CHAPTER 4

Section 4-2

4-1. $$E\left(\overline{X}_1\right) = E\left(\frac{\sum_{i=1}^{2n} X_i}{2n}\right) = \frac{1}{2n}E\left(\sum_{i=1}^{2n} X_i\right) = \frac{1}{2n}(2n\mu) = \mu$$

$$E\left(\overline{X}_2\right) = E\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{1}{n}E\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n}(n\mu) = \mu ,$$ $\overline{X}_1$ and $\overline{X}_2$ are unbiased estimators of μ.

The variances are $V\left(\overline{X}_1\right) = \frac{\sigma^2}{2n}$ and $V\left(\overline{X}_2\right) = \frac{\sigma^2}{n}$; compare the MSE (variance in this case),

$$\frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} = \frac{\sigma^2 / 2n}{\sigma^2 / n} = \frac{n}{2n} = \frac{1}{2}$$

Since both estimators are unbiased, examination of the variances would conclude that $\overline{X}_1$ is the "better" estimator with the smaller variance.

4-2. $$E\left(\hat{\theta}_1\right) = \frac{1}{9}\left[E(X_1) + E(X_2) + \cdots + E(X_9)\right] = \frac{1}{9}(9E(X)) = \frac{1}{9}(9\mu) = \mu$$

$$E\left(\hat{\theta}_2\right) = \frac{1}{2}\left[E(3X_1) - E(X_6) + E(2X_4)\right] = \frac{1}{2}[3\mu - \mu + 2\mu] = \mu$$

a) Both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimates of μ since the expected values of these statistics are equivalent to the true mean, μ.

b) $$V\left(\hat{\theta}_1\right) = V\left[\frac{X_1 + X_2 + \ldots + X_9}{9}\right] = \frac{1}{9^2}\left(V(X_1) + V(X_2) + \cdots + V(X_9)\right) = \frac{1}{81}(9\sigma^2) = \frac{1}{9}\sigma^2$$

$$V(\hat{\theta}_1) = \frac{\sigma^2}{9}$$

$$V\left(\hat{\theta}_2\right) = V\left[\frac{3X_1 - X_6 + 2X_4}{2}\right] = \frac{1}{2^2}\left(V(3X_1) + V(X_6) + V(2X_4)\right) = \frac{1}{4}(9V(X_1) + V(X_6) + 4V(X_4))$$

$$= \frac{1}{4}\left(9\sigma^2 + \sigma^2 + 4\sigma^2\right)$$

$$= \frac{1}{4}(14\sigma^2)$$

$$V(\hat{\theta}_2) = \frac{7\sigma^2}{2}$$

Since both estimators are unbiased, the variances can be compared to decide which is the better estimator. The variance of $\hat{\theta}_1$ is smaller than that of $\hat{\theta}_2$, $\hat{\theta}_1$ is the better estimator.

4-3. Since both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased, the variances of the estimators can be examined to determine which is the "better" estimator. The variance of $\hat{\theta}_1$ is smaller than that of $\hat{\theta}_2$ thus $\hat{\theta}_1$ may be the better estimator.

Relative Efficiency $= \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} = \frac{V(\hat{\theta}_1)}{V(\hat{\theta}_2)} = \frac{2}{4} = 0.5$

4-4. Since both estimators are unbiased:

$$\text{Relative Efficiency} = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} = \frac{V(\hat{\theta}_1)}{V(\hat{\theta}_2)} = \frac{\sigma^2/9}{7\sigma^2/2} = \frac{2}{63}$$

4-5. $$\frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} = \frac{V(\hat{\theta}_1)}{V(\hat{\theta}_2)} = \frac{2}{4} = 0.5$$

4-6. $E(\hat{\theta}_1) = \theta \quad E(\hat{\theta}_2) = \theta/2$

$$\begin{aligned} \text{Bias} &= E(\hat{\theta}_2) - \theta \\ &= \frac{\theta}{2} - \theta \\ &= -\frac{\theta}{2} \end{aligned}$$

$V(\hat{\theta}_1) = 10 \qquad V(\hat{\theta}_2) = 4$

For unbiasedness, use $\hat{\theta}_1$ since it is the only unbiased estimator. As for minimum variance and efficiency we have:

$$\text{Relative Efficiency} = \frac{(V(\hat{\theta}_1) + \text{Bias}^2)_1}{(V(\hat{\theta}_2) + \text{Bias}^2)_2}$$ where, Bias for θ_1 is 0.

Thus,

$$\text{Relative Efficiency} = \frac{(10+0)}{\left(4 + \left(\frac{-\theta}{2}\right)^2\right)} = \frac{40}{\left(16+\theta^2\right)}$$

If the relative efficiency is less than or equal to 1, $\hat{\theta}_1$ is the better estimator.

Use $\hat{\theta}_1$, when $\frac{40}{(16+\theta^2)} \le 1$

$$40 \le (16+\theta^2)$$
$$24 \le \theta^2$$
$$\theta \le -4.899 \text{ or } \theta \ge 4.899$$

If $-4.899 < \theta < 4.899$ then use $\hat{\theta}_2$.

For unbiasedness, use $\hat{\theta}_1$. For efficiency, use $\hat{\theta}_1$ when $\theta \le -4.899$ or $\theta \ge 4.899$ and use $\hat{\theta}_2$ when $-4.899 < \theta < 4.899$.

4-7.

$E(\hat{\theta}_1) = \theta$	No bias	$V(\hat{\theta}_1) = 12 = MSE(\hat{\theta}_1)$	
$E(\hat{\theta}_2) = \theta$	No bias	$V(\hat{\theta}_2) = 10 = MSE(\hat{\theta}_2)$	
$E(\hat{\theta}_3) \neq \theta$	Bias	$V(\hat{\theta}_3) = 6$	includes (bias^2)

To compare the three estimators, calculate the relative efficiencies:

$\frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} = \frac{12}{10} = 1.2$, since rel. eff. > 1 use $\hat{\theta}_2$ as the estimator for θ

$\frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_3)} = \frac{12}{6} = 2$, since rel. eff. > 1 use $\hat{\theta}_3$ as the estimator for θ

$\frac{MSE(\hat{\theta}_2)}{MSE(\hat{\theta}_3)} = \frac{10}{6} = 1.8$, since rel. eff. > 1 use $\hat{\theta}_3$ as the estimator for θ

Conclusion: $\hat{\theta}_3$ is the most efficient estimator with bias. $\hat{\theta}_2$ is would be the best "unbiased" estimator.

4-8. $n_1 = 20$, $n_2 = 10$, $n_3 = 8$

Show that S^2 is unbiased:

$$E\left(S^2\right) = E\left(\frac{20S_1^2 + 10S_2^2 + 8S_3^2}{38}\right)$$

$$= \frac{1}{38}\left(E\left(20S_1^2\right) + E\left(10S_2^2\right) + E\left(8S_3^2\right)\right)$$

$$= \frac{1}{38}\left(20\sigma_1^2 + 10\sigma_2^2 + 8\sigma_3^2\right)$$

$$= \frac{1}{38}\left(38\sigma^2\right)$$

$$= \sigma^2$$

$\therefore$ S^2 is an unbiased estimator of σ^2.

4-9. a) Show that $\dfrac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n}$ is a biased estimator of σ^2:

$$E\left(\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n}\right)$$

$$= \frac{1}{n}E\left(\sum_{i=1}^{n}X_i - n\overline{X}\right)^2$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n}E\left(X_i^2\right) - nE\left(\overline{X}^2\right)\right)$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n}\left(\mu^2 + \sigma^2\right) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right)$$

$$= \frac{1}{n}\left(n\mu^2 + n\sigma^2 - n\mu^2 - \sigma^2\right)$$

$$= \frac{1}{n}((n-1)\sigma^2)$$

$$= \sigma^2 - \frac{\sigma^2}{n}$$

$\therefore$ $\dfrac{\sum\left(X_i - \overline{X}\right)^2}{n}$ is a biased estimator of σ^2.

b) $\text{Bias} = E\left[\dfrac{\sum\left(X_i^2 - n\overline{X}\right)^2}{n}\right] - \sigma^2 = \sigma^2 - \dfrac{\sigma^2}{n} - \sigma^2 = -\dfrac{\sigma^2}{n}$

c) Bias decreases as n increases.

4-10. Show that $\overline{X}^2$ is a biased estimator of μ.

Using $E\left(X^2\right) = V(X) + \left[E(X)\right]^2$

$$E\left(\overline{X}^2\right) = \frac{1}{n^2} E\left(\sum_{i=1}^{n} X_i\right)^2$$

$$= \frac{1}{n^2}\left(V\left(\sum_{i=1}^{n} X_i\right) + \left[E\left(\sum_{i=1}^{n} X_i\right)\right]^2\right)$$

$$= \frac{1}{n^2}\left(n\sigma^2 + \left(\sum_{i=1}^{n} \mu\right)^2\right)$$

$$= \frac{1}{n^2}\left(n\sigma^2 + (n\mu)^2\right)$$

$$= \frac{1}{n^2}\left(n\sigma^2 + n^2\mu^2\right)$$

$$E\left(\overline{X}^2\right) = \frac{\sigma^2}{n} + \mu^2$$

$\therefore$ $\overline{X}^2$ is a biased estimator of μ.

b) Bias = $E\left(\overline{X}^2\right) - \mu^2 = \frac{\sigma^2}{n} + \mu^2 - \mu^2 = \frac{\sigma^2}{n}$

c) Bias decreases as *n* increases.

Section 4-3

4-11. a) α = P(reject H_0 when H_0 is true)

$= P(\overline{X} \le 13.7 \text{ when } \mu = 14) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \le \frac{13.7-14}{0.3/\sqrt{5}}\right)$

$= P(Z \le -2.23)$

$= 0.0129.$

The probability of rejecting the null hypothesis when it is true is 0.0129.

b) β = P(accept H_0 when μ = 13.5) = $P\left(\overline{X} > 13.7 \text{ when } \mu = 13.5\right) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{13.7-13.5}{0.3/\sqrt{5}}\right)$

$P(Z > 1.49) = 1 - P(Z \le 1.49) = 1 - 0.931888 = 0.0681$

The probability of accepting the null hypothesis when it is false is 0.0681.

c) $1 - \beta = 1 - 0.0681 = 0.9319$

4-12. a) $\alpha = P(\overline{X} \le 13.7 \text{ when } \mu = 14) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \le \frac{13.7-14}{0.3/\sqrt{16}}\right) = P(Z \le -4) = 0.$

The probability of rejecting the null, when the null is true, is 0 with a sample size of 16.

b) $\beta = P(\overline{X} > 13.7 \text{ when } \mu = 13.5) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{13.7-13.5}{0.3/\sqrt{16}}\right) = P(Z > 2.67)$

$= 1 - P(Z \le 2.67) = 1 - 0.99621 = 0.00379.$

The probability of accepting the null hypothesis when it is false is 0.00379.

c) $1 - \beta = 1 - 0.00379 = 0.99621$

4-13. Find the boundary of the critical region if $\alpha = 0.01$:

a) $0.01 = P\left(Z \le \frac{c-14}{0.3/\sqrt{5}}\right)$

What Z value will give a probability of 0.01? Using Table 1 in the appendix, Z value is –2.33.

Thus, $\frac{c-14}{0.3/\sqrt{5}} = -2.33$, c = 13.687

b) $\beta = P(\overline{X} > 13.687 \text{ when } \mu = 13.5) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{13.687-13.5}{0.3/\sqrt{16}}\right) = P(Z > 2.49)$

$= 1 - P(Z \le 2.49) = 1 - 0.99361 = 0.00639.$

The probability of accepting the null hypothesis when it is false is 0.00639.

c) $1 - \beta = 1 - 0.00639 = 0.99361$

4-14. $0.05 = P\left(Z \le \frac{c-14}{0.3/\sqrt{16}}\right)$

What Z value will give a probability of 0.05? Using Table 1 in the appendix, Z value is –1.65.

Thus, $\frac{c-14}{0.3/\sqrt{16}} = -1.65$, c = 13.876

b) $\beta = P(\overline{X} > 13.876 \text{ when } \mu = 13.5) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{13.876-13.5}{0.3/\sqrt{16}}\right) = P(Z > 5.01)$

$= 1 - P(Z \le 5.01) = 1 - 1 = 0.$

The probability of accepting the null hypothesis when it is false is 0.

c) $1 - \beta = 1 - 0 = 1$

4-15. a) $\alpha = P(\overline{X} \le 98.5) + P(\overline{X} \ge 101.5)$

$= P\left(\frac{\overline{X}-100}{2/\sqrt{9}} \le \frac{98.5-100}{2/\sqrt{9}}\right) + P\left(\frac{\overline{X}-100}{2/\sqrt{9}} \ge \frac{101.5-100}{2/\sqrt{9}}\right)$

$= P(Z \le -2.25) + P(Z \ge 2.25)$

$= P(Z \le -2.25) + (1 - P(Z \le 2.25))$

$= 0.01222 + 0.01222 = 0.0244$

b) $\beta = P(98.5 \le \overline{X} \le 101.5 \text{ when } \mu = 103)$

$= P\left(\frac{98.5-103}{2/\sqrt{9}} \le \frac{\overline{X}-103}{2/\sqrt{9}} \le \frac{101.5-103}{2/\sqrt{9}}\right)$

$= P(-6.75 \le Z \le -2.25)$

$= P(Z \le -2.25) - P(Z \le -6.75)$

$= 0.01222 - 0 = 0.0122$

c) $\beta = P(98.5 \le \overline{X} \le 101.5 \text{ when } \mu = 105)$

$= P\left(\frac{98.5-105}{2/\sqrt{9}} \le \frac{\overline{X}-105}{2/\sqrt{9}} \le \frac{101.5-105}{2/\sqrt{9}}\right)$

$= P(-9.75 \le Z \le -5.25)$

$= P(Z \le -5.25) - P(Z \le -9.75)$

$= 0.$

The probability of accepting the null hypothesis when it is actually false is smaller in part c since the true mean, $\mu = 105$, is further from the acceptance region. A larger difference exists.

