CHAPTER 5

Section 5-2

5-1. a) 1) The parameter of interest is the difference in fill volume, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 16.015 \quad \bar{x}_2 = 16.005 \quad \delta = 0$

$\sigma_1 = 0.02 \quad \sigma_2 = 0.025$

$n_1 = 10 \quad n_2 = 10$

$$z_0 = \frac{(16.015 - 16.005) - 0}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}} = 0.99$$

8) since -1.96 < 0.99 < 1.96, do not reject the null hypothesis and conclude there is no evidence that the two machine fill volumes differ at $\alpha = 0.05$.

b) P-value $= 2(1 - \Phi(0.99)) = 2(1 - 0.8389) = 0.3222$

c) Power $= 1 - \beta$, where

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}}\right) - \Phi\left(-1.96 - \frac{0.04}{\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}}\right)$$

$$= \Phi(1.96 - 3.95) - \Phi(-1.96 - 3.95) = \Phi(-1.99) - \Phi(-5.91)$$

$= 0.0233 - 0$

$= 0.0233$

Power $= 1 - 0.0233 = 0.977$

d) $$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) - 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}} \leq \mu_1 - \mu_2 \leq (16.015 - 16.005) + 1.96\sqrt{\frac{(0.02)^2}{10} + \frac{(0.025)^2}{10}}$$

$$-0.0098 \leq \mu_1 - \mu_2 \leq 0.0298$$

With 95% confidence, we believe the true difference in the mean fill volumes is between –0.0098 and 0.0298. Since 0 is contained in this interval, we can conclude there is no significant difference between the means.

e) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.01$, and $\Delta = 0.04$

$$n \cong \frac{(z_{\alpha/2}+z_\beta)^2(\sigma_1^2+\sigma_2^2)}{\delta^2} = \frac{(1.96+2.33)^2((0.02)^2+(0.025)^2)}{(0.04)^2} = 2.08, \quad n = 11.79, \text{ use } n_1 = n_2 = 12$$

5-2. 1) The parameter of interest is the difference in breaking strengths, $\mu_1 - \mu_2$ and $\Delta_0 = 10$

2) $H_0: \mu_1 - \mu_2 = 10$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 10$ or $\mu_1 > \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 > z_\alpha = 1.645$

7) $\bar{x}_1 = 162.7$ $\bar{x}_2 = 155.4$ $\delta = 10$

$\sigma_1 = 1.0$ $\sigma_2 = 1.0$

$n_1 = 10$ $n_2 = 12$

$$z_0 = \frac{(162.7 - 155.4) - 10}{\sqrt{\frac{(1.0)^2}{10} + \frac{(1.0)^2}{12}}} = -6.31$$

8) Since $-6.31 < 1.645$ do not reject the null hypothesis and conclude there is insufficient evidence to support the use of plastic 1 at $\alpha = 0.05$.

5-3. a) 1) The parameter of interest is the difference in mean burning rate, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 18.02$ $\bar{x}_2 = 24.37$ $\delta = 0$

$\sigma_1 = 3$ $\sigma_2 = 3$

$n_1 = 20$ $n_2 = 20$

$$z_0 = \frac{(18.02 - 24.37) - 0}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}} = -6.70$$

8) Since $-6.70 < -1.96$ reject the null hypothesis and conclude the mean burning rates do not differ significantly at $\alpha = 0.05$.

b) P-value $= 2(\Phi(-6.70)) = 2(0) = 0$

c) $$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}}\right) - \Phi\left(-1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}}\right)$$

$$= \Phi(1.96 - 2.64) - \Phi(-1.96 - 2.64) = \Phi(-0.68) - \Phi(-4.6)$$

$$= 0.2483 - 0$$

$$= 0.2483$$

d) $$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(18.02 - 24.37) - 1.96\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}} \le \mu_1 - \mu_2 \le (18.02 - 24.37) + 1.96\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}$$

$$-8.21 \le \mu_1 - \mu_2 \le -4.49$$

We are 95% confident that the mean burning rate for solid fuel propellant 2 exceeds that of propellant 1 by between 4.49 and 8.21 cm/s.

5-4. $\bar{x}_1 = 30.61$ $\bar{x}_2 = 30.34$

$\sigma_1 = 0.10$ $\sigma_2 = 0.15$

$n_1 = 12$ $n_2 = 10$

a) 90% two-sided confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.61 - 30.34) - 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.61 - 30.34) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.179 \le \mu_1 - \mu_2 \le 0.361$$

We are 90% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.179 and 0.361 fl. oz.

b) 95% two-sided confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.61 - 30.34) - 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.61 - 30.34) + 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.161 \le \mu_1 - \mu_2 \le 0.379$$

We are 95% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.161 and 0.379 fl. oz.

Comparison of parts a and b:

As the level of confidence increases, the interval width also increases (with all other values held constant).

c) 95% upper-sided confidence interval:

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (30.61 - 30.34) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$
$$\mu_1 - \mu_2 \le 0.361$$

With 95% confidence, we believe the fill volume for machine 1 exceeds the fill volume of machine 2 by no more than 0.361 fl. oz.

5-5. a) 1) The parameter of interest is the difference in mean fill volume, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 30.61$ $\bar{x}_2 = 30.34$ $\Delta_0 = 0$

$\sigma_1 = 0.10$ $\sigma_2 = 0.15$

$n_1 = 12$ $n_2 = 10$

$$z_0 = \frac{(30.61 - 30.34) - 0}{\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}} = 2.92$$

8) Since 2.92 > 1.96 reject the null hypothesis and conclude the mean fill volumes of machine 1 and machine 2 differ significantly at $\alpha = 0.05$.

b) P-value $= 2(1 - \Phi(2.92)) = 2(1 - 0.9982) = 0.0036$

c) Assume the sample sizes are to be equal, use $\alpha = 0.05$, $\beta = 0.10$, and $\Delta = 0.20$

$$n \cong \frac{(z_{\alpha/2} + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2} = \frac{(1.96 + 1.28)^2((0.10)^2 + (0.15)^2)}{(-0.20)^2} = 8.53, \quad n = 9, \text{ use } n_1 = n_2 = 9$$

5-6. $\bar{x}_1 = 88.85$ $\bar{x}_2 = 92.54$

$\sigma_1^2 = 1.5$ $\sigma_2^2 = 1.2$

$n_1 = 15$ $n_2 = 20$

a) 95% confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
$$(88.85 - 92.54) - 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}} \le \mu_1 - \mu_2 \le (88.85 - 92.54) + 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}}$$
$$-4.474 \le \mu_1 - \mu_2 \le -2.906$$

With 95% confidence, we believe the mean road octane number for formulation 2 exceeds that of formulation 1 by between 2.906 and 4.474.

b) 1) The parameter of interest is the difference in mean road octane number, $\mu_1 - \mu_2$ and $\Delta_0 = 0$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha = -1.645$

7) $\bar{x}_1 = 88.85 \quad \bar{x}_2 = 92.54$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

$$z_0 = \frac{(88.85 - 92.54) - 0}{\sqrt{\frac{(1.5)}{15} + \frac{(1.2)}{20}}} = -9.225$$

8) Since –9.225 < -1.645 reject the null hypothesis and conclude the mean road octane number for formulation 2 exceeds that of formulation 1 using $\alpha = 0.05$.

c) P-value = $P(z \le -9.225) = 0$

5-7. 99% level of confidence, E = 4, and $z_{0.005} = 2.575$

$$n \cong \left(\frac{z_{0.005}}{E}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right) = \left(\frac{2.575}{4}\right)^2 (9+9) = 7.46,\ n = 8,\ \text{use } n_1 = n_2 = 8$$

5-8. 95% level of confidence, E = 1, and $z_{0.025}$ =1.96

$$n \cong \left(\frac{z_{0.025}}{E}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right) = \left(\frac{1.96}{1}\right)^2 (1.5+1.2) = 10.37,\ n = 11,\ \text{use } n_1 = n_2 = 11$$

5-9.

Case 1: Before Process Change	Case 2: After Process Change
μ_1 = mean batch viscosity before change	μ_2 = mean batch viscosity after change
$\bar{x}_1 = 750.2$	$\bar{x}_2 = 756.88$
$\sigma_1 = 20$	$\sigma_2 = 20$
$n_1 = 15$	$n_2 = 8$

90% confidence on $\mu_1 - \mu_2$, the difference in mean batch viscosity before and after process change:

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(750.2 - 756.88) - 1.645\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}} \le \mu_1 - \mu_2 \le \left(750.2 - 756/88\right) + 1.645\sqrt{\frac{(20)^2}{15} + \frac{(20)^2}{8}}$$

$$-21.08 \le \mu_1 - \mu_2 \le 7.72$$

We are 90% confident that the difference in mean batch viscosity before and after the process change lies within –21.08 and 7.72. Since 0 is contained in this interval we can conclude with 90% confidence that the mean batch viscosity was unaffected by the process change.

5-10.

Catalyst 1	Catalyst 2
$\bar{x}_1 = 63.56$	$\bar{x}_2 = 67.81$
$\sigma_1 = 3$	$\sigma_2 = 3$
$n_1 = 10$	$n_2 = 10$

a) 95% confidence interval on $\mu_1 - \mu_2$, the difference in mean active concentration

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(63.56-67.81)-1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}} \le \mu_1-\mu_2 \le (63.56-67.81)+1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}$$
$$-6.88 \le \mu_1-\mu_2 \le -1.62$$

b) Yes, since the 95% confidence interval did not contain the value 0, we would conclude the mean active concentration depends on the choice of catalyst.

5-11. a) 1) The parameter of interest is the difference in mean batch viscosity before and after the process change, $\mu_1-\mu_2$

2) H_0: $\mu_1-\mu_2 = 10$

3) H_1: $\mu_1-\mu_2 < 10$

4) $\alpha = 0.10$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1-\bar{x}_2)-\Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha$ where $z_{0.1} = -1.28$

7) $\bar{x}_1 = 750.2$ $\quad\bar{x}_2 = 756.88$ $\quad\Delta_0 = 10$

$\sigma_1 = 20$ $\quad\sigma_2 = 20$

$n_1 = 15$ $\quad n_2 = 8$

$$z_0 = \frac{(750.2-756.88)-10}{\sqrt{\frac{(20)^2}{15}+\frac{(20)^2}{8}}} = -1.90$$

8) Since $-1.90 < -1.28$ reject the null hypothesis and conclude the process change has increased the mean by less than 10.

b) P-value = $P(z \le -1.90) = 1-P(z \le 1.90) = 1-0.97128 = 0.02872$

c) Parts a and b above give evidence that the mean batch viscosity change is less than 10. This conclusion is also seen by the confidence interval of 5-9 since the interval did not contain the value 10. Since the upper endpoint is 7.72, then this also gives evidence that the difference is less than 10.

5-12. 1) The parameter of interest is the difference in mean active concentration, $\mu_1-\mu_2$

2) H_0 : $\mu_1-\mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1 : $\mu_1-\mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1-\bar{x}_2)-\Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 63.56$ $\quad\bar{x}_2 = 67.81$ $\quad\delta = 0$

$\sigma_1 = 3$ $\quad\sigma_2 = 3$

$n_1 = 10$ $\quad n_2 = 10$

$$z_0 = \frac{(63.56-67.81)-0}{\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}} = -3.17$$

8) Since $-3.17 < -1.96$ reject the null hypothesis and conclude the mean active concentrations do differ significantly at $\alpha = 0.05$.

P-value = $2(1-\Phi(3.17)) = 2(1-0.99924) = 0.00152$

The conclusions reached by the confidence interval of problem 5-10 and the test of hypothesis conducted here are the same. A two-sided confidence interval can be thought of as representing the "acceptance region" of a hypothesis test, given that the level of significance is the same for both procedures. Thus if the value δ falls outside the confidence interval, it is the same result as rejecting the null hypothesis.

Section 5-3

5-13. a) 1) The parameter of interest is the difference in mean rod diameter, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,30} = -2.042$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,30} = 2.042$

7)) $\bar{x}_1 = 8.73$ $\quad \bar{x}_2 = 8.68$ $\quad \Delta_0 = 0$

$s_1^2 = 0.35$ $\quad s_2^2 = 0.40$

$n_1 = 15$ $\quad n_2 = 17$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{14(0.35) + 16(0.40)}{30}} = 0.614$$

$$t_0 = \frac{(8.73 - 8.68) - 0}{0.614\sqrt{\frac{1}{15} + \frac{1}{17}}} = 0.230$$

8) Since $-2.042 < 0.230 < 2.042$, do not reject the null hypothesis and conclude the two machines do not produce rods with significantly different mean diameters at $\alpha = 0.05$.

b) P-value $= 2P\left(t > 0.230\right) > 2(0.40)$, P-value > 0.80

c) 95% confidence interval: $t_{0.025,30} = 2.042$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(8.73 - 8.68) - 2.042(0.614)\sqrt{\frac{1}{15} + \frac{1}{17}} \le \mu_1 - \mu_2 \le \left(8.73 - 8.68\right) + 2.042(0.614)\sqrt{\frac{1}{15} + \frac{1}{17}}$$

$$-0.394 \le \mu_1 - \mu_2 \le 0.494$$

Since zero is contained in this interval, we are 95% confident that machine 1 and machine 2 do not produce rods whose diameters are significantly different.

5-14. Assume the populations follow normal distributions and $\sigma_1^2 = \sigma_2^2$. The assumption of equal variances may be permitted in this case since it is known that the t-test and confidence intervals involving the t-distribution are robust to this assumption of equal variances when sample sizes are equal.

Case 1: AFCC	Case 2: ATC
μ_1 = mean foam expansion for AFCC	μ_2 = mean foam expansion for ATC
$\bar{x}_1 = 4.34$	$\bar{x}_2 = 7.091$
$s_1 = 0.508$	$s_2 = 0.430$
$n_1 = 5$	$n_2 = 5$

a) 1) The parameter of interest is the difference in mean foam expansion, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.10$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.05,8} = -1.86$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.05,8} = 1.86$

7) $\Delta_0 = 0$ $$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{4(0.508)^2 + 4(0.430)}{8}} = 0.4706$$

$$t_0 = \frac{(4.34 - 7.091) - 0}{0.4706\sqrt{\frac{1}{5} + \frac{1}{5}}} = -9.235$$

8) Since $-9.235 < -1.86$, reject the null hypothesis and conclude there is a statistically significant different in mean foam expansion between the two agents at $\alpha = 0.10$.

b) P-value = $2P(t < -9.235) \cong 0$

c) 90% confidence interval: $t_{0.05,8} = 1.86$ $$s_p = \sqrt{\frac{4(0.508)^2 + 4(0.430)^2}{8}} = 0.471$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(4.34 - 7.091) - 1.86(0.471)\sqrt{\frac{1}{5} + \frac{1}{5}} \le \mu_1 - \mu_2 \le (4.34 - 7.091) + 1.86(0.471)\sqrt{\frac{1}{5} + \frac{1}{5}}$$

$$-3.305 \le \mu_1 - \mu_2 \le -2.197$$

Yes, with 95% confidence, we believe the mean foam expansion for ATC exceeds that of AFCC by between 2.197 and 3.305.

