approximately normally distributed.

a) $P(\overline{X} < 8) = P\left(Z < \frac{8 - 8.2}{0.2143}\right) = P(Z < -0.93) = 1 - 0.82381 = 0.1762$

b) $P(8 < \overline{X} < 9) = P\left(\frac{8 - 8.2}{0.2143} < Z < \frac{9 - 8.2}{0.2143}\right)$

$$= P(Z < 3.73) - P(Z < -0.93) = 0.99990 - 0.17619 = 0.8237$$

c) $P(\overline{X} < 7.5) = P\left(Z < \frac{7.5 - 8.2}{0.2143}\right) = P(Z < -3.27) = 1 - 0.99946 = 0.0005$

3-166. n = 36

$E(X) = 1\frac{1}{3} + 2\frac{1}{3} + 3\frac{1}{3} = 2$

$V(X) = (1-2)^2 \frac{1}{3} + (2-2)^2 \frac{1}{3} + (3-2)^2 \frac{1}{3} = \frac{2}{3}$

$\mu_{\overline{X}} = 2, \sigma_{\overline{X}} = \frac{\sqrt{2/3}}{\sqrt{36}} = \frac{\sqrt{2/3}}{6}$

$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

Using the central limit theorem:

$$P(2.1 < \overline{X} < 2.5) = P\left(\frac{2.1 - 2}{\frac{\sqrt{2/3}}{6}} < Z < \frac{2.5 - 2}{\frac{\sqrt{2/3}}{6}}\right)$$

$$= P(0.7348 < Z < 3.6742)$$

$$= P(Z < 3.6742) - P(Z < 0.7348)$$

$$= 1 - 0.7688 = 0.2312$$

3-167. $X \sim N(20, 0.25)$

a) $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{40}} = 0.0791$

b) $P(\overline{X} \ge 20.1)$

$$P(\overline{X} \geq 20.1) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \geq \frac{20.1-20}{0.0791}\right)$$

$$= P(Z \geq 1.26) = 1 - P(Z \leq 1.26)$$

$$= 1 - 0.89617 = 0.1038$$

c) $P(\overline{X} \geq 20.1)$

$$P(\overline{X} \geq 20.1) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \geq \frac{20.1-20}{0.5/\sqrt{20}}\right)$$

$$= P(Z \geq 0.89) = 1 - P(Z \leq 0.89)$$

$$= 1 - 0.81327 = 0.1867$$

d) The probability in part c) is greater than the probability in part b). This inequality occurs due to the decrease in sample size which contributes to the increase in variability.

3-168. Assume $\overline{X}$ is approximately normally distributed.

$$P(\overline{X} > 4985) = 1 - P(\overline{X} \leq 4985) = 1 - P\left(Z \leq \frac{4985-5500}{100/\sqrt{9}}\right)$$

$$= 1 - P(Z \leq -15.45) = 1 - 0 = 1$$

Supplemental Exercises

3-169. a) $P(X \leq 1.5) = \int_0^{1.5} e^{-x}dx = 0.7769$

b) $P(X < 1.5) = P(X \leq 1.5) = \int_0^{1.5} e^{-x}dx = 0.7769$

c) $P(1.5 < X < 3) = \int_{1.5}^{3} e^{-x}dx = 0.1733$

d) $P(X = 3) = 0$

e) $P(X > 3) = 1 - P(X \leq 3) = 1 - \int_0^{3} e^{-x}dx = 0.0498$

3-170. a) $\int_x^{\infty} e^{-\frac{x}{2}}dx = 0.2$

$$e^{-\frac{x}{2}}\Big|_x^{\infty} = 0 - \left(-2e^{-\frac{x}{2}}\right) = 0.2$$

$2e^{-\frac{x}{2}} = 0.2$

$x = 4.6052$

b) $\int_x^{\infty} e^{-\frac{x}{2}}dx = 0.75$

$$e^{-\frac{x}{2}}\Big|_x^{\infty} = 0 - \left(-2e^{-\frac{x}{2}}\right) = 0.75$$

$$2e^{-\frac{x}{2}} = 0.75$$

$$x = 1.9617$$

3-171. a) $P(X \le 3) = 0.2 + 0.4 = 0.6$

b) $P(X > 2.5) = 0.4 + 0.3 + 0.1 = 0.8$

c) $P(2.7 < X < 5.1) = 0.4 + 0.3 = 0.7$

d) $E(X) = 2(0.2) + 3(0.4) + 5(0.3) + 8(0.1) = 3.9$

e) $V(X) = (2-3.9)^2 0.2 + (3-3.9)^2 0.4 + (5-3.9)^2 0.3 + (8-3.9)^2 0.1 = 3.09$

3-172. $(X < 36000) = P\left(Z < \frac{36000 - 40000}{\sigma}\right) = 0.04$

Where $\frac{36000 - 40000}{\sigma}$ = -1.75, thus, σ = 2285.71

3-173. Standard fluorescent tube:

$$P(X > 9000) = P\left(\frac{x-\mu}{\sigma} > \frac{9000 - 7000}{1000}\right)$$
$$= P(Z > 2)$$
$$= 1 - P(Z < 2)$$
$$= 1 - 0.97725$$
$$= 0.023$$

Competitor's tube:

$$P(X > 9000) = P\left(\frac{x-\mu}{\sigma} > \frac{9000 - 7500}{1200}\right)$$
$$= P(Z > 1.25)$$
$$= 1 - P(Z < 1.25)$$
$$= 1 - 0.89435$$
$$= 0.106$$

The competitor's tube is more likely to have a life length greater than 9000 hours since it has a higher probability of lasting longer.

