CHAPTER 7

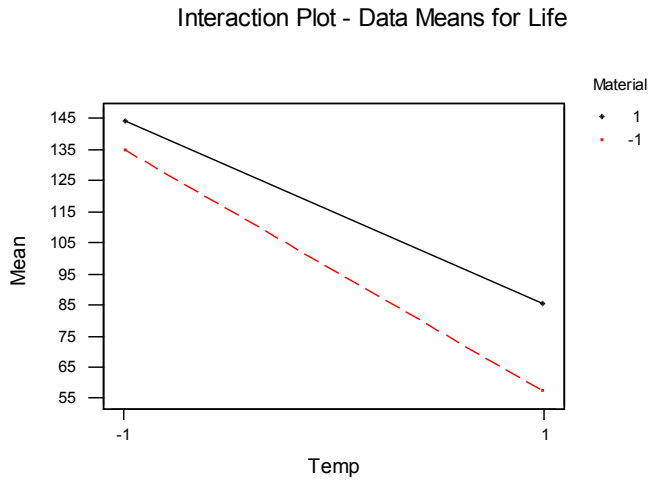
Notes to the Instructor: The normal probability plots are constructed in Minitab using the command 'Normality Test' under "Basic Statistics".
The analysis of variance is carried out using "ANOVA" under 'Stat' in Minitab. In cases where there are exactly two levels for each factor of interest, use "DOE" under 'Stat'.
Hierarchial models are used throughout.
To use "Regression" to find an appropriate model, the interactions must be entered by hand. The data files on disk contain the design shown in each problem and columns for the corresponding interactions.

Section 7-4

7-1. a)

Predictor	Coef	StDev	T	P
Constant	105.438	7.730	13.64	0.000
Material	9.313	7.730	1.20	0.252
Temp	-33.938	7.730	-4.39	0.001
Mat*Temp	4.687	7.730	0.61	0.556

b)



The interaction plot does not indicate a strong interaction between temperature and material.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that temperature is significant, but material and the interaction material*temperature are not at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where
effect = 2(coefficient). The coefficient is given in the Minitab output of part a.
s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Temperature: effect = 2(coefficient) = 2(-33.938) = -67.876
s.e.(effect) = 2(7.73) = 15.46
Approximate 95% confidence interval on the effect of Temperature:

-67.876 ± 2(15.46)
(–98.796, -36.956)

Material: effect = 2(coefficient) = 2(9.313) = 18.626
s.e.(effect) = 2(7.73) = 15.46
Approximate 95% confidence interval for the effect of Material:

18.626 ± 2(15.46)
(-12.294, 49.546)

Material*Temperature: effect = 2(coefficient) = 2(4.687) = 9.374
s.e.(effect) = 2(7.73) = 15.46
Approximate 95% confidence interval for the effect of Material*Temperature:

9.374 ± 2(15.46)
(–21.546, 40.294)

e)

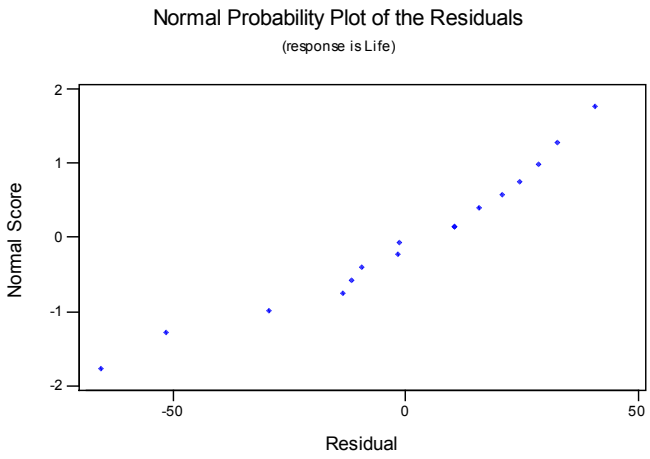
```
Life = 105 + 9.31 Material - 33.9 Temp + 4.69 Mat*Temp

Predictor         Coef       StDev          T        P
Constant       105.438       7.730      13.64    0.000
Material         9.313       7.730       1.20    0.252
Temp           -33.938       7.730      -4.39    0.001
Mat*Temp         4.687       7.730       0.61    0.556
```

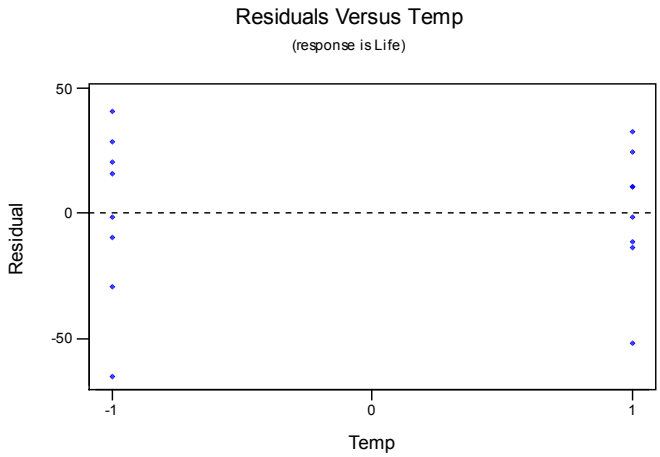
Based on the regression analysis, only temperature appears to be the significant factor. This result is equivalent to that obtained in part c.