4-16. Use n = 5, everything else held constant:

a) $P(\overline{X} \le 98.5) + P(\overline{X} \ge 101.5)$

$= P\left(\frac{\overline{X}-100}{2/\sqrt{5}} \le \frac{98.5-100}{2/\sqrt{5}}\right) + P\left(\frac{\overline{X}-100}{2/\sqrt{5}} \ge \frac{101.5-100}{2/\sqrt{5}}\right)$

$= P(Z \le -1.68) + P(Z \ge 1.68)$
$= 0.093$

b) $\beta = P(98.5 \le \overline{X} \le 101.5 \text{ when } \mu = 103)$

$$= P\left(\frac{98.5 - 103}{2/\sqrt{5}} \le \frac{\overline{X} - 103}{2/\sqrt{5}} \le \frac{101.5 - 103}{2/\sqrt{5}}\right)$$

$= P(-5.03 \le Z \le -1.68)$
$= P(Z \le -1.68) - P(Z \le -5.03)$
$= 0.04648 - 0$
$= 0.04648$

c) $\beta = P(98.5 \le \overline{x} \le 101.5 \text{ when } \mu = 105)$

$$= P\left(\frac{98.5 - 105}{2/\sqrt{5}} \le \frac{\overline{X} - 105}{2/\sqrt{5}} \le \frac{101.5 - 105}{2/\sqrt{5}}\right)$$

$= P(-7.27 \le Z \le -3.91)$
$= P(Z \le -3.91) - P(Z \le -7.27)$
$= 0.00005 - 0$
$= 0.00005$

4-17. a) $\alpha = P(\overline{X} > 185 \text{ when } \mu = 175)$

$$= P\left(\frac{\overline{X} - 175}{20/\sqrt{10}} > \frac{185 - 175}{20/\sqrt{10}}\right)$$

$= P(Z > 1.58)$
$= 1 - P(Z \le 1.58)$
$= 1 - 0.94295$
$= 0.057$

b) $\beta = P(\overline{X} \le 185 \text{ when } \mu = 200)$

$$= P\left(\frac{\overline{X} - 200}{20/\sqrt{10}} \le \frac{185 - 200}{20/\sqrt{10}}\right)$$

$= P(Z \le -2.37)$
$= 0.00889.$

c) $1 - \beta = 1 - 0.00889 = 0.99111$

4-18. a) Reject the null hypothesis and conclude that the mean foam height is greater than 175 mm.

b) $P(\overline{X} > 190 \text{ when } \mu = 175)$

$$= P\left(\frac{\overline{X} - 175}{20/\sqrt{10}} > \frac{190 - 175}{20/\sqrt{10}}\right)$$

$= P(Z > 2.37) = 1 - P(Z \le 2.37)$
$= 1 - 0.99111$
$= 0.0089.$

The probability that a value of at least 190 mm would be observed (if the true mean height is 175 mm) is only 0.00889. Thus, the sample value of $\overline{x} = 190$ mm would be an unusual result.

4-19. Using $n = 16$:

a) $\alpha = P(\overline{X} > 185 \text{ when } \mu = 175)$

$$= P\left(\frac{\overline{X} - 175}{20/\sqrt{16}} > \frac{185 - 175}{20/\sqrt{16}}\right)$$

$= P(Z > 2)$
$= 1 - P(Z \le 2)$
$= 1 - 0.97725$

$= 0.0228$

b) $\beta = P(\overline{X} \le 185 \text{ when } \mu = 200)$

$$= P\left(\frac{\overline{X} - 200}{20/\sqrt{16}} \le \frac{185 - 200}{20/\sqrt{16}}\right)$$

$= P(Z \le -3)$
$= 0.00135$

c) Power $= 1 - \beta = 1 - 0.00135 = 0.99865$

4-20. n = 16:

a) $0.0571 = P\left(\overline{X} > c \text{ when } \mu = 175\right) = P\left(Z > \frac{c - 175}{20/\sqrt{16}}\right) = P(Z \ge 1.58)$

Thus, $1.58 = \frac{c - 175}{20/\sqrt{16}}$, and $c = 182.9$

b) If the true mean foam height is 195 mm, then

$\beta = P(\overline{X} \le 182.9 \text{ when } \mu = 195)$

$$= P\left(Z \le \frac{182.9 - 195}{20/\sqrt{16}}\right)$$

$= P(Z \le -2.42)$
$= 1 - P(Z \le 2.42)$
$= 0.0078.$

c) For the same level of α, with the increased sample size, β is reduced. With an increased sample size, the power has also increased.

4-21. a) $\alpha = P(\overline{X} \le 8.85 \text{ when } \mu = 9) + P(\overline{X} \ge 9.15 \text{ when } \mu = 9)$

$$= P\left(\frac{\overline{X} - 9}{0.25/\sqrt{8}} \le \frac{8.85 - 9}{0.25/\sqrt{10}}\right) + P\left(\frac{\overline{X} - 9}{0.25/\sqrt{10}} > \frac{9.15 - 9}{0.25/\sqrt{10}}\right)$$

$= P(Z \le -1.9) + P(Z \ge 1.9)$
$= P(Z \le -1.9) + (1 - P(Z \le 1.9))$
$= 0.028717 + 0.028717$
$= 0.0574.$

b) Power $= 1 - \beta$

$\beta = P(8.85 \le \overline{X} \le 9.15 \text{ when } \mu = 9.1)$

$$= P\left(\frac{8.85 - 9.1}{0.25/\sqrt{10}} \le \frac{\overline{X} - 5.1}{0.25/\sqrt{10}} \le \frac{9.15 - 9.1}{0.25/\sqrt{10}}\right)$$

$= P(-3.16 \le Z \le 0.63)$
$= P(Z \le 0.63) - P(Z \le -3.16)$
$= 0.735653 - 0.000789$
$= 0.7349$

$1 - \beta = 0.265.$

4-22. Using n = 16:

a) $\alpha = P(\overline{X} \le 8.85 \text{ when } \mu = 9) + P(\overline{X} > 9.15 \text{ when } \mu = 9)$

$$= P\left(\frac{\overline{X} - 9}{0.25/\sqrt{16}} \le \frac{8.85 - 9}{0.25/\sqrt{16}}\right) + P\left(\frac{\overline{X} - 9}{0.25/\sqrt{16}} > \frac{9.15 - 9}{0.25/\sqrt{16}}\right)$$

$= P(Z \le -2.4) + P(Z > 2.4)$
$= P(Z \le -2.4) + (1 - P(Z \le 2.4))$

$= 0.0082 + 0.0082$
$= 0.0164.$

b) $\beta = P(8.85 \le \overline{X} \le 9.15 \text{ when } \mu = 9.1)$

$$= P\left(\frac{8.85-9.1}{0.25/\sqrt{16}} \le \frac{\overline{X}-9.1}{0.25/\sqrt{16}} \le \frac{9.15-9.1}{0.25/\sqrt{16}}\right)$$

$= P(-4 \le Z \le 0.8) = P(Z \le 0.8) - P(Z \le -4)$
$= 0.7881 - 0)$
$= 0.7881 - 0$
$= 0.7881$
$1 - \beta = 0.2119$

4-23. $\alpha = P\left(\overline{X} \le c_L\right) + P\left(\overline{X} > c_H\right)$

$0.025 = P\left(\overline{X} \le c_L\right),\ 0.025 = P\left(\overline{X} > c_H\right)$

$P\left(\frac{c_L - 9}{0.25/\sqrt{10}}\right) = 0.025$, thus, $-1.96 = \frac{c_L - 9}{0.25/\sqrt{10}}$; $c_L = 8.85$

$P\left(\frac{c_H - 9}{0.25/\sqrt{10}}\right) = 0.025$, thus, $1.96 = \frac{c_L - 9}{0.25/\sqrt{10}}$; $c_H = 9.16$

$P\left(8.845 \le \overline{X} \le 9.155\right)$

Section 4-4

4-24. a) 1) The parameter of interest is the true mean breaking strength, μ.
2) $H_0 : \mu = 100$
3) $H_1 : \mu > 100$
4) $\alpha = 0.05$
5) $z_0 = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$
6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$
7) $\bar{x} = 100.6$, $\sigma = 2$

$$z_0 = \frac{100.6 - 100}{2/\sqrt{9}} = 0.9$$

8) Since $0.9 < 1.65$ do not reject H_0 and conclude that the fiber would not be judged acceptable at $\alpha = 0.05$.

b) P-value $= P(Z \ge 0.9) = 1 - P(Z < 0.9) = 0.18406$

c) For $\alpha = 0.05$, accept H_0 if $\overline{X} < 100 + 1.65\left(\frac{2}{\sqrt{9}}\right) = 101.1$

$$P\left(\bar{X} \le 101.1 \text{ when } \mu = 102\right) = P\left(Z \le \frac{101.1 - 102}{2/\sqrt{9}}\right) = P\left(Z \le -1.35\right) = 0.088508$$

The probability is 0.088508 of accepting the null hypothesis if the true mean breaking strength is 102 psi, with a level of significance of $\alpha = 0.05$.

d) $z_\alpha = z_{0.05} = 1.645$

$$\bar{x} - z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu$$

$$100.6 - 1.645\left(\frac{2}{\sqrt{9}}\right) \le \mu$$

$99.50 \le \mu$

With 95% confidence, we believe the true mean breaking strength is at least 99.50 psi.

e) Since the value of 100 falls within the confidence interval, we conclude that the null hypothesis cannot be rejected.

f) $n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(1.645 + 1.645)^2 2^2}{0.4^2} = 270.60$, n = 271

4-25. a) 1) The parameter of interest is the true mean yield, μ.

2) $H_0 : \mu = 90$

3) $H_1 : \mu \neq 90$

4) $\alpha = 0.05$

5) $z_0 = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\bar{x} = 90.48$, $\sigma = 3$

$$z_0 = \frac{90.48 - 90}{3 / \sqrt{5}} = 0.36$$

8) Since $-1.96 < 0.36 < 1.96$ do not reject H_0 and conclude the yield is not significantly different from 90% at $\alpha = 0.05$.

b) P-value $= 2[1 - \Phi(0.36)] = 2[1 - 0.64058] = 0.7188$

c) $n = \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \sigma^2}{\delta^2} = \frac{\left(z_{0.025} + z_{0.05}\right)^2 3^2}{(85-90)^2} = \frac{(1.96 + 1.65)^2 9}{(-5)^2} = 4.67$

$n \cong 5$.

d) $\beta = \Phi\left(z_{0.025} - \frac{92-90}{3/\sqrt{5}}\right) - \Phi\left(-z_{0.025} - \frac{92-90}{3/\sqrt{5}}\right)$

$= \Phi(1.96 - 1.49) - \Phi(-1.96 - 1.49)$

$= \Phi(0.47) - \Phi(-3.45)$

$= 0.680822 - 0.000280$

$= 0.680542$

e) For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$90.48 - 1.96\left(\frac{3}{\sqrt{5}}\right) \le \mu \le 90.48 + 1.96\left(\frac{3}{\sqrt{5}}\right)$$

$$87.85 \le \mu \le 93.11$$

With 95% confidence, we believe the true mean yield of the chemical process is between 87.85% and 93.11%.

f) Based on the confidence interval obtained, the null hypothesis cannot be rejected since the value, 90%, lies within this interval.

4-26. a) 1) The parameter of interest is the true mean ppm of benzene, μ.

2) H_0: $\mu = 7980$

3) H_1: $\mu < 7980$

4) $\alpha = 0.01$

5) $z_0 = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_{0.01} = -2.33$

7) $\overline{x} = 7906$, $\sigma = 80$

$$z_0 = \frac{7906 - 7980}{80/\sqrt{10}} = -2.925$$

8) Since -2.925 < -2.33, reject the null hypothesis and conclude manufacturers exit water meets federal regulation at $\alpha = 0.01$.

b) P-value = $\Phi(-2.925) \cong 0.00172$

c) $\beta = \Phi\left(-z_{0.01} + \frac{7920 - 7980}{80/\sqrt{10}}\right) = \Phi(-4.70) = 0$

d) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.01} + z_{0.10})^2 \sigma^2}{(7920 - 7980)^2} \cong \frac{(2.33 + 1.29)^2 (80)^2}{(-60)^2} = 23.29,$$

$n \cong 24.$

e) For $\alpha = 0.01$, $z_\alpha = z_{0.01} = 2.33$

$$\mu \le \bar{x} + z_{0.01}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\mu \le 7906 + 2.33\left(\frac{80}{\sqrt{10}}\right)$$

$\mu \le 7965$

f) The confidence interval constructed does not contain the value 7980, thus the manufacturer's exit water meets federal regulation using a 99% level of confidence.

4-27. a) 1) The parameter of interest is the true mean distance, μ.

2) H_0: $\mu = 2.00$

3) H_1: $\mu > 2.00$

4) $\alpha = 0.01$

5) $z_0 = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$

6) Reject H_0 $z_0 > z_\alpha$ where $z_{0.01} = 2.33$

7) $\overline{x} = 2.02$, $\sigma = 0.05$

$$z_0 = \frac{2.02 - 2.00}{0.05/\sqrt{20}} = 1.79$$

8) Since 1.79 < 2.33, do not reject the null hypothesis and conclude the true mean distance of the foil to the edge is not at least 2.00 cm.

b) P-value = $1 - \Phi(1.79) = 1 - 0.96327 = 0.03673$

Since the P-value is less than the level of significance, α, we would not reject the null hypothesis.

c) $\beta = \Phi\left(z_{0.01} - \frac{2.03 - 2.00}{0.05/\sqrt{20}}\right) = \Phi(-0.35) = 0.363169$

d) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.01} + z_{0.10})^2 0.05^2}{(2.03 - 2.00)^2} \cong \frac{(2.33 + 1.29)^2 (0.05)^2}{(0.03)^2} = 36.4,$$

$n \cong 36.$

e) For $\alpha = 0.01$, $z_\alpha = z_{0.01} = 2.33$

$$\bar{x} - z_{0.01}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu$$

$$2.02 - 2.33\left(\frac{0.05}{\sqrt{20}}\right) \le \mu$$

$$1.994 \le \mu$$

f) The confidence interval constructed contains the value 2.00, thus the average distance of the foil to the edge of the inflator is not at least 2.00 using a 99% level of confidence.

4-28. a) 1) The parameter of interest is the true mean life, μ.
2) H_0: $\mu = 540$
3) H_1: $\mu > 540$
4) $\alpha = 0.05$
5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
6) Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$
7) $\bar{x} = 551.33$, $\sigma = 20$

$$z_0 = \frac{551.33 - 540}{20/\sqrt{15}} = 2.19$$

8) Since 2.19 > 1.65, reject the null hypothesis and conclude there is sufficient evidence to support the claim the life exceeds 540 hrs at $\alpha = 0.05$.

b) P-value = $P(Z > 2.19) = 1 - P(Z \le 2.19) = 1 - \Phi(2.19) = 0.0143$.

c) $\beta = \Phi\left(z_{0.05} - \dfrac{(560 - 540)\sqrt{15}}{20}\right) = \Phi(1.65 - 3.873) = \Phi(-2.223) = 0.0131$

d) $n = \dfrac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.10})^2 \sigma^2}{(560 - 540)^2} = \dfrac{(1.65 + 1.29)^2 (20)^2}{(20)^2} = 8.644$,

$n \cong 9$.

e) $\bar{x} - z_{0.050}\dfrac{\sigma}{\sqrt{n}} \le \mu$

$$551.33 - 1.645\frac{20}{\sqrt{15}} \le \mu$$

$$542.83 \le \mu$$

With 95% confidence, the true mean life is at least 542.83 hrs.

f) Since 540 does not fall within this interval, we can reject the null hypothesis in favor of the alternative.

4-29. a) 1) The parameter of interest is the true mean compressive strength, μ.
2) $H_0 : \mu = 3500$
3) $H_1 : \mu \ne 3500$
4) $\alpha = 0.01$
5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$
7) $\bar{x} = 3255.42$, $\sigma = 31.62$

$$z_0 = \frac{3255.42 - 3500}{31.62/\sqrt{12}} = -26.79$$

8) Since $-26.79 < -2.58$, reject the null hypothesis and conclude the true mean compressive strength is significantly different from 3500 at $\alpha = 0.01$.

b) Smallest level of significance = P-value = $2[1 - \Phi(26.84)] = 2[1 - 1] = 0$
The smallest level of significance at which we are willing to reject the null hypothesis is 0.

c) $z_{\alpha/2} = z_{0.025} = 1.96$

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3255.42 - 1.96\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3255.42 + 1.96\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3237.53 \le \mu \le 3273.31$$

With 95% confidence, we believe the true mean compressive strength is between 3237.53 psi and 3273.31psi.

d) $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3255.42 - 2.58\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3255.42 + 2.58\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3231.96 \le \mu \le 3278.88$$

With 99% confidence, we believe the true mean compressive strength is between 3231.96 psi and 3278.88 psi.
The 99% confidence interval is wider than the 95% confidence interval. The confidence interval with the larger level of confidence will always result in a wider confidence interval when $\bar{x}$, σ^2, and *n* are held constant.