3-174. $P(X < x) = = P\left(Z < \frac{x - 10}{1}\right) = 0.03$

Where $\frac{x-10}{1}$ = -1.88, thus, x = 8.12

Therefore, the manufacturer could have the guarantee in effect for 8.12 years.

3-175. Let X denote the number of calls that are answered in 15 seconds or less. Then, X is a binomial random variable with p = 0.85.

a) $P(X = 7) = \binom{10}{7} 0.85^7 \times 0.15^3 = 0.1298$

b) $P(X \ge 16) = P(X=16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)$

$$= \binom{20}{16} 0.85^{16}(0.15)^4 + \binom{20}{17} 0.85^{17}(0.15)^3 + \binom{20}{18} 0.85^{18}(0.15)^2 + \binom{20}{19} 0.85^{19}(0.15)^1 + \binom{20}{20} 0.85^{20}(0.15)^0$$
$$= 0.8972$$

c) $E(X) = 50\ (0.85) = 42.5$

d) $E(X) = 10(0.85) = 8.5$ $\quad V(X) = np(1 - p) = 10(0.85)(0.15) = 1.275$

$$\begin{aligned} P(X = 7) &= P(6.5 \le X \le 7.5) \\ &= P\left(\frac{6.5 - 8.5}{\sqrt{1.275}} \le Z \le \frac{7.5 - 8.5}{\sqrt{1.275}}\right) \\ &= P(-1.77 \le Z \le -0.88) \\ &= P(Z \le -0.88) - P(Z \le -1.77) \\ &= 0.18943 - 003836 \\ &= 0.151 \end{aligned}$$

$E(X) = 20(0.85) = 17$ $\quad V(X) = np(1 - p) = 20(0.85)(0.15) = 2.55$

$$P(X \ge 16) = P\left(Z \ge \frac{16 - 17}{\sqrt{2.55}}\right) = P(Z \ge -0.63) = 1 - P(Z < -0.63) = 1 - 0.2644 = 0.7356$$

3-176. a) Let X denote the number of messages sent in one hour. $\quad P(X = 5) = \dfrac{e^{-5}5^5}{5!} = 0.1755$

b) Let Y denote the number of messages sent in 1.5 hours. Then, Y is a Poisson random variable with $\lambda = 7.5$.

$$P(Y = 10) = \frac{e^{-7.5}(7.5)^{10}}{10!} = 0.0858$$

c) Let W denote the number of messages sent in one-half hour. Then, W is a Poisson random variable with $\lambda = 2.5$.
$P(W < 2) = P(W = 0) + P(W = 1) = 0.2873$.

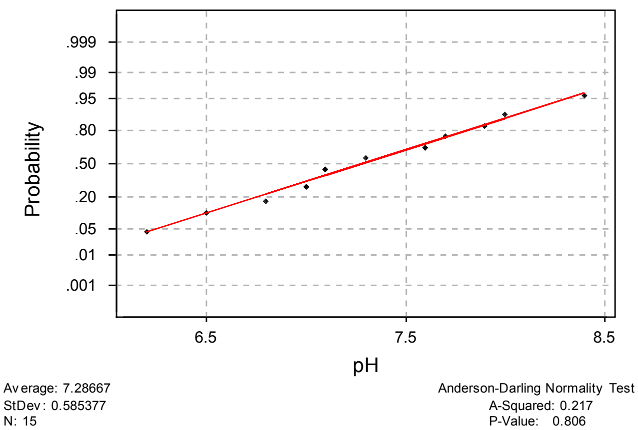
3-177. a) The distribution of Y is exponential; the mean of Y is 60/5 = 12 min
b) $P(Y > 15) = 1 - 0.7135 = 0.2865$
c) $P(Y < 5) = 0.341$
d) $P(Y > 10) = 1 - 0.565402 = 0.436$

3-178. $X \sim \text{Poisson}(\lambda = 0.01)$, $X_{100} \sim \text{Poisson}(\lambda = 1)$
a) $P(X_{100} \le 3) = 0.981$
b) $P(X_{100} \ge 4) = 1 - 0.9810 = 0.0190$
c) $X_{200} \sim \text{Poisson}(\lambda = 2)$
$P(X_{200} \le 3) = 0.8571$

3-179. a) $Y \sim$ exponential $\lambda = 1/100 \quad \rightarrow \quad$ mean = 100
b) $P(Y < 100) = 0.632$
c) $P(Y > 200) = 1 - 0.8647 = 0.1353$
d) $P(Y > 50) = 1 - 0.3935 = 0.6065$

3-180.

Normal Probability Plot



According to the normal probability plot of the pH readings, it is reasonable to model these data using a normal distribution, since the data appear to fall along a straight line.