The final regression analysis and model are

```
Life = 105 - 33.9 Temp

Predictor         Coef       StDev          T        P
Constant       105.438       7.680      13.73    0.000
Temp           -33.938       7.680      -4.42    0.001

S = 30.72       R-Sq = 58.2%     R-Sq(adj) = 55.3%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1       18428       18428     19.53    0.001
Residual Error    14       13212         944
Total             15       31640
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value = 0.001.



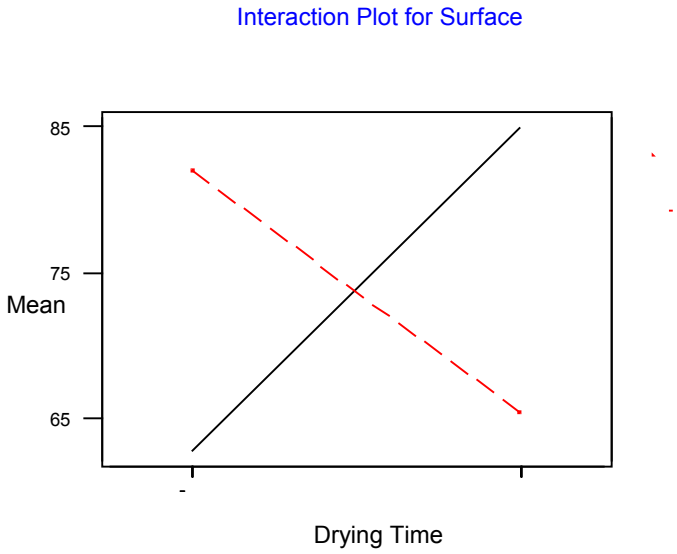


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The assumption of constant variance does not appear to be violated. The residuals appear to have the same spread for both levels of Temperature.

7-2. a)

Predictor	Coef	StDev	T	P
Constant	73.750	3.955	18.65	0.000
Paint	-0.083	3.955	-0.02	0.984
Time	1.417	3.955	0.36	0.729
Paint*Ti	-9.750	3.955	-2.47	0.039

b)



The interaction plot indicates an interaction between drying time and paint type.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that the interaction of Paint type*Drying Time is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate $\pm$ 2(s.e.(effect))
where
effect = 2(coefficient). The coefficient is given in the Minitab output of part a.
s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Paint Type: effect = 2(coefficient) = 2(-0.083) = -0.167
s.e.(effect) = 2(3.955) = 7.91
Approximate 95% confidence interval on the the effect of Paint Type:

$-0.167 \pm 2(7.91)$
$(-15.987, 15.653)$

Drying Time: effect = 2(coefficient) = 2(1.417) = 2.834
s.e.(effect) = 2(3.955) = 7.91
Approximate 95% confidence interval for the effect of Drying Time:

$2.834 \pm 2(7.91)$
$(-12.986, 18.654)$

Paint Type*Drying Time: effect = 2(coefficient) = 2(-9.750) = -19.50
s.e.(effect) = 2(3.955) = 7.91
Approximate 95% confidence interval for the effect of Paint Type*Drying Time:

$-19.50 \pm 2(7.91)$
$(-35.32, -3.68)$

e) The regression analysis and, in this case, the final model are

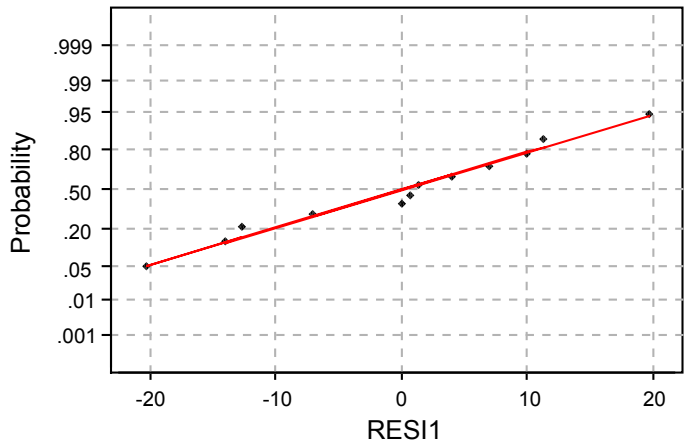
```
Surface Finish = 73.8 - 0.08 Paint Type + 1.42 Drying Time
                 - 9.75 Paint Type*Drying Time

Predictor       Coef       StDev          T        P
Constant      73.750       3.955      18.65    0.000
Paint Ty      -0.083       3.955      -0.02    0.984
Drying T       1.417       3.955       0.36    0.729
Paint*Drying  -9.750       3.955      -2.47    0.039

S = 13.70       R-Sq = 43.7%     R-Sq(adj) = 22.6%
```

Based on the regression analysis, only the interaction appears to be significant. Since we have adopted the procedure of using hierarchical models, then this result is equivalent to that obtained in part c.

```
Analysis of Variance

Source       DF          SS          MS         F        P
Regression    3      1164.9       388.3      2.07    0.183
Error         8      1501.3       187.7
Total        11      2666.2

Source       DF      Seq SS
Paint Ty      1         0.1
Drying T      1        24.1
Paint Ty      1      1140.7
```

The analysis of variance may indicate the final regression model is inadequate for this set of data. The adequacy may be influenced by the inclusion of the insignificant main effects to maintain hierarchy.

Normal Probability Plot

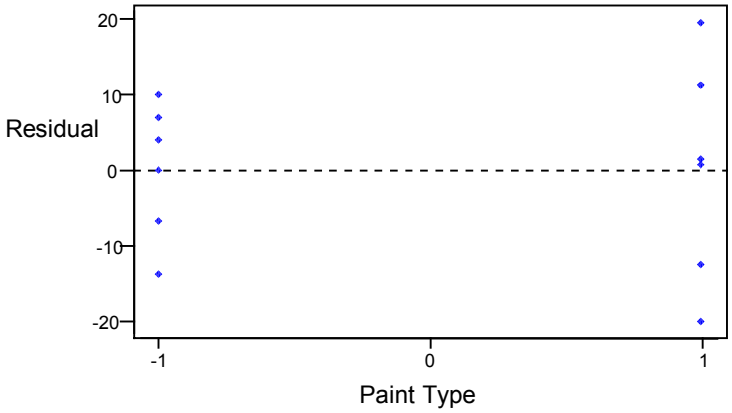


Average: -0.0000000
StDev: 11.6827
N: 12

Anderson-Darling Normality Test
A-Squared: 0.200
P-Value: 0.846

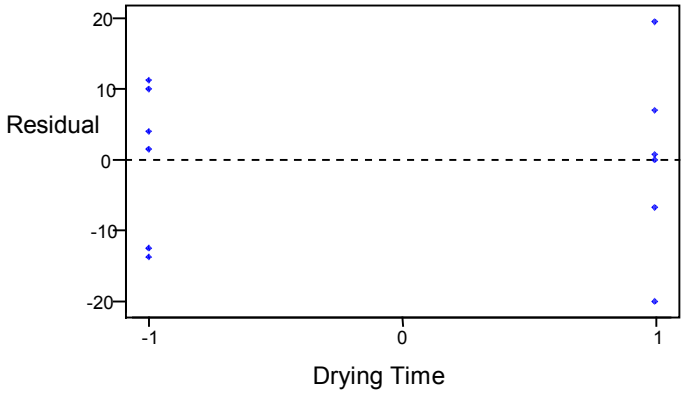
Residuals Versus Paint Type

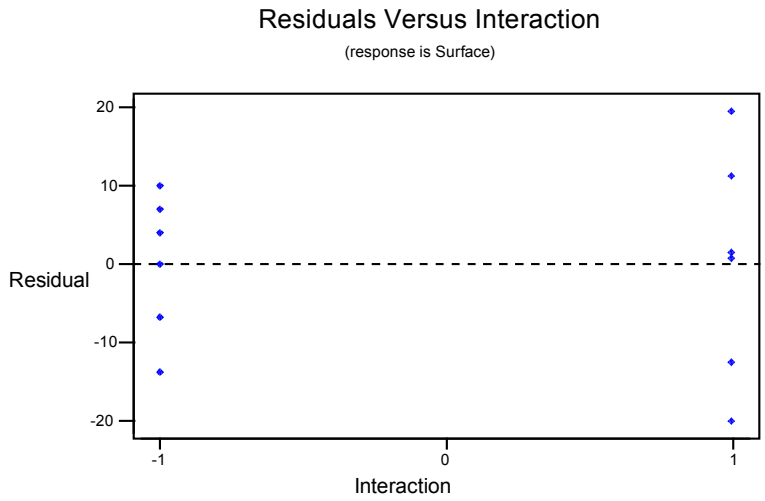
(response is Surface Finish)



Residuals Versus Drying Time

(response is Surface Finish)



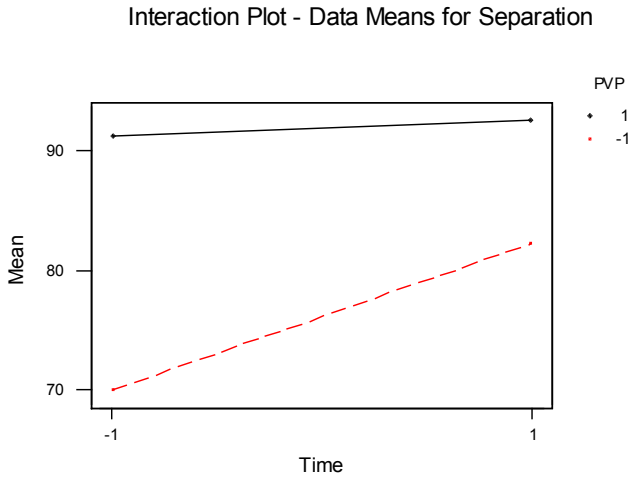


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The assumption of constant variance does not appear to be seriously violated. The residuals for the high level of the interaction appear to be slightly more spread out than those for the low level.

7-3. a)

Predictor	Coef	StDev	T	P
Constant	83.9500	0.5744	146.16	0.000
PVP	7.8250	0.5744	13.62	0.000
Time	3.3875	0.5744	5.90	0.000
PVP*Time	-2.7125	0.5744	-4.72	0.000

b)



The interaction plot indicates a significant interaction between PVP and Time.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that both PVP, Time, and the interaction PVP*Time are significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate $\pm$ 2(s.e.(effect))
where
effect = 2(coefficient). The coefficient is given in the Minitab output of part a.
s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

PVP: effect = 2(coefficient) = 2(7.825) = 15.65
s.e.(effect) = 2(0.5744) = 1.149
Approximate 95% confidence interval on the effect of PVP:

$15.65 \pm 2(1.149)$
(13.352, 17.948)

Time: effect = 2(coefficient) = 2(3.3875) = 6.775
s.e.(effect) =2(0.5744) = 1.149
Approximate 95% confidence interval for the effect of Time:

$6.775 \pm 2(1.149)$
(4.477, 9.073)

PVP*Time: effect = 2(-2.7125) = -5.425
s.e.(effect) =2(0.5744) = 1.149
Approximate 95% confidence interval for the effect of PVP*TIme:

$-5.425 \pm 2(1.149)$
(–7.723, –3.127)

e)

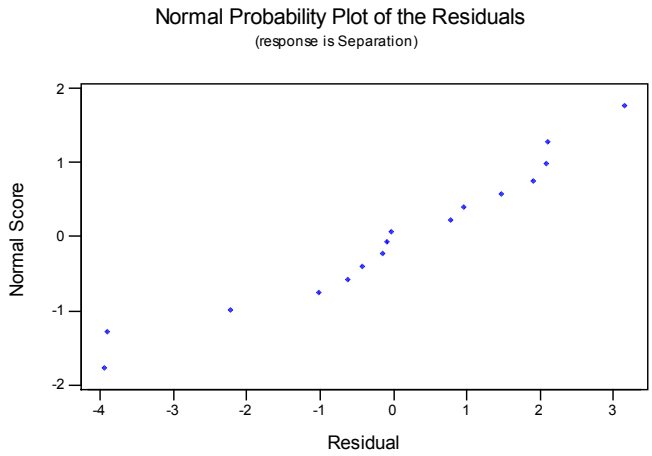
```
The regression equation is
current = 265 - 30.0 Glass - 8.33 Phosphor - 1.67 Glass*Phosphor