4-30. $z_{\alpha/2} = z_{0.025} = 1.96$, $E = 5$

$$n \cong \left[\frac{20(1.96)}{5}\right]^2 = 61.4656, \; n \cong 62.$$

4-31. $z_{\alpha/2} = z_{0.025} = 1.96$, $E = 0.01$

$$n \cong \left[\frac{0.05(1.96)}{0.01}\right]^2 = 96.04, \; n \cong 97.$$

4-32. $z_{\alpha/2} = z_{0.005} = 2.58$, $E = 15$

$$n = \left[\frac{31.62(2.58)}{15}\right]^2 = 29.58, \; n \cong 30.$$

Section 4-5

4-33. a) 1) The parameter of interest is the true mean life, μ.
2) $H_0 : \mu = 60000$
3) $H_1 : \mu > 60000$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,9} = 1.833$
7) $\bar{x} = 61492$ $s = 3035$ $n = 10$

$$t_0 = \frac{61492 - 60000}{3035/\sqrt{10}} = 1.55$$

8) Since 1.55 < 1.833, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the mean life of this new tire is in excess of 60,000 km at $\alpha = 0.05$.

b) $d = \frac{\delta}{\sigma} = \frac{\mu - \mu_0}{\sigma} = \frac{61000 - 60000}{3035} = 0.3295$

Using the OC curve for $\alpha = 0.05$, $d = 0.3295$, and $n = 10$, we get $\beta \cong 0.78$ and power of $1 - 0.78 = 0.22$. With the power being smaller than the acceptable level, 10 is not an adequate sample size for detecting a difference with probability of at least 0.90.

c) $t_{\alpha,n-1} = t_{0.05,9} = 1.833$

$$\bar{x} - t_{0.05,9}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$61492 - 1.833\left(\frac{3035}{\sqrt{10}}\right) \le \mu$$

$59732.78 \le \mu$

d) Since 60,000 km falls within this confidence interval, we cannot reject the null hypothesis or support the alternative.

4-34. In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true Izod impact strength, μ.
2) H_0: $\mu = 1.0$
3) H_1: $\mu > 1.0$
4) $\alpha = 0.01$
5) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,19} = 2.539$
7) $\bar{x} = 1.121$ $s = 0.328$ $n = 20$

$$t_0 = \frac{1.121 - 1.0}{0.328/\sqrt{20}} = 1.65$$

8) Since 1.65 < 2.539, do not reject the null hypothesis and conclude there is not sufficient evidence to indicate that the true Izod impact strength is greater than 1.0 ft-lb/in at $\alpha = 0.01$.

4-35. In order to use *t* statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

a) 1) The parameter of interest is the mean life in hours, μ.
2) H_0: $\mu = 5500$
3) H_1: $\mu > 5500$
4) $\alpha = 0.05$
5) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 $t > t_{0.05,n-1}$ where $t_{0.05,14} = 1.761$
7) $\bar{x} = 5625.1$ $s = 226.1$ $n = 15$

$$t_0 = \frac{5625.1 - 5500}{226.1/\sqrt{15}} = 2.14$$

8) Since 2.14 > 1.761, reject the null hypothesis and conclude there is sufficient evidence to indicate that the amount of current necessary is not 300 microamps at $\alpha = 0.05$.

b) P-value = $P(t > 2.14)$: for degrees of freedom of 14 we obtain $0.025 < \text{P-value} < 0.05$;

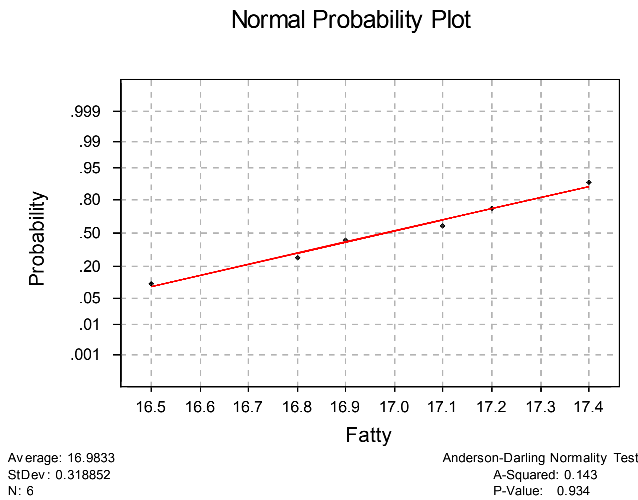
c) $\bar{x} - t_{0.05,14}\left(\frac{s}{\sqrt{n}}\right) \le \mu$

$$5625.1 - 1.761\left(\frac{226.1}{\sqrt{15}}\right) \le \mu$$

$$5522.3 \le \mu$$

d) We are 95% confident that the true mean life is at least 5522.3. Therefore, the confidence interval also indicates that the null hypothesis is rejected since 5500 is not contained in this interval.

4-36.



The normality assumption appears to be satisfied. This is evident by the fact that the data fall along a straight line.

a) 1) The parameter of interest is the true mean level of polyunsaturated fatty acid, μ.

2) H_0: $\mu = 17$

3) H_1: $\mu \ne 17$

4) $\alpha = 0.01$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-1}$ where $-t_{0.005,5} = -4.032$ or $t > t_{\alpha/2,n-1}$ where $t_{0.005,5} = 4.032$

7) $\bar{x} = 16.98$ $s = 0.319$ $n = 6$

$$t_0 = \frac{16.98 - 17}{0.319/\sqrt{6}} = -0.154$$

8) Since $-4.032 < -0.154 < 4.032$, do not reject the null hypothesis and conclude the true mean level is not significantly different from 17% at $\alpha = 0.01$.

b) P-value = $2P(t > 0.154)$: for degrees of freedom of 5 we obtain

$$2(0.40) < \text{P-value}$$
$$0.80 < \text{P-value}$$

c) Using the OC curves on Chart IIIb, with $d = \dfrac{0.5}{0.319} = 1.567$, $n = 10$, when $\beta \cong 0.1$. Therefore, the current sample size of 6 is inadequate.

d) For $\alpha = 0.01$, $t_{\alpha/2,n-1} = t_{0.005,5} = 4.032$

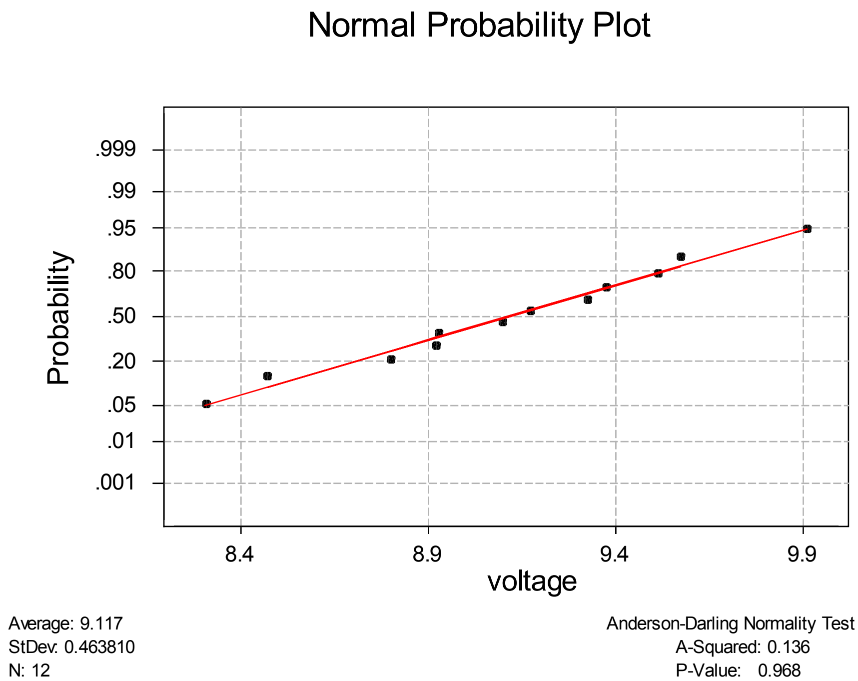
$$\bar{x} - t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,5}\left(\frac{s}{\sqrt{n}}\right)$$

$$16.98 - 4.032\left(\frac{0.319}{\sqrt{6}}\right) \le \mu \le 16.98 + 4.032\left(\frac{0.319}{\sqrt{6}}\right)$$

$$16.455 \le \mu \le 17.505$$

With 99% confidence, we believe the true mean level of polyunsaturated fatty acid is between 16.455% and 17.505%.

4-37. a) According to the normal probability plot, the data appear to follow a normal distribution. This is evident by the fact that the data fall along a straight line.



b) 1) The parameter of interest is the true mean breakdown voltage, μ.

2) $H_0 : \mu = 9$

3) $H_1 : \mu < 9$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 < -t_{\alpha,n-1}$ where $-t_{0.05,11} = -1.796$

7) $\overline{x} = 9.117$ $s = 0.464$ $n = 12$

$$t_0 = \frac{9.117 - 9}{0.464/\sqrt{12}} = 0.8735$$

8) Since $0.8735 > -1.796$, do not reject the null hypothesis and conclude there is insufficient evidence to indicate that the true mean breakdown voltage is less than 9 volts at $\alpha = 0.05$.

c) For $\alpha = 0.05$ and $n = 12$, $t_{\alpha,n-1} = t_{0.05,11} = 1.796$

$$\mu \le \overline{x} + t_{0.05,11}\left(\frac{s}{\sqrt{n}}\right)$$

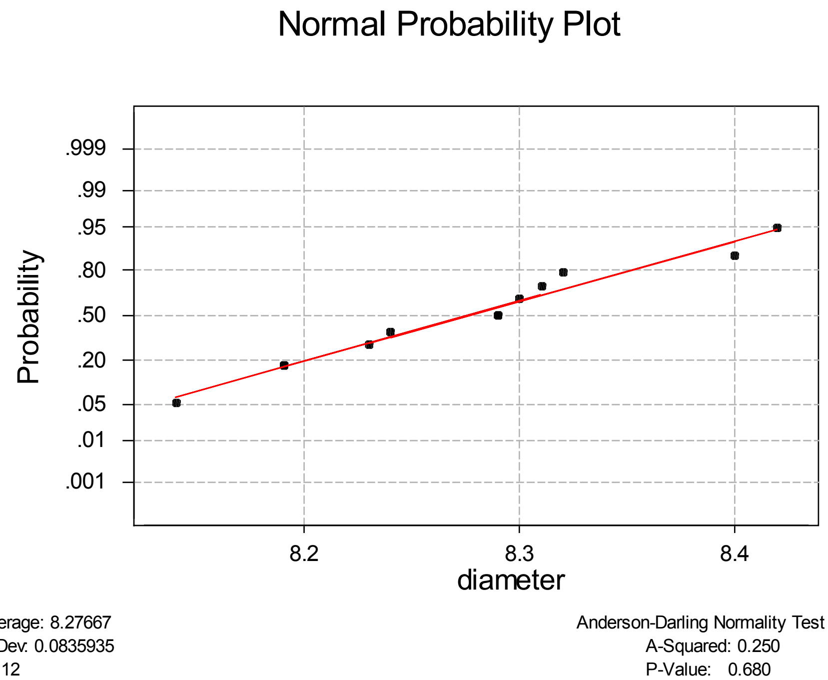
$$\mu \le 9.117 + 1.796\left(\frac{0.464}{\sqrt{12}}\right)$$

$$\mu \le 9.358$$

d) Since 9 volts lies within this confidence interval, we cannot reject the null hypothesis.

e) Using the OC curves on Chart IIIb, with $d = \dfrac{8.8 - 9}{0.464} = 0.431$, when $\beta \cong 0.05$, we have $n \cong 60$.

4-38. a) According to the normal probability plot there does not seem to be a severe deviation from normality for this data. This is evident by the fact that the data appears to fall along a straight line.



b) 1) The parameter of interest is the true mean rod diameter, μ.

2) H_0: $\mu = 8.2$

3) H_1: $\mu \neq 8.2$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-1}$ $t_0 > t_{\alpha/2,n-1}$ where $t_{0.025,11} = 2.201$

7) $\overline{x} = 8.2767$ $s = 0.0836$ $n = 12$

$$t_0 = \frac{8.2767 - 8.2}{0.0836 / \sqrt{12}} = 3.18$$

8) Since 3.18 > 2.201, reject the null hypothesis and conclude the true mean rod diameter is not equal to 8.2 mm at the 0.05 level of significance.

c) P-value = $2 \cdot P(t > 3.18)$ for degrees of freedom of 11:

$$0.005 < \text{P-value} < 0.01$$

d) For $\alpha = 0.05$ and $n = 12$, $t_{\alpha/2,n-1} = t_{0.025,11} = 2.201$

$$\overline{x} - t_{0.025,11}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \overline{x} + t_{0.025,11}\left(\frac{s}{\sqrt{n}}\right)$$

$$8.2767 - 2.201\left(\frac{0.0836}{\sqrt{12}}\right) \le \mu \le 8.2767 + 2.201\left(\frac{0.0836}{\sqrt{12}}\right)$$

$$8.226 \le \mu \le 8.33$$

With 95% confidence, we believe the true mean rod diameter is between 8.226 mm and 8.33 mm. Since this interval does not contain the hypothesized value of 8.2, we reject the null hypothesis.

4-39. a) 1) The parameter of interest is the true mean wall thickness, μ.

2) H_0: $\mu = 4.0$

3) H_1: $\mu > 4.0$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,24} = 1.711$

7) $\overline{x} = 4.058$ $s = 0.081$ $n = 25$

$$t_0 = \frac{4.058 - 4.0}{0.081/\sqrt{25}} = 3.58$$

8) Since 3.58 > 1.711, reject the null hypothesis and conclude there is sufficient evidence to indicate that the true mean wall thickness is greater than 4.0 mm at $\alpha = 0.05$.

$$\text{P-value} = P(t > 3.58);\ 0.0005 < \text{P-value} < 0.001$$

b) $t_{\alpha/2,n-1} = t_{0.05,24} = 1.711$

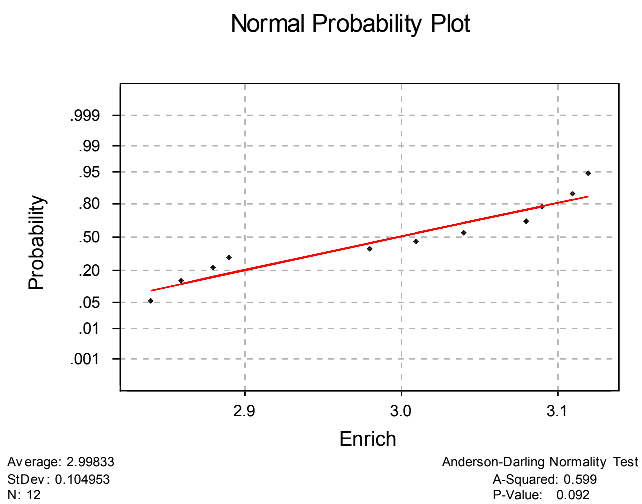
$$\bar{x} - t_{0.05,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$4.058 - 1.711\left(\frac{0.081}{\sqrt{25}}\right) \le \mu$$

$4.030 \le \mu$

With 95% confidence, we believe the true mean wall thickness is at least 4.03 mm

4-40. a)



The normality assumption appears to be reasonable. This is evident by the fact that the data appear to fall along a straight line.

b) 1) The parameter of interest is the true mean percent enrichment, μ.