3-181. Let X_1 , X_2, ..., X_6 denote the lifetimes of the six components, respectively.

a) Because of independence,

$P(X_1 > 5000, X_2 > 5000, ..., X_6 > 5000) = P(X_1 > 5000)P(X_2 > 5000) \cdots P(X_6 > 5000)$

If X_i is exponentially distributed with mean θ, then $\lambda = 1/\theta$ and $P(X_i > x) = e^{-x/\theta}$. Then

$P(X_1 > 5000, X_2 > 5000, ..., X_6 > 5000) = e^{-5/8}e^{-0.5}e^{-0.5}e^{-0.25}e^{-0.25}e^{-0.2} = 0.0978$

b) $P(X_1 < 5000, X_2 < 5000, ..., X_6 < 5000) = P(X_1 < 5000)P(X_2 < 5000) \cdots P(X_6 < 5000)$

If X_i is exponentially distributed with mean θ, then $\lambda = 1/\theta$ and $P(X_i \le x) = 1 - e^{-x/\theta}$.
Then

$$P(X_1{<}5000, X_2{<}5000, ..., X_6{<}5000) = (1-e^{-5/8})(1-e^{-0.5})(1-e^{-0.5})(1-e^{-0.25})(1-e^{-0.25})(1-e^{-0.2}) = 0.0006$$

c) $P(X_1 < 3000, X_2 < 3000, ..., X_6 < 3000) = P(X_1 < 3000)P(X_2 < 3000) \cdots P(X_6 < 3000)$

If X_i is exponentially distributed with mean θ, then $\lambda = 1/\theta$ and $P(X_i \le x) = 1 - e^{-x/\theta}$.
Then

$$P(X_1{<}3000, X_2{<}3000, ..., X_6{<}3000) = (1-e^{-3/8})(1-e^{-0.3})(1-e^{-0.3})(1-e^{-0.15})(1-e^{-0.15})(1-e^{-0.12}) = 0.00005$$

3-182. a)
$$\begin{aligned}P(47 \le \overline{X} < 53) &= P\left(\frac{47-50}{12/\sqrt{36}} \le Z \le \frac{53-50}{12/\sqrt{36}}\right)\\ &= P(-1.5 \le Z \le 1.5)\\ &= P(Z \le 1.5) - P(Z \le -1.5)\\ &= 0.93319 - 0.06681\\ &= 0.86638\end{aligned}$$

b)
$$P(47 \le \overline{X} < 53) = P\left(\frac{47-50}{50/\sqrt{36}} \le Z \le \frac{53-50}{50/\sqrt{36}}\right)$$

$= P(-0.36 \le Z \le 0.36)$
$= P(Z \le 0.36) - P(Z \le -0.36)$
$= 0.64058 - 0.35942$
$= 0.28116$

c) $P(47 \le \overline{X} < 53) = P\left(\frac{47-50}{50/\sqrt{36}} \le Z \le \frac{53-50}{50/\sqrt{36}}\right)$
$= P(-0.36 \le Z \le 0.36)$
$= P(Z \le 0.36) - P(Z \le -0.36)$
$= 0.64058 - 0.35942$
$= 0.28116$

d) The values for the Poisson and exponential differ from the normal because the variances are different.

3-183. $\mu = 5500$ psi $\sigma = 100$ psi $n = 9$

a) $\sigma_{\overline{X}} = \frac{100}{\sqrt{9}} = 33.3$ psi

b) $\sigma_{\overline{X}} = \frac{100}{\sqrt{20}} = 22.36$ psi

c) The results of parts a and b do differ. By increasing the sample size (with all other values held constant), the variation of the sampling distribution of the sample mean will decrease.

3-184. Let $\overline{X}$ denote the mean weight of the 25 bricks in the sample. Then, E($\overline{X}$) = 3 and $\sigma_{\overline{X}} = \frac{0.25}{\sqrt{25}} = 0.05$.

Then, P($\overline{X}$ < 2.95) = P(Z < $\frac{2.95-3}{0.05}$) = P (Z < –1) = 0.159.

3-185. W ~ N(120, 0.25)
X ~ N(20, 0.01)
Y ~ N(100, 0.16)

a) (W + X + Y) ~ N(240, 0.42)
E(W + X + Y) = 120 + 20 + 100 = 240
V(W + X + Y) = 0.25 + 0.01 + 0.16 = 0.42

b) $P(W + X + Y > 242) = P\left(Z > \frac{242-240}{0.42}\right)$
$= P(Z > 3.08)$
$= 1 - P(Z < 3.08)$
$= 1 - 0.998965$
$= 0.0010$

3-186. Let $\overline{X}$ denote the average time to locate 10 parts. Then, E($\overline{X}$) =45 and $\sigma_{\overline{X}} = \frac{30}{\sqrt{10}}$

a) $P(\overline{X} > 60) = P\left(Z > \frac{60-45}{30/\sqrt{10}}\right) = P(Z > 1.58) = 0.057$

b) Let Y denote the total time to locate 10 parts. Then, Y > 600 if and only if $\overline{X}$ > 60. Therefore, the answer is the same as part a.

3-187. a) Let Y denote the weight of an assembly. Then, E(Y) = 4 + 5.5 + 10 + 8 = 27.5 and

$V(Y) = 0.4^2 + 0.5^2 + 0.2^2 + 0.5^2 = 0.7$.

$P(Y > 29.5) = P\left(Z > \frac{29.5-27.5}{\sqrt{0.7}}\right) = P(Z > 2.39) = 0.0084$

b) Let $\overline{X}$ denote the mean weight of 8 independent assemblies.
Then, $E(\overline{X}) = 27.5$ and $V(\overline{X}) = 0.7/8 = 0.0875$. Also,

$P(\overline{X} > 29) = P\left(Z > \frac{29-27.5}{\sqrt{0.0875}}\right) = P(Z > 5.07) = 0$.