Predictor       Coef       StDev          T        P
Constant     265.000       2.357     112.43    0.000
Glass        -30.000       2.357     -12.73    0.000
Phosphor      -8.333       2.357      -3.54    0.008
Glass*Ph      -1.667       2.357      -0.71    0.500
```

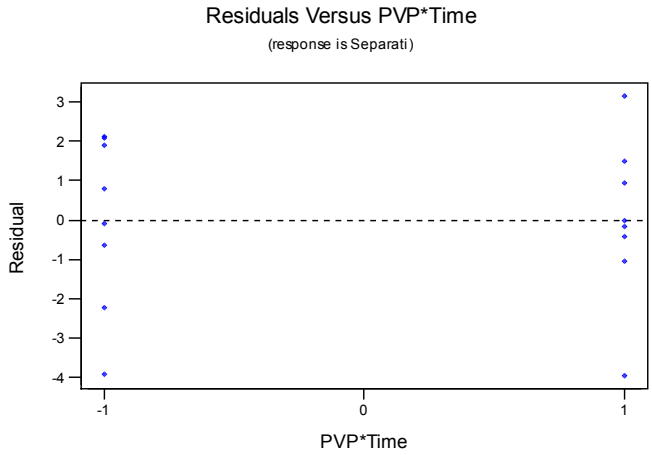
Based on the regression analysis, both Glass type and Phosphor type appear to be the significant factors. This result is equivalent to that obtained in part c.

The regression analysis and final model are:

```
Separation = 84.0 + 7.82 PVP + 3.39 Time - 2.71 PVP*Time

Predictor        Coef       StDev          T        P
Constant      83.9500      0.5744     146.16    0.000
PVP            7.8250      0.5744      13.62    0.000
Time           3.3875      0.5744       5.90    0.000
PVP*Time      -2.7125      0.5744      -4.72    0.000

S = 2.298       R-Sq = 95.3%     R-Sq(adj) = 94.1%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         3     1281.01      427.00     80.89    0.000
Residual Error    12       63.34        5.28
Total             15     1344.36
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value $\cong 0.000$.



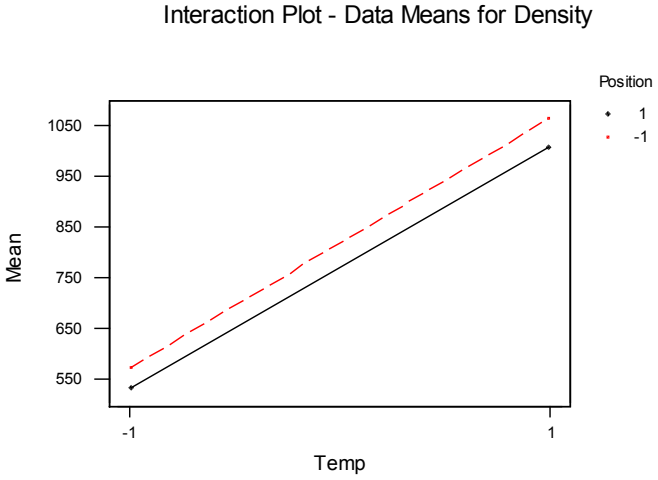


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The constant variance assumption appears to be satisfied also.