2) $H_0 : \mu = 2.95$

3) $H_1 : \mu \neq 2.95$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-1}$ where $-t_{0.025,11} = -2.201$ or $t > t_{\alpha/2,n-1}$ where $t_{0.025,11} = 2.201$

7) $\overline{x} = 2.998$ $s = 0.105$ $n = 12$

$$t_0 = \frac{2.998 - 2.95}{0.105/\sqrt{12}} = 1.58$$

8) Since $-2.201 < 1.58 < 2.201$, do not reject the null hypothesis and conclude there is no strong evidence to indicate that the true mean percent enrichment is significantly different from 2.95 at $\alpha = 0.05$.

c) For $\alpha = 0.01$ and n = 12, $t_{\alpha/2,n-1} = t_{0.005,11} = 3.106$

$$\bar{x} - t_{0.005,11}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.005,11}\left(\frac{s}{\sqrt{n}}\right)$$

$$2.998 - 3.106\left(\frac{0.105}{\sqrt{12}}\right) \le \mu \le 2.998 + 3.106\left(\frac{0.105}{\sqrt{12}}\right)$$

$$2.904 \le \mu \le 3.092$$

With 99% confidence, we believe the true mean percent enrichment is between 2.904% and 3.092%. Since the interval contains the value 2.95% with a large level of confidence, we conclude that the mean percent enrichment is not significantly different 2.95%.

4-41. a) 1) The parameter of interest is the true mean quantity of syrup, μ.

2) $H_0 : \mu = 1.0$

3) $H_1 : \mu \ne 1.0$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-1}$ where $t_{0.025,24} = -2.064$ or $t > t_{\alpha/2,n-1}$ where $t_{0.025,24} = 2.064$

7) $\bar{x} = 1.098$ $s = 0.016$ $n = 25$

$$t_0 = \frac{1.098 - 1.0}{0.016/\sqrt{25}} = 30.625$$

8) Since 30.625 > 2.201, reject the null hypothesis and conclude the true mean quantity of syrup is significantly different from 1.0 fl. oz. at $\alpha = 0.05$

b) 2) H_0: $\mu = 1.0$

3) H_1: $\mu > 1.0$

4) $\alpha = 0.05$

5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

6) Reject H_0 if $t > t_{\alpha/n-1}$ where $t_{0.05,24} = 1.711$

7) $t_0 = 30.625$

8) Since 30.625 > 1.711, reject the null hypothesis and conclude there is sufficient evidence to support the claim that the mean quantity of syrup dispensed exceeds 1.0 fl. oz. at $\alpha = 0.05$.

c) Using the OC curve in Chart V e), with d = 0.05/0.016 = 3.125 and n = 25 we get $\beta \cong 0$. Thus the power is given by $1 - \beta = 1$. The sample size is adequate for the experiment.

d) For $\alpha = 0.05$ and n = 25, $t_{\alpha/2,n-1} = t_{0.025,24} = 2.064$

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$1.098 - 2.064\left(\frac{0.016}{\sqrt{25}}\right) \le \mu \le 1.098 + 2.064\left(\frac{0.016}{\sqrt{25}}\right)$$

$$1.091 \le \mu \le 1.105$$

With 99% confidence, we believe the true mean quantity is between 1.091 fl. oz. and 1.105 fl. oz.

4-42. The parameter of interest is the true mean natural frequency, μ.

For $\alpha = 0.10$ and n = 5, $t_{\alpha/2,n-1} = t_{0.05,4} = 2.132$

$$\bar{x} - t_{0.05,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.05,4}\left(\frac{s}{\sqrt{n}}\right)$$

$$231.67 - 2.132\left(\frac{1.53}{\sqrt{5}}\right) \le \mu \le 231.67 + 2.132\left(\frac{1.53}{\sqrt{5}}\right)$$

$230.21 \le \mu \le 233.13$

With 90% confidence, we believe the true mean frequency is between 230.21 Hz and 233.13 Hz. The claim that the mean natural frequency is 253 Hz cannot be supported with a level of confidence of 90% since this interval did not contain the value 253.

Section 4-6

4-43. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1)The parameter of interest is the true standard deviation of the diameter, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0: $\sigma^2 = 0.0004$

3) H_1: $\sigma^2 > 0.0004$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$ where $\chi_{0.05,14}^2 = 23.685$

7) n = 15, s = 0.016

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(0.016)^2}{0.0004} = 8.96$$

8) Since 8.96 < 23.685 do not reject H_0 and conclude there is insufficient evidence to indicate the true standard deviation of the diameter exceeds 0.02 at $\alpha = 0.05$.

b) P-value = $P(\chi^2 > 8.96)$ for 14 degrees of freedom:

$0.5 < \text{P-value} < 0.9$

c) 95% lower confidence interval on σ^2:

For $\alpha = 0.05$ and n = 15, $\chi_{\alpha,n-1}^2 = \chi_{0.05,14}^2 = 23.68$

$$\frac{14(0.016)^2}{23.68} < \sigma^2$$

$0.00015 < \sigma^2$

With 95% confidence, we believe the true variance of the hole diameter is greater than 0.00015 mm^2. With 95% confidence, we believe the true standard deviation of the hole diameter is greater than 0.012 mm

d) Based on the lower confidence bound, we cannot reject the null hypothesis.

4-44. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true variance of the sugar content, σ^2.

2) H_0: $\sigma^2 = 18$

3) H_1: $\sigma^2 \neq 18$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha/2,n-1}^2$ where $\chi_{0.975,9} = 2.70$ or $\chi_0^2 > \chi_{\alpha,2,n-1}^2$ where $\chi_{0.025,9} = 19.02$

7) n = 10, $s^2 = 16$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(16)}{18} = 8$$

8) Since 2.70 < 8 < 19.02 do not reject H_0 and conclude the evidence indicates the true variance of the

sugar content is not significantly different from 18 mg^2 at $\alpha = 0.05$.

b) P-value = $2P(\chi^2 > 8) \cong 1$ for 9 degrees of freedom

c) 95% confidence interval for σ:
First find a confidence interval for σ^2:
For $\alpha = 0.05$ and n = 10, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,9} = 19.02$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,9} = 2.70$

$$\frac{9(16)}{19.02} \le \sigma^2 \le \frac{9(16)}{2.70}$$
$$7.57 \le \sigma^2 \le 53.33$$

Take the square root of the endpoints of this interval to find the approximate confidence interval for σ:
$2.75 \le \sigma \le 7.30$
With 95% confidence, we believe the true standard deviation of the sugar content is between 2.75 mg and 7.30 mg.

d) Since the hypothesized value lies within this confidence interval, the null hypothesis cannot be rejected.

4-45. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the standard deviation of tire life, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) H_0: $\sigma^2 = 3000^2$
3) H_1: $\sigma^2 > 3000^2$
4) $\alpha = 0.05$
5) $\chi^2_0 = \dfrac{(n-1)s^2}{\sigma^2}$
6) Reject H_0 if $\chi^2_0 > \chi^2_{\alpha,n-1}$ where $\chi^2_{0.05,9} = 16.919$
7) n = 10, $s^2 = (3035)^2$

$$\chi^2_0 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(3035)^2}{3000^2} = 9.2112$$

8) Since 9.2112 < 16.919 do not reject H_0 and conclude there is no evidence to indicate the true standard deviation of tire life exceeds 3000 km at $\alpha = 0.05$.

b) P-value = $P(\chi^2 > 9.2112)$ for 9 degrees of freedom, 0.10 < P-value < 0.50.

c) 95% lower confidence interval for σ^2:
For $\alpha = 0.05$ and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.05,9} = 16.919$

$$\frac{9(3035)^2}{16.919} < \sigma^2$$
$$4899877.36 < \sigma^2$$

With 95% confidence, we believe the true variance of tire life is greater than 4,899,877.36 km^2. With 95% confidence, we believe the true standard deviation of tire life is greater than 2213.57 km.

d) Since the hypothesized value falls within this interval, we cannot reject the null hypothesis.

4-46. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of Izod strength, σ. However, the answer can be found by performing a hypothesis test on σ^2.
2) H_0: $\sigma^2 = 0.1$
3) H_1: $\sigma^2 \neq 0.1$
4) $\alpha = 0.01$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.995,19} = 6.84$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.005,19} = 38.58$

7) n = 20, s = 0.328

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(0.328)^2}{0.1} = 20.441$$

8) Since 6.84 < 20.441 < 38.58 we would not reject H_0 and conclude the true variance of Izod strength is not significantly different from 0.10 ft-lb/in at $\alpha = 0.01$.

b) P-value = $2P(\chi^2 > 20.441)$ for 19 degrees of freedom

$$0.20 < 2P(\chi^2 > 20.441) < 1$$

c) 99% confidence interval for σ^2 :

For $\alpha = 0.01$ and n = 20, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.005,19} = 38.58$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.995,19} = 6.84$

$$\frac{19(0.328)^2}{38.58} \le \sigma^2 \le \frac{19(0.328)^2}{6.84}$$

$0.053 \le \sigma^2 \le 0.299$

With 99% confidence, we believe the true variance of Izod strength is between 0.053 $(\text{ft-lb/in})^2$ and 0.299 (ft-lb/in).

4-47. a) In order to use χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of titanium percentage, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0: $\sigma^2 = (0.35)^2$

3) H_1: $\sigma^2 \ne (0.35)^2$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\chi^2_{0.975,50} = 32.36$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\chi^2_{0.025,50} = 71.42$

7) n = 51, s = 0.37

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{50(0.37)^2}{(0.35)^2} = 55.88$$

8) Since 32.36 < 55.88 < 71.42 we would not reject H_0 and conclude there is insufficient evidence to indicate the true standard deviation of titanium percentage is significantly different from 0.35 at $\alpha = 0.01$.

b) $2P(\chi^2 > 55.88)$ for 50 degrees of freedom: $0.20 < 2P(\chi^2 > 55.88) < 1$

c) 95% confidence interval for σ:

First find the confidence interval for σ^2 :

For $\alpha = 0.05$ and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{(71.42)^2} \le \sigma^2 \le \frac{50(0.37)^2}{(32.36)^2}$$

$0.096 \le \sigma^2 \le 0.2115$

Taking the square root of the endpoints of this interval we obtain, $0.31 < \sigma < 0.46$

With 95% confidence, we believe the true standard deviation of titanium percentage is between 0.31 and 0.46.

d) Since the hypothesized value falls within this confidence interval, we cannot reject the null hypothesis.

Section 4-7

4-48. a) 1) The parameter of interest is the true proportion of deaths from lung cancer, p.

2) H_0: p = 0.85

3) H_1: p ≠ 0.85

4) $\alpha = 0.05$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) x = 823 n = 1000 $\hat{p} = \frac{823}{1000} = 0.823$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{823 - 1000(0.85)}{\sqrt{1000(0.85)(0.15)}} = -2.39$$

8) Since -2.39 < -1.96, reject the null hypothesis and conclude the percentage of deaths resulting from lung cancer is significantly different from 85%.

b) $\hat{p} = \frac{823}{1000} = 0.823$ n = 1000 For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

$$0.823 - 1.96\sqrt{\frac{0.823(0.177)}{1000}} \le p \le 0.823 + 1.96\sqrt{\frac{0.823(0.177)}{1000}}$$

$0.799 \le p \le 0.847$

With 95% confidence, we believe the true proportion of deaths resulting from cancer is between 0.799 and 0.847.

c) E = 0.03, $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$ and $\hat{p} = 0.823$ as the initial estimate of p,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.03}\right)^2 0.823(1-0.823) = 621.79,$$

$n \cong 622$.

4-49. n = 30, x = 11, $\hat{p} = \frac{11}{30} = 0.367$, $\alpha = 0.10$, $z_\alpha = z_{0.10} = 1.28$

a) 1) The parameter of interest is the true proportion of rollovers, p.

2) H_0 : p = 0.25

3) H_1 : p > 0.25

4) $\alpha = 0.10$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.10} = 1.28$

7) x = 11 n = 30 $\hat{p} = \frac{11}{30} = 0.367$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.367 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{30}}} = 1.48$$

8) Since 1.48 > 1.28, reject the null hypothesis and conclude the percentage of rollovers exceeds 25%

b) $$\beta = \Phi\left[\frac{p_0 - p + z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right] = \Phi\left[\frac{0.25 - 0.35 + 1.28\sqrt{0.25(1-0.25)/30}}{\sqrt{0.35(1-0.35)/30}}\right] = \Phi[0.0137]$$

$\beta \cong 0.5055$

c) $n = \left[\dfrac{z_\alpha \sqrt{p_0(1-p_0)} + z_\beta \sqrt{p(1-p)}}{p - p_0}\right]^2 = \left[\dfrac{1.28\sqrt{0.25(1-0.25)} + 1.28\sqrt{0.35(1-0.35)}}{0.35 - 0.25}\right]^2$

n = 135.67, $n \cong 136$

d) $0.367 - 1.28\sqrt{\dfrac{0.367(0.633)}{30}} \le p$

$0.254 \le p$

e) Since the hypothesized value does not fall within this interval, we can reject the null hypothesis.

f) $n = \left(\dfrac{z_\alpha}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\dfrac{1.645}{0.02}\right)^2 0.367(1-0.367) = 1571.598$, $n \cong 1572$

4-50. n = 50, x = 18, $\hat{p} = \dfrac{18}{50} = 0.36$, $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

a) 1) The parameter of interest is the true proportion of damage, p.
 2) $H_0: p = 0.30$
 3) $H_1: p \neq 0.30$
 4) $\alpha = 0.05$
 5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
 6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$
 7) x = 18 n = 50 $\hat{p} = \dfrac{18}{50} = 0.36$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{0.36 - 0.30}{\sqrt{\dfrac{(0.30)(0.70)}{50}}} = 0.926$$

 8) Since -1.96 < 0.926 < -1.96, do not reject the null hypothesis and conclude the percentage of helmets damaged is not significantly different from 30%.

b) $0.36 - 1.96\sqrt{\dfrac{0.36(0.64)}{50}} \le p \le 0.36 + 1.96\sqrt{\dfrac{0.36(0.64)}{50}}$

$0.23 \le p \le 0.49$

c) P-value = $2 \cdot P(Z > 0.93) = 2 \cdot (0.176185) = 0.35237$

With 95% confidence, we believe the true proportion of helmets damaged lies within 0.23 and 0.49.

d) $n = \left(\dfrac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\dfrac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76$, $n \cong 2213$

e) $n = \left(\dfrac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\dfrac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$, $n \cong 2401$

4-51. The worst case would be for p = 0.5, thus with E = 0.05 and $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$ we obtain a sample size of:

$n = \left(\dfrac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\dfrac{2.58}{0.05}\right)^2 0.5(1-0.5) = 665.64$, $n \cong 666$

4-52. $\hat{p} = \frac{10}{800} = 0.0125$, n = 800,

a) 1) The parameter of interest is the true fraction defective, p.