3-188. Let X denote the diameter of the maximum diameter bearing. Then, $P(X > 1.6) = 1 - P(X \le 1.6)$. Also, $X \le 1.6$ if and only if all the diameters are less than 1.6. Let Y denote the diameter of a bearing. Then, by independence $P(X \le 1.6) = [P(Y \le 1.6)]^{10} = \left[P(Z \le \frac{1.6-1.5}{0.025})\right]^{10} = 1^{10} = 1$.
Then, $P(X > 1.6) = 0$.

3-189. a) $P(\mu_0 - 6\sigma < X < \mu_0 + 6\sigma) = P(-6 < Z < 6) = 1.8\times10^{-9} = 0.0018$ or 18 ppm

b) $P(\mu_0 - 6\sigma < X < \mu_0 + 6\sigma) = P(-7.5 < \frac{X-(\mu_0+1.5\sigma)}{\sigma} < 4.5) = 3.4\times10^{-6} = 3.4$ppm

3-190. mean = 206.04 $\hat{\sigma} = 11.57$
$P(X \le 210) = 0.6339$
$P(X \le 220) = 0.8862$

a) $P(210 \le X \le 220) = 0.8862 - 0.6339 = 0.2523$

b) $P(X \ge 228) = 1 - 0.9712 = 0.0288$

c) $P(X \ge x) = 0.02$

$P(X \ge x) = P\left(\frac{X - 206.04}{11.57} \ge \frac{x - 206.04}{11.57}\right) = P(Z \ge z) = 0.02$

Thus, $\frac{x - 206.04}{11.57} = z$

The value of z for $P(Z \ge z) = 0.02$ is found to be approximately 2.05. Solving
$\frac{x - 206.04}{11.57} = 2.05$ for x, we get x = 9.7585.

3-191. $\overline{x}_x = 100.27$ $s_x = 2.28$
$\overline{x}_y = 100.11$ $s_y = 7.58$

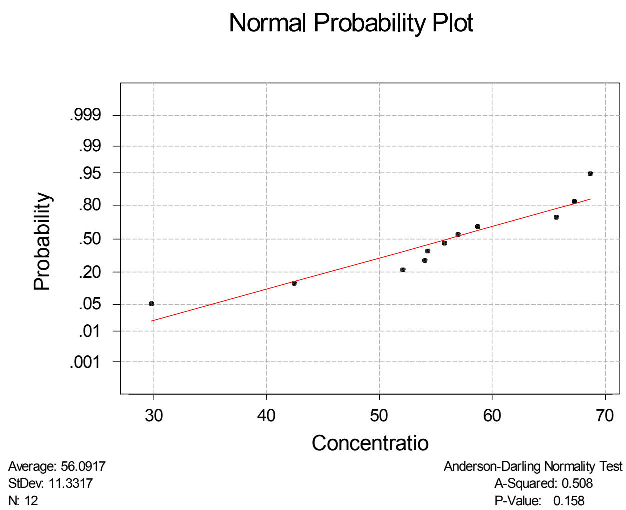
a) $P(96 \le X \le 104) = 0.9491 - 0.0305 = 0.919$

b) $P(96 \le Y \le 104) = 0.6961 - 0.2938 = 0.402$

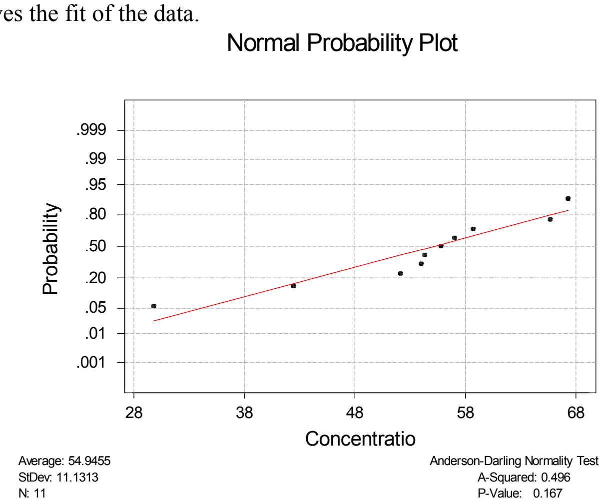
c) Since machine 1 has a higher probability of producing acceptable parts, we prefer machine 1.

d) $\overline{x}_y = 105.39$ $s_y = 2.08$
$P(96 \le Y \le 104) = 0.252 - 0 = 0.252$; Adjusting machine 2 did not improve its overall performance. The probability of producing an acceptable part has decreased.

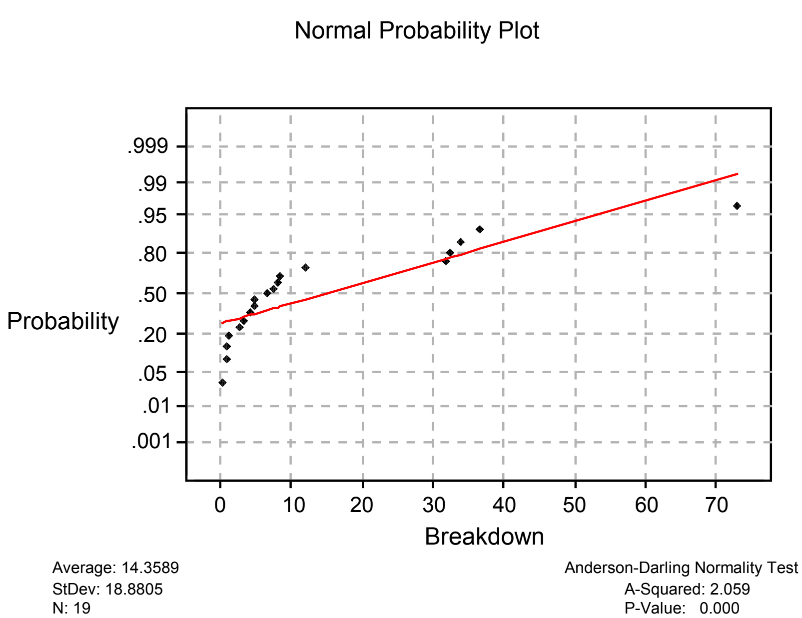
3-192. a) The normal distribution does not appear to be a very good fit.