7-4. a)

Predictor	Coef	StDev	T	P
Constant	793.167	4.505	176.05	0.000
Position	-24.167	4.505	-5.36	0.001
Temp	240.833	4.505	53.45	0.000
Pos*Temp	-3.833	4.505	-0.85	0.420

b)



The interaction plot does not indicate a significant interaction between temperature and position.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that both Position and Temperature are significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where
effect = 2(coefficient). The coefficient is given in the Minitab output of part a.
s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Position: effect = 2(-24.167) = -48.334
s.e.(effect) = 2(4.505) = 9.01
Approximate 95% confidence interval on the effect of Position:

–48.334 ± 2(9.01)
(–66.354, –30.314)

Temperature: effect = 2(240.833)= 481.666
s.e.(effect) =2(4.505) = 9.01
Approximate 95% confidence interval for the effect of Temperature:

481.666 ± 2(9.01)
(463.646, 499.686)

Position*Temperature: effect = 2(-3.833) = -7.666
s.e.(effect) =2(4.505) = 9.01
Approximate 95% confidence interval for the effect of Position*Temperature:

-7.666 ± 2(9.01)
(–25.686, 10.354)

e)

```
Density = 793 - 24.2 Position + 241 Temp - 3.83 Pos*Temp

Predictor        Coef       StDev          T        P
Constant      793.167       4.505     176.05    0.000
Position      -24.167       4.505      -5.36    0.001
Temp          240.833       4.505      53.45    0.000
Pos*Temp       -3.833       4.505      -0.85    0.420

S = 15.61       R-Sq = 99.7%      R-Sq(adj) = 99.6%
```

Based on the regression analysis, both Position and Temperature appear to be the significant factors. This result is equivalent to that obtained in part c.

Density = 793 - 24.2 Position + 241 Temp

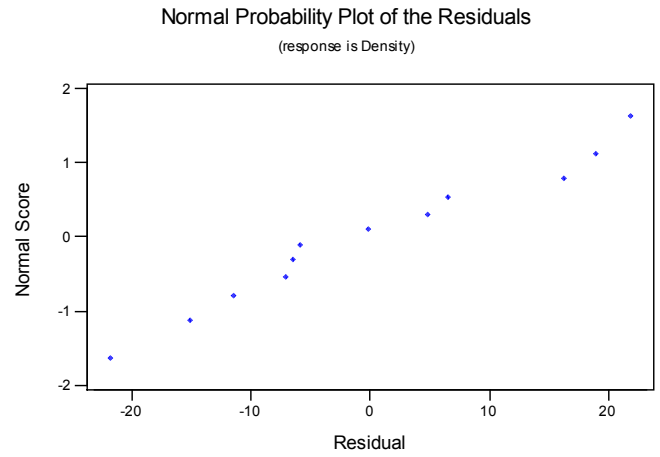
Predictor	Coef	StDev	T	P
Constant	793.167	4.436	178.81	0.000
Position	-24.167	4.436	-5.45	0.000
Temp	240.833	4.436	54.29	0.000

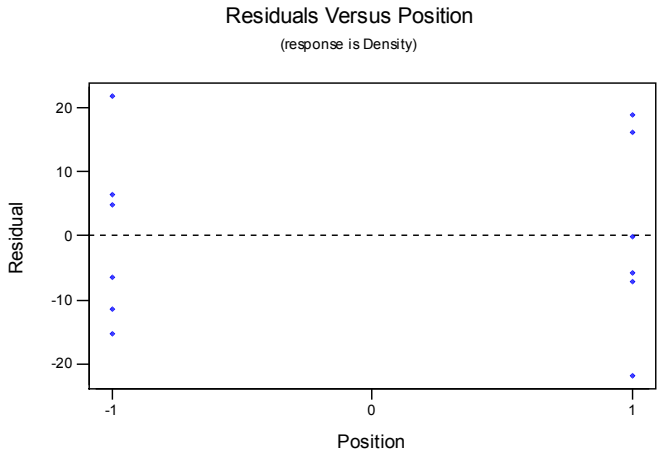
S = 15.37 R-Sq = 99.7% R-Sq(adj) = 99.6%

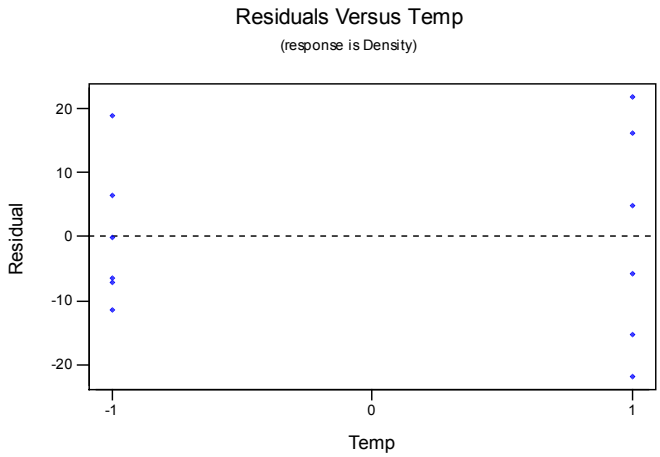
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	703017	351508	1488.74	0.000
Residual Error	9	2125	236		
Total	11	705142			

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.





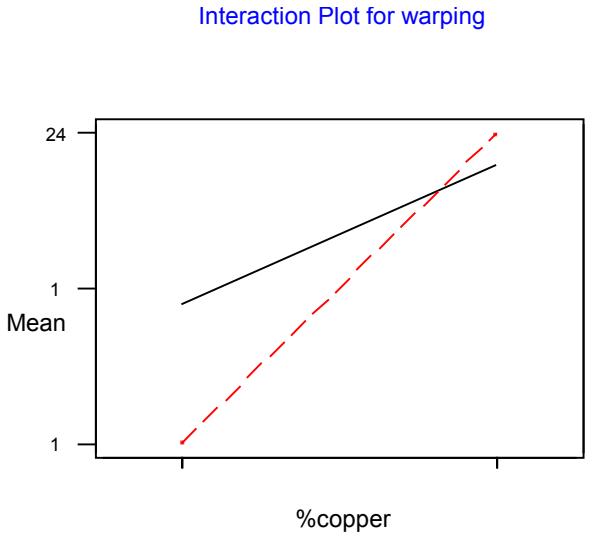


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The assumption of constant variance does not appear to be violated. The residuals appear to have approximately the same spread for both levels of Temperature and both levels of Position.