2) H_0: p = 0.01

3) H_1: p > 0.01

4) $\alpha = 0.05$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.05} = 1.65$

7) x = 10 n = 800 $\hat{p} = \frac{10}{800} = 0.0125$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0125 - 0.01}{\sqrt{\frac{(0.01)(0.99)}{800}}} = 0.711$$

8) Since 0.711 < 1.65, do not reject the null hypothesis and conclude the proportion of defective calculators does not exceed 0.01.

b) $\beta = \Phi\left[\frac{0.01 - 0.02 + z_{0.05}\sqrt{(0.01)(0.99)/800}}{\sqrt{(0.02)(0.98)/800}}\right] = \Phi(-0.85) = 0.197662$

c) Use equation in part b) to solve for n when β = 0.10, α = 0.05, n = 800, $z_{0.05}$ = 1.65, p = 0.02, and p_0 = 0.01. Using MINITAB, n = 1178 with Power = 0.9002.

4-53. E = 0.02, $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$$

4-54. a) 1) The parameter of interest is the true fraction of defective integrated circuits, p.

2) H_0: p = 0.04

3) H_1: $p \neq 0.04$

4) $\alpha = 0.05$

5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) x = 18 n = 300 $\hat{p} = \frac{18}{300} = 0.06$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{18 - 300(0.04)}{\sqrt{300(0.04)(0.96)}} = 1.77$$

8) Since −1.96 < 1.77 < 1.96, do not reject the null hypothesis and conclude the true fraction of defective integrated circuits is not significantly different from 0.04, at $\alpha = 0.05$.

b) P-value = 2(1 - Φ(1.77)) = 0.0767.
Again since the P-value is greater than the level of significance, we would not reject the null hypothesis.

c) $\hat{p}$ = 18/300 = 0.06, α = 0.05, n = 300, $z_{0.05}$ = 1.65

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.06-1.96\sqrt{\frac{0.06(0.94)}{300}} < p < 0.06+1.96\sqrt{\frac{0.06(0.94)}{300}}$$

$0.0331 < p < 0.0869$

d) Because $p_0 = 0.04$ is in the interval, we fail to reject H_0 and conclude that the true fraction of the defective circuits is not significantly different from 0.04.

4-55. a) $$\beta=\Phi\left(\frac{0.04-0.05+1.96\sqrt{\frac{0.04(0.96)}{300}}}{\sqrt{\frac{0.05(0.95)}{300}}}\right)-\Phi\left(\frac{0.04-0.05-1.96\sqrt{\frac{0.04(0.96)}{300}}}{\sqrt{\frac{0.05(0.95)}{300}}}\right)$$

$\beta = \Phi(0.97) - \Phi(-2.56)$
$= 0.8340 - 0.0052$
$= 0.8288$

b) $$n=\left(\frac{1.96\sqrt{0.04(0.96)}+1.28\sqrt{0.05(0.95)}}{0.05-0.04}\right)^2$$

$n = 4396.3$, $n = 4397$

4-56. a) 1) The parameter of interest is the true proportion of engineers who continue their education, p.
2) H_0: $p = 0.50$
3) H_1: $p \neq 0.50$
4) $\alpha = 0.05$
5) $z_0=\frac{x-np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$
7) $x = 117$ $n = 484$ $\hat{p}=\frac{117}{484}=0.242$

$$z_0=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}=\frac{0.242-0.50}{\sqrt{\frac{0.5(1-0.5)}{484}}}=-11.352$$

8) Since $-11.352 < -1.96$, reject the null hypothesis and conclude the data from "Engineering Horizons" yield results significantly different from the claim reported by "Fortune", at $\alpha = 0.05$.

b) P-value $= 2(1 - \Phi(11.352)) = 2(1 - 1) = 0$

4-57. a) 1) The parameter of interest is the true percentage of polished lenses that contain surface defects, p.
2) $H_0 : p = 0.04$
3) $H_1 : p < 0.04$
4) $\alpha = 0.05$
5) $z_0=\frac{x-np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.05} = -1.65$
7) $x = 14$ $n = 300$ $\hat{p}=\frac{14}{300}=0.047$

$$z_0=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}=\frac{0.047-0.04}{\sqrt{\frac{0.04(1-0.04)}{300}}}=0.619$$

8) Since $0.619 > -1.65$ do not reject the null hypothesis and conclude the machine cannot be qualified at the 0.05 level of significance.

b) P-value = $\Phi(0.619) = 0.7320$

= 0..2776

c) $\beta = 1 - \Phi\left(\frac{0.04 - 0.02 - 1.65\sqrt{\frac{0.04(0.96)}{300}}}{\sqrt{\frac{0.02(0.98)}{300}}} \right) = 1 - \Phi(0.1648) = 0.4345$

d) $n = \left(\frac{1.65\sqrt{0.04(0.96)} + 1.65\sqrt{0.02(0.98)}}{0.02 - 0.04} \right)^2 = 763.56$, n = 764

4-58. a) 1) The parameter of interest is the true percentage of football helmets that have flaws, p.
2) H_0: p = 0.10
3) H_1: p > 0.10
4) $\alpha = 0.01$
5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 > z_\alpha$ where $z_\alpha = z_{0.01} = 2.33$
7) x = 24 n = 200 $\hat{p} = \frac{24}{200} = 0.12$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{24 - 200(0.10)}{\sqrt{200(0.10)(0.90)}} = 0.943$$

8) Since 0.943 < 2.33 do not reject the null hypothesis and conclude there is insufficient evidence to support the claim that at least 10% of all football helmets have manufacturing flaws that could potentially cause injury to the wearer, at $\alpha = 0.01$.

b) P-value = $1 - \Phi(0.943) = \Phi(0.943) = 0.1728$.

4-59. The problem statement implies H_0: p = 0.6, H_1: p > 0.6 and defines an acceptance region as $\hat{p} \le \frac{315}{500} = 0.63$ and rejection region as $\hat{p} > 0.63$

a) $\alpha = P(\hat{P} \ge 0.63 \text{ when } p = 0.6) = P\left(Z \ge \frac{0.63 - 0.6}{\sqrt{\frac{0.6(0.4)}{500}}} \right) = P(Z \ge 1.37) = 1 - P(Z < 1.37) = 0.08535.$

b) $\beta = P(\hat{P} \le 0.63 \text{ when } p = 0.75) = P(Z \le -5.4772) = 0.$

4-60. 1) The parameter of interest is the true percentage of batteries that will fail during the warranty period, p.
2) H_0 : p = 0.002
3) H_1 : p < 0.002
4) $\alpha = 0.01$
5) $z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_\alpha$ where $-z_\alpha = -z_{0.01} = -2.33$
7) x = 4 n = 2000 $\hat{p} = \frac{4}{2000} = 0.002$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{4 - 2000(0.002)}{\sqrt{2000(0.002)(0.998)}} = 0$$

8) Since 0 > –2.33 do not reject the null hypothesis and conclude there is insufficient evidence to support the

claim that less than 0.2 percent of the company's batteries will fail during the warranty period, with proper charging procedures at $\alpha = 0.01$.

<u>Section 4-8</u>

4-61. a) $\bar{x} - t_{\alpha/2,n-1} s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2,n-1} s\sqrt{1+\frac{1}{n}}$

$$61492 - 2.262(3035)\sqrt{1+\frac{1}{10}} \le X_{n+1} \le 61492 + 2.262(3035)\sqrt{1+\frac{1}{10}}$$

54291.75, 68692.25

b) $\bar{x} - ks, \bar{x} + ks$, $k = 2.839$

61492 – 3035(2.839), 61492 + 3035(2.839)

52875.64, 70108.37

4-62. a) A 90% PI is

$$\bar{x} \pm t_{0.1/2,20-1} \cdot s\sqrt{1+\frac{1}{n}} = 1.121 \pm (1.729)(0.328)\sqrt{1+\frac{1}{20}} = (1.0628,\ 1.1791)$$

b) $\bar{x} - ks, \bar{x} + ks$, $k = 2.752$

1.121 ± (2.752)(0.328) = (0.2183, 2.0237)

4-63. a) $\bar{x} - t_{\alpha/2,n-1} s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2,n-1} s\sqrt{1+\frac{1}{n}}$

$$5625.1 - 2.977(226.1)\sqrt{1+\frac{1}{15}} \le X_{n+1} \le 5625.1 + 2.977(226.1)\sqrt{1+\frac{1}{15}}$$

4929.93, 6320.27

b) $\bar{x} - ks, \bar{x} + ks$, $k = 3.562$

5625.1 – 226.1(3.562), 5625.1 + 226.1(3.562)

4819.73, 6430.47

4-64. a) A 95% PI is

$$\bar{x} \pm t_{0.05/2,6-1} \cdot s\sqrt{1+\frac{1}{n}} = 16.9833 \pm (2.571)(0.3189)\sqrt{1+\frac{1}{6}} = (16.0977,\ 17.8689)$$

b) $\bar{x} - ks, \bar{x} + ks$, $k = 6.345$

16.9833 ± (6.345)(0.3189) = (14.9599, 19.0067)

4-65. a) $\bar{x} - t_{\alpha/2,n-1} s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2,n-1} s\sqrt{1+\frac{1}{n}}$

$$9.117 - 3.106(0.464)\sqrt{1+\frac{1}{12}} \le X_{n+1} \le 9.117 + 3.106(0.464)\sqrt{1+\frac{1}{12}}$$

7.617, 10.617

b) $\bar{x} - ks, \bar{x} + ks$, $k = 5.079$

9.117 – 0.464(5.079), 9.117 + 0.464(5.079)

6.760, 11.474

4-66. a) A 90% PI is

$$\bar{x} \pm t_{0.05/2,12-1} \cdot s\sqrt{1+\frac{1}{n}} = 8.2767 \pm (1.796)(0.0836)\sqrt{1+\frac{1}{12}} = (8.1204,\ 8.4330)$$

b) $\bar{x} - ks, \bar{x} + ks$, $k = 2.404$

8.2767 ± (2.404)(0.0836) = (8.0757, 8.4777)

Section 4-10

4-67.

Value	0	1	2	3	4	5
Observed Frequency	8	25	23	21	16	7
Expected Frequency	9.07	21.77	26.13	20.9	12.54	6.02

The degrees of freedom are k – p – 1 = 6 – 1 – 1 = 4

a) 1) The variable of interest is the form of the distribution for X.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,4}^2 = 9.49$
7)

$$\chi_0^2 = \frac{(8 - 9.07)^2}{9.07} + \cdots + \frac{(7 - 9.59)^2}{9.59} = 2.094$$

8) Since 2.0936 < 9.49 do not reject H_0. We are unable to reject the null hypothesis that the distribution of X is Poisson.

b) P-value = 0.7185 (found using Minitab)

4-68. Estimated mean = 4.907

Value	1	2	3	4	5	6	7	8
Observed Frequency	1	11	8	13	11	12	10	9
Expected Frequency	2.721	6.677	10.921	13.398	13.149	10.753	7.538	4.624

Since the first category has an expected frequency less than 3, combine it with the next category:

Value	1-2	3	4	5	6	7	8
Observed Frequency	12	8	13	11	12	10	9
Expected Frequency	9.398	10.921	13.398	13.149	10.753	7.538	4.624

The degrees of freedom are k – p – 1 = 7 – 1 – 1 = 5

a) 1) The variable of interest is the form of the distribution for the number of flaws.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.01$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.01,5}^2 = 15.09$
7)

$$\chi_0^2 = \frac{(12-9.3984)^2}{9.3984} + \cdots + \frac{(9-4.6237)^2}{4.6237} = 6.955$$

8) Since 6.955 < 15.09 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of flaws is Poisson.

b) P-value = 0.2237 (found using Minitab)

4-69. Estimated mean = 9.6

Value	5	7	8	9	10	11	12	13	14	15
Rel. Freq	0.133	0.133	0.133	0.167	0.033	0.100	0.100	0.033	0.133	0.033
Observed (Days)	4	4	4	5	1	3	3	1	4	1
Expected (Days)	1.381	3.029	3.635	3.878	3.723	3.249	2.599	1.919	1.316	0.842

Since there are several cells with expected frequencies less than 3, the revised table could be:

Value	5-7	8	9	10	11	12-15
Observed (Days)	8	4	5	1	3	9
Expected (Days)	4.41	3.635	3.878	3.723	3.249	6.676

The degrees of freedom are k – p – 1 = 6 – 1 – 1 = 4

a) 1) The variable of interest is the form of the distribution for the number of calls arriving to a switchboard from noon to 1pm during business days.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,4}^2 = 9.49$

7) $\chi_0^2 = 5.79$

8) Since 5.79 < 9.49 do not reject H_0. We are unable to reject the null hypothesis that the distribution for the number of calls is Poisson.

b) P-value = 0.2154 (found using Minitab)

4-70. Estimated mean = 49.6741
All expected frequencies are greater than 3.
The degrees of freedom are k – p – 1 = 26 – 1 – 1 = 24

a) 1) The variable of interest is the form of the distribution for the number of cars passing through the intersection.
2) H_0: The form of the distribution is Poisson
3) H_1: The form of the distribution is not Poisson
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,24}^2 = 36.42$

7) Estimated mean = 49.6741

$$\chi_0^2 = 769.57$$

8) Since 769.57 >>> 36.42, reject H_0. We can conclude that the distribution is not Poisson at $\alpha = 0.05$.

b) P-value = 0 (found using Minitab)

4-71. Mean = np = 6(0.25) = 1.5

Value	0	1	2	3	4
Observed	4	21	10	13	2
Expected	8.899	17.798	14.832	6.592	1.648

The expected frequency for value 4 is less than 3. Combine this cell with value 3:

Value	0	1	2	3-4
Observed	4	21	10	15
Expected	8.899	17.798	14.832	8.24

The degrees of freedom are k – p – 1 = 4 – 0 – 1 = 3

a) 1) The variable of interest is the form of the distribution for the random variable X.
2) H_0: The form of the distribution is binomial with n = 6 and p = 0.25
3) H_1: The form of the distribution is not binomial with n = 6 and p = 0.25
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.05,3}^2 = 7.81$
7)

$$\chi_0^2 = \frac{(4-8.899)^2}{8.899} + \cdots + \frac{(15-8.24)^2}{8.24} = 10.39$$

8) Since 10.39 > 7.81 reject H_0. We can conclude that the distribution is not binomial with n = 6 and p = 0.25 at $\alpha = 0.05$.

b) P-value = 0.0155 (found using Minitab)

4-72. The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{\text{sample}}$

sample mean = [0(21) + 1(30) + 2(22) + 3(6) + 4(1)]/80 = 1.2

$$\hat{p}_{sample} = \frac{\text{sample mean}}{n} = \frac{1.2}{12} = 0.10$$

Value	0	1	2	3	4
Observed	21	30	22	6	1
Expected	22.59436	30.12582	18.41022	6.818601	1.70465

Since value 4 has an expected frequency less than 3, combine this category with that of value 3:

Value	0	1	2	3-4
Observed	21	30	22	7
Expected	22.59436	30.12582	18.41022	8.523251

The degrees of freedom are k – p – 1 = 4 – 1 – 1 = 2

a) 1) The variable of interest is the form of the distribution for the number of underfilled cartons, X.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial

4) $\alpha = 0.10$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_o^2 > \chi_{0.1,2}^2 = 4.61$

7) $\chi_0^2 = 1.2026$

8) Since 1.2026 < 4.61 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of underfilled cartons is binomial at $\alpha = 0.10$.

b) P-value = 0.5481 with d.f. = 2 (found using Minitab)

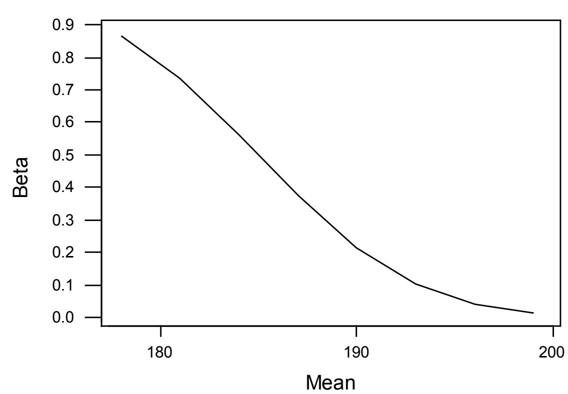
Supplemental Exercises

4-73. Operating characteristic curve:
$\bar{x} = 185$

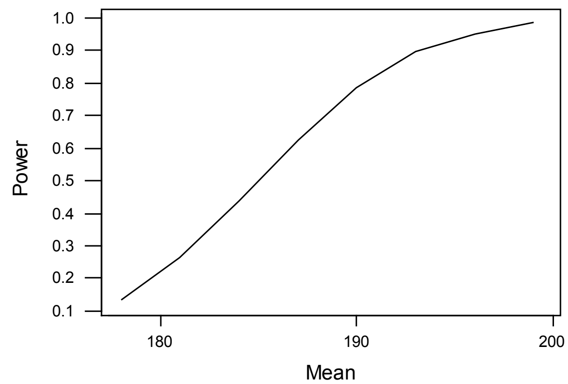
$$\beta = P\left(Z \le \frac{\bar{x} - \mu}{20/\sqrt{10}}\right) = P\left(Z \le \frac{185 - \mu}{20/\sqrt{10}}\right)$$

The probabilities given in the following table were found using Minitab.