5-15. a) 1) The parameter of interest is the difference in mean battery life, $\mu_A - \mu_B$

2) H_0: $\mu_A - \mu_B = 0$

3) H_1: $\mu_A - \mu_B > 0$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{\alpha,n1+n2-2}$ where $t_{0.01,22} = 2.508$

7) $\bar{x}_A = 36.51$ $\bar{x}_B = 34.21$ $\Delta_0 = 0$ $$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$s_A = 1.43$ $s_B = 0.93$ $$= \sqrt{\frac{11(1.43)^2 + 11(0.93)^2}{22}} = 1.206$$

$n_1 = 12$ $n_2 = 12$

$$t_0 = \frac{(36.51 - 34.21) - 0}{1.206\sqrt{\frac{1}{12} + \frac{1}{12}}} = 4.67$$

8) Since 4.67 > 2.508, reject the null hypothesis and conclude that the mean battery life of Type A significantly exceeds that of Type B at $\alpha = 0.01$.

b) P-value = $P(t > 4.67) < 0.0005$

c) 99% lower-side confidence bound: $t_{0.01,22} = 2.508$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_A - \mu_B$$

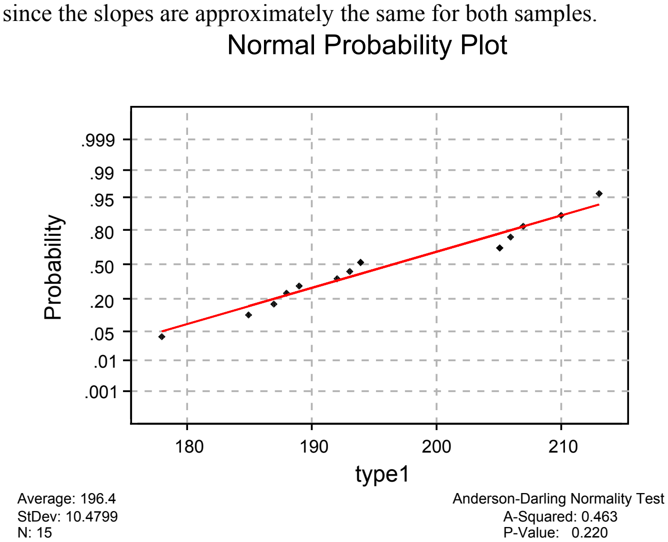
$$(36.51 - 34.21) - 2.508(1.206)\sqrt{\frac{1}{12}+\frac{1}{12}} \le \mu_A - \mu_B$$

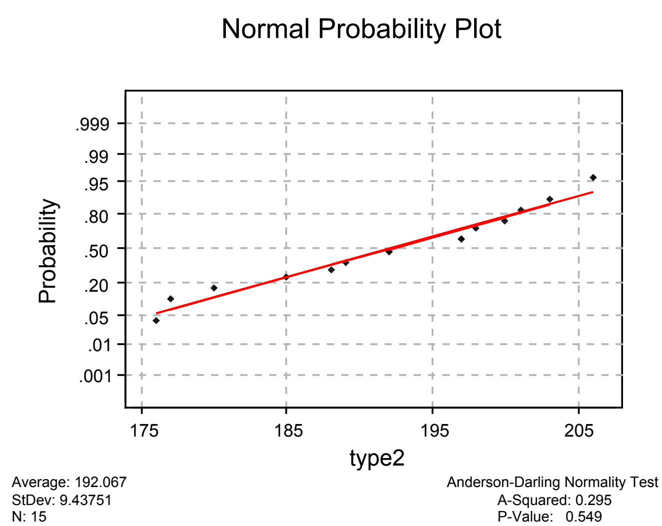
$$1.065 \le \mu_A - \mu_B$$

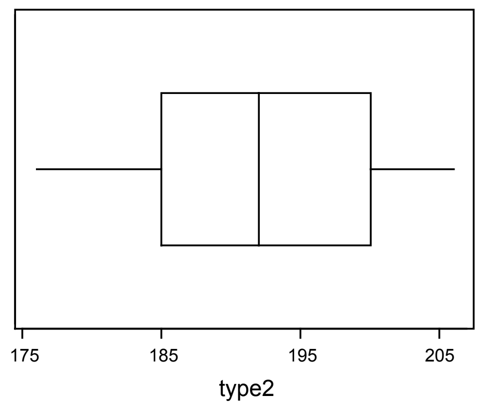
Since zero is not contained in this interval, we conclude that the null hypothesis can be rejected and the alternative accepted. The life of Type A is significantly longer than that of Type B.

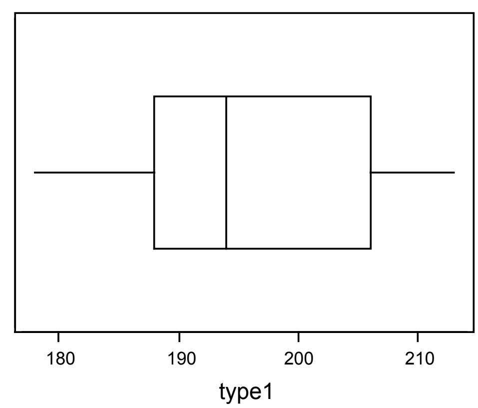
d) Yes, the sample sizes of 12 are adequate. From Minitab, the power of this test was found to 0.9889, which exceeds the required power of 0.95.

5-16. a) According to the normal probability plots, the assumption of normality appears to be met since the data fall approximately along a straight line. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.









b) 1) The parameter of interest is the difference in deflection temperature under load, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha, n_1+n_2-2}$ where $-t_{0.05,28} = -1.701$

7) Type 1 Type 2

$\overline{x}_1 = 196.4$ $\overline{x}_2 = 192.067$ $\Delta_0 = 0$

$s_1 = 10.48$ $s_2 = 9.44$

$n_1 = 15$ $n_2 = 15$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{14(10.48)^2 + 14(9.44)^2}{28}} = 9.97$$

$$t_0 = \frac{(196.4 - 192.067) - 0}{9.97\sqrt{\frac{1}{15} + \frac{1}{15}}} = 1.19$$

8) Since $1.19 > -1.701$ do not reject the null hypothesis and conclude the mean deflection temperature under load for type 2 does not significantly exceed the mean deflection temperature under load for type 1 at the 0.05 level of significance.

c) P-value = $2P\left(t > 1.19\right)$ $\quad 0.2 <$ p-value < 0.5

d) $\Delta = 5$ Use s_p as an estimate of σ:

$$d = \frac{\mu_2 - \mu_1}{2s_p} = \frac{5}{2(9.97)} = 0.251$$

Using Chart V g) with $\beta = 0.10$, $d = 0.251$ we get $n \cong 85$. So, $n_1 = n_2 = 85$; Therefore, the sample sizes of 15 are inadequate.

5-17. a) 1) The parameter of interest is the difference in mean etch rate, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2,n_1+n_2-2}$ where $-t_{0.025,18} = -2.101$ or $t_0 > t_{\alpha/2,n_1+n_2-2}$ where $t_{0.025,18} = 2.101$

7) $\overline{x}_1 = 9.97$ $\quad \overline{x}_2 = 10.4$ $\quad \Delta_0 = 0$

$s_1 = 0.422$ $\quad s_2 = 0.231$

$n_1 = 10$ $\quad n_2 = 10$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{10(0.422)^2 + 10(0.231)^2}{18}} = 0.340$$

$$t_0 = \frac{(9.97 - 10.4) - 0}{0.340\sqrt{\frac{1}{10} + \frac{1}{10}}} = -2.82$$

8) Since $-2.82 < -2.101$ reject the null hypothesis and conclude the two machines mean etch rates do significantly differ at $\alpha = 0.05$.

b) P-value = $2P\left(t < -2.82\right)$ $\quad 2(0.005) <$ P-value $< 2(0.010) = 0.010 <$ P-value < 0.020

c) 95% confidence interval: $t_{0.025,18} = 2.101$

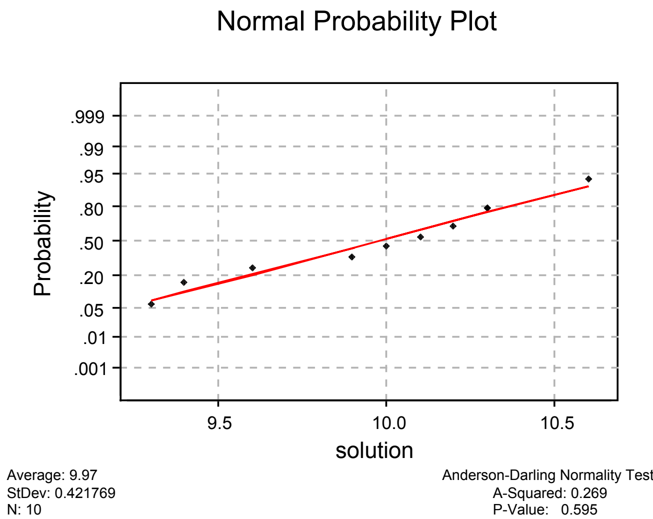
$$\left(\overline{x}_1 - \overline{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\overline{x}_1 - \overline{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

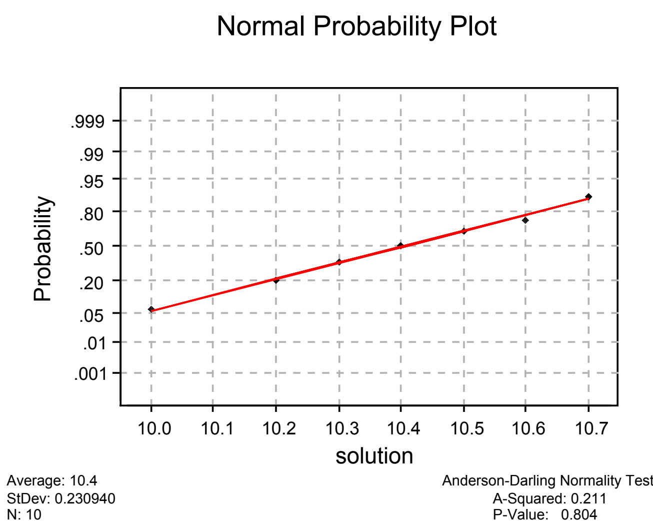
$$(9.97 - 10.4) - 2.101(0.34)\sqrt{\frac{1}{10} + \frac{1}{10}} \le \mu_1 - \mu_2 \le \left(9.97 - 10.4\right) + 2.101(0.34)\sqrt{\frac{1}{10} + \frac{1}{10}}$$

$$-0.749 \le \mu_1 - \mu_2 \le -0.111$$

We are 95% confident that the mean etch rate for solution 2 exceeds that for solution 1 by between 0.111 and 0.749.

d) According to the normal probability plots, the assumption of normality appears to be met since the data fall approximately along a straight line. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.





5-18. a) 1) The parameter of interest is the difference in mean impact strength, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,18} = -1.734$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 18.23$$

$\nu \cong 18$
(truncated)

7) $\bar{x}_1 = 289.30 \quad \bar{x}_2 = 321.50$

$s_1 = 22.5 \quad s_2 = 21$
$n_1 = 10 \quad n_2 = 16$

$$t_0 = \frac{(289.30 - 321.5) - 0}{\sqrt{\frac{(22.5)^2}{10} + \frac{(21)^2}{16}}} = -3.65$$

8) Since –3.65 < –1.734 reject the null hypothesis and conclude that supplier 2 provides gears with higher mean impact strength at the 0.05 level of significance.

b) P-value = P(t < –3.65): P-value < 0.0005

c) 1) The parameter of interest is the difference in mean impact strength, $\mu_2 - \mu_1$
2) $H_0 : \mu_2 - \mu_1 = 25$
3) $H_1 : \mu_2 - \mu_1 > 25$ or $\mu_2 > \mu_1 + 25$
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{(\bar{x}_2 - \bar{x}_1) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{\alpha,\nu} = 1.734$ where

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\frac{(22.5)^2}{10} + \frac{(21)^2}{16}\right)^2}{\frac{\left(\frac{(22.5)^2}{10}\right)^2}{10-1} + \frac{\left(\frac{(21)^2}{16}\right)^2}{16-1}} = 18.23$$

$\nu \cong 18$

7) $\bar{x}_1 = 289.30 \quad \bar{x}_2 = 321.5 \quad \Delta_0 = 25$

$s_1 = 22.5 \quad s_2 = 21$
$n_1 = 10 \quad n_2 = 16$

$$t_0 = \frac{(321.5 - 289.3) - 25}{\sqrt{\frac{(22.5)^2}{10} + \frac{(21)^2}{16}}} = 0.814$$

8) Since 0.814 < 1.734, do not reject the null hypothesis and conclude that the mean impact strength from supplier 2 is not at least 25 ft-lb higher that supplier 1 using $\alpha = 0.05$.

d) Construct a 95% lower confidence bound on $\mu_2 - \mu_1$

$$\left(\bar{x}_2 - \bar{x}_1\right) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_2 - \mu_1$$

$$\left(321.5 - 289.3\right) - 1.734\sqrt{\frac{(22.5)^2}{10} + \frac{(21)^2}{16}} \le \mu_2 - \mu_1$$

$16.87 \le \mu_2 - \mu_1$

since 25 > 16.87, we would not reject the null hypothesis and would reach the same conclusion as in part c.

5-19. a) 1) The parameter of interest is the difference in mean speed, $\mu_1 - \mu_2$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$

4) $\alpha = 0.10$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{\alpha, n_1+n_2-2}$ where $t_{0.10,14} = 1.345$

7) Case 1: 25 mil Case 2: 20 mil

$\overline{x}_1 = 1.179$ $\overline{x}_2 = 1.0362$ $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 0.088$ $s_2 = 0.093$ $= \sqrt{\frac{7(0.088)^2 + 7(0.093)^2}{14}} = 0.091$

$n_1 = 8$ $n_2 = 8$

$$t_0 = \frac{(1.179 - 1.0362) - 0}{0.091\sqrt{\frac{1}{8} + \frac{1}{8}}} = 3.14$$

8) Since 3.14 > 1.345 reject the null hypothesis and conclude reducing the film thickness from 25 mils to 20 mils significantly increases the mean speed of the film at the 0.10 level of significance (Note: since increase in film speed will result in *lower* values of observations).

b) P-value = $P(t > 3.14)$ $0.0025 < \text{P-value} < 0.005$

c) 95% confidence interval: $t_{0.025,14} = 2.145$

$$(\overline{x}_1 - \overline{x}_2) - t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le (\overline{x}_1 - \overline{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$(1.179 - 1.0362) - 2.145(0.091)\sqrt{\frac{1}{8} + \frac{1}{8}} \le \mu_1 - \mu_2 \le (1.179 - 1.0362) + 2.145(0.091)\sqrt{\frac{1}{8} + \frac{1}{8}}$$

$$0.045 \le \mu_1 - \mu_2 \le 0.240$$

We are 95% confident the mean speed of the film at 20 mil exceeds the mean speed for the film at 25 mil by between 0.045 and 0.240 $\mu J/in^2$.

5-20. a) 1) The parameter of interest is the difference in mean melting point, $\mu_1 - \mu_2$

2) H_0 : $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1 : $\mu_1 - \mu_2 \ne 0$ or $\mu_1 \ne \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025,40} = -2.021$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.025,40} = 2.021$

7) $\overline{x}_1 = 420.48$ $\overline{x}_2 = 425$, $\Delta_0 = 0$ $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$s_1 = 2.34 \qquad s_2 = 2.5 \qquad = \sqrt{\dfrac{20(2.34)^2 + 20(2.5)^2}{40}} = 2.42$

$n_1 = 21 \qquad n_2 = 21$

$$t_0 = \frac{(420.48 - 425) - 0}{2.42\sqrt{\frac{1}{20} + \frac{1}{20}}} = -5.99$$

8) Since –5.99 < –2.021 reject the null hypothesis and conclude that the data do not support the claim that both alloys have the same melting point at $\alpha = 0.05$

b) P-value = $2P\left(t < -5.424\right)$ P-value < 0.0010

5-21. $d = \dfrac{|\mu_1 - \mu_2|}{2\sigma} = \dfrac{3}{2(4)} = 0.375$

Using Chart V e) with $\beta = 0.10$ and $\alpha = 0.05$ we have: $n^* = 75$, so $n = \dfrac{n^* + 1}{2} = 38$,

$n_1 = n_2 = 38$

5-22. a) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,26}$ where $-t_{0.025,26} = -2.056$ or $t_0 > t_{0.025,26}$ where $t_{0.025,26} = 2.056$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(\frac{(1.9)^2}{25} + \frac{(7.9)^2}{25}\right)^2}{\frac{\left(\frac{(1.9)^2}{25}\right)^2}{24} + \frac{\left(\frac{(7.9)^2}{25}\right)^2}{24}} = 26.77$$

$\nu \cong 26$
(truncated)

7) $\overline{x}_1 = 20.12 \quad \overline{x}_2 = 11.64 \quad \Delta_0 = 0$

$s_1 = 1.9 \quad s_2 = 7.9$

$n_1 = 25 \quad n_2 = 25$

$$t_0 = \frac{(20.12 - 11.64) - 0}{\sqrt{\frac{(1.9)^2}{25} + \frac{(7.9)^2}{25}}} = 5.22$$

8) Since 5.22 > 2.056 reject the null hypothesis and conclude that the data support the claim that the two companies produce material with significantly different wear at the 0.05 level of significance.

b) P-value = 2P(t > 3.03), 2(0.0025) < P-value < 2(0.005)

0.005 < P-value < 0.010

c) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$
3) $H_1: \mu_1 - \mu_2 > 0$
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,26}$ where $t_{0.05,26} = 1.706$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 26.77$$

$$\nu \cong 26$$

7) $\bar{x}_1 = 20.12$ $\bar{x}_2 = 11.64$ $\Delta_0 = 0$

$s_1 = 1.9$ $s_2 = 7.9$
$n_1 = 25$ $n_2 = 25$

$$t_0 = \frac{(20.12 - 11.64) - 0}{\sqrt{\frac{(1.9)^2}{25} + \frac{(7.9)^2}{25}}} = 5.22$$

8) Since 5.22 > 1.706 reject the null hypothesis and conclude that the data support the claim that the material from company 1 has a higher mean wear than the material from company 2 using a 0.05 level of significance.