7-5. a)

Predictor	Coef	StDev	T	P
Constant	19.8750	0.7181	27.68	0.000
Temp	-0.8750	0.7181	-1.22	0.290
Copper	3.6250	0.7181	5.05	0.007
Temp*Cop	1.3750	0.7181	1.91	0.128

b)



The interaction plot indicates there may be a slightly significant interaction between %Copper and Temperature.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that only %Copper is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Temperature: effect = 2(-0.8750) = -1.750

s.e.(effect) = 2(0.7181) = 1.436

Approximate 95% confidence interval on the effect of Temperature:

$-1.75 \pm 2(1.436)$
$(-4.622, 1.122)$

%Copper: effect = 2(3.6250) = 7.25
s.e.(effect) =2(0.7181) = 1.436

Approximate 95% confidence interval for the effect of %Copper:

$7.25 \pm 2(1.436)$
$(4.378, 10.122)$

Temperature*%Copper: effect = 2(1.3750) = 2.75
s.e.(effect) =2(0.7181) = 1.436

Approximate 95% confidence interval for the effect of Temperature*%Copper:

$2.75 \pm 2(1.436)$
$(-0.122, 5.622)$

e)

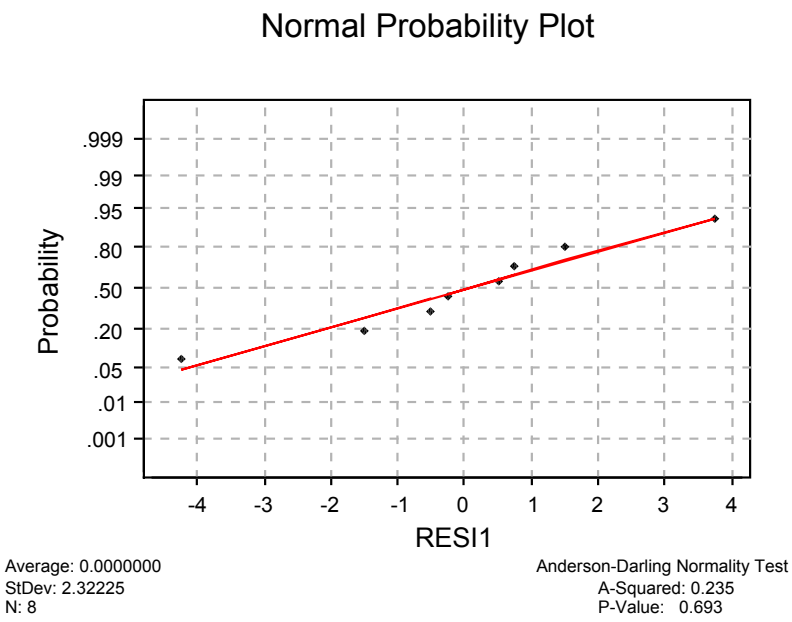
```
The regression equation is
warping = 19.9 - 0.875 temp + 3.63 %copper + 1.38 Temp*%Copper

Predictor       Coef       StDev          T        P
Constant     19.8750      0.7181      27.68    0.000
temp         -0.8750      0.7181      -1.22    0.290
%copper       3.6250      0.7181       5.05    0.007
Temp*%Co      1.3750      0.7181       1.91    0.128
```

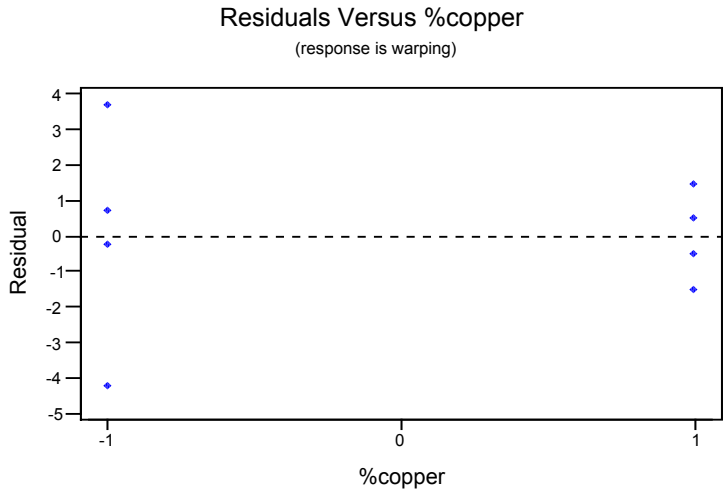
Based on the regression analysis, only %Copper appears to be the significant factor. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
warping = 19.9 + 3.62 %copper

Predictor       Coef       StDev          T        P
Constant     19.8750      0.8868      22.41    0.000
%copper       3.6250      0.8868       4.09    0.006

S = 2.508       R-Sq = 73.6%     R-Sq(adj) = 69.2%

Analysis of Variance

Source       DF          SS          MS          F        P
Regression    1      105.13      105.13     16.71    0.006
Error         6       37.75        6.29
Total         7      142.88
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value $\cong 0.006$.



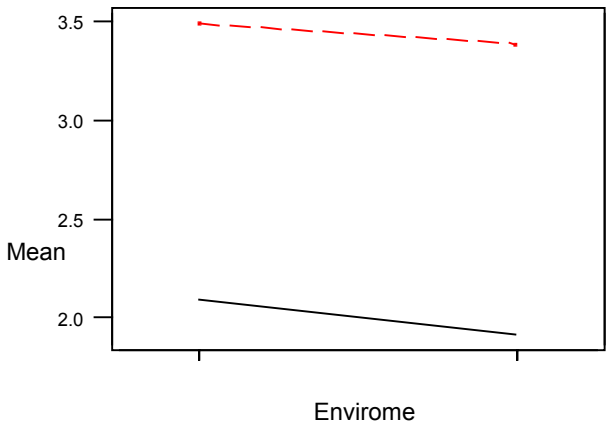


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The residuals for the low level of %Copper are more spread out than for the high level. The constant variance assumption may be a concern.

7-6. a)

Predictor	Coef	StDev	T	P
Constant	2.72187	0.07126	38.20	0.000
Freq	0.72062	0.07126	10.11	0.000
Environm	-0.07188	0.07126	-1.01	0.333
Freq*Env	0.01938	0.07126	0.27	0.790

b)



The interaction plot does not indicate a significant interaction between Enviroment and Frequency.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that only Frequency is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Frequency: s.e.(effect) = 2(0.07126) = 0.1425
Approximate 95% confidence interval on the effect of Frequency:

1.44125 ± 2(0.1425)
(1.299,1.584)

Enviroment: s.e.(effect) =2(0.07126) = 0.1425
Approximate 95% confidence interval for the effect of Enviroment:

–.14375± 2(0.1425)
(–0.428, 0.142)

Frequency*Enviroment: s.e.(effect) =2(0.07126) = 0.1425
Approximate 95% confidence interval for the effect of Frequency*Enviroment:

0.03875 ± 2(0.1425)
(–0.246, 0.324)

e)