μ	$P\left(Z \le \frac{185-\mu}{20/\sqrt{10}}\right) =$	β	$1 - \beta$
178	$P(Z \le 1.11) =$	0.8643	0.1357
181	$P(Z \le 0.63) =$	0.7357	0.2643
184	$P(Z \le 0.16) =$	0.5636	0.4364
187	$P(Z \le -0.32) =$	0.3745	0.6255
190	$P(Z \le -0.79) =$	0.2148	0.7852
193	$P(Z \le -1.26) =$	0.1038	0.8962
196	$P(Z \le -1.74) =$	0.0409	0.9491
199	$P(Z \le -2.21) =$	0.0136	0.9864



4-74.



4-75. Symmetric confidence interval: $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$, since $z_{0.025} = 1.96$

The length of this interval is $2\left(1.96\frac{\sigma}{\sqrt{n}}\right) = 3.92\left(\frac{\sigma}{\sqrt{n}}\right)$

Asymmetric confidence interval: $\bar{x} - 2.33\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + 1.76\frac{\sigma}{\sqrt{n}}$, since $-z_{0.01} = -2.325$ and $z_{0.04} = 1.75$

The length of this interval is $(2.33 + 1.76)\frac{\sigma}{\sqrt{n}} = 4.09\left(\frac{\sigma}{\sqrt{n}}\right)$

The symmetric confidence interval is the shorter of the two; the advantage to a symmetric confidence interval is the fact that in general they are shorter than an asymmetric confidence interval.

4-76. a) H_0: $p = 0.9995$, H_1: $p < 0.995$
b) H_0: $\mu = 45$, H_1: $\mu > 45$
c) H_0: $p = 0.05$, H_1: $p < 0.05$
d) H_0: $\mu = 90$, H_1: $\mu < 90$
e) H_0: $\sigma = 10$, H_1: $\sigma < 10$
f) H_0: $\mu = 2160$, H_1: $\mu > 2160$

4-77. $\mu = 50$ $\quad \sigma^2 = 5$ $\quad$ Find $P\left(s^2 \ge 7.44\right)$ and $P\left(s^2 \le 2.56\right)$

a) $n = 16$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \ge \frac{15(7.44)}{5}\right) = P\left(\chi_{15}^2 \ge 22.32\right) \approx 0.10$$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \le \frac{15(2.56)}{5}\right) = P\left(\chi_{15}^2 \le 7.68\right) = 1 - P\left(\chi_{15}^2 \ge 7.68\right); \qquad 0.05 < P\left(\chi_{15}^2 \le 7.68\right) < 0.10$$

b) $n = 30$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \ge \frac{29(7.44)}{5}\right) = P\left(\chi_{29}^2 \ge 43.152\right); \quad 0.025 < P\left(\chi_{29}^2 \ge 43.152\right) < 0.05$$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \le \frac{29(2.56)}{5}\right) = P\left(\chi_{29}^2 \le 14.85\right) = 1 - P\left(\chi_{29}^2 \ge 14.85\right); \quad 0.01 < P\left(\chi_{29}^2 \le 14.85\right) < 0.025$$

c) $n = 71$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \geq \frac{70(7.44)}{5}\right) = P\left(\chi_{70}{}^2 \geq 104.16\right); \quad 0.005 < P\left(\chi_{70}{}^2 \geq 104.16\right) \approx 0.005$$

$$P\left(\frac{(n-1)s^2}{\sigma^2} \leq \frac{70(2.56)}{5}\right) = P\left(\chi_{70}{}^2 \leq 35.84\right) = 1 - P\left(\chi_{70}{}^2 \geq 35.84\right); \quad P\left(\chi_{70}{}^2 \leq 35.84\right) < 0.005$$

d) As the sample size increases with all other values held constant, the probability $P\left(s^2 \geq 7.44\right)$ decreases because the right tail of the χ^2 distribution becomes relatively shorter.

e) As the sample size increases with all other values held constant, the probability $P\left(s^2 \leq 2.56\right)$ decreases because the left tail of the χ^2 distribution becomes relatively shorter.

4-78. a) The data appear to follow a normal distribution based on the normal probability plot since the data fall along a straight line.

b) It is important to check for normality of the distribution underlying the sample data since the confidence intervals to be constructed should have the assumption of normality for the results to be reliable (especially since the sample size is less than 30 and the central limit theorem does not apply).

c) No, with 95% confidence, we cannot infer that the true mean could be 14. 5 since this value is not contained within the given 95% confidence interval.

d) As with part b, to construct a confidence interval on the variance, the normality assumption must hold for the results to be reliable.

e) Yes, it is reasonable to infer that the variance could be 0.35 since the 95% confidence interval on the variance contains this value.

f) i) & ii) No, doctors and children would represent two completely different populations not represented by the population of Canadian Olympic hockey players. Since doctors nor children were the target of this study or part of the sample taken, the results should not be extended to these groups.

g) A 95% PI for is

$$\bar{x} \pm t_{0.05/2,20-1} \cdot s\sqrt{1+\frac{1}{n}} = 15.33 \pm (2.093)(0.6182)\sqrt{1+\frac{1}{20}} = (14.0042, 16.6558)$$

b) $\bar{x} - ks, \bar{x} + ks$, $k = 2.31$

$15.33 \pm (2.31)(0.6182) = (13.9020, 16.7580)$

4-79. a) The data appear to follow a normal distribution based on the normal probability plot since the data fall along a straight line.

b) $\bar{x} = 25.12 \quad s = 8.42 \quad n = 9$; Use the t-distribution to construct the confidence intervals; $\alpha = 1 - 0.99$

$t_{\alpha,n-1} = t_{0.01,8} = 2.896$

$$\bar{x} - t_{\alpha,n-1}\left(\frac{s}{\sqrt{n}}\right) \leq \mu$$

$$25.12 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \leq \mu$$

$$16.99 \leq \mu$$

With 99% confidence, we believe the true mean compressive strength is at least 16.99 Mpa.

c) $t_{\alpha/2,n-1} = t_{0.01,8} = 2.896$

$$\bar{x} - t_{\alpha,n-1}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{\alpha,n-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$25.12 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \le \mu \le 25.12 + 2.896\left(\frac{8.42}{\sqrt{9}}\right)$$

$$16.99 \le \mu \le 33.25$$

With 98% confidence, we believe the true mean compressive strength is between 16.99 and 33.25 Mpa.

The lower endpoint of the one-sided confidence interval is the same as that of the two-sided confidence interval due to the level of confidence used. In both cases the probability in the left tail is 0.01.

d) $\chi^2_{1-\alpha,n-1} = \chi^2_{0.99,8} = 1.65$

$$\sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha.n-1}}$$

$$\sigma^2 \le \frac{8(8.42)^2}{1.65}$$

$$\sigma^2 \le 343.76$$

With 99% confidence, we believe the true variance of compressive strength is at most 343.76 $(\text{Mpa})^2$.

e) $\chi^2_{\alpha/2,8} = 20.09$ $\chi^2_{1-(\alpha/2),8} = 1.65$ with $\alpha = 0.02$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,8}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-(\alpha/2),8}}$$

$$\frac{8(8.42)^2}{20.09} \le \sigma^2 \le \frac{8(8.42)^2}{1.65}$$

$$28.23 \le \sigma^2 \le 343.74$$

With 98% confidence, we believe the true variance of compressive strength is between 28.23 and 343.74 $(\text{Mpa})^2$. The upper endpoint of the one-sided confidence interval is the same as that of the two-sided confidence interval due to the level of confidence. In both cases the probability in the right tail is 0.01.

f) Change 40.2 to 20.4

$\bar{x} = 22.9$ and $s^2 = 39.83$

98% confidence on μ:

$$22.9 - 2.896\left(\frac{6.3}{\sqrt{9}}\right) \le \mu \le 22.9 + 2.896\left(\frac{6.3}{\sqrt{9}}\right)$$

$$16.82 \le \mu \le 28.98$$

With 98% confidence, we believe the true mean compressive strength is between 16.82 and 28.98 Mpa.

98% confidence interval on σ^2:

$$\frac{8(6.3)^2}{20.09} \le \sigma^2 \le \frac{8(6.3)^2}{1.65}$$

$$15.81 \le \sigma^2 \le 192.44$$

With 98% confidence, we believe the true variance of compressive strength is between 15.81 and 192.44 $(\text{Mpa})^2$.

Comparison of intervals:

Confidence interval on μ: The confidence interval now covers a region slightly less than the original interval, that is the length of the second interval is shorter than the original.

Confidence interval on σ^2 : The confidence interval covers a region slightly less than the original; the length of the second interval is shorter than the original.

Effects: We see by correcting the value, the length of the intervals have become shorter and the mean and variance have decreased. In particular, the sample standard deviation as decreased by about 25% causing the confidence intervals to decrease substantially.

g) Change 25.8 to 24.8

$\bar{x} = 25$ and $s^2 = 70.84$

98% confidence on μ:

$$25 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \le \mu \le 25 + 2.896\left(\frac{8.42}{\sqrt{9}}\right)$$

$$16.88 \le \mu \le 33.14$$

With 98% confidence, we believe the true mean compressive strength is between 16.88 and 33.14.

98% confidence interval on σ^2 :

$$\frac{8(8.42)^2}{20.09} \le \sigma^2 \le \frac{8(8.42)^2}{1.65}$$

$$28.23 \le \sigma^2 \le 343.74$$

With 98% confidence, we believe the true variance of compressive strength is between 28.23 and 343.74 $(\text{Mpa})^2$.

Comparison of intervals parts f and g:

Confidence interval on μ: There is very little difference when the data value is changed only slightly.

Confidence interval on σ^2: The confidence interval has changed very little with a slight change in one data value.

Effects: The sample mean, sample variance, and confidence intervals have changed very little.

h) There are two cases that the above exercises illustrate: 1) when a value is changed and the new value lies far from the sample mean; and 2) when a value is changed and the new value lies near the sample mean. When the situation is case 1, the variance has changed dramatically resulting in a smaller confidence interval. The mean is less sensitive to this change. When the situation is case 2, the mean and variance have changed less than in case 1. Notice the widths of the confidence intervals for the original data, case 1, and case 2 in the table below.

	Width of Confidence Interval		
Parameter of Interest	**Original Data**	**Case 1**	**Case 2**
Mean, μ	16.26	12.16	16.26
Variance, σ^2	315.51	176.63	315.55

i) $\bar{x} - t_{\alpha/2,n-1}s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2,n-1}s\sqrt{1+\frac{1}{n}}$

$$25.12 - 3.355(8.42)\sqrt{1+\frac{1}{9}} \le X_{n+1} \le 25.12 + 3.355(8.42)\sqrt{1+\frac{1}{9}}$$

-4.657, 54.897

j) $\bar{x} - ks, \bar{x} + ks$, $k = 4.550$

25.12 – 8.42(4.550), 25.12 + 8.42(4.550)

-13.191, 63.431

4-80. With $\sigma = 8$, the 95% confidence interval on the mean has length of at most 5; the error is then E = 2.5.

a) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 8^2 = \left(\frac{1.96}{2.5}\right)^2 64 = 39.34 = 40$

b) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 6^2 = \left(\frac{1.96}{2.5}\right)^2 36 = 22.13 = 23$

As the standard deviation decreases, with all other values held constant, the sample size necessary to maintain the acceptable level of significance and the length of the interval, decreases.

c) We would want to have a relatively large sample size, $n \geq 30$. With a sample size of at least 30, the central limit theorem can apply.

4-81. Sample Mean = $\hat{p}$ Sample Variance = $\frac{\hat{p}(1-\hat{p})}{n}$

	Sample Size, n	Sampling Distribution	Sample Mean	Sample Variance
a.	60	Normal	$\hat{p}$	$\frac{\hat{p}(1-\hat{p})}{60}$
b.	70	Normal	$\hat{p}$	$\frac{\hat{p}(1-\hat{p})}{70}$
c.	100	Normal	$\hat{p}$	$\frac{\hat{p}(1-\hat{p})}{100}$

d) As the sample size increases, the variance of the sampling distribution deccreases.

4-82. $z_{\alpha/2} = 1.96$ $E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

a) $n = 60$ $\hat{p} = 0.1$

$$E = 1.96\sqrt{\frac{0.1(1-0.1)}{60}} = 0.0759$$

b) $n = 70$ $\hat{p} = 0.1$

$$E = 1.96\sqrt{\frac{0.1(1-0.1)}{70}} = 0.0702$$

c) $n = 100$ $\hat{p} = 0.1$

$$E = 1.96\sqrt{\frac{0.1(1-0.1)}{100}} = 0.0588$$

d) As the sample size increases and all other values held constant, the error decreases.

e) $z_{\alpha/2} = 2.575$ $E = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$n = 60$

$$E = 2.575\sqrt{\frac{0.1(1-0.1)}{60}} = 0.0997$$

$n = 70$

$$E = 2.575\sqrt{\frac{0.1(1-0.1)}{70}} = 0.0923$$

$n = 100$

$$E = 2.575\sqrt{\frac{0.1(1-0.1)}{100}} = 0.077$$

As the sample size increase and all other values held constant, the error decreases.

f) When the confidence level is increased, the error in estimating the true value of p will also increase. This can be seen by comparing values between parts d and e. For n = 60 we see that E = 0.0759 with a 95% level of confidence, while E = 0.0997 with a 99% level of confidence for the same sample size.