5-23. a) 1) The parameter of interest is the difference in mean coating thickness, $\mu_1 - \mu_2$.
2) $H_0: \mu_1 - \mu_2 = 0$
3) $H_1: \mu_1 - \mu_2 > 0$
4) $\alpha = 0.01$
5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{0.01,14}$ where $t_{0.01,14} = 2.624$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{1117.15}{81.03} = 13.787$$

$$\nu \cong 14$$

7) $\bar{x}_1 = 101.28$ $\bar{x}_2 = 101.7$

$s_1 = 5.08$ $s_2 = 20.1$
$n_1 = 11$ $n_2 = 13$

$$t_0 = \frac{(101.28 - 101.7) - 0}{\sqrt{\frac{(5.08)^2}{11} + \frac{(20.1)^2}{13}}} = -0.07$$

8) Since –0.07 < 2.539, do not reject the null hypothesis and conclude that increasing the temperature does not significantly reduce the mean coating thickness at $\alpha = 0.01$.

b) P-value = P(t > 0.602), 0.40 < P-value

5-24. 95% confidence interval:

$t_{0.025,26} = 2.056$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(20.12 - 11.64) - 2.056\sqrt{\frac{(1.9)^2}{25} + \frac{(7.9)^2}{25}} \le \mu_1 - \mu_2 \le (20.12 - 11.64) + 2.056\sqrt{\frac{(1.9)^2}{25} + \frac{(7.9)^2}{25}}$$

$5.14 \le \mu_1 - \mu_2 \le 11.82$

95% lower one-sided confidence interval:

$t_{0.05,27} = 1.706$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2$$

$$(20.12 - 11.64) - 1.706\sqrt{\frac{(1.9)^2}{25} + \frac{(7.9)^2}{25}} \le \mu_1 - \mu_2$$

$5.71 \le \mu_1 - \mu_2$

For part a):

We are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by between 5.14 and 11.82 mg/1000.

For part c):

We are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by at least 5.71 mg/1000.

5-25. If $\alpha = 0.01$, construct a 99% lower one-sided confidence interval on the difference to answer this question.

$t_{0.01,14} = 2.624$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2$$

$$(101.28 - 101.7) - 2.624\sqrt{\frac{(5.08)^2}{11} + \frac{(20.1)^2}{13}} \le \mu_1 - \mu_2$$

$-15 \le \mu_1 - \mu_2$.

Since the interval covers the value 0, we are 99% confident there is no difference in the mean coating Thickness between the two temperatures; that is, raising the process temperature does not significantly reduce the mean coating thickness.

Section 5-4

5-26. $\bar{d} = 0.2736$ $s_d = 0.1356$, n = 9

95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

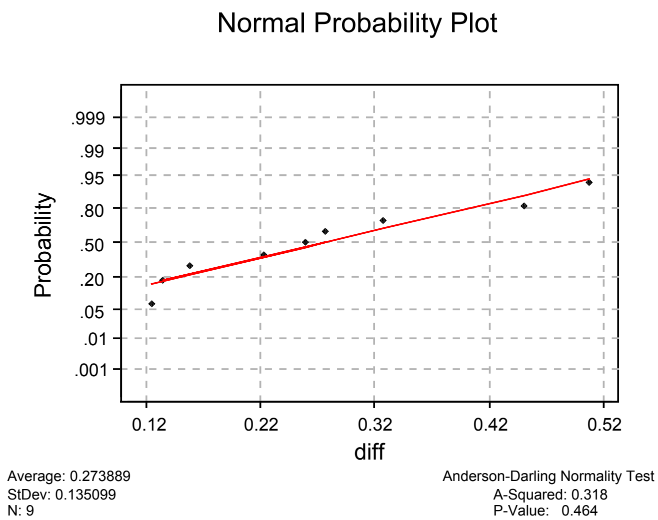
$$0.2736 - 2.306\left(\frac{0.1356}{\sqrt{9}}\right) \le \mu_d \le 0.2736 + 2.306\left(\frac{0.1356}{\sqrt{9}}\right)$$

$0.1694 \le \mu_d \le 0.3778$

With 95% confidence, we believe the mean shear strength of Karlsruhe method exceeds the mean shear strength of the Lehigh method by between 0.1714 and 0.3758. Since 0 is not included in this interval, the interval is consistent with rejecting the null hypothesis that the means are the same.

The 95% confidence interval is directly related to a test of hypothesis with 0.05 level of significance, and the conclusions reached are identical.

5-27. It is only necessary for the differences to be normally distributed for the paired t-test to be appropriate and reliable.



5-28. 1) The parameter of interest is the difference between the mean parking times, μ_d.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) $\alpha = 0.10$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hpothesis if $t_0 < -t_{0.05,13}$ where $-t_{0.05,13} = -1.771$ or $t_0 > t_{0.05,13}$ where $t_{0.05,13} = 1.771$

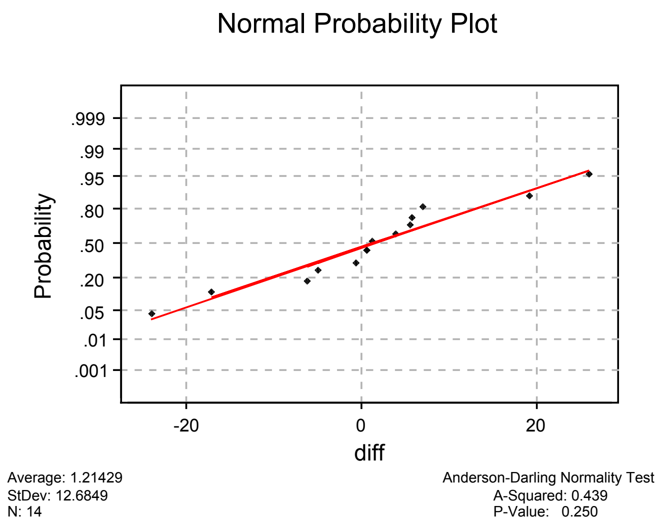
7) $\bar{d} = 1.21$
$s_d = 12.68$
$n = 14$

$$t_0 = \frac{1.21}{12.68 / \sqrt{14}} = 0.357$$

8) Since $-1.771 < 0.357 < 1.771$ do not reject the null and conclude the data do not support the claim that the two cars have different mean parking times at the 0.10 level of significance. The result is consistent with

the confidence interval constructed since 0 is included in the 90% confidence interval.

5-29. According to the normal probability plots, the assumption of normality does not appear to be violated since the data fall approximately along a straight line.



5-30. $\bar{d} = 868.375 \quad s_d = 1290,\ n = 8$ where d_i = brand 1 - brand 2

99% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$868.375 - 3.499\left(\frac{1290}{\sqrt{8}}\right) \le \mu_d \le 868.375 + 3.499\left(\frac{1290}{\sqrt{8}}\right)$$

$$-727.46 \le \mu_d \le 2464.21$$

Since zero is contained within this interval, we are 99% confident there is no significant difference between the two brands of tire.

5-31. a) $\bar{d} = 0.667 \quad s_d = 2.964,\ n = 12$

95% confidence interval:

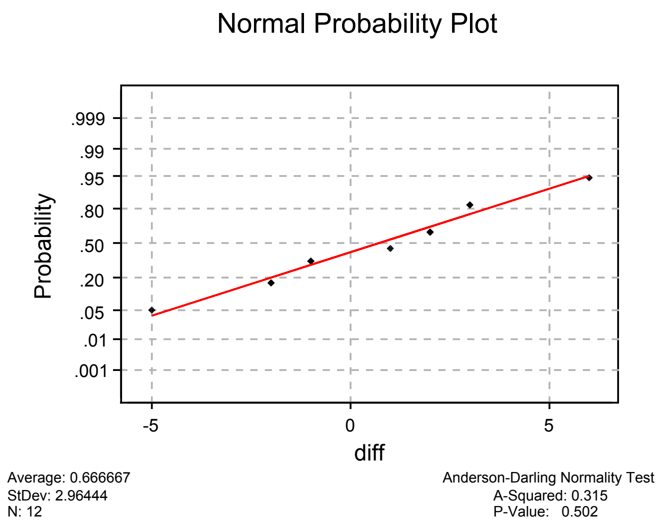
$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$0.667 - 2.201\left(\frac{2.964}{\sqrt{12}}\right) \le \mu_d \le 0.667 + 2.201\left(\frac{2.964}{\sqrt{12}}\right)$$

$$-1.216 \le \mu_d \le 2.55$$

Since zero is contained within this interval, we are 95% confident there is no significant indication that one design language is preferable.

b) According to the normal probability plots, the assumption of normality does not appear to be violated since the data fall approximately along a straight line.



5-32. a) 1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before – After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,14}$ where $t_{0.05,14} = 1.761$

7) $\bar{d} = 26.867$

$s_d = 19.04$

$n = 15$

$$t_0 = \frac{26.867}{19.04 / \sqrt{15}} = 5.465$$

8) Since 5.465 > 1.761 reject the null and conclude the data support the claim that low the mean difference in cholesterol levels is significantly less after fat diet and aerobic exercise program at the 0.05 level of significance.

b) $P(T > 5.465) < 0.0005$

5-33. a) 1) The parameter of interest is the mean difference in natural vibration frequencies, μ_d where d_i = Finite Element – Equivalent Plate.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,6}$ where $-t_{0.025,6} = -2.447$ or $t_0 > t_{0.025,6}$ where $t_{0.025,6} = 2.447$

7) $\bar{d} = -5.49$

$s_d = 5.924$

$n = 7$

$$t_0 = \frac{-5.49}{5.924 / \sqrt{7}} = -2.45$$

8) Since –2.45 < -2.447, reject the null and conclude the data suggest that the two methods do produce significantly different mean values for natural vibration frequency at the 0.05 level of significance.

b) $P(T < -2.45) = P(T > 2.45) \cong 0.025$; The p-value is then $\cong 0.05$

c) 95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$-5.49 - 2.447\left(\frac{5.924}{\sqrt{7}}\right) \le \mu_d \le -5.49 + 2.447\left(\frac{5.924}{\sqrt{7}}\right)$$

$$-10.97 \le \mu_d \le -0.011$$

With 95% confidence, we believe that the mean difference between the natural vibration frequency from the equivalent plate method and the natural vibration frequency from the finite element method is between –10.97 and –0.011 cycle/s. Since 0 is not contained in this interval, we would reject the null hypothesis.

5-34. 1) The parameter of interest is the difference in tensile strength, μ_d where d_i =Strength Before - Strength After.

2) H_0: $\mu_d = 0$

3) H_1: $\mu_d \neq 0$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.005,9}$ or $t_0 < -t_{0.005,9}$ where $t_{0.005,9} = 3.250$

7) $\bar{d} = 9.5$

$s_d = 1.841$

$n = 10$

$$t_0 = \frac{9.5}{1.841/\sqrt{10}} = 16.32$$

8) Since 16.32 > 3.250 reject the null. There is evidence to indicate that the tensile strength before the aging process is not the same as the tensile strength after the aging process at the 0.01 level of significance.

b) P-value < 0.0005

c) (7.608, 11.392); Since 0 is not contained in this interval, evidence suggests that the two tensile strengths differ.

5-35. 1) The parameter of interest is the mean difference in impurity level, μ_d where d_i = Test 1 – Test 2.

2) H_0: $\mu_d = 0$

3) H_1: $\mu_d \neq 0$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.005,7}$ where $-t_{0.005,7} = -3.499$ or $t_0 > t_{0.005,7}$ where $t_{0.005,7} = 3.499$

7) $\bar{d} = -0.2125$

$s_d = 0.1727$

$n = 8$

$$t_0 = \frac{-0.2125}{0.1727 / \sqrt{8}} = -3.48$$

8) Since –3.48 < 2.365 reject the null and conclude the tests give significantly different impurity levels at $\alpha = 0.05$. But do not reject at $\alpha = 0.01$.

5-36. 1) The parameter of interest is the difference in tensile strength, μ_d where d_i =Strength Before - Strength After.
2) H_0: $\mu_d = 5$
3) H_1: $\mu_d > 5$
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$
7) $\bar{d} = 9.50$
$s_d = 1.841$
$n = 10$

$$t_0 = \frac{9.5 - 5}{1.841/\sqrt{10}} = 7.73$$

8) Since 7.73 > 1.833 reject the null and conclude there is evidence to support the claim that the accelerated life test will result in a mean loss of at least 5 psi at the 0.05 level of significance.

5-37. From Minitab, the 99% two-sided confidence interval was found to be (-0.4262, 0.0012). Since 0 is contained in this interval, we conclude there is no significant difference between the two testing procedures.

Section 5-5

5-38. a) $f_{0.25,5,10} = 1.59$

d) $f_{0.75,5,10} = \frac{1}{f_{0.25,10,5}} = \frac{1}{1.89} = 0.529$

b) $f_{0.10,24,9} = 2.28$

e) $f_{0.90,24,9} = \frac{1}{f_{0.10,9,24}} = \frac{1}{1.91} = 0.524$

c) $f_{0.05,8,15} = 2.64$

f) $f_{0.95,8,15} = \frac{1}{f_{0.05,15,8}} = \frac{1}{3.22} = 0.311$

5-39. a) $f_{0.25,7,15} = 1.47$

d) $f_{0.75,7,15} = \frac{1}{f_{0.25,15,7}} = \frac{1}{1.68} = 0.595$

b) $f_{0.10,10,12} = 2.19$

e) $f_{0.90,10,12} = \frac{1}{f_{0.10,12,10}} = \frac{1}{2.28} = 0.439$

c) $f_{0.01,20,10} = 4.41$

f) $f_{0.99,20,10} = \frac{1}{f_{0.01,10,20}} = \frac{1}{3.37} = 0.297$

5-40. 1) The parameters of interest are the variances of concentration, σ_1^2, σ_2^2
2) H_0: $\sigma_1^2 = \sigma_2^2$
3) H_1: $\sigma_1^2 \neq \sigma_2^2$
4) $\alpha = 0.05$
5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,10,9}$ where $f_{0.975,10,9} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,10,9} = 3.96$
7) $n_1 = 11$ $n_2 = 10$

$s_1 = 2.77 \qquad s_2 = 2.41$

$$f_0 = \frac{(2.77)^2}{(2.41)^2} = 1.32$$

8) Since 0.265 < 1.32 < 3.96 do not reject the null hypothesis and conclude there is insufficient evidence to indicate the two population variances differ significantly at the 0.05 level of significance.

5-41. 1) The parameters of interest are the etch rate variances, σ_1^2, σ_2^2.

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,9} = 0.248$ or $f_0 > f_{0.025,9,9} = 4.03$

7) $n_1 = 10 \qquad n_2 = 10$

$s_1 = 0.422 \qquad s_2 = 0.231$

$$f_0 = \frac{(0.422)^2}{(0.231)^2} = 3.34$$

8) Since 0.248 < 3.34 < 4.03 do not reject the null hypothesis and conclude the etch rate variances do not differ at the 0.05 level of significance.

5-42. a) 90% confidence interval for the ratio of variances: $s_1^2 = 0.35 \qquad s_2^2 = 0.40$

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_2-1,n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{(0.35)}{(0.40)} 0.412 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{(0.35)}{(0.40)} 2.33$$

$$0.361 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 2.039$$

$$0.601 \leq \frac{\sigma_1}{\sigma_2} \leq 1.43$$

Since the interval contains 1, we are 90% confident the standard deviations for the rod diameters are not significantly different.

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_2-1,n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{(0.35)}{(0.40)} 0.355 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{(0.35)}{(0.40)} 2.92$$

$$0.310 \leq \frac{\sigma_1^2}{\sigma_2^2} \leq 2.555$$

$$0.556 \leq \frac{\sigma_1}{\sigma_2} \leq 1.60$$

We are 95% confident the standard deviations for the rod diameters are not significantly different.