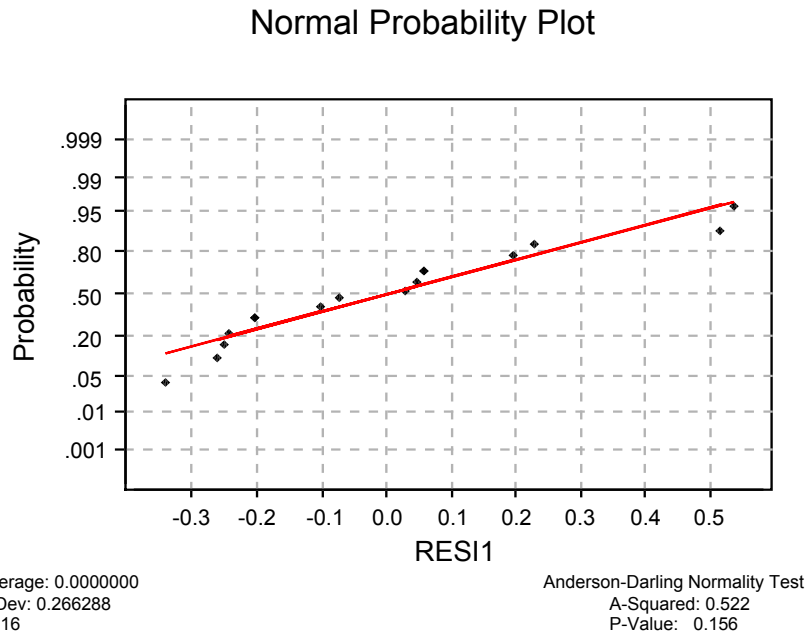
```
The regression equation is
Fatigue = 2.72 + 0.721 Frequency - 0.0719 Enviroment + 0.0194
Freq*Envir

Predictor       Coef       StDev          T        P
Constant     2.72187     0.07126      38.20    0.000
Frequenc     0.72063     0.07126      10.11    0.000
Envirome    -0.07187     0.07126      -1.01    0.333
Freq*Env     0.01938     0.07126       0.27    0.790
```

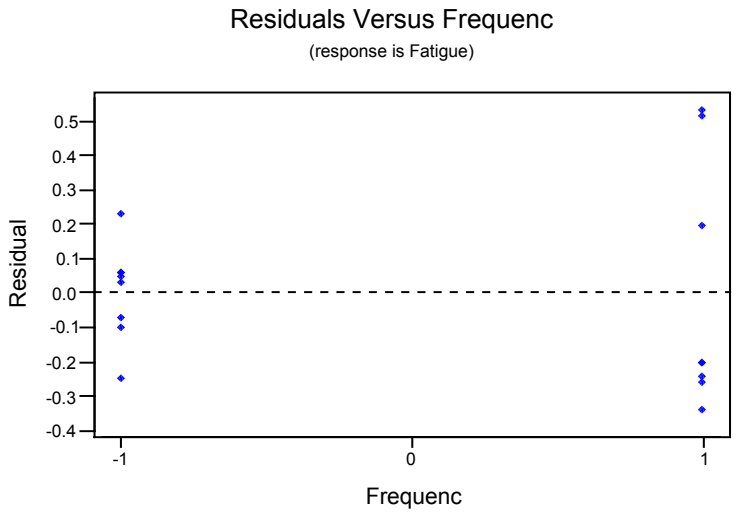
Based on the regression analysis, only Frequency appears to be the significant factor. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
Fatigue = 2.72 + 0.721 Frequency

Predictor       Coef       StDev          T        P
Constant     2.72187     0.06891      39.50    0.000
Frequenc     0.72063     0.06891      10.46    0.000

S = 0.2756      R-Sq = 88.7%     R-Sq(adj) = 87.8%

Analysis of Variance

Source       DF          SS          MS          F        P
Regression    1      8.3088      8.3088     109.36    0.000
Error        14      1.0636      0.0760
Total        15      9.3724

Unusual Observations
Obs  Frequenc    Fatigue        Fit  StDev Fit   Residual    St Resid
 10      1.00     3.9600     3.4425     0.0975     0.5175       2.01R
 12      1.00     3.9800     3.4425     0.0975     0.5375       2.08R

R denotes an observation with a large standardized residual
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.





There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The residuals for the high level of Frequency are more spread out than for the low level. The constant variance assumption may be a concern.

7-7. a)

Predictor	Coef	StDev	T	P
Constant	9.20500	0.07622	120.77	0.000
Doping	-0.24250	0.07622	-3.18	0.033
Anneal	1.01500	0.07622	13.32	0.000
Doping*A	0.16750	0.07622	2.20	0.093

b)

The interaction plot does not indicate a significant interaction between Polysilicon Doping and Anneal.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that both Polysilicon doping and Anneal are significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate $\pm$ 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Polysilicon Doping: s.e.(effect) = 2(0.07622) = 0.1524

Approximate 95% confidence interval on the the effect of Polysilicon Doping:

$-0.485 \pm 2(0.1524)$
$(-0.7898, -0.1802)$

Anneal: s.e.(effect) $=2(0.07622) = 0.1524$
Approximate 95% confidence interval for the effect of Anneal:

$2.03 \pm 2(0.1524)$
$(1.725, 2.34)$

Polysilicon doping*Anneal: s.e.(effect) $=2(0.07622) = 0.1524$
Approximate 95% confidence interval for the effect of Polysilicon doping*Anneal:

$0.335 \pm 2(0.1524)$
$(0.0302, 0.6398)$

e)

```
The regression equation is
Current = 9.21 - 0.242 Polysilicon + 1.02 Anneal + 0.167 Poly*Anneal

Predictor       Coef       StDev          T        P
Constant     9.20500     0.07622     120.77    0.000
Polysili    -0.24250     0.07622      -3.18    0.033
Anneal       1.01500     0.07622      13.32    0.000
Poly*Ann     0.16750     0.07622       2.20    0.093
```

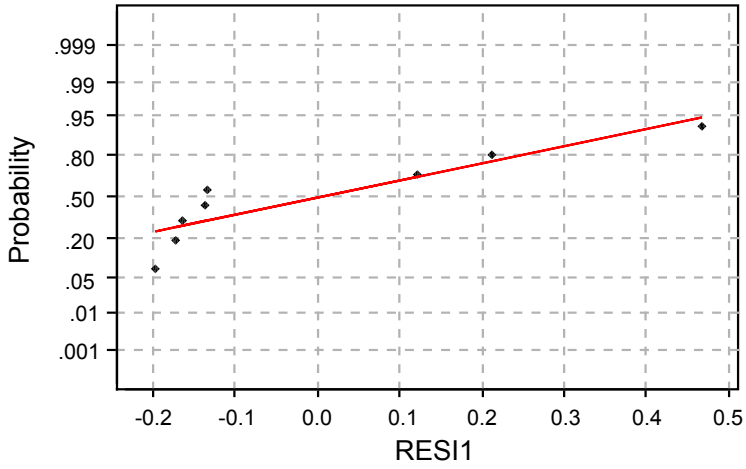
Based on the regression analysis, both Polysilicon and Anneal appear to be the significant factors. This Result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
Current = 9.21 - 0.242 Polysilicon + 1.02 Anneal

Predictor       Coef       StDev          T        P
Constant      9.2050      0.1013      90.88    0.000
Polysili     -0.2425      0.1013      -2.39    0.062
Anneal        1.0150      0.1013      10.02    0.000

S = 0.2865      R-Sq = 95.5%     R-Sq(adj) = 93.7%

Analysis of Variance

Source       DF          SS          MS          F        P
Regression    2      8.7123      4.3561     53.08    0.000
Error         5      0.4103      0.0821
Total         7      9.1226

Source       DF      Seq SS
Polysili      1      0.4704
Anneal        1      8.2418
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value $\cong 0.000$.