b) Yes, removing the largest observation slightly improves the fit of the data.

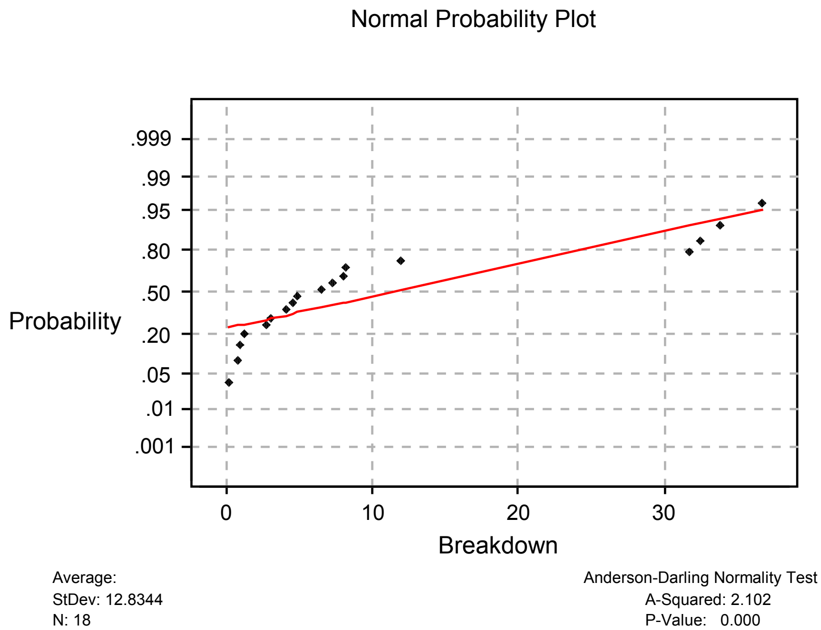


3-193. a)



No, the data does not appear to follow a normal distribution.

b)



No, removing the largest data observation did not improve the fit.

3-194. $n = (0.03/0.005)^2 = 36$

3-195. $[(3\text{-}c) - \mu]/\sigma = z_{0.005}$, where $z_{0.005} = -2.575$; $[(3\text{-}c) - 3]/0.12 = -2.575$, solving for $c = 0.309$

3-196. $P(X < 10) = P(Z < z) = 0.01$ where $z = \dfrac{10-\mu_x}{0.05} = -2.33$.

Thus $\mu_x = 10 + 2.33(0.05) = 10.1165$.

3-197. Let T = thickness

a) $T = W + X + Y + Z$

b) $E(T) = E(W + X + Y + Z) = E(W) + E(X) + E(Y) + E(Z) = 10 + 50 + 5 + 8 = 73$ mm

c) $V(T) = V(W + X + Y + Z) = V(W) + V(X) + V(Y) + V(Z) = 2^2 + 10^2 + 1^2 + 1^2 = 106$ mm^2

d) $P(T > 75) = (Z > 0.19) = 1 - P(Z < 0.19) = 0.57534$

3-198. a) $P(X > 165) = P(Z > \frac{165-150}{7}) = P(Z > 2.14) = P(Z < -2.14) = 0.016177$

b) $P(X > 165) = P(Z > \frac{165-144}{9}) = P(Z > 2.33) = P(Z < -2.33) = 0.009903$

c)

System	*Probability of well funct.*	*Total unit*	*Benefit*	*Total Benefit*
Old	1 – 0.016177 = 0.98382:	1,000	\$1,200 – \$1,000 = 200	0.983823×1,000×200 = \$196,764.60
New	1 – 0.009903 = 0.99009'	1,000	\$1,200 – \$1,050 = 150	0.990097×1,000×150 = \$148,514.77

Although new fan is less likely to malfunction but it costs much more to manufacture, it turns out that original system generate more revenue.

3-199. $n = 8$, $p = 0.10$

a) $P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.9619$

b) $P(X = 0) = 0.4305$

3-200. Let X be a life of the keyboard (in years).