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right)f_{1-\alpha,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\frac{(0.35)}{(0.40)}0.51 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.446 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.668 \le \frac{\sigma_1}{\sigma_2}$$

5-43. a) 90% confidence interval for the ratio of variances: $s_1 = 0.508$

$s_2 = 0.430$

$$\left(\frac{s_1^2}{s_2^2}\right)f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right)f_{\alpha/2,n_2-1,n_1-1}$$

$$\left(\frac{(0.508)^2}{(0.430)^2}\right)0.156 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.508)^2}{(0.430)^2}\right)6.39$$

$$0.2177 \le \frac{\sigma_1^2}{\sigma_2^2} \le 8.92$$

Since the interval contains 1, we are 90% confident the catalyst variances are not significantly different.

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right)f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right)f_{\alpha/2,n_2-1,n_1-1}$$

$$\left(\frac{(0.508)^2}{(0.430)^2}\right)0.104 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.508)^2}{(0.430)^2}\right)9.60$$

$$0.145 \le \frac{\sigma_1^2}{\sigma_2^2} \le 13.40$$

We are 95% confident the catalyst variances are not significantly different.

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right)f_{1-\alpha,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.508)^2}{(0.430)^2}\right)0.243 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.339 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.582 \le \frac{\sigma_1}{\sigma_2}$$

5-44. 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.02$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.99,7,7}$ where $f_{0.99,7,7} = 0.143$ or $f_0 > f_{0.01,7,7}$ where $f_{0.01,7,7} = 6.99$

7) $n_1 = 8$ $n_2 = 8$

$s_1 = 0.088$ $s_2 = 0.093$

$$f_0 = \frac{(0.088)^2}{(0.093)^2} = 0.895$$

8) Since $0.143 < 0.895 < 6.99$ do not reject the null hypothesis and conclude the thickness variances do not significantly differ at the 0.02 level of significance.

5-45. 1) The parameters of interest are the strength variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$

7) $n_1 = 10$ $n_2 = 16$

$s_1 = 22.5$ $s_2 = 21$

$$f_0 = \frac{(22.5)^2}{(21)^2} = 1.15$$

8) Since $0.265 < 1.15 < 3.12$ do not reject the null hypothesis and conclude the population variances do not significantly differ at the 0.05 level of significance.

5-46. 1) The parameters of interest are the melting variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,20,20}$ where $f_{0.975,20,20} = 0.4065$ or $f_0 > f_{0.025,20,20}$ where $f_{0.025,20,20} = 2.46$

7) $n_1 = 21$ $n_2 = 21$

$s_1 = 2.34$ $s_2 = 2.5$

$$f_0 = \frac{(2.34)^2}{(2.5)^2} = 0.876$$

8) Since $0.4065 < 0.876 < 2.46$ do not reject the null hypothesis and conclude the population variances do not significantly differ at the 0.05 level of significance.

5-47. 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 < \sigma_2^2$

4) $\alpha = 0.10$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.10,10,12}$ where $f_{0.10,10,12} = 1/f_{0.10,12,10} = 1/(2.28) = 0.4386$

7) $n_1 = 11$ $n_2 = 13$

$s_1 = 5.08$ $s_2 = 20.1$

$$f_0 = \frac{(5.08)^2}{(20.1)^2} = 0.064$$

8) Since 0.064 < 0.4386 reject the null hypothesis and conclude the thickness variance for the 125° F process is less than the thickness variance for the 125° F process.

5-48. 1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 < \sigma_2^2$

4) $\alpha = 0.01$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.99,24,20} = 1/f_{0.99,20,24} = 1/(2.74) = 0.3650$

7) $n_1 = 25$ $n_2 = 21$

$s_1 = 0.914$ $s_2 = 1.093$

$$f_0 = \frac{(0.914)^2}{(1.093)^2} = 0.699$$

8) Since 0.699 > 0.3650, do not reject the null hypothesis and conclude there is no evidence to support the claim that men have less repeatability than women for this assembly task at the 0.02 level of significance.

5-49. 99% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha, n_2-1, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.245 \le \frac{\sigma_1^2}{\sigma_2^2}$$

where group 1 represent the men, and group 2 represent the women. Since the value 1 is contained within this interval, we are 99% confident there is no significant difference between the repeatability of mean and women for the assembly task.

Section 5-6

5-50. 1) the parameters of interest are the proportion of defective parts, p_1 and p_2

2) $H_0: p_1 = p_2$

3) $H_1: p_1 \ne p_2$

4) $\alpha = 0.05$
5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $n_1 = 300$ $\quad n_2 = 300$

$x_1 = 15$ $\quad x_2 = 8$

$\hat{p}_1 = 0.05$ $\quad \hat{p}_2 = 0.0267$ $\quad \hat{p} = \frac{15+8}{300+300} = 0.0383$

$$z_0 = \frac{0.05 - 0.0267}{\sqrt{0.0383(1-0.0383)\left(\frac{1}{300} + \frac{1}{300}\right)}} = 1.49$$

8) Since $-1.96 < 1.49 < 1.96$ do not reject the null hypothesis and conclude that yes the evidence indicates that there is not a significant difference in the fraction of defective parts produced by the two machines at the 0.05 level of significance.

P-value = $2(1-P(z < 1.49)) = 0.13622$

5-51. a) Power = $1 - \beta$

$$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} - (p_1 - p_2)}{\hat{\sigma}_{\hat{p}_1 - \hat{p}_2}}\right) - \Phi\left(\frac{-z_{\alpha/2}\sqrt{\overline{pq}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} - (p_1 - p_2)}{\hat{\sigma}_{\hat{p}_1 - \hat{p}_2}}\right)$$

$\overline{p} = \frac{300(0.05) + 300(0.01)}{300 + 300} = 0.03$ $\quad \overline{q} = 0.97$

$$\hat{\sigma}_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{0.05(1-0.05)}{300} + \frac{0.01(1-0.01)}{300}} = 0.014$$

$$\beta = \Phi\left(\frac{1.96\sqrt{0.03(0.97)\left(\frac{1}{300} + \frac{1}{300}\right)} - (0.05 - 0.01)}{0.014}\right) - \Phi\left(\frac{-1.96\sqrt{0.03(0.97)\left(\frac{1}{300} + \frac{1}{300}\right)} - (0.05 - 0.01)}{0.014}\right)$$

$$= \Phi(-0.91) - \Phi(-4.81) = 0.18141 - 0 = 0.18141$$

Power $\cong 1 - 0.18141 = 0.819$

$$\text{b) } n = \frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1 + p_2)(q_1 + q_2)}{2}} + z_\beta\sqrt{p_1q_1 + p_2q_2}\right)^2}{(p_1 - p_2)^2}$$

$$= \frac{\left(1.96\sqrt{\frac{(0.05 + 0.01)(0.95 + 0.99)}{2}} + 1.29\sqrt{0.05(0.95) + 0.01(0.99)}\right)^2}{(0.05 - 0.01)^2} = 382.11$$

$n \cong 383$

5-52. a) $$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\overline{pq}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right) - \Phi\left(\frac{-z_{\alpha/2}\sqrt{\overline{pq}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}-(p_1-p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$$\overline{p} = \frac{300(0.05)+300(0.02)}{300+300} = 0.035 \qquad \overline{q} = 0.965$$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.02(1-0.02)}{300}} = 0.015$$

$\beta =$

$$\Phi\left(\frac{1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.02)}{0.015}\right) - \Phi\left(\frac{-1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)}-(0.05-0.02)}{0.015}\right)$$

$= \Phi(-0.04) - \Phi(-3.96) = 0.48405 - 0.00004 = 0.48401$

Power = 1 – 0.48401 = 0.51599

b) $$n = \frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}} + z_\beta\sqrt{p_1q_1+p_2q_2}\right)^2}{(p_1-p_2)^2}$$

$$= \frac{\left(1.96\sqrt{\frac{(0.05+0.02)(0.95+0.98)}{2}} + 1.29\sqrt{0.05(0.95)+0.02(0.98)}\right)^2}{(0.05-0.02)^2} = 790.67$$

n = 791

5-53. 1) the parameters of interest are the proportion of rollover accidents, p_A and p_B

2) H_0: $p_A = p_B$

3) H_1: $p_A > p_B$

4) $\alpha = 0.05$

5) Test statistic is

$$z_0 = \frac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{where}$$

$$\hat{p} = \frac{x_1+x_2}{n_1+n_2}$$

6) Reject the null hypothesis if $z_0 > z_{0.05}$ where $z_{0.05} = 1.645$

7) $n_1 = 100$ $\quad n_2 = 100$

$x_1 = 35$ $\quad x_2 = 41$

$\hat{p}_A = 0.35 \qquad \hat{p}_B = 0.25 \qquad \hat{p} = \frac{35+25}{100+100} = 0.30$

$$z_0 = \frac{0.35-0.25}{\sqrt{0.30(1-0.30)\left(\frac{1}{100}+\frac{1}{100}\right)}} = 1.543$$

8) Since 1.543 < 1.645 do not reject the null hypothesis and conclude that Manufacturer A does not have a significantly higher rollover rate than Manufacturer B for $\alpha = 0.05$.

b) P-value = P(Z > 1.543) = 0.06142
c) From Minitab, the power of this test is found to be 0.4592
d) From Minitab, the sample size is found to be n = 166 with $p_1 = 0.4$, $p_2 = 0.25$, $\alpha = 0.05$, Power = 0.90.

5-54. 95% confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2 \le (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.05 - 0.0267) - 1.96\sqrt{\frac{0.05(1-0.05)}{300} + \frac{0.0267(1-0.0267)}{300}} \le p_1 - p_2 \le (0.05 - 0.0267) + 1.96\sqrt{\frac{0.05(1-0.05)}{300} + \frac{0.0267(1-0.0267)}{300}}$$

$$-0.0074 \le p_1 - p_2 \le 0.054$$

Since this interval contains the value zero, we are 95% confident there is no significant difference in the fraction of defective parts produced by the two machines and that the difference in proportions is between –0.0074 and 0.054.

5-55. 95% lower confidence bound on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2$$

$$(0.35 - 0.25) - 1.64\sqrt{\frac{0.35(1-35)}{100} + \frac{0.25(1-0.25)}{100}} \le p_1 - p_2$$

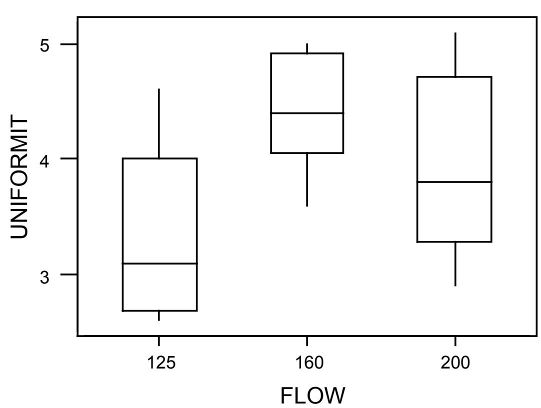
$$-0.00565 \le p_1 - p_2$$

Since this interval contains the value zero, we are 95% confident there is no significant difference in the rollover rate.

Section 5-8

5-56. a)
```
Analysis of Variance for Uniformity
Source     DF        SS        MS       F       P
Flow        2     3.648     1.824    3.59   0.053
Error      15     7.630     0.509
Total      17    11.278
```

Reject H_0 at $\alpha = 0.1$. C_2F_6 flow rate does appear to affect etch uniformity.

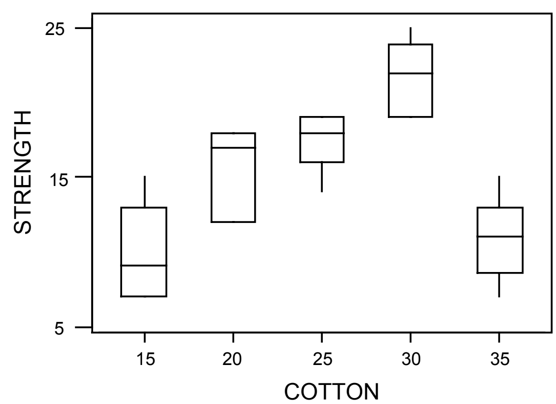


b) Examining the box plots, the 125 and 160 mean levels seem to be different.

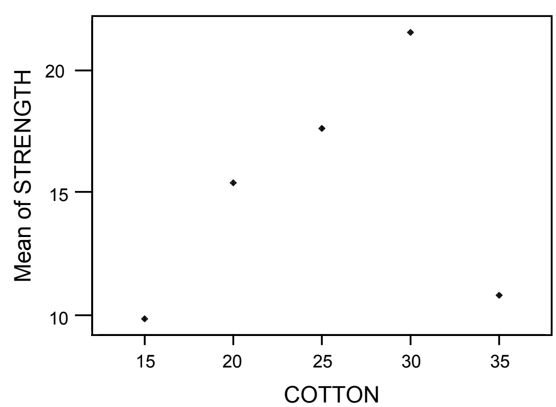
5-57. a) Analysis of Variance for Breaking Strength

Source	DF	SS	MS	F	P
Cotton%	4	475.76	118.94	14.76	0.000
Error	20	161.20	8.06		
Total	24	636.96			

Because the P-value = 0, reject H_0 and conclude that cotton percentage affects breaking strength.



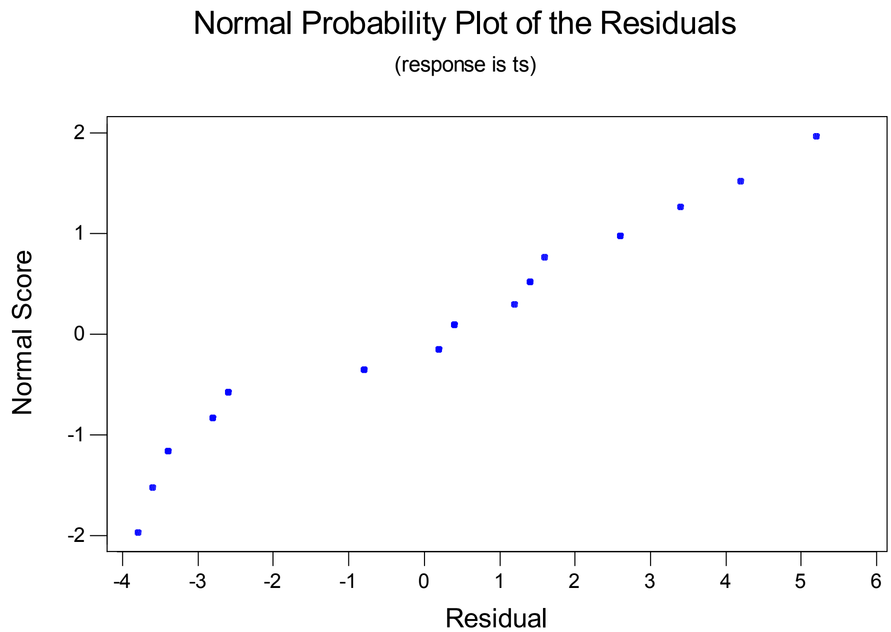
b)