Normal Probability Plot

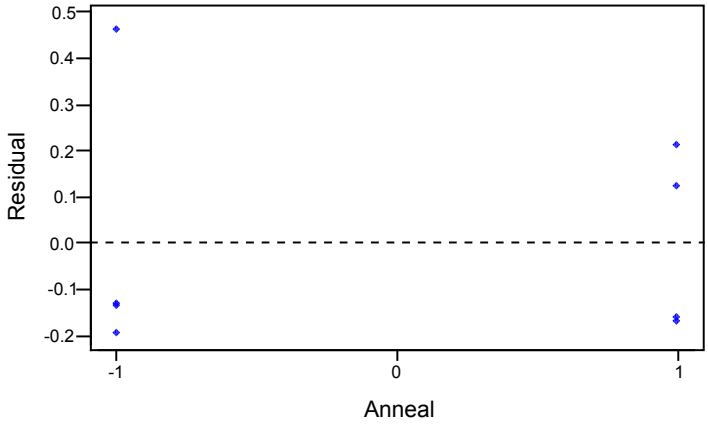


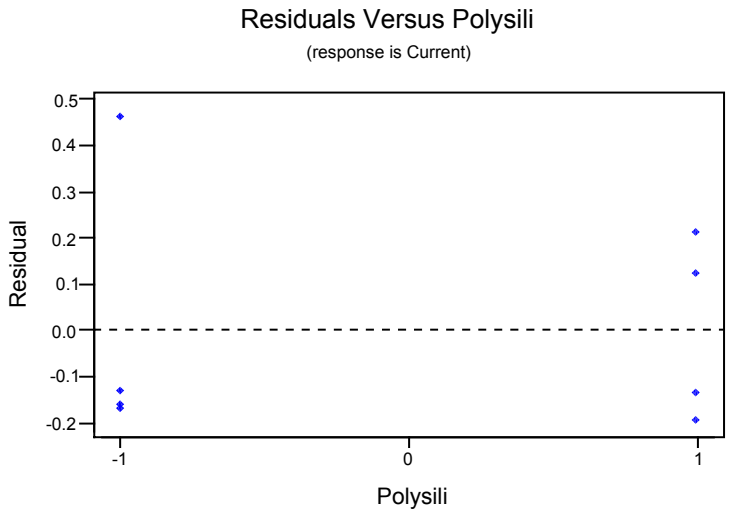
Average: 0
StDev: 0.242119
N: 8

Anderson-Darling Normality Test
A-Squared: 0.747
P-Value: 0.030

Residuals Versus Anneal

(response is Current)





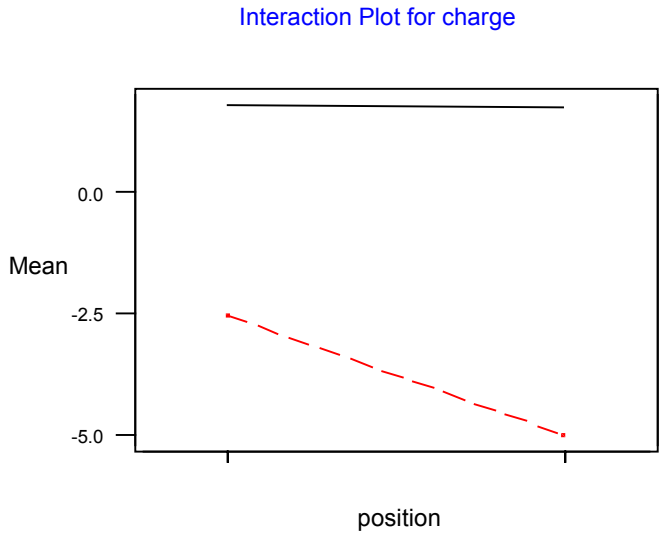
There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The residuals for the high level of Anneal and for Polysilicon are more spread out than for the low levels of each. The constant variance assumption may be a concern.

7-8. a)

Estimated Effects and Coefficients for charge

Term	Coef	StDev Coef	T	P
Constant	-1.000	0.4462	-2.24	0.055
method	-2.797	0.4462	-6.27	0.000
position	-0.640	0.4462	-1.43	0.189
method*position	-0.610	0.4462	-1.37	0.209

b)



The interaction plot does not indicate a significant interaction between Method and Position.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that only Method is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate $\pm$ 2(s.e.(effect))

where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Method: s.e.(effect) = 2(0.4462) = 0.8924
Approximate 95% confidence interval on the the effect of Method:

$-5.593 \pm 2(0.8924)$
$(-7.38, -0.3.81)$

Position: s.e.(effect) = 2(0.4462) = 0.8924
Approximate 95% confidence interval for the effect of Position:

$-1.28 \pm 2(0.8924)$
$(-3.065, 0.505)$

Method*Position: s.e.(effect) = 2(0.4462) = 0.8924
Approximate 95% confidence interval for the effect of Method*Position:

$-1.22 \pm 2(0.8924)$
$(-3.005, 0.565)$

e)

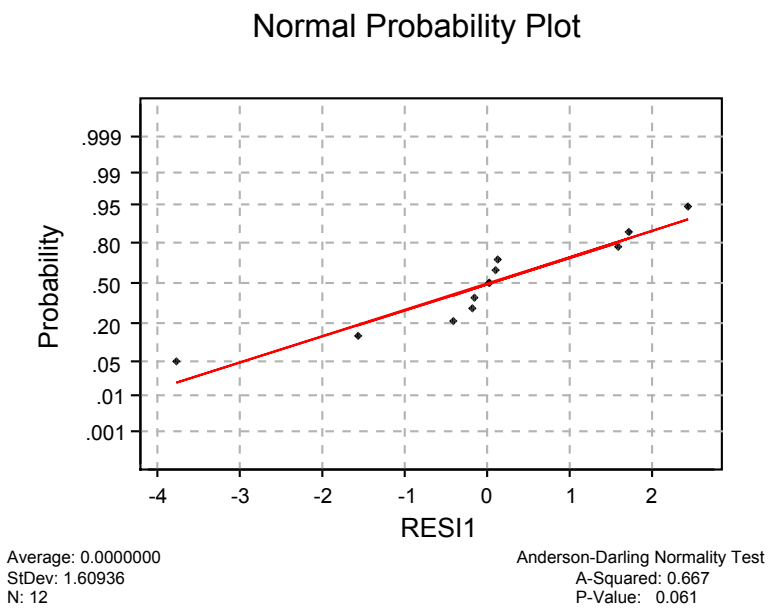
```
The regression equation is
charge = - 1.00 - 2.80 method - 0.640 position - 0.610 meth*pos

Predictor       Coef       StDev          T        P
Constant     -1.0000      0.4462      -2.24    0.055
method       -2.7967      0.4462      -6.27    0.000
position     -0.6400      0.4462      -1.43    0.189
meth*pos     -0.6100      0.4462      -1.37    0.209
```