X follows an exponential distribution with $E(X) = 5 = 1/\lambda$ and $V(X) = 25 = 1/\lambda^2$.

a) P(2 < X <4) = $\int_2^4 \frac{1}{5} e^{-x/5} dx = e^{-x/5} |_2^4 = -e^{-4/5} + e^{-2/5} = 0.2210$

b) P(X > 1) = $\int_1^\infty \frac{1}{5} e^{-x/5} dx = e^{-x/5} |_1^\infty = -e^{-\infty/5} + e^{-1/5} = 0.8187$

c) P(X < 0.5) = $\int_0^{0.5} \frac{1}{5} e^{-x/5} dx = e^{-x/5} |_0^{0.5} = -e^{-0.5/5} + e^{-0/5} = 0.0952$

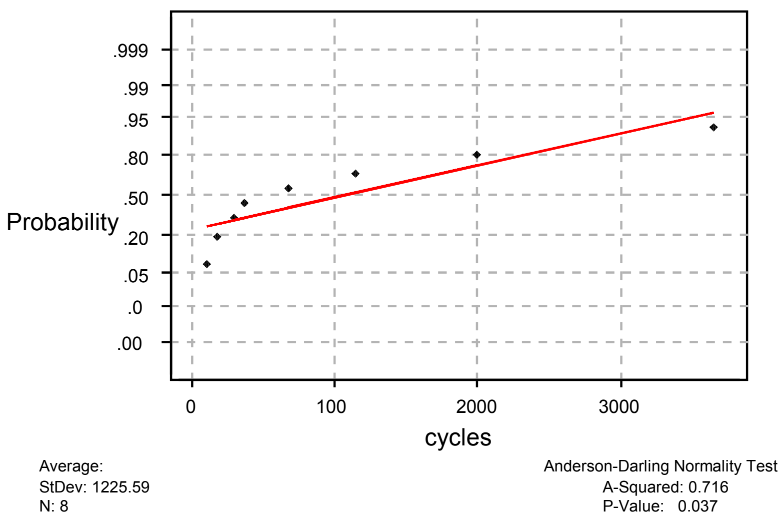
3-201. a) E(X) = 5(0.04) + 6(0.19) + 7(0.61) + 8(0.13) + 9(0.03) = 6.92
b) P(X > 6) = P(X ≥ 7) = 0.61 + 0.13 + 0.03 = 0.77
c) n = 10, p = 0.77 (from part c), use the binomial distribution: P(X = 9) = 0.2188

3-202. a) P(system operates) = $P(C_1)P(C_2 \text{ or } C_3)P(C_4) = (0.9)(1 - P(C_2')P(C_3'))(0.95)$
= (0.9)(1-(0.1)(0.05))(0.95)
= 0.8507
b) Components in parallel do not fail.
$P(C_1'$ or C_2' or both fail) = $1 - P(C_1)P(C_4) = 1 - 0.855 = 0.145$
c) P(Both fail) = $P(C_2')P(C_3') = (0.1)(0.05) = 0.005$
d) 0.145(0.995) + 0.9(0.95)(0.1)(0.05) + 0.145(0.005) = 0.1493
e) Fail due to series and not to parallel components
Fail due to parallel and not series components
Fail due to both series and parallel components
f) P(system fails) = 1 – 0.8507 = 0.1493

3-203. a) Recompute parts a, b, c, and f from 3-202
a) (0.95)(0.995)(0.95) = 0.8980
b) $1 - 0.95^2 = 0.0975$
c) 0.005
f) 1 – 0.8980 = 0.1020
b) Recompute parts a, b, c, and f
a) $0.9(1-0.5^2)(0.95) = 0.8529$
b) 0.1450
c) $0.05^2 = 0.0025$
f) 1 – 0.8529 = 0.1471
c) Increase reliability of series component since improvement in series reduced the probability of failure from 0.1493 to 0.102, whereas the improvement in parallel component only resulted in a decrease to 1471.

3-204. a)

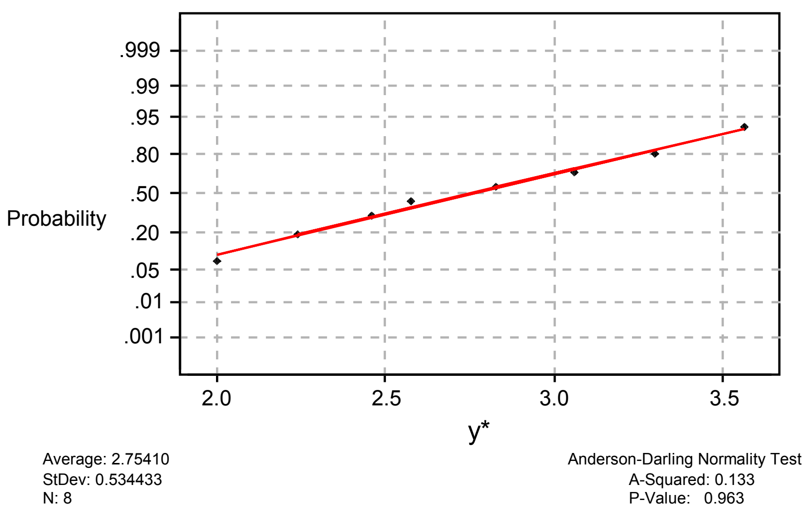
Normal Probability Plot



The data do not appear to follow a normal distribution. The fit is not adequate.

b)