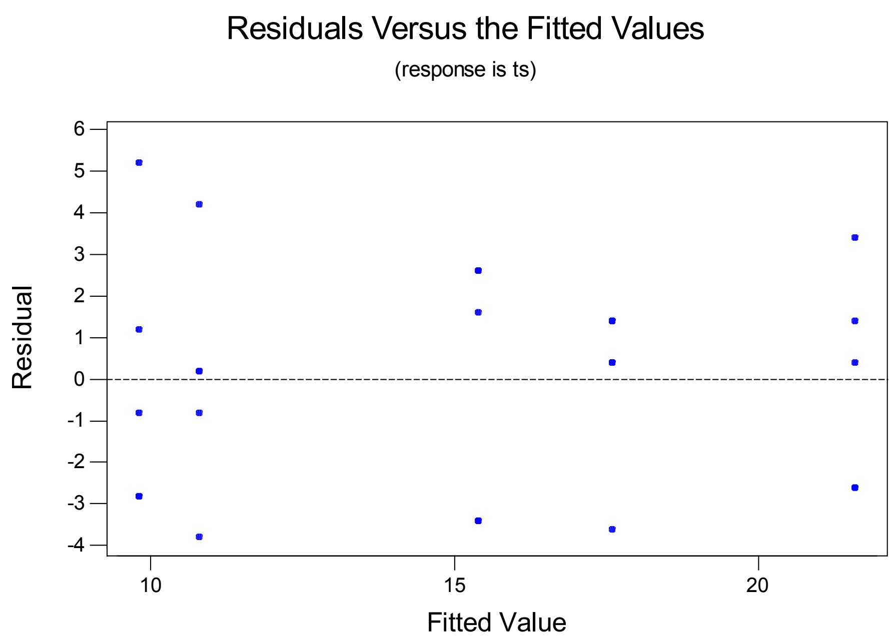


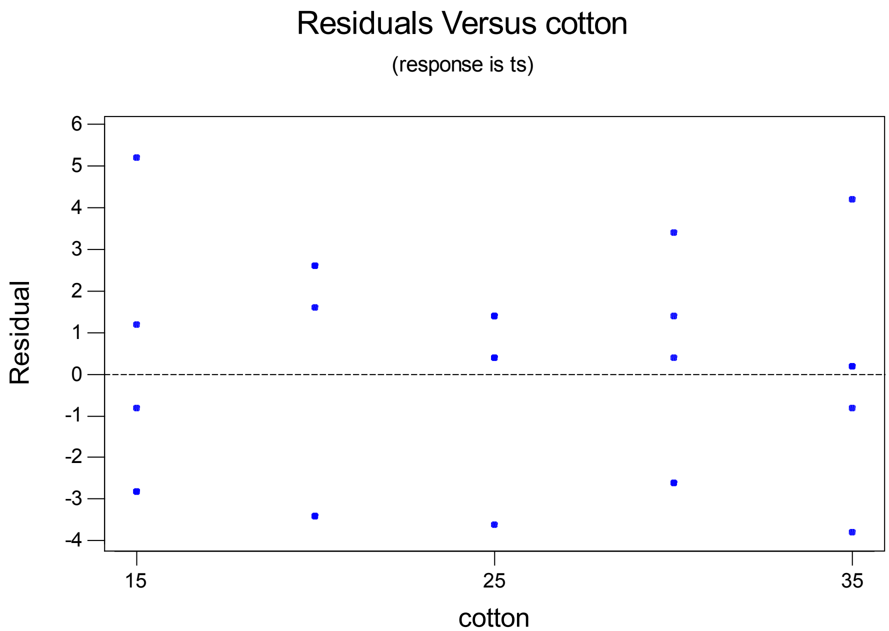
Tensile strength seems to increase to 30% cotton and declines at 35% cotton.

c) It appears that tensile strength due to 15% cotton and 35% cotton are different from that of 20%, 25%, and 30%, but not each other. There also seems to be a difference between 20% and 30% cotton and also 25% and 30% cotton.

d) Residual analysis


Normal Probability Plot of the Residuals
(response is ts)
Normal Score
Residual


Residuals Versus the Fitted Values
(response is ts)
Residual
Fitted Value



5-58. a) Analysis of Variance for Density

Source	DF	SS	MS	F	P
Firing T	3	0.1391	0.0464	2.62	0.083
Error	18	0.3191	0.0177		
Total	21	0.4582			

Do not reject H_0. There is insignificant evidence to indicate the four firing temperatures affect the density of the brick.

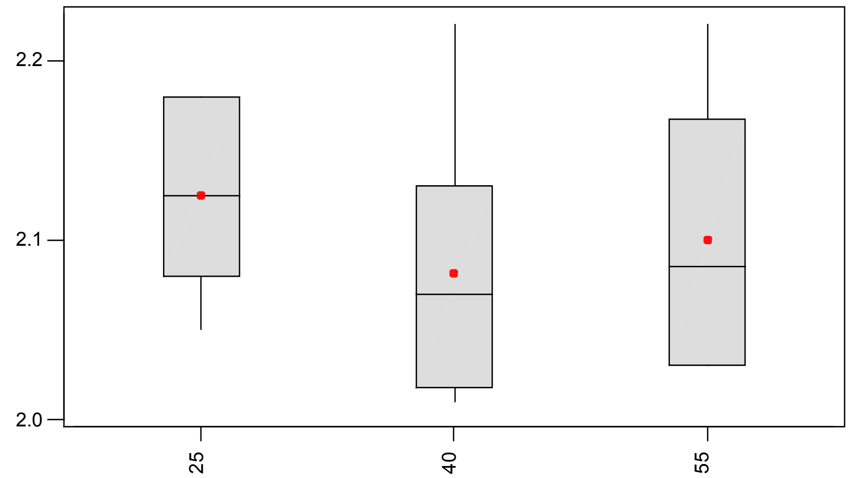
b) P-value = 0.0827

5-59. a) Analysis of Variance

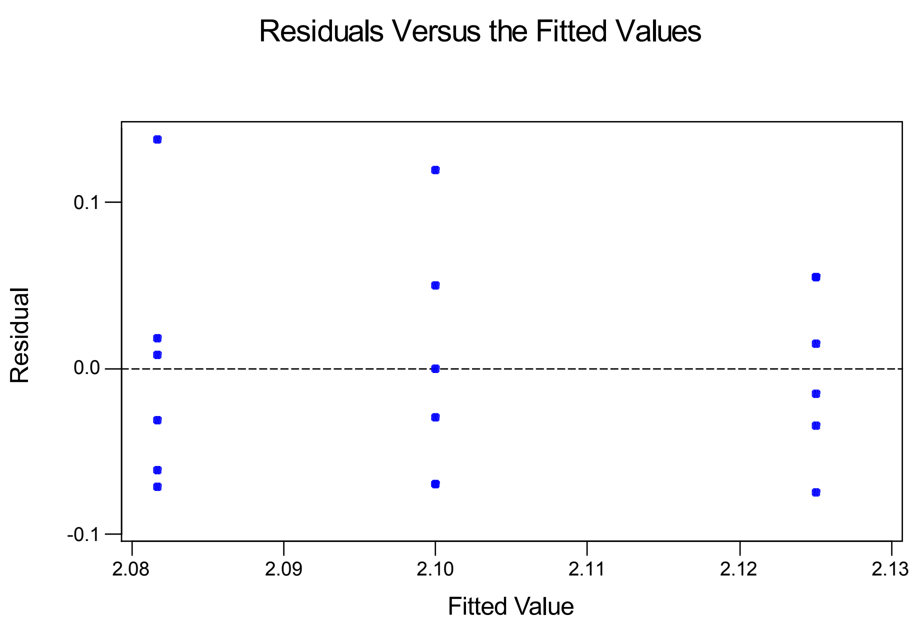
Source	DF	SS	MS	F	P
Factor	2	0.00568	0.00284	0.60	0.559
Error	15	0.07043	0.00470		
Total	17	0.07611			

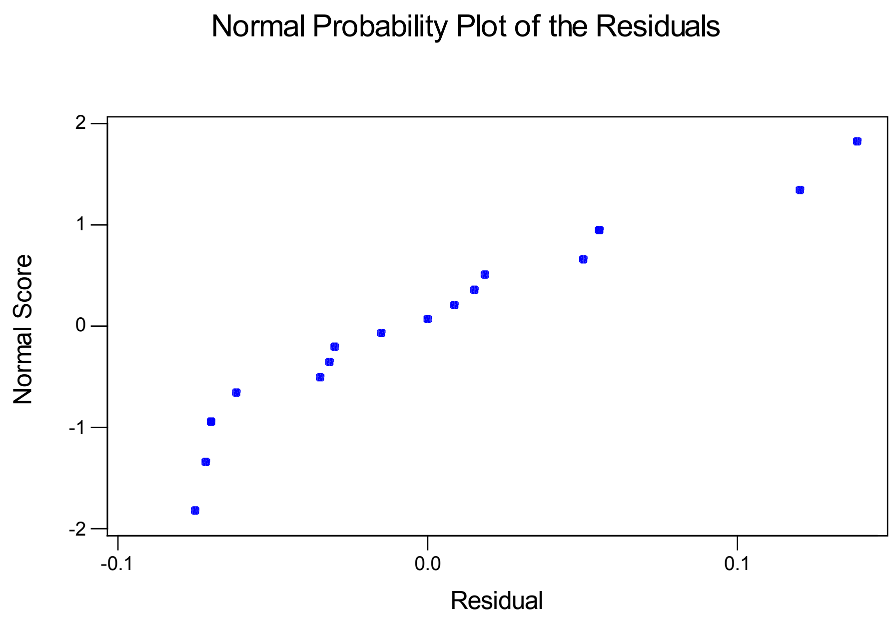
Do not reject H_0. The curing temperatures do not appear to affect the strength of the silicon rubber.

b) P-value = 0.559
c)


2.2
2.1
2.0
25
40
55

d)


Residuals Versus the Fitted Values
Residual
0.1
0.0
-0.1
2.08
2.09
2.10
2.11
2.12
2.13
Fitted Value



There appears to be a slight departure from normality. The residuals versus fitted values indicate a possible problem with nonconstant variance.

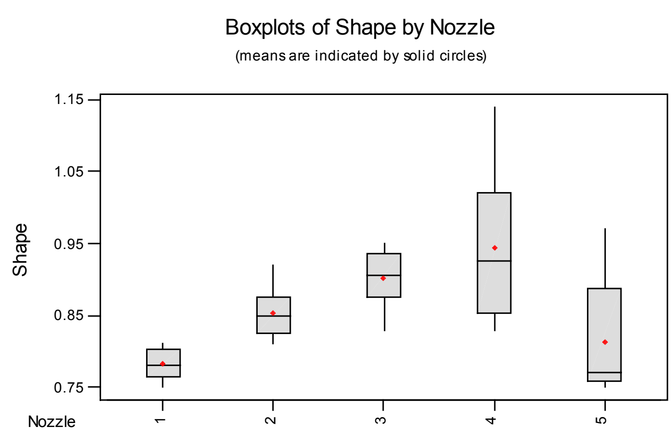
5-60.

Analysis of Variance for Conductivity

Source	DF	SS	MS	F	P
Coating	4	1060.5	265.1	16.35	0.000
Error	15	243.3	16.2		
Total	19	1303.7			

Reject H_0. There appears to be a significant difference among the five coating types in their effect on conductivity.

5-61. a)



Based on the box plots, there appears to be some differences among the nozzle types. To determine if this is true, conduct an analysis of variance.

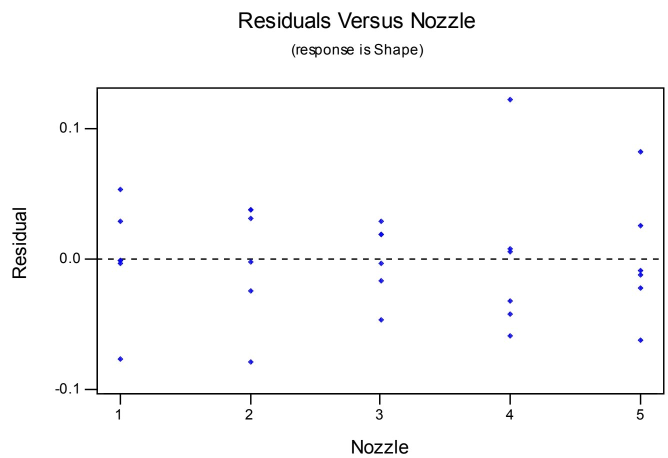
Analysis of Variance for Shape

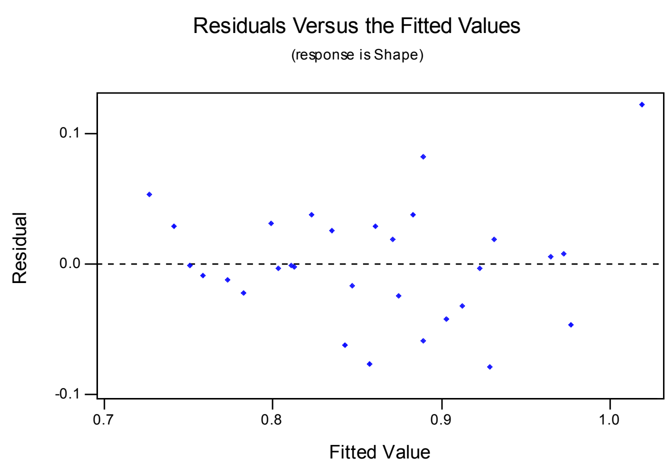
Source	DF	SS	MS	F	P
Nozzle	4	0.10218	0.02555	8.92	0.000
Velocity	5	0.06287	0.01257	4.39	0.007
Error	20	0.05730	0.00287		
Total	29	0.22235			

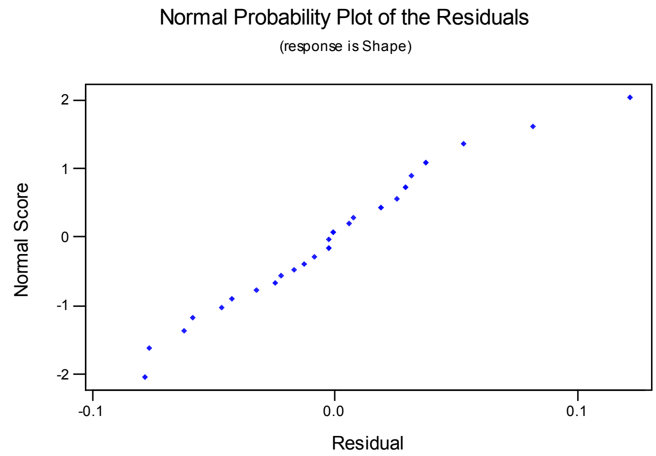
Nozzle type does significantly affect the shape measurement.

b) The standard deviation for the normal distribution will be $\sqrt{MSE/b} = \sqrt{0.00287/6} = 0.022$. The types found to be different are 1 and 3, 1 and 4, 2 and 4, 3 and 5, 4 and 5.

c)







No severe departure from the assumptions of normality or constant variance.

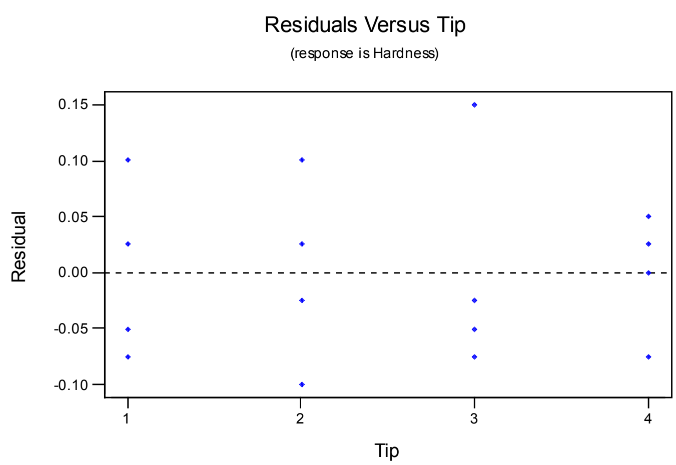
5-62. a) Analysis of Variance for Hardness

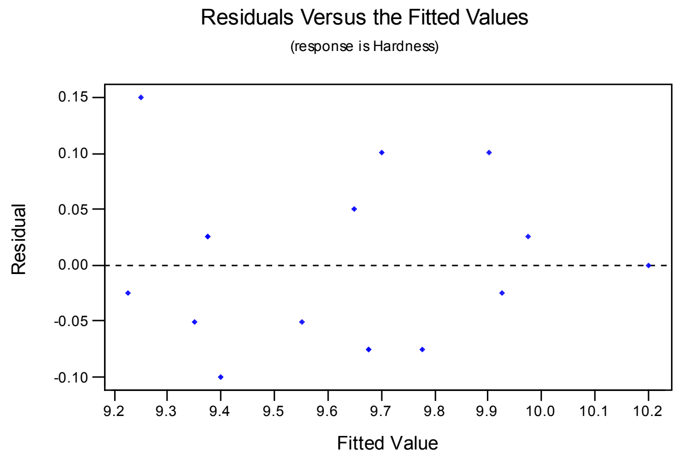
Source	DF	SS	MS	F	P
Tip	3	0.38500	0.12833	14.44	0.001
Specimen	3	0.82500	0.27500	30.94	0.000
Error	9	0.08000	0.00889		
Total	15	1.29000			

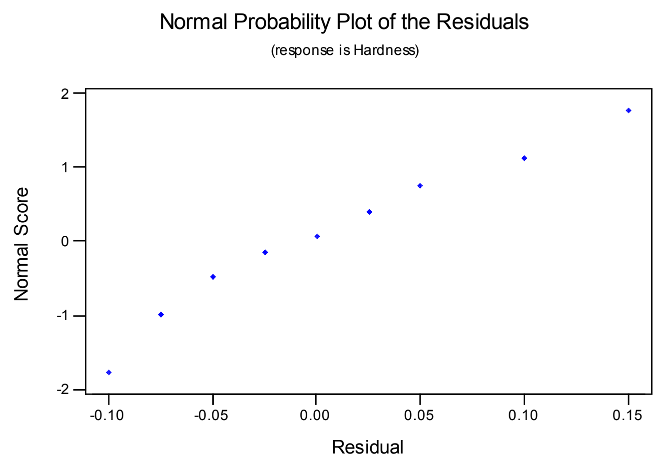
There is a significant difference in hardness measurements between tips.

b) The standard deviation for the normal distribution will be $\sqrt{MSE/b} = \sqrt{0.00889/4} = 0.1491$. The tips found to be different are 3 and 4.

c)







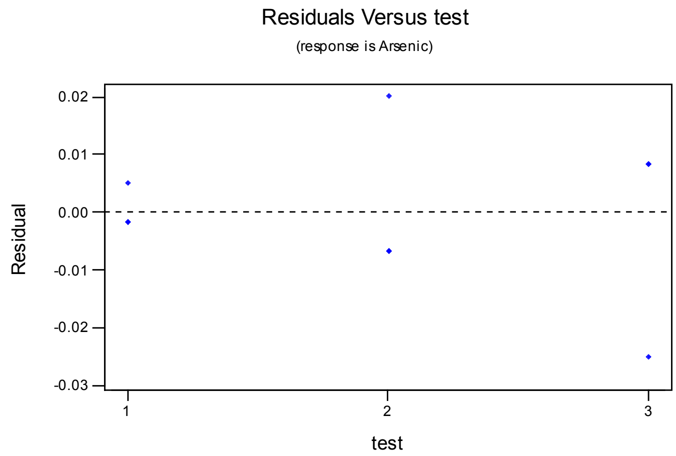
No severe departure from the assumptions of normality or constant variance.