4-83. $\sigma = 12$, $\delta = 205 - 200 = 5$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$, using eq. (4-24)

a) n = 25: $\beta = \Phi\left(1.96 - \frac{5\sqrt{25}}{12}\right) - \Phi\left(-1.96 - \frac{5\sqrt{25}}{12}\right) = \Phi(-0.12) - \Phi(-4.04) = 0.4522 - 0.00003 = 0.452$

b) n = 60: $\beta = \Phi\left(1.96 - \frac{5\sqrt{60}}{12}\right) - \Phi\left(-1.96 - \frac{5\sqrt{60}}{12}\right) = \Phi(-1.27) - \Phi(-5.19) = 0.102 - 0 = 0.102$

c) n = 100: $\beta = \Phi\left(1.96 - \frac{5\sqrt{100}}{12}\right) - \Phi\left(-1.96 - \frac{5\sqrt{100}}{12}\right) = \Phi(-2.21) - \Phi(-6.13) = 0.01355 - 0 = 0.014$

d) β, which is the probability of a Type II error, decreases as the sample size increases because the variance of the sample mean decreases. Consequently, the probability of observing a sample mean in the acceptance region centered about the incorrect value of 200 ml/h decreases with larger n.

4-84. $\sigma = 14$, $\delta = 205 - 200 = 5$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$, using eq. (4-24)

a) n = 20: $\beta = \Phi\left(1.96 - \frac{5\sqrt{20}}{14}\right) - \Phi\left(-1.96 - \frac{5\sqrt{20}}{14}\right) = \Phi(0.36) - \Phi(-3.56) = 0.6406 - 0.0002 = 0.6404$

b) n = 50: $\beta = \Phi\left(1.96 - \frac{5\sqrt{50}}{14}\right) - \Phi\left(-1.96 - \frac{5\sqrt{50}}{14}\right) = \Phi(-0.565) - \Phi(-4.49) = 0.2860 - 0 = 0.2860$

c) n = 100: $\beta = \Phi\left(1.96 - \frac{5\sqrt{100}}{14}\right) - \Phi\left(-1.96 - \frac{5\sqrt{100}}{14}\right) = \Phi(-1.61) - \Phi(-5.53) = 0.0537 - 0 = 0.054$

d) The probability of a Type II error increases with an increase of the standard deviation.

4-85. a) 1) The parameter of interest is the true mean life of a heating element, μ.

2) H_0: $\mu = 550$

3) H_1: $\mu > 550$

4) $\alpha = 0.05$

5) $t_0 = \frac{\overline{x} - \mu}{s / \sqrt{n}}$

6) Reject H_0 if $t > t_{\alpha/2,n-1}$ where $t_{0.05,14} = 1.761$

7) $\overline{x} = 598.14$ $s = 16.93$ $n = 15$

$$t_0 = \frac{598.14 - 550}{16.93 / \sqrt{15}} = 11.01$$

8) Since 11.01 > 1.761, reject H_0 and conclude the true mean life of a heating element is greater than 550 hours.

b) P-value = $P(t > 11.01)$: for degrees of freedom of 14 we obtain P-value < 0.0005

c) $598.14 - 1.645\frac{16.93}{\sqrt{15}} \le \mu$

$590.95 \le \mu$

d) $\frac{14(16.93)^2}{26.12} \le \sigma^2 \le \frac{14(16.93)^2}{5.63}$

$153.63 \le \sigma^2 \le 712.74$

4-86. H_0: $\mu = 85$ $\sigma = 16$ the true mean is 86

a) $\beta = P(\overline{x} \le 85 \text{ when } \mu = 86) = P\left(\frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \le \frac{85 - 86}{16 / \sqrt{n}}\right) = P(z \le -0.0625\sqrt{n})$

n = 25: $\beta = P(z \le -0.0625\sqrt{25}) = 0.3783$

n = 100: $\beta = P(z \le -0.0625\sqrt{100}) = 0.2643$

n = 400: $\beta = P(z \le -0.0625\sqrt{400}) = 0.1056$

n = 2500: $\beta = P(z \le -0.0625\sqrt{2500}) = 0.0009$

b) P-value = $P(\bar{x} \ge 86) = P\left(\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \ge \frac{86-85}{16/\sqrt{n}}\right) = P(z \ge 0.0625\sqrt{n})$

n = 25: P-value = $P(z \ge 0.0625\sqrt{25}) = 0.3783$

n = 100: P-value = $P(z \ge 0.0625\sqrt{100}) = 0.2643$

n = 400: P-value = $P(z \ge 0.0625\sqrt{400}) = 0.1056$

n = 2500: P-value = $P(z \ge 0.0625\sqrt{2500}) = 0.0009$; only sample that is statistically significant at $\alpha = 0.01$.

c) As the sample size increases, the probability of committing an error, Type I or Type II, decreases.

4-87. a) Rejecting a null hypothesis provides *stronger evidence* than not rejecting a null hypothesis. Therefore, place what we are trying to prove in the alternative hypothesis.

Assume the data follow a normal distribution.

b) 1) the parameter of interest is the mean weld strength, μ.
2) $H_0 : \mu = 150$
3) $H_1 : \mu > 150$
4) Not given
5) The test statistic is:

$$t_0 = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$

6) Since no critical value is given, we will calculate the P-value
7) $\bar{x} = 153.7$

$$t_0 = 1.33$$

P-value = $P(t \ge 1.33) = 0.05 < \text{p-value} < 0.10$

8) If we used $\alpha = 0.05$, we would not reject the null hypothesis, thus the claim would not be supported.

4-88. $H_0 : p = 0.5$ versus $H_1 : p \ne 0.5$

a) Find the Power, $1-\beta$ for 95% confidence

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{\frac{p_0(1-p_0)}{n}}}{\sqrt{\frac{p(1-p)}{n}}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$$= \Phi\left(\frac{0.5 - 0.6 + 1.96\sqrt{\frac{0.5(1-0.5)}{n}}}{\sqrt{\frac{0.6(1-0.6)}{n}}}\right) - \Phi\left(\frac{0.5 - 0.6 - 1.96\sqrt{\frac{0.6(1-0.6)}{n}}}{\sqrt{\frac{0.6(1-0.6)}{n}}}\right)$$

n = 100: $\beta = \Phi(-0.041) - \Phi(-4.04) = 0.48405 - 0 = 0.48405$

$1-\beta = 1-0.48405 = 0.516$

n = 150: $\beta = \Phi(-0.50) - \Phi(-4.5) = (1-0.69146) - 0 = 0.30856$

$1-\beta = 1-0.30856 = 0.691$

n = 300: $\beta = \Phi(-1.54) - \Phi(-5.5) = (1-0.93822) - 0 = 0.06178$
$1-\beta = 1-0.06178 = 0.938$

Power increases as sample size increases, when all other values are held constant.

b) Find the Power, $1-\beta$ for 99% confidence

n = 100: $\beta = \Phi(0.59) - \Phi(-4.67) = 0.7224 - 0 = 0.7224$
$1-\beta = 1-0.7224 = 0.278$

n = 150: $\beta = \Phi(0.13) - \Phi(-5.12) = 0.55172 - 0 = 0.55172$
$1-\beta = 1-0.55172 = 0.448$

n = 300: $\beta = \Phi(-0.91) - \Phi(-6.16) = 1-0.81859 - 0 = 0.18141$
$1-\beta = 1-0.18141 = 0.819$

The power of the test is greater for the larger values of α, since the larger α value results in a smaller acceptance region.

c) n = 100 p = 0.8 α = 0.05

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{\frac{p_0(1-p_0)}{n}}}{\sqrt{\frac{p(1-p)}{n}}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}}{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$$= \Phi\left(\frac{.05 - 0.8 + 1.96\sqrt{\frac{0.5(1-0.5)}{n}}}{\sqrt{\frac{0.8(1-0.8)}{n}}}\right) - \Phi\left(\frac{0.5 - 0.8 - 1.96\sqrt{\frac{0.8(1-0.8)}{n}}}{\sqrt{\frac{0.8(1-0.8)}{n}}}\right)$$

$$= \Phi(-5.05) - \Phi(-9.95) = 0$$

Power = 1 – 0 = 1.

As the difference between the hypothesized value of p and the true value of p increases, the more powerful the test.

d) $\alpha = 0.01$, $\beta = 0.05$, $z_{\alpha/2} = 2.575$, $z_\beta = 1.645$

p = 0.6:

$$n = \left(\frac{2.575(\sqrt{0.5(0.5)}) + 1.645(\sqrt{0.6(0.4)})}{0.5-0.6}\right)^2 = 438.2$$

n = 439

p = 0.8:

$$n = \left(\frac{2.575(\sqrt{0.5(0.5)}) + 1.645(\sqrt{0.8(0.2)})}{0.5-0.8}\right)^2 = 48.69$$

n = 49

A smaller sample size is required when the true proportion lies further from the hypothesized value, since not as large of a sample will be necessary to detect this difference.

4-89. a) 1) the parameter of interest is the standard deviation, σ

2) H_0: $\sigma^2 = 78400$
3) H_1: $\sigma^2 < 78400$
4) Not given

5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) Since no critical value is given, we will calculate the p-value
7) n = 15, s = 226.1

$$\chi_0^2 = \frac{14(226.1)^2}{78400} = 9.129$$

P-value = $P(\chi^2 < 9.129)$; $0.50 < \text{P-value} < 0.90$

8) The P-value is greater than any acceptable significance level, α, therefore we do not reject the null hypothesis. There is insufficient evidence to support the claim that the standard deviation is less than 280 hours.

b) 7) n = 51, s = 226.1

$$\chi_0^2 = \frac{50(226.1)^2}{78400} = 32.603$$

P-value = $P(\chi^2 < 32.603)$; $0.025 < \text{P-value} < 0.05$

8) The P-value is less than 0.05, therefore we can reject the null hypothesis and conclude that the standard deviation is significantly less than 280 hours.

c) Increasing the sample size increases the test statistic χ_0^2 and therefore decreases the P-value, providing more evidence against the null hypothesis.

4-90. n = 6

a) 1) The parameter of interest is the standard deviation, σ.
2) H_0: $\sigma^2 = 1.0$
3) H_1: $\sigma^2 \neq 1.0$
4) $\alpha = 0.01$

5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha/2,n-1}^2$ where $\chi_{0.995,5}^2 = 0.41$ or $\chi_0^2 > \chi_{\alpha/2,n-1}^2$ where $\chi_{0.005,5}^2 = 16.75$

7) n = 6, s = 0.319

$$\chi_0^2 = \frac{5(0.319)^2}{1.0} = 0.5088$$

8) Since 0.41 < 0.5088 < 16.75, do not reject the null hypothesis and conclude the true variance of fatty acid for diet margarine is not significantly different from 1.0 at $\alpha = 0.05$.

b) n = 51
2) H_0 : $\sigma^2 = 1.0$
3) H_1 : $\sigma^2 \neq 1.0$
4) $\alpha = 0.01$

5) The test statistic is: $\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha/2,n-1}^2$ where $\chi_{0.995,50}^2 = 27.99$ or $\chi_0^2 > \chi_{\alpha/2,n-1}^2$ where $\chi_{0.005,50}^2 = 79.49$

7) n = 51, s = 0.319

$$\chi_0^2 = \frac{50(0.319)^2}{1.0} = 5.09$$

8) Since 5.09 < 27.99, reject the null hypothesis and conclude the true variance of fatty acid for diet margarine is significantly different from 1.0 at $\alpha = 0.05$.

c) The increased sample size changes the degrees of freedom of the test statistic and therefore the acceptance region; the larger sample size actually decreases the acceptable difference between the sample variance and actual.

4-91. Assume the data follow a normal distribution.

a) 1) The parameter of interest is the standard deviation, σ.
2) $H_0 : \sigma^2 = (0.00002)^2$
3) $H_1 : \sigma^2 < (0.00002)^2$
4) $\alpha = 0.01$

5) The test statistic is: $$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$$

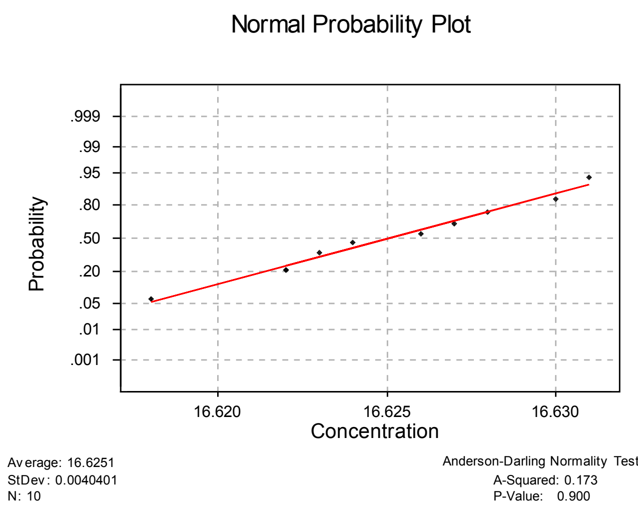
6) $\chi^2_{0.99,7} = 1.24$ reject H_0 if $\chi_0^2 < 1.24$
7) s = 0.00001 and $\alpha = 0.01$

$$\chi_0^2 = \frac{7(0.00001)^2}{(0.00002)^2} = 1.75$$

1.75 > 1.24, do not reject the null hypothesis; that is, there is insufficient evidence to conclude the standard deviation is at most 0.00002 mm.

b) Although the sample standard deviation is less than the hypothesized value of 0.00002, it is *not significantly less* (when $\alpha = 0.01$) than 0.00002 to conclude the standard deviation is at most 0.00002 mm. The value of 0.00001 could have occurred as a result of sampling variation.

4-92.



According to the normal probability plot, we can assume the underlying distribution is normal. This is evident by the fact that the data fall along a straight line. The assumption of normality should be satisfied in order to perform a hypothesis test using a χ^2 test statistic.

1) The parameter of interest is the standard deviation, σ.
2) $H_0 : \sigma^2 = 16$
3) $H_1 : \sigma^2 < 16$
4) Not given

5) The test statistic is: $$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2}$$

6) Since no critical value is given, we will calculate the p-value

7) n = 10

$s^2 = 0.004$

$$\chi_0^2 = \frac{9(0.004)^2}{16} = 0.000009$$

P-value = $P\left(\chi^2 \le 0.000009\right)$; P-value < 0.005

8) A P-value of less than 0.005 is highly significant evidence to conclude the standard deviation is less than 4 g/l.

4-93. a) 1) The parameter of interest is the true proportion, p.

2) H_0: p = 0.01

3) H_1: p < 0.01

4) $\alpha = 0.01$

5) The test statistic is: $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$

6) $-z_{0.01} = -2.33$, reject H_0 if $z_0 < -2.33$

7) $\hat{p} = \dfrac{8}{1200} = 0.0067$

$$z_0 = \frac{0.0067 - 0.01}{\sqrt{\dfrac{0.01(1-0.01)}{1200}}} = -1.15$$

8) $-1.15 > -2.33$, do not reject the null hypothesis and conclude that there is insufficient evidence to support the claim that the true proportion is less than 1%.

P-value = $P(z \le -1.15) = 0.1251$

b) Although the sample proportion is less than the hypothesized value of 0.01, it is *not significantly less* than 0.01 to conclude that the proportion is at less than 0.01. The value of 0.0067 could have occurred as a result of sampling variation.