Normal Probability Plot

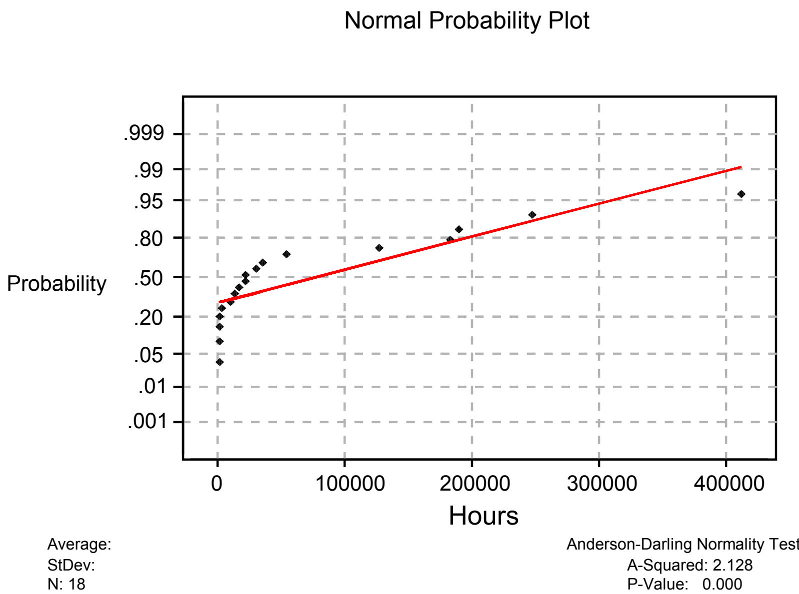


The data appear to follow a normal distribution. The fit appears adequate using logarithms.

c) $\overline{y}^* = 2.7541$ $\quad s_{y^*} = 0.534433$

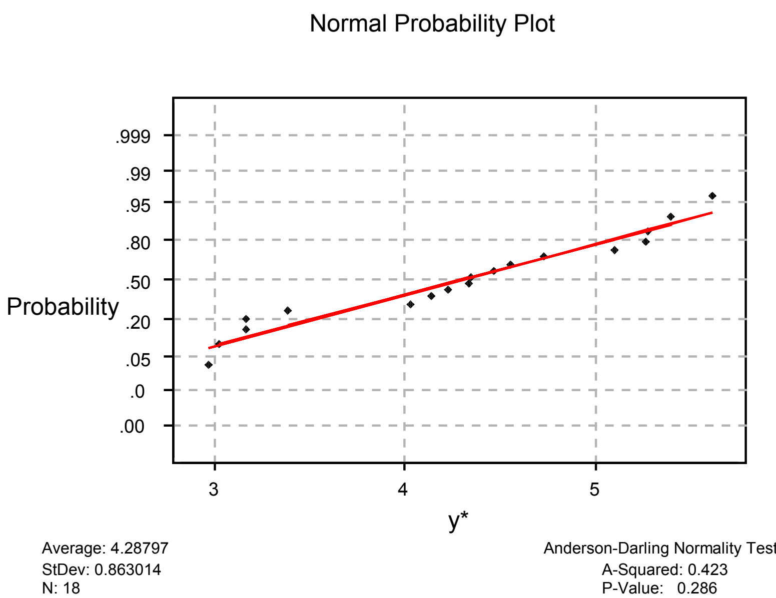
$$P(Y > 200) = P(Y^* > 2.3010) = P\left(Z > \frac{2.3010 - 2.7541}{0.534433}\right) = P(Z > -0.8478) = 1 - 0.1977 = 0.8023$$

3-205. a)



No, the fit does not appear to be adequate.

b)



The transformed data appear to follow a normal distribution.

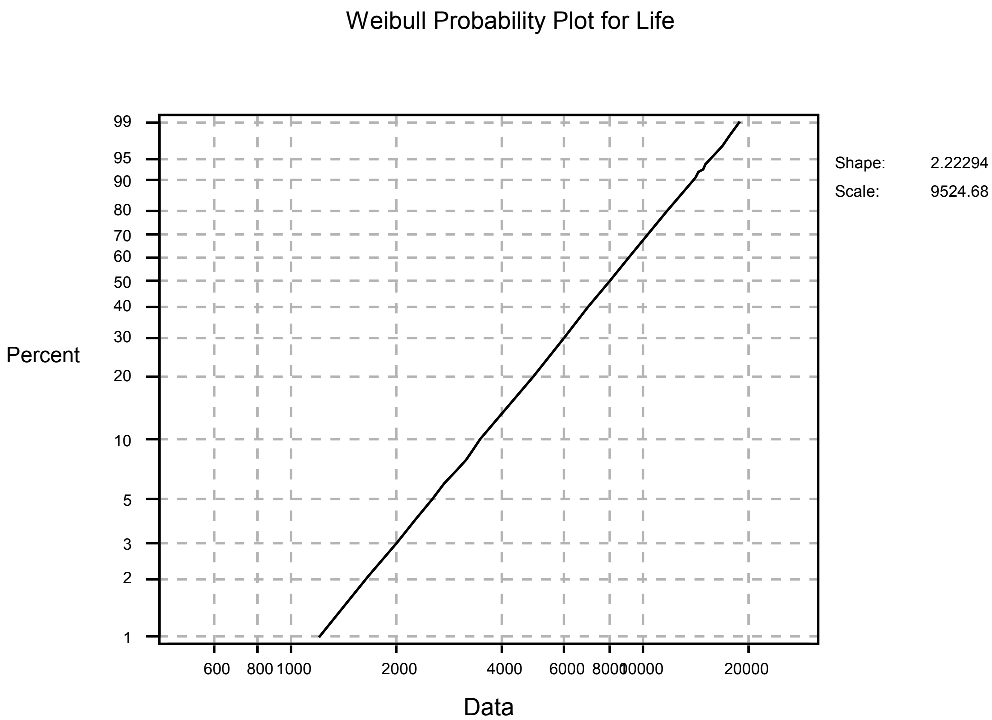
c) $\overline{y}^* = 4.2773 \qquad s_{y^*} = 0.8650$

$$P(Y^* < y^*) = P\left(Z < \frac{y^* - 4.2773}{0.8650}\right) = 0.02 \rightarrow -2.06 = \frac{y^* - 4.2773}{0.8650}$$