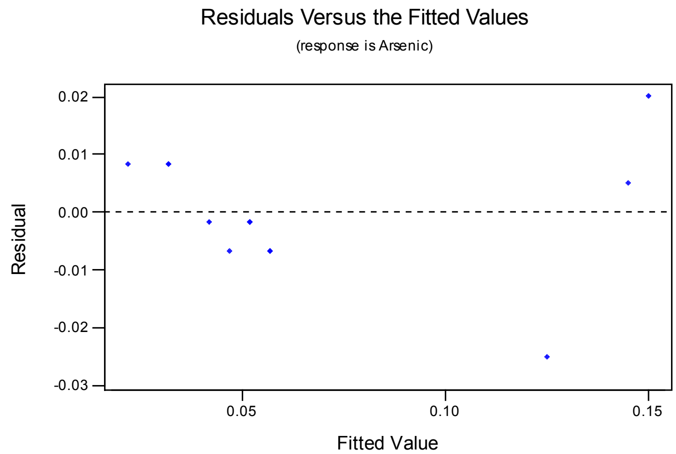
5-63. a) Analysis of Variance for Arsenic

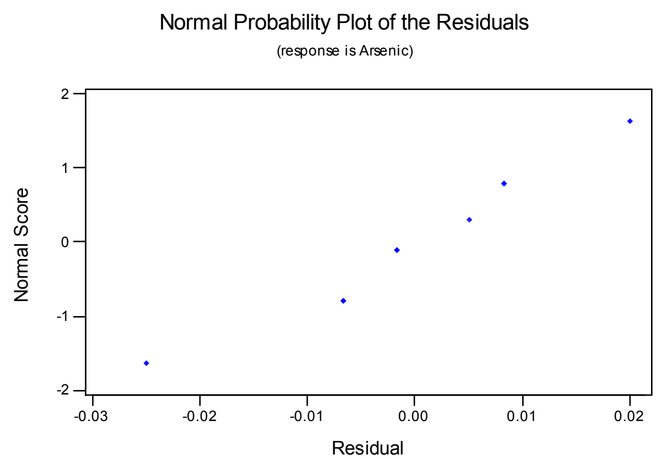
Source	DF	SS	MS	F	P
test	2	0.001400	0.000700	3.00	0.125
subject	3	0.021225	0.007075	30.32	0.001
Error	6	0.001400	0.000233		
Total	11	0.024025			

There is no difference in the arsenic test procedure.

b)







There may be a problem with the constant variance assumption as displayed in the residual plot versus fitted values. There does not appear to be a severe departure from normality.

5-64. a)

```
Analysis of Variance for Protopectin
Source       DF        SS        MS        F       P
Storage       3   1972652    657551     4.33   0.014
```

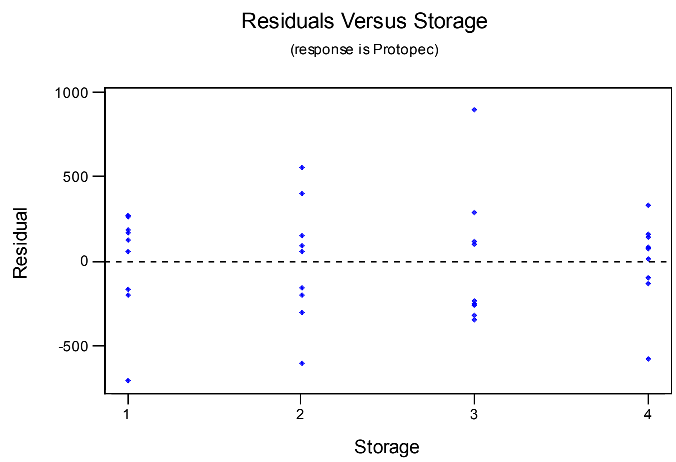
Lot	8	1980499	247562	1.63	0.169
Error	24	3647150	151965		
Total	35	7600300			

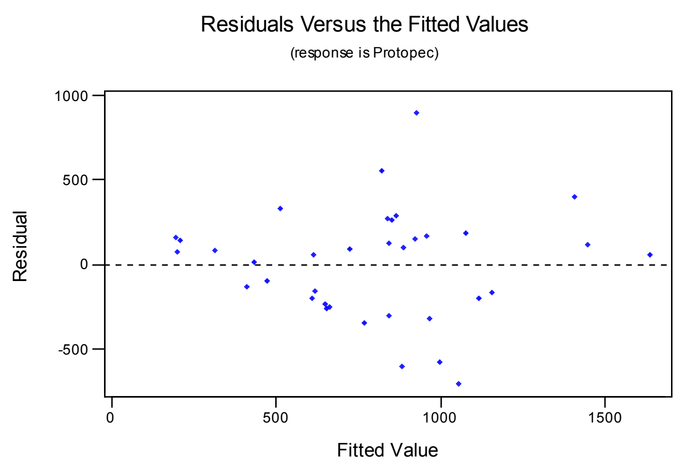
There is a significant difference in mean protopectin content at different storage times.

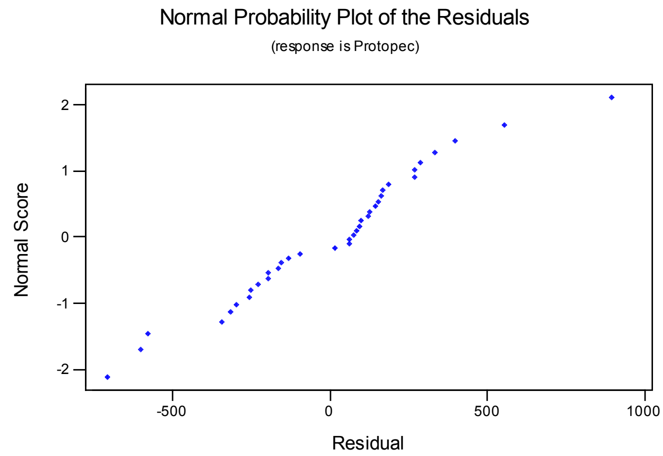
b) P-value = 0.014 (for storage since that is our factor of interest.)

c) The results from using four storage days appears to differ than the results using any of the other levels of storage (1, 2, or 3 days).

d)







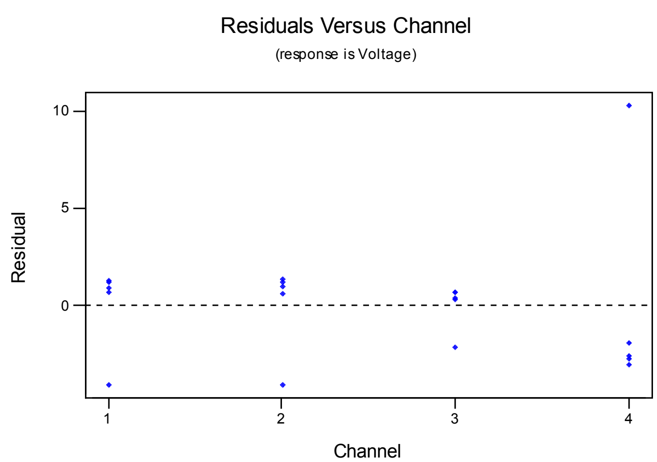
There may be a departure from the constant variance assumption as can be seen by the patterns on the residual plots versus storage and fitted values. No severe departure from normality.

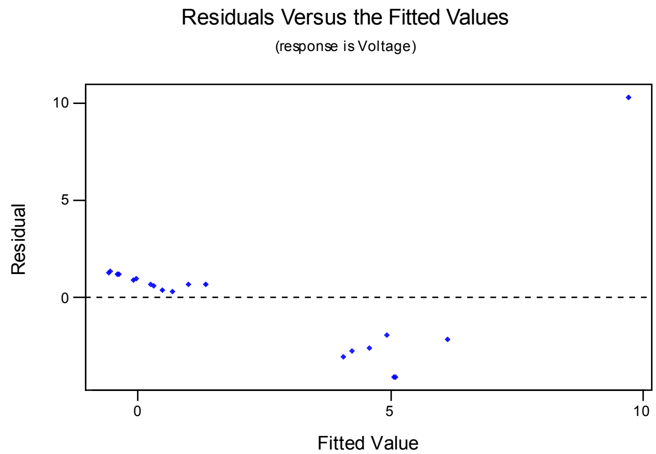
5-65. a) Analysis of Variance for Voltage

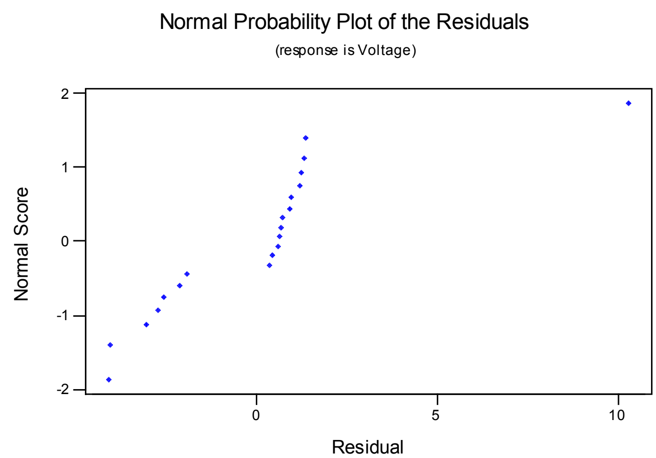
Source	DF	SS	MS	F	P
Channel	3	72.7	24.2	1.61	0.240
Width	4	90.5	22.6	1.50	0.263
Error	12	180.8	15.1		
Total	19	344.0			

Mean leakage voltage does not depend on the channel length.

b)







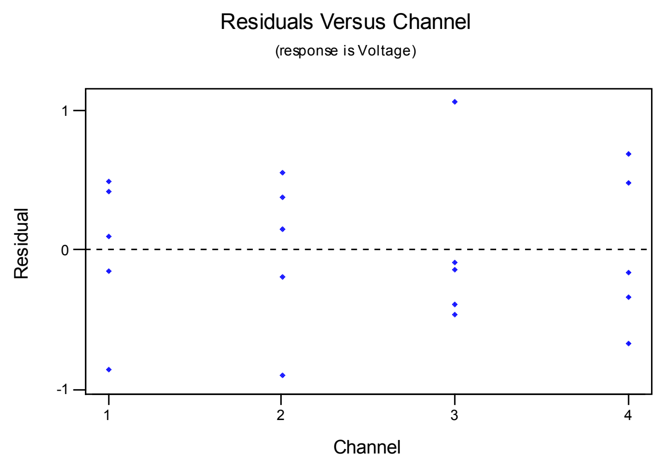
There appears to be an outlier that is casuing the residual plots to display patterns and giving the impression that the assumptions of constant variance and normality are not valid. Perhaps removing this point and running an analysis on an unbalanced block design will give better results.

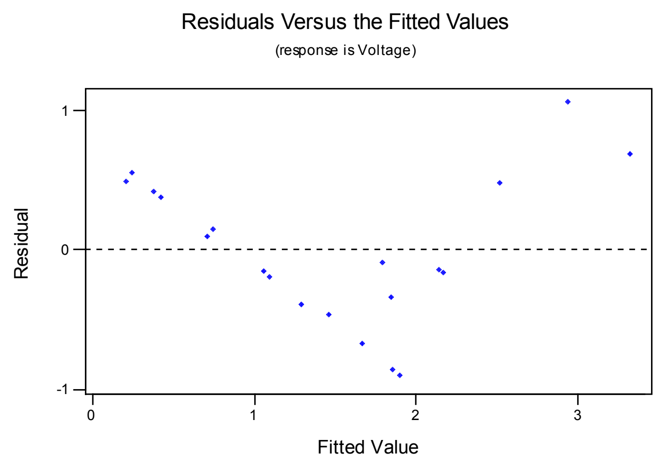
5-66. a) Analysis of Variance for Voltage

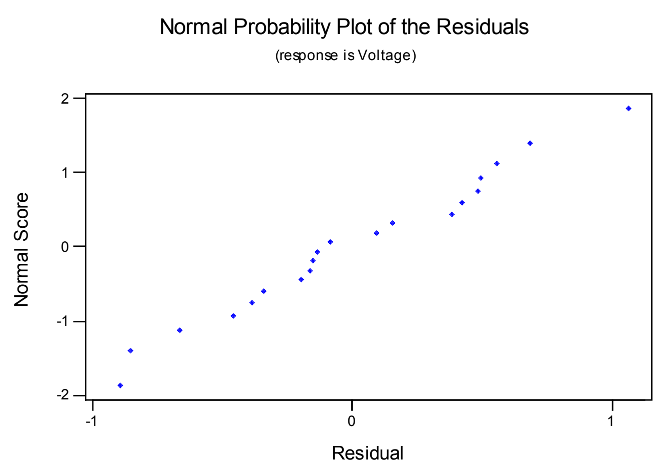
Source	DF	SS	MS	F	P
Channel	3	8.178	2.726	6.16	0.009
Width	4	6.838	1.709	3.86	0.031
Error	12	5.310	0.443		
Total	19	20.325			

With the corrected value, we now see that mean leakage voltage does depend on the channel length.

b)



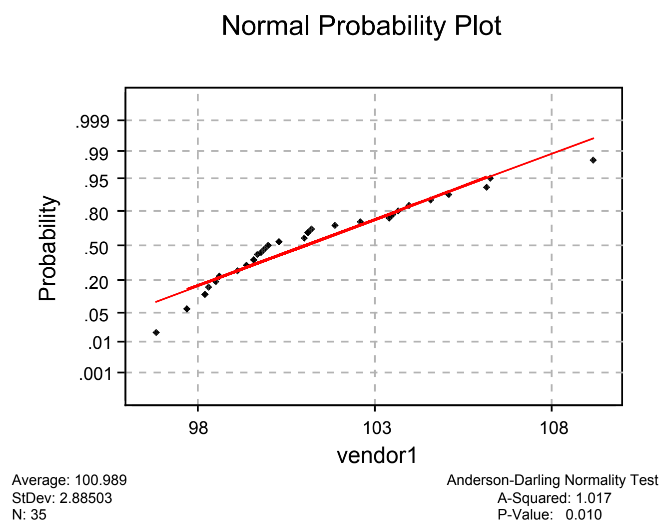


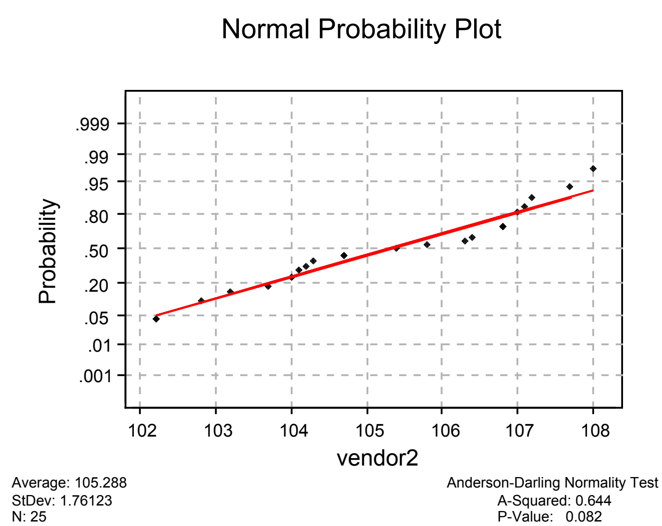


The residual plots are much more satisfactory although the residuals versus fitted plots still appear to have some pattern.