4-94. $\hat{p} = \dfrac{8}{1600} = 0.005$

a) 99% confidence interval: $z_{\alpha/2} = 2.575$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$0.00045 \le p \le 0.0095$

With 99% confidence, we believe the true proportion of aircraft that have wiring errors lies between 0.00045 and 0.0095.

b) $n = \left(\dfrac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\dfrac{2.575}{0.008}\right)^2 (0.005)(1-0.005) = 515.427$

n = 516

c) Use $\hat{p} = 0.5$

$n = \left(\dfrac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\dfrac{2.575}{0.008}\right)^2 (0.5)(1-0.5) = 25{,}900$

n = 25,900

d) Preliminary information can significantly decrease the computed needed sample size. Without this information, we must use p = 0.5 in the computations resulting the worst case size.

4-95. $\hat{p} = \frac{64}{100} = 0.64$

a) 90% confidence interval; $z_{\alpha/2} = 1.645$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.554 \le p \le 0.720$$

With 90% confidence, we believe the true proportion of graduates completing the test within 40 minutes lies between 0.554 and 0.720.

b) 95% confidence interval; $z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.538 \le p \le 0.734$$

With 95% confidence, we believe the true proportion of graduates completing the test within 40 minutes lies between 0.538 and 0.734.

c) Comparison of parts a and b:

The 95% confidence interval is larger than the 90% confidence interval. Higher confidence always yields larger intervals, all other values held constant.

d) Yes, since neither interval contain the value 0.75, thus the true proportion cannot be considered significantly different from 0.75.

4-96. $X \sim bin(15, 0.4)$ H_0: $p = 0.4$ and H_1: $p \neq 0.4$

$4/15 = 0.267$ $8/15 = 0.533$

Accept Region: $0.267 \le \hat{p} \le 0.533$

Reject Region: $\hat{p} \le 0.267$ or $\hat{p} > 0.533$

Use the normal approximation for parts a) and b)

a) When $p = 0.4$, $\alpha = P(\hat{p} \le 0.267) + P(\hat{p} > 0.533)$

$$= P\left(Z \le \frac{0.267-0.4}{\sqrt{\frac{0.4(0.6)}{15}}}\right) + P\left(Z > \frac{0.533-0.4}{\sqrt{\frac{0.4(0.6)}{15}}}\right)$$

$$= P(Z \le -1.05) + P(Z > 1.05)$$

$$= P(Z \le -1.05) + (1 - P(Z < 1.05))$$

$$= 0.14686 + 0.14686$$

$$= 0.2937$$

b) When $p = 0.2$, $\beta = P(0.267 < \hat{p} \le 0.533) = P\left(\frac{0.267-0.2}{\sqrt{\frac{0.2(0.8)}{15}}} < Z \le \frac{0.533-0.2}{\sqrt{\frac{0.2(0.8)}{15}}}\right)$

$$= P(0.65 < Z \le 3.22)$$

$$= P(Z \le 3.22) - P(Z < 0.65)$$

$$= 0.99936 - 0.74215$$

$$= 0.2572$$

4-97. $X \sim bin(10, 0.3)$ Implicitly, H_0: $p = 0.3$ and H_1: $p < 0.3$

$n = 20$

Accept region: $\hat{p} > 0.1$
Reject region: $\hat{p} \le 0.1$

Use the normal approximation for parts a), b) and c):

a) When $p = 0.3$ $\alpha = P(\hat{p} < 0.1) = P\left(Z \le \frac{0.1 - 0.3}{\sqrt{\frac{0.3(0.7)}{20}}}\right)$

$$= P(Z \le -1.95)$$
$$= 0.0256$$

b) When $p = 0.2$ $\beta = 1 - P(\hat{p} > 0.1) = 1 - P\left(Z > \frac{0.1 - 0.2}{\sqrt{\frac{0.2(0.8)}{20}}}\right)$

$$= 1 - P(Z > -1.12)$$
$$= P(Z < -1.12)$$
$$= 0.1314$$

c) Power $= 1 - \beta = 1 - 0.1314 = 0.8686$

4-98. [Note: This problem refers to Exercise 2-48, not Exercise 2-45 as given in the textbook]. Create a table for the number of nonconforming coil springs (value) and the observed number of times the number appeared. One possible table is:

Value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Obs	0	0	0	1	4	3	4	6	4	3	0	3	3	2	1	1	0	2	1	2

The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(0) + 1(0) + 2(0) + \cdots + 19(2)}{40} = 9.325$$

$$\hat{p}_{sample} = \frac{\text{sample mean}}{n} = \frac{9.325}{50} = 0.1865$$

Value	Observed	Expected
0	0	0.00132
1	0	0.01511
2	0	0.08486
3	1	0.31128
4	4	0.83853
5	3	1.76859
6	4	3.04094
7	6	4.38212
8	4	5.39988
9	3	5.77713
10	0	5.43022
11	3	4.52695

12	3	3.37296
13	2	2.26033
14	1	1.36952
15	1	0.75353
16	0	0.37789
17	2	0.17327
18	1	0.07283
19	2	0.02812

Since several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	Observed	Expected
0-5	8	3.01969
6	4	3.04094
7	6	4.38212
8	4	5.39988
9	3	5.77713
10	0	5.43022
11	3	4.52695
12	3	3.37296
≥13	9	5.03548

The degrees of freedom are $k - p - 1 = 9 - 1 - 1 = 7$

a) 1) The variable of interest is the form of the distribution for the number of nonconforming coil springs.

2) H_0: The form of the distribution is binomial

3) H_1: The form of the distribution is not binomial

4) $\alpha = 0.05$

5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_0^2 > \chi_{0.05,7}^2 = 14.07$

7)

$$\chi_0^2 = 19.9195$$

8) Since 19.9195 > 14.07 reject H_0. We are able to conclude the distribution of nonconforming springs is not binomial at $\alpha = 0.05$.

b) P-value = 0.0057 (found using Minitab)

4-99. [Note: This problem refers to Exercise 2-49, not Exercise 2-46 as given in the textbook]. Create a table for the number of errors in a string of 1000 bits (value) and the observed number of times the number appeared. One possible table is:

Value	0	1	2	3	4
Obs	3	7	4	5	1

The value of p must be estimated. Let the estimate be denoted by $\hat{p}_{sample}$

$$\text{sample mean} = \frac{0(3) + 1(7) + 2(4) + 3(5) + 4(1) + 5(0)}{20} = 1.7$$

$$\hat{p}_{sample} = \frac{\text{sample mean}}{n} = \frac{1.7}{1000} = 0.0017$$

Value	0	1	2	3	4	5
Observed	3	7	4	5	1	0
Expected	3.64839	6.21282	5.28460	2.99371	1.27067	0.43103

Since several of the expected values are less than 3, some cells must be combined resulting in the following table:

Value	0	1	2	≥3
Observed	3	7	4	6
Expected	3.64839	6.21282	5.28460	4.69541

The degrees of freedom are $k - p - 1 = 4 - 1 - 1 = 2$

a) 1) The variable of interest is the form of the distribution for the number of errors in a string of 1000 bits.
2) H_0: The form of the distribution is binomial
3) H_1: The form of the distribution is not binomial
4) $\alpha = 0.05$
5) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

6) Reject H_0 if $\chi_0^2 > \chi_{0.05,2}^2 = 5.99$
7)

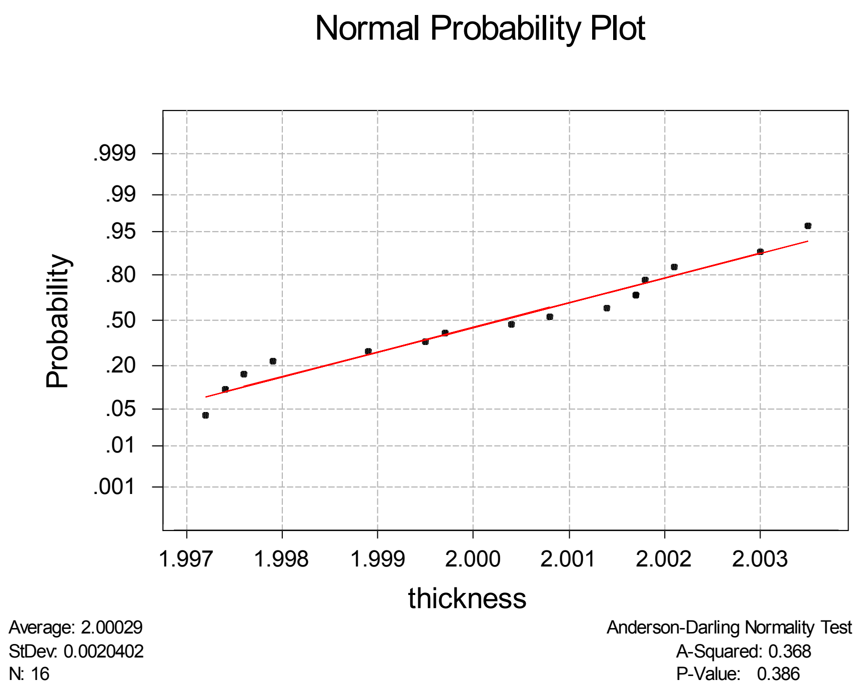
$$\chi_0^2 = \frac{(3 - 3.64839)^2}{3.64839} + \cdots + \frac{(6 - 4.69541)^2}{4.69541} = \ 0.88971$$

8) Since 0.88971 < 9.49 do not reject H_0. We are unable to reject the null hypothesis that the distribution of the number of errors is binomial at $\alpha = 0.05$.

b) P-value = 0.641 (found using Minitab)

4-100. a) H_0: $\mu = 5000$, H_1: $\mu > 5000$ Upper-tailed
b) H_0: $\mu = 60{,}000$, H_1: $\mu > 60{,}000$ Upper-tailed
c) H_0: $\sigma = 2$, H_1: $\sigma < 2$, Lower-tailed
d) H_0: $p = 0.60$, H_1: $p > 0.60$ Upper-tailed
e) H_0: $\mu = 42{,}000$, H_1: $\mu > 42{,}000$ Upper-tailed
f) H_0: $\sigma = 0.02$, H_1: $\sigma < 0.02$, Lower-tailed
g) H_0: $\sigma^2 = 0.05$, H_1: $\sigma^2 < 0.05$, Lower-tailed

4-101. a)



The data appear to be normally distributed.

b) Using Minitab, the results are

```
Test of mu = 2.001 vs mu not = 2.001

Variable            N       Mean     StDev   SE Mean
C1                 16    2.00029   0.00204   0.00051

Variable                 95.0% CI            T      P
C1              ( 1.99920, 2.00137)     -1.40  0.183
```

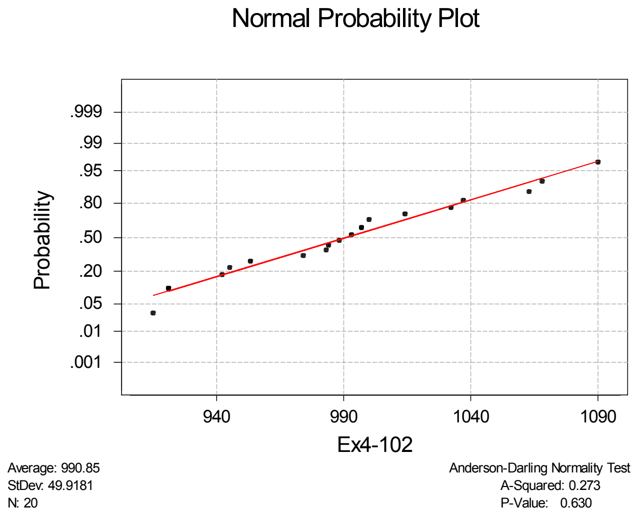
Based on the t-test, we fail to reject the null hypothesis and concluded that the mean could be 2.001 mm.

c) Using Minitab, the results are

```
Test of p = 0.1 vs p > 0.1

Sample       X      N  Sample p  95.0% Lower Bound  Z-Value  P-Value
1            2     16  0.125000           0.000000     0.33    0.369
```

Based on this result, we would not reject the null hypothesis. There is no evidence that the proportion exceeds 0.10.

4-102. a)



The data appear to be normally distributed.

b) Using Minitab, the results are

```
Test of mu = 1000 vs mu < 1000

Variable            N      Mean     StDev   SE Mean
Ex4-102            20     990.9      49.9      11.2

Variable       95.0% Upper Bound         T       P
Ex4-102                   1010.2     -0.82   0.211
```

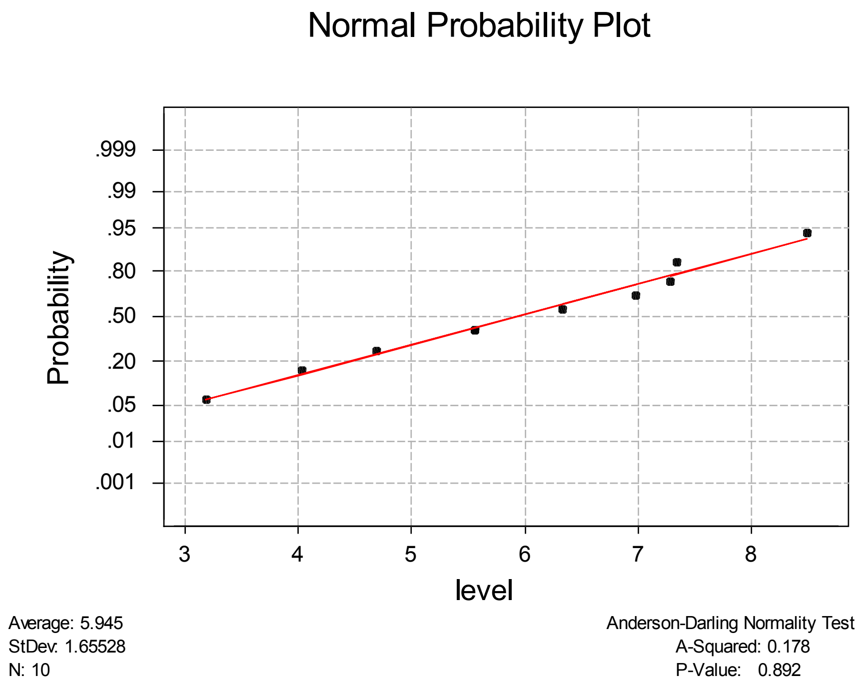
Based on the t-test, we fail to reject the null hypothesis and concluded that the mean could be less than 1000 hrs.

c) Using Minitab, the results are

```
Test of p = 0.2 vs p < 0.2

Success = 1

Variable      X    N    Sample p    95.0% Upper Bound  Z-Value  P-Value
Ex4-102c      3   20    0.150000             0.281331    -0.56    0.288

* NOTE * The normal approximation may be inaccurate for small samples.
```

Based on this result, we would not reject the null hypothesis. There is no evidence that the proportion less than 0.20.

4-103. a)



The data appear to be normally distributed.

b) 1) The parameter of interest is the standard deviation of CO level, σ.

2) H_0: $\sigma = 2.0$

3) H_1: $\sigma > 2.0$

4) $\alpha = 0.05$

5) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

6) Reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$ where $\chi_{0.05,9}^2 = 16.919$

7) $n = 10$, $s^2 = (1.6553)^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9(1.6553)^2}{4.0} = 6.165$$

8) Since $6.165 < 16.919$ do not reject H_0 and conclude there is no evidence to indicate the true standard deviation of CO level exceeds 2.0 ppm at $\alpha = 0.05$.