$y^* = 2.4953$

$y = 10^{2.4953}$
$= 312.825$ hours

3-206. a)

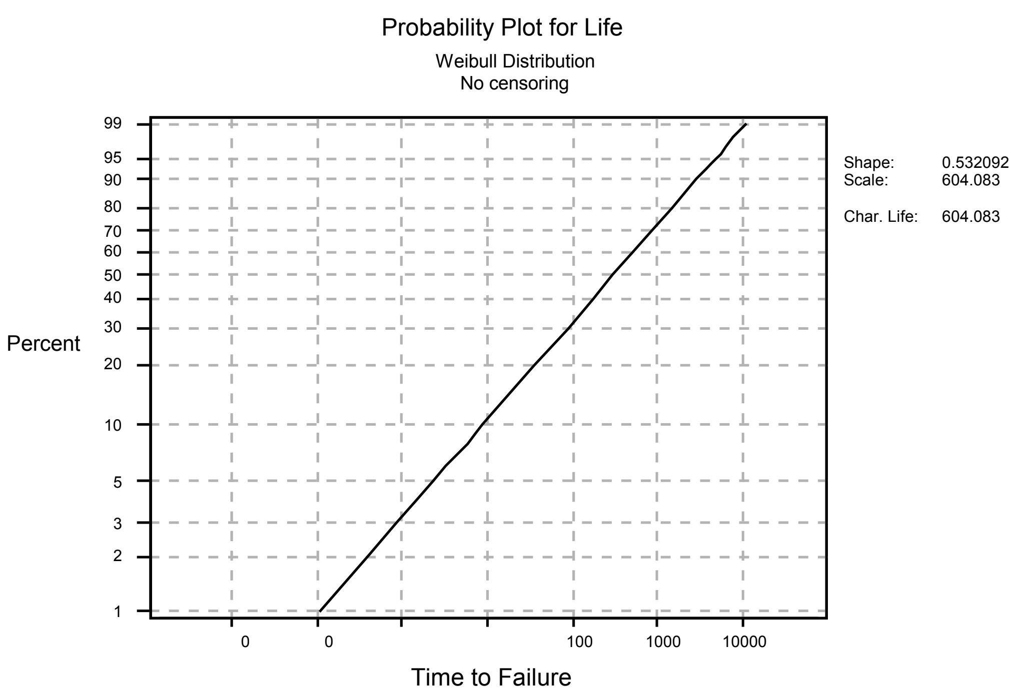


The fit appears to be adequate.

b) $P(Y \geq 7500) = 1 - F(7500) = 1 - (1 - e^{-(7500/9525)^{2.2}}) = e^{-0.591} = 0.5538$

c) $[P(Y \geq 7500)]^5 = 0.052$

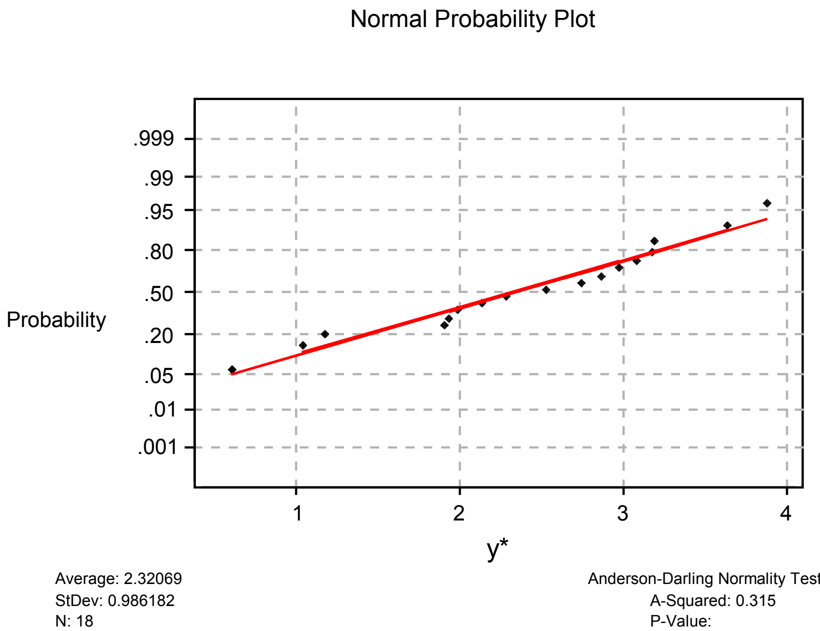
3-207. a)



b) $P(Y \leq 150) = 1 - e^{-(150/604)^{0.53}} = 1 - e^{-0.4779} = 1 - 0.62 = 0.38$

c) Find y such that $P(Y \le y) = 0.1$
$y = 8.65$

3-208. a)



The fit appears to be adequate.

b) $\overline{y}^* = 2.32069$ $s_{y^*} = 0.986182$

$$P(Y \le 150) = P(Y^* \le \ln(150))$$

$$= P(Y^* \le 5.0106) = P\left(Z \le \frac{5.0106 - 2.32069}{0.986182}\right) = P(Z \le 2.73)$$

$$= 0.99683$$

The transformed data resulted in a higher probability that the disk fails before 150 hours.