Supplemental Exercises

5-67. a) The assumption of normality is necessary to test the claim. According to the normal probability plots, the assumption of normality does not appear to be violated. This is evident from the fact that the data appear to fall along a straight line.





b) 1) the parameters of interest are the variances of resistance of products, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject H_0 if $f_0 < f_{0.975,24,34}$ where $f_{0.975,24,34} = \frac{1}{f_{0.025,34,24}} = \frac{1}{2.18} = 0.459$

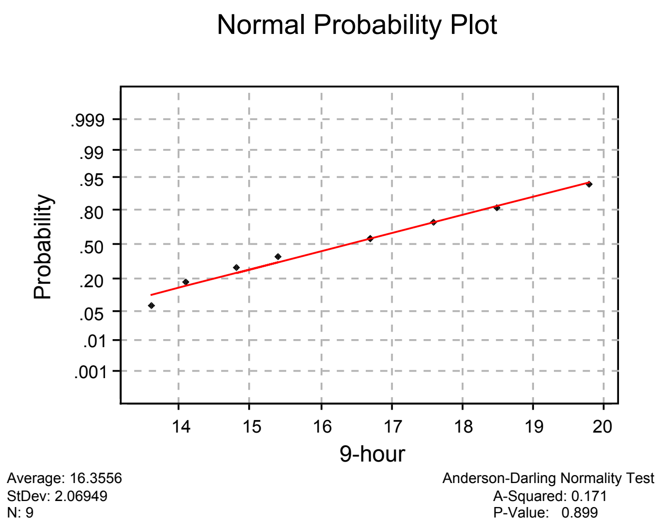
or $f_0 > f_{0.025,24,34}$ where $f_{0.025,24,34} = 2.07$

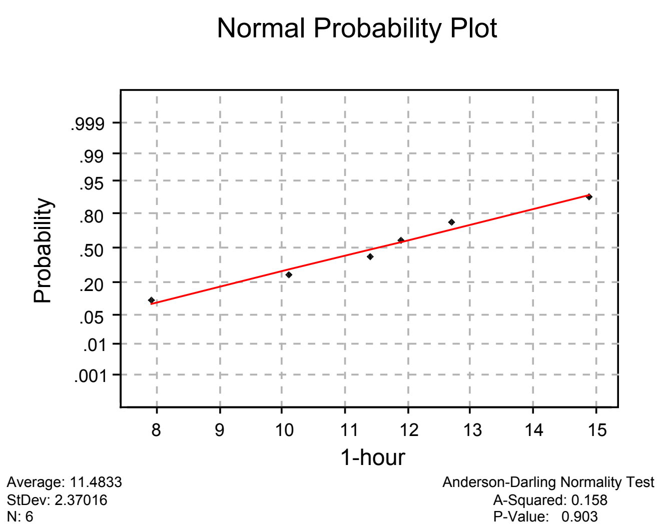
7) $s_1 = 1.53$ $s_2 = 1.96$
$n_1 = 25$ $n_2 = 35$

$$f_0 = \frac{(1.53)^2}{(1.96)^2} = 0.609$$

8) Since 0.601 > 0.459, do not reject H_0 and conclude the variances are not significantly different at $\alpha = 0.05$.

5-68. a) Normality and equality of variances appears to be reasonable, see normal probability plot. The data appear to fall along a straight line and the slopes appear to be the same.





b) From Minitab, the test statistic is found to be $T_0 = 4.22$, The p-value is 0.001

c) From Minitab, the p-value is found to be 0.001

d) $\bar{x}_1 = 16.36$ $\quad \bar{x}_2 = 11.48$

$s_1 = 2.07$ $\quad s_2 = 2.37$

$n_1 = 9$ $\quad n_2 = 6$

99% confidence interval: $t_{\alpha/2, n_1+n_2-2} = t_{0.005,13}$ where $t_{0.005,13} = 3.012$

$$s_p = \sqrt{\frac{8(2.07)^2 + 5(2.37)^2}{13}} = 2.19$$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha/2,n_1+n_2-2}\left(s_p\right)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\le \mu_1-\mu_2\le \left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha/2,n_1+n_2-2}\left(s_p\right)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$(16.36-11.48)-3.012(2.19)\sqrt{\frac{1}{9}+\frac{1}{6}}\le \mu_1-\mu_2\le (16.36-11.48)+3.012(2.19)\sqrt{\frac{1}{9}+\frac{1}{6}}$$

$$1.40\le \mu_1-\mu_2\le 8.36$$

We are 99% confident the results from the first test condition exceed the results of the second test condition by between 1.40 and 8.36 ($\times 10^6$ PA), 0 is not contained in the interval.

5-69. a) 1) The parameters of interest are the variances in deposit thickness, $\sigma^2_{old}, \sigma^2_{new}$

2) H_0: $\sigma^2_{old} = \sigma^2_{new}$

3) H_1: $\sigma^2_{old} > \sigma^2_{new}$

4) $\alpha = 0.10$

5) The test statistic is

$$f_0=\frac{s^2_{old}}{s^2_{new}}$$

6) Reject the null hypothesis if $f_0 > f_{0.10,20,15} = 1.92$

7) $n_1 = 22$ $\quad n_2 = 20$

$s_{old} = 0.094$ $\quad s_{new} = 0.047$

$$f_0=\frac{(0.094)^2}{(0.047)^2}=4$$

8) Since 4 > 1.92, reject the null hypothesis and conclude there is evidence to support the claim that the new system results in a variance that is significantly less than the old system at $\alpha = 0.10$.

b) P-value < 0.01 from Table IV

c) 90% confidence interval:

$$\left(\frac{s^2_{old}}{s^2_{new}}\right)f_{1-\alpha,n_2-1,n_1-1}\le\frac{\sigma^2_{old}}{\sigma^2_{new}}$$

$$2.17\le\frac{\sigma_1^2}{\sigma_2^2}$$

d) Since the value 1 is not contained within this interval, we are 90% confident that the new system results in a variance that is significantly less than the old system.

5-70. a) 1) The parameter of interest is the mean weight loss, μ_d where d_i = Initial Weight – Final Weight.

2) $H_0 : \mu_d = 3$

3) $H_1 : \mu_d > 3$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0=\frac{\bar{d}-\Delta_0}{s_d/\sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$.

7) $\bar{d} = 4.125$

$s_d = 1.246$

$n = 8$

$$t_0=\frac{4.125-3}{1.246/\sqrt{8}}=2.554$$

8) Since 2.554 > 1.895, reject the null hypothesis and conclude the average weight loss is significantly greater than 3 at $\alpha = 0.05$.

b) 2) $H_0 : \mu_d = 3$
3) $H_1 : \mu_d > 3$
4) $\alpha = 0.01$
5) The test statistic is

$$t_0 = \frac{\overline{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,7} = 2.998$.
7) $\overline{d} = 4.125$
$s_d = 1.246$
$n = 8$

$$t_0 = \frac{4.125 - 3}{1.246 / \sqrt{8}} = 2.554$$

8) Since 2.554 <2.998, do not reject the null hypothesis and conclude the average weight loss is not significantly greater than 3 at $\alpha = 0.01$.

c) 2) $H_0 : \mu_d = 5$
3) $H_1 : \mu_d > 5$
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{\overline{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$.
7) $\overline{d} = 4.125$
$s_d = 1.246$
$n = 8$

$$t_0 = \frac{4.125 - 5}{1.246 / \sqrt{8}} = -1.986$$

8) Since −1.986 < 1.895, do not reject the null hypothesis and conclude the average weight loss is not significantly greater than 5 at $\alpha = 0.05$.

Using $\alpha = 0.01$

2) $H_0 : \mu_d = 5$
3) $H_1 : \mu_d > 5$
4) $\alpha = 0.01$
5) The test statistic is

$$t_0 = \frac{\overline{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.01,7} = 2.998$.
7) $\overline{d} = 4.125$
$s_d = 1.246$
$n = 8$

$$t_0 = \frac{4.125 - 5}{1.246 / \sqrt{8}} = -1.986$$

8) Since −1.986 < 2.998, do not reject the null hypothesis and conclude the average weight loss is not significantly greater than 5 at $\alpha = 0.01$.

5-71. $$\left(\bar{x}_1-\bar{x}_2\right)-z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\le \mu_1-\mu_2\le \left(\bar{x}_1-\bar{x}_2\right)+z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

a) 90% confidence interval: $z_{\alpha/2} = 1.65$

$$\left(88-91\right)-1.65\sqrt{\frac{5^2}{20}+\frac{4^2}{20}}\le \mu_1-\mu_2\le \left(88-91\right)+1.65\sqrt{\frac{5^2}{20}+\frac{4^2}{20}}$$

$$-5.362\le \mu_1-\mu_2\le -0.638$$

Yes, with 90% confidence, the data indicate that the mean breaking strength of the yarn of manufacturer 2 exceeds that of manufacturer 1 by between 0.638 and 5.362.

b) 98% confidence interval: $z_{\alpha/2} = 2.33$

$$\left(88-91\right)-2.33\sqrt{\frac{5^2}{20}+\frac{4^2}{20}}\le \mu_1-\mu_2\le \left(88-91\right)+2.33\sqrt{\frac{5^2}{20}+\frac{4^2}{20}}$$

$$-6.340\le \mu_1-\mu_2\le 0.340$$

Yes, we are 98% confident manufacturer 2 produces yarn with higher breaking strength by between 0.340 and 6.340 psi.

c) The results of parts a) and b) are different because the confidence level or z-value used is different. Which one is used depends upon the level of confidence considered acceptable.

5-72. a) $\alpha = 0.10$ $z_{\alpha/2} = 1.65$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2+\sigma_2^2\right)}{(E)^2}\cong \frac{(1.65)^2(25+16)}{(1.5)^2} = 49.61\,,\quad n = 50$$

b) $\alpha = 0.10$ $z_{\alpha/2} = 2.33$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2+\sigma_2^2\right)}{(E)^2}\cong \frac{(2.33)^2(25+16)}{(1.5)^2} = 98.93\,,\quad n = 99$$

c) As the confidence level increases, sample size will also increase.

d) $\alpha = 0.10$ $z_{\alpha/2} = 1.65$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2+\sigma_2^2\right)}{(E)^2}\cong \frac{(1.65)^2(25+16)}{(0.75)^2} = 198.44\,,\quad n = 199$$

b) $\alpha = 0.10$ $z_{\alpha/2} = 2.33$

$$n \cong \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2+\sigma_2^2\right)}{(E)^2}\cong \frac{(2.33)^2(25+16)}{(0.75)^2} = 395.70\,,\quad n = 396$$

e) As the error decreases, the required sample size increases.

5-73. a) 1) The parameters of interest are the proportions of children who contract polio, p_1 , p_2
2) $H_0 : p_1 = p_2$
3) $H_1 : p_1 \neq p_2$
4) $\alpha = 0.05$
5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 1.96$

7) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{110}{201299} = 0.00055$ (Placebo) $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.000356$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{33}{200745} = 0.00016$ (Vaccine)

$$z_0 = \frac{0.00055 - 0.00016}{\sqrt{0.000356(1-0.000356)\left(\frac{1}{201299} + \frac{1}{200745}\right)}} = 6.55$$

8) Since 6.55 > 1.96 reject H_0 and conclude the proportion of children who contracted polio is significantly different at $\alpha = 0.05$.

b) $\alpha = 0.01$
Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 2.57$
$z_0 = 6.55$

Since 6.55 > 2.57, reject H_0 and conclude the proportion of children who contracted polio is different at $\alpha = 0.05$.

c) The conclusions are the same since z_0 is so large it exceeds $z_{\alpha/2}$ in both cases.

5-74. a) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{942}{1095} = 0.86$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{850}{1042} = 0.816$

2) H_0: $p_1 = p_2$
3) H_1: $p_1 \neq p_2$
4) $\alpha = 0.05$
5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $\hat{p} = \frac{942 + 850}{1095 + 1042} = 0.839$

$$z_0 = \frac{0.86 - 0.816}{\sqrt{0.839(1-0.839)\left(\frac{1}{1095} + \frac{1}{1042}\right)}} = 3.35$$

8) Since 3.35 > 1.96, reject H_0 and conclude that there is a difference in accuracy in these two sources.

b) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{473}{550} = 0.86$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{451}{550} = 0.82$

2) H_0: $p_1 = p_2$

3) H_1: $p_1 \neq p_2$

4) $\alpha = 0.05$

5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $\hat{p} = \frac{473 + 451}{550 + 550} = 0.84$

$$z_0 = \frac{0.86 - 0.82}{\sqrt{0.84(1-0.84)\left(\frac{1}{550} + \frac{1}{550}\right)}} = 1.81$$

8) Since 1.81 < 1.96, do not reject H_0 and conclude that there is not a difference in accuracy in these two sources.

c) The estimated accuracy percentages were nearly identical for both studies, but the results of hypothesis testing in parts a) and b) differ. The conclusions differ because of the differences in sample size from the first study to the second study. The sample sizes play a significant role in the standard error for difference in proportion:

First Study: standard error = 0.016
Second Study: standard error = 0.022

d) $\left(\hat{p}_1 - \hat{p}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$, $\quad z_{\alpha/2} = z_{0.025} = 1.96$

First Study:

$$\left(0.86 - 0.819\right) \pm 1.96\sqrt{\frac{0.86(0.14)}{1095} + \frac{0.819(0.181)}{1042}}$$

$0.0099 \leq p_1 - p_2 \leq 0.0721$

Since zero is not contained in this interval, we are 95% confident there is a significant difference in the accuracy of the two sources.

Second Study:

$$\left(0.86 - 0.82\right) \pm 1.96\sqrt{\frac{0.86(0.14)}{550} + \frac{0.82(0.18)}{550}}$$

$-0.0033 \leq p_1 - p_2 \leq 0.0833$

Since zero is contained in this interval, we are 95% confident there is not a significant difference in the accuracy of the two sources.

The lengths of the confidence intervals are different due to the sample sizes and subsequent standard errors.

5-75. a) 1) The parameters of interest are the proportions of those residents who wear a seat belt regularly, p_1 , p_2

2) H_0 : $p_1 = p_2$

3) H_1 : $p_1 \neq p_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{165}{200} = 0.825$ $\qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.807$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{198}{250} = 0.792$

$$z_0 = \frac{0.825 - 0.792}{\sqrt{0.807(1-0.807)\left(\frac{1}{200} + \frac{1}{250}\right)}} = 0.88$$

8) Since $-1.96 < 0.88 < 1.96$ do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.05$.

b) $\alpha = 0.10$
Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.05} = 1.65$
$z_0 = 0.88$

Since $-1.65 < 0.88 < 1.65$, do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.10$.

c) The conclusions are the same, but with different levels of confidence.

d) $n_1 = 400$, $n_2 = 500$

$\alpha = 0.05$
Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

$$z_0 = \frac{0.825 - 0.792}{\sqrt{0.807(1-0.807)\left(\frac{1}{400} + \frac{1}{500}\right)}} = 1.25$$

Since $-1.96 < 1.25 < 1.96$ do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.05$.

$\alpha = 0.10$
Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.05} = 1.65$
$z_0 = 1.012$

Since $-1.65 < 1.012 < 1.65$, do not reject H_0 and conclude that evidence is insufficient to claim that there is a difference in seat belt usage $\alpha = 0.10$.

As the sample size increased, the test statistic has also increased, since the denominator of z_0 decreased. However, the decrease (or sample size increase) was not enough to change our conclusion.

5-76. a) Yes, there could be some bias in the results due to the survey and subsequent collection of the data.

b) If it could be shown that these populations are similar to the respondents, the results may be extended.

5-77. a) 1) The parameters of interest are the proportion of lenses that are unsatisfactory after tumble-polishing, p_1, p_2
2) $H_0 : p_1 = p_2$
3) $H_1 : p_1 \neq p_2$
4) $\alpha = 0.01$
5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = 2.58$

7) x_1 =number of defective lenses

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{47}{300} = 0.1567 \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.2517$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{104}{300} = 0..3467$$

$$z_0 = \frac{0.1567 - 0.3467}{\sqrt{0.2517(1-0.2517)\left(\frac{1}{300} + \frac{1}{300}\right)}} = -5.36$$

8) Since $-5.36 < -2.58$ reject H_0 and conclude there is strong evidence to support the claim that the two polishing fluids are different.

b) The conclusions are the same whether we analyze the data using the proportion unsatisfactory or proportion satisfactory. The proportions of defectives are different for the two fluids.

5-78. The parameter of interest is $\mu_1 - 2\mu_2$

$$H_0: \mu_1 = 2\mu_2 \qquad H_0: \mu_1 - 2\mu_2 = 0$$
$$\rightarrow$$
$$H_1: \mu_1 > 2\mu_2 \qquad H_1: \mu_1 - 2\mu_2 > 0$$

Let n_1 = size of sample 1 $\qquad \overline{X}_1$ estimate for μ_1

Let n_2 = size of sample 2 $\qquad \overline{X}_2$ estimate for μ_2

$\overline{X}_1 - 2\overline{X}_2$ is an estimate for $\mu_1 - 2\mu_2$

The variance is $V(\overline{X}_1 - 2\overline{X}_2) = V(\overline{X}_1) + V(2\overline{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{4\sigma_2^2}{n_2}$

The test statistic for this hypothesis would then be:

$$Z_0 = \frac{(\overline{X}_1 - 2\overline{X}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{4\sigma_2^1}{n_2}}}$$

We would reject the null hypothesis if $z_0 > z_{\alpha/2}$ for a given level of significance.
The P-value would be $P(Z \geq z_0)$.

5-79. $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$n_1 = n_2 = n$

$\beta = 0.10$

$\alpha = 0.05$

Assume normal distribution and $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$\mu_1 = \mu_2 + \sigma$

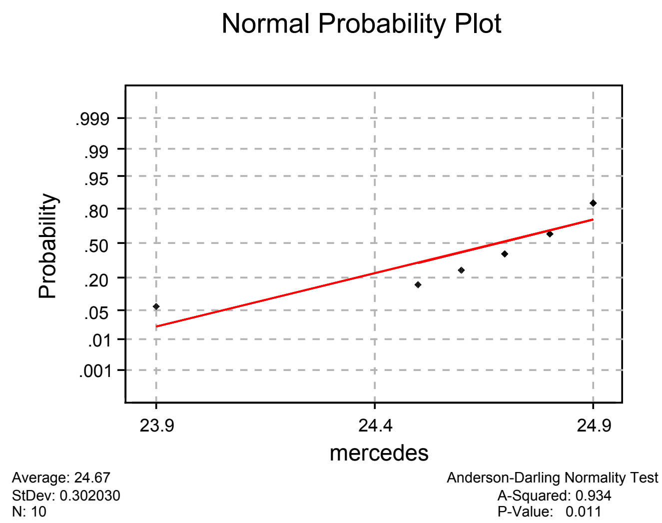
$$d = \frac{|\mu_1 - \mu_2|}{2\sigma} = \frac{\sigma}{2\sigma} = \frac{1}{2}$$

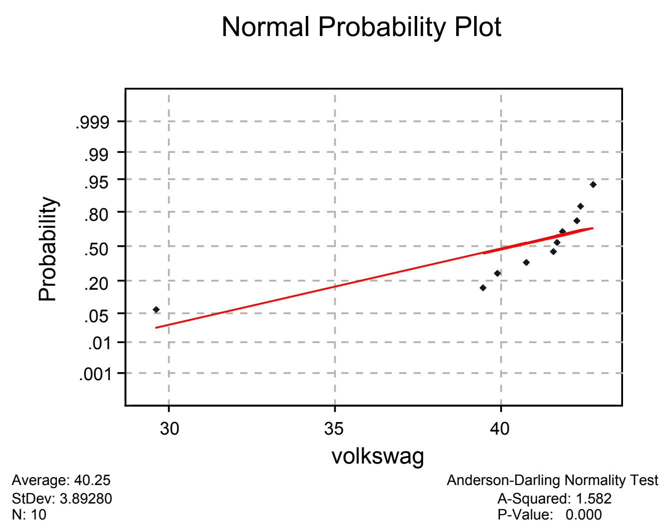
From Chart V e), $n^* = 42$

$$n = \frac{n^* + 1}{2} = \frac{44 + 1}{2} = 22.5$$

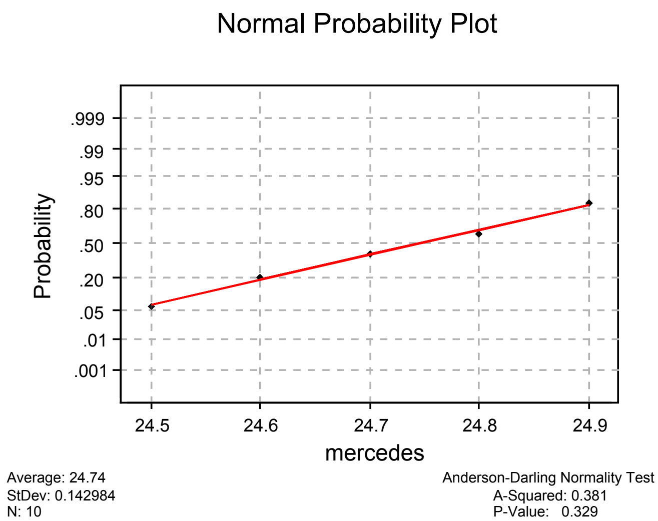
$n_1 = n_2 = 23$

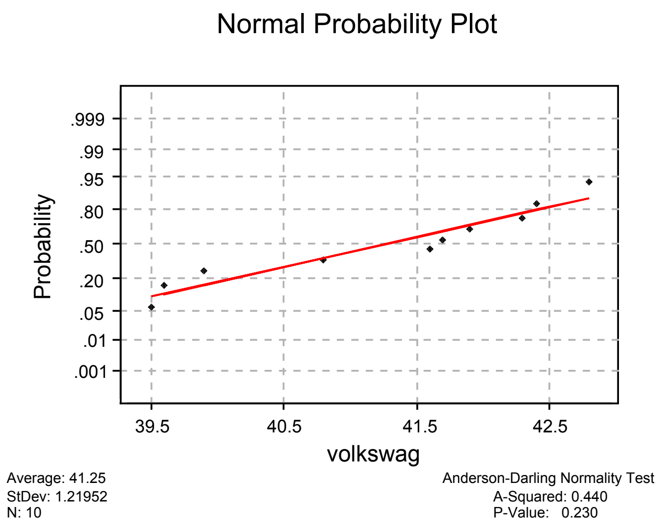
5-80. a) No.





b) The normal probability plots indicate that the data follow normal distributions since the data appear to fall along a straight line. The plots also indicate that the variances could be equal since the slopes appear to be the same.





c) By correcting the data points, it is more apparent the data follow normal distributions. Apparently, one observation can cause an analyst to reject the normality assumption.

d) 95% confidence interval on the ratio of the variances, σ_V^2 / σ_M^2

$s_V^2 = 1.49 \qquad f_{9,9,0.025} = 4.03$

$s_M^2 = 0.0204 \qquad f_{9,9,0.975} = \dfrac{1}{f_{9,9,0.025}} = \dfrac{1}{4.03} = 0.248$

$$\left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.975} < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{s_V^2}{s_M^2}\right) f_{9,9,0.025}$$

$$\left(\frac{1.49}{0.0204}\right) 0.248 < \frac{\sigma_V^2}{\sigma_M^2} < \left(\frac{1.49}{0.0204}\right) 4.03$$

$$18.124 < \frac{\sigma_V^2}{\sigma_M^2} < 294.35$$

Since the interval covers a range larger than 1 and not including 1, we are 95% confident that there is evidence to support the claim that the variability in mileage performance is greater for a Volkswagen than for a Mercedes.

5-81. a) Assume normal distribution and $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$\mu_1 = \mu_2 + 1.5$

$$d = \frac{|\mu_1 - \mu_2|}{2\sigma} = \frac{1.5}{2(0.471)} = 1.6$$

Using Chart Va with d = 1.6 and $\beta = 0.05$ ($\beta = 1 - 0.95$), we find $n^* = 7$ resulting in n = (7 + 1)/2 = 4. We would need n = 4 to reject the null hypothesis that the two agents differ by 1.5 with probability of at least 0.95

b) The original size of n = 5 in Exercise 5-14 was appropriate to detect the difference since it is necessary to only have a sample size of 4 to reject the null hypothesis that the two agents differ by 1.5 with probability of at least 0.95.

5-82. a) Answers given in bold.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-value
Factor	4.1408	**2**	**2.0704**	**60.538**	**< 0.01**
Error	**0.7188**	21	**0.0342**		
Total	4.8596	**23**			

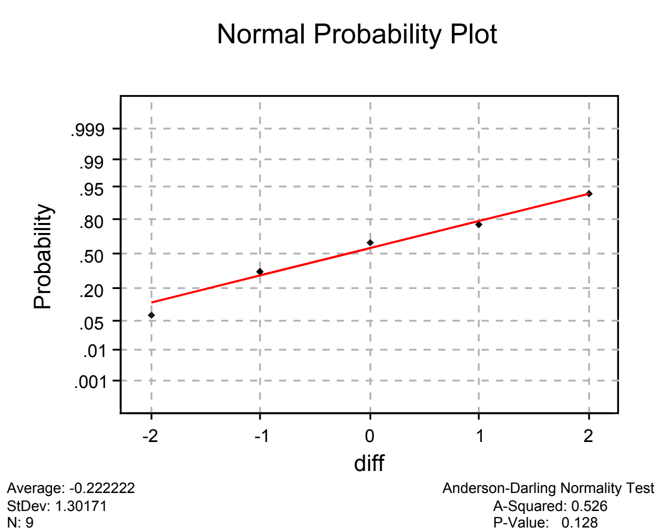
b) Using the information found in part a) with a p-value less than 0.01 (thus less than $\alpha = 0.10$), we would reject the null hypothesis that all weights are the same. Therefore, there is a significant difference among the alloy types.

5-83. a) Answers given in bold.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	95.129	**4**	**23.782**	**4.112**
Error	**86.752**	**15**	**5.783**	
Total	**181.881**	19		

b) $f_{0.05,4,15} = 3.06$; Since $F_0 = 4.112 > 3.06$, reject H_0 and conclude that there is a significant difference among the five types of foam pads.

5-84. a) The assumption of normality appears to be valid. This is evident by the fact that the data lie along a straight line in the normal probability plot.



b) 1) The parameter of interest is the mean difference in tip hardness, μ_d

2) $H_0 : \mu_d = 0$

3) $H_1 : \mu_d \neq 0$

4) No significance level, calculate P-value

5) The test statistic is

$$t_0 = \frac{\overline{d}}{s_d / \sqrt{n}}$$

6) Reject H_0 if the P-value is significantly small.

7) $\overline{d} = -0.222$

$s_d = 1.30$

$n = 9$

$$t_0 = \frac{-0.222}{1.30 / \sqrt{9}} = -0.512$$

8) P-value = 2P(T < -0.512) = 2P(T > 0.512)   2(0.25) < P-value < 2(0.40)
0.50 < P-value < 0.80

Since the P-value is larger than any acceptable level of significance, do not reject H_0 and conclude there is no difference in mean tip hardness.

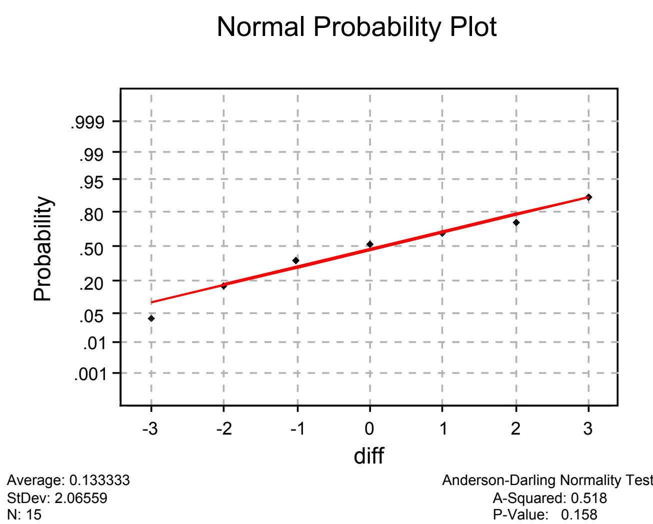
c) $\beta = 0.10$

$\mu_d = 1$

$$d = \frac{1}{\sigma_d} = \frac{1}{1.3} = 0.769$$

From Chart V e) with $\alpha = 0.01$, n = 50 coupons

5-85. a) According to the normal probability plot the data appear to follow a normal distribution. This is evident by the fact that the data fall along a straight line.



b) 1) The parameter of interest is the mean difference in depth using the two gauges, μ_d

2) $H_0 : \mu_d = 0$

3) $H_1 : \mu_d \neq 0$

4) No significance level, calculate p-value

5) The test statistic is

$$t_0 = \frac{\overline{d}}{s_d / \sqrt{n}}$$

6) Reject H_0 if the P-value is significantly small.

7) $\overline{d} = 0.133$

$s_d = 2.065$
$n = 15$

$$t_0 = \frac{0.133}{2.065/\sqrt{15}} = 0.25$$

8) P-value = 2P(T > 0.25)   2(0.40) < P-value
0.80 < P-value

Since the P-value is larger than any acceptable level of significance, do not reject H_0 and conclude there is no difference in mean depth measurements for the two gauges.

c) $1 - \beta = 0.80$
$\mu_d = 1.65$
$$d = \frac{1.65}{2\sigma_d} = \frac{1.65}{2(2.066)} = 0.40$$

From Chart V with $\alpha = 0.01$ and $1 - \beta = 0.80$, we find n = 22. [Note: if using Minitab, use One-sample t-test to find the sample size with power = 0.80 and st. dev. = 2.066].

5-86. a)

Analysis of Variance

Source	DF	SS	MS	F	P
Factor	3	22.124	7.375	14.85	0.000
Error	16	7.948	0.497		
Total	19	30.072			

Reject H_0. There is significant evidence to indicate the presence of oxygen during preparation affects the mean transition temperature

b) P-value = 0

5-87. a) Analysis of Variance for Strength

Source	DF	SS	MS	F	P
AirVoids	2	1230.2	615.1	8.30	0.002
Error	21	1555.8	74.1		
Total	23	2786.0			

The different levels of air voids significantly affect mean retained strength.

b) P-value = 0.002

5-88. a)

Analysis of Variance for radon

Source	DF	SS	MS	F	P
Diameter	5	1133.37	226.67	30.85	0.000
Error	18	132.25	7.35		
Total	23	1265.63			

Reject H_0. There is significant evidence to indicate the size of the orifice affect the mean percentage of radon released.

b) P-value = 0

5-89. a) The ANOVA table from Minitab is

Analysis of Variance for response time

Source	DF	SS	MS	F	P
percent	3	0.0683	0.0228	1.57	0.228
Error	20	0.2900	0.0145		
Total	23	0.3583			

b) The P-value was found to be 0.228. Since 0.228 > 0.05, do not reject the null hypothesis and conclude that percentage of the server allocated to memory does not change the response time of the server.

Example Team Problem

5-90. The requested result can be obtained from data in which the pairs are very different.
Example:

pair	1	2	3	4	5
sample 1	100	10	50	20	70
sample 2	110	20	59	31	80

$\bar{x}_1 = 50$ $\bar{x}_2 = 60$

$s_1 = 36.74$ $s_2 = 36.54$

$$s_p = \sqrt{\frac{4(36.74)^2 + 4(36.54)^2}{8}} = 36.64$$

Two-Sample t-test:

$$t_0 = \frac{(50-60)}{36.64\sqrt{\frac{1}{5}+\frac{1}{5}}} = -0.43$$

P-value = $2P(T < -0.43) = 2P(T > 0.43)$ $2(0.25) < \text{P-value} < 2(0.40)$

$0.50 < \text{P-value} < 0.80$

Paired t-test:

$\bar{d} = -10$

$s_d = 0.707$

$n = 5$

$$t_0 = \frac{-10}{0.707/\sqrt{5}} = -31.62$$

P-value = $2P(T < -31.62) = 2P(T > 31.62) = 2(0) = 0$