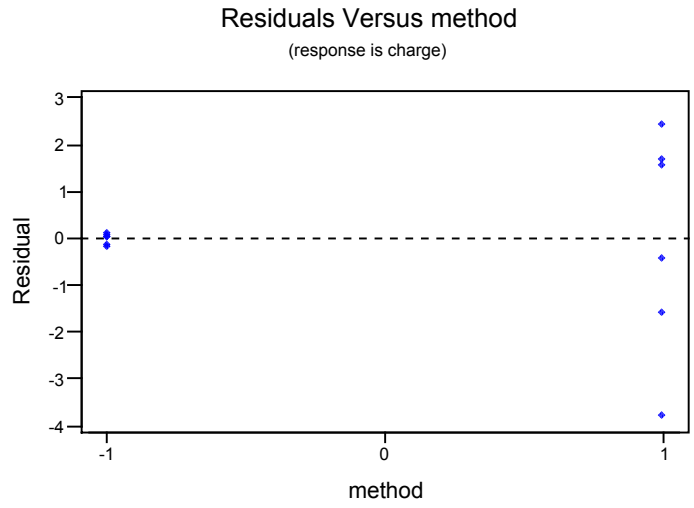
Based on the regression analysis, only Method appears to be the significant factor. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
charge = - 1.00 - 2.80 method

Predictor       Coef       StDev          T        P
Constant     -1.0000      0.4873      -2.05    0.067
method       -2.7967      0.4873      -5.74    0.000

S = 1.688       R-Sq = 76.7%     R-Sq(adj) = 74.4%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1      93.856      93.856     32.94    0.000
Error        10      28.490       2.849
Total        11     122.347

Unusual Observations
Obs    method     charge        Fit  StDev Fit   Residual    St Resid
  4      1.00     -7.580     -3.797      0.689     -3.783      -2.46R

R denotes an observation with a large standardized residual
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by P-value $\cong$ 0.000.





There appears to be a departure from normality shown in the normal probability plot of the residuals. The residuals for the high level of method are more spread out than for the low level. The constant variance assumption may be a concern.

Sections 7-5, 7-6, and 7-7

7-9. a)

Predictor	Coef	StDev	T	P
Constant	413.13	12.41	33.30	0.000
A	9.12	12.41	0.74	0.483
B	42.12	12.41	3.40	0.009
C	35.88	12.41	2.89	0.020
ab	-5.63	12.41	-0.45	0.662
ac	-59.63	12.41	-4.81	0.001
bc	-12.13	12.41	-0.98	0.357
abc	-17.38	12.41	-1.40	0.199

Based on the t-ratios and associated p-values, the significant factors are metal hardness, cutting angle,

Interaction cutting speed and cutting angle.

b)

The regression equation is
Tool life = 413 + 9.1 A + 42.1 B + 35.9 C - 59.6 AC

Predictor	Coef	StDev	T	P
Constant	413.12	12.47	33.12	0.000
A	9.12	12.47	0.73	0.480
B	42.12	12.47	3.38	0.006
C	35.88	12.47	2.88	0.015
AC	-59.62	12.47	-4.78	0.000

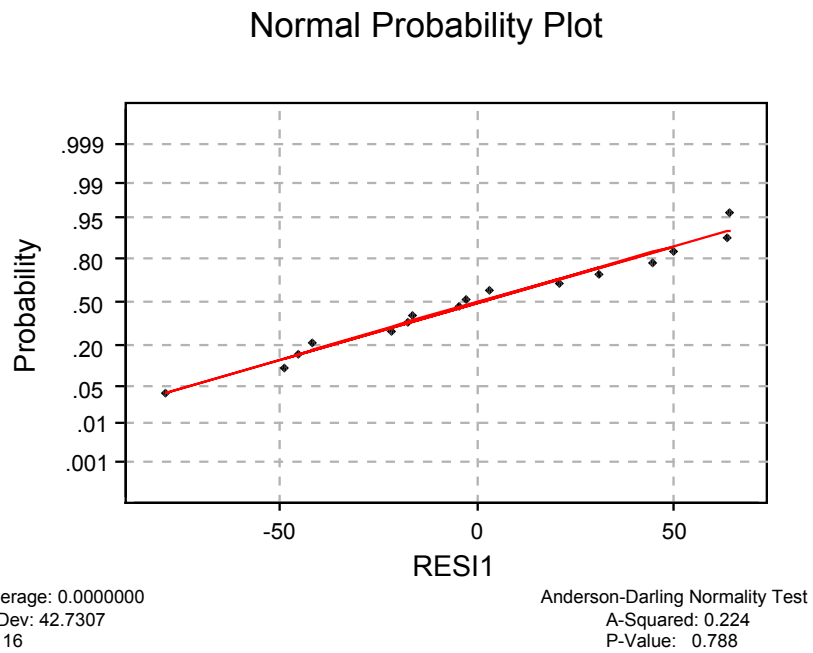
S = 49.90 R-Sq = 79.6% R-Sq(adj) = 72.2%

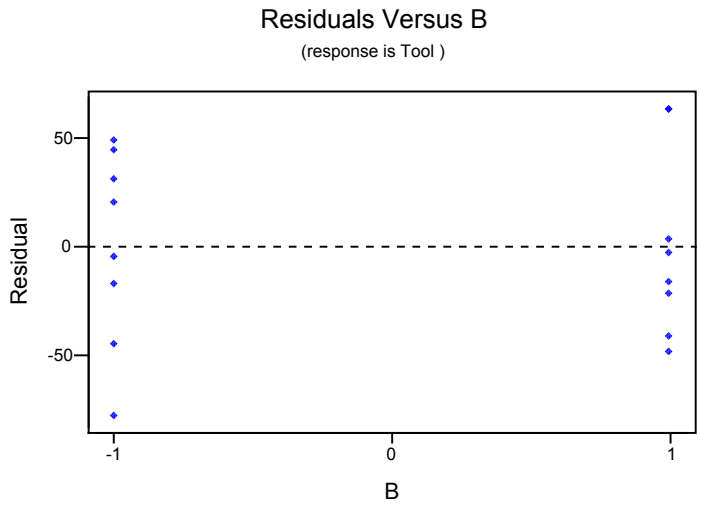
Analysis of Variance

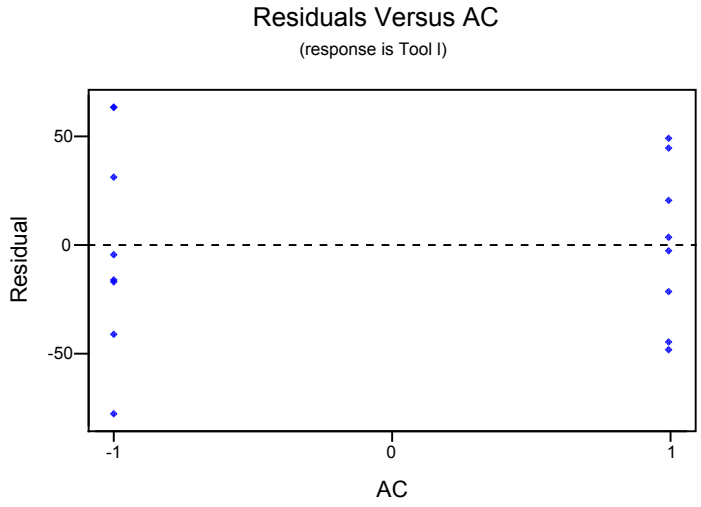
Source	DF	SS	MS	F	P
Regression	4	107199	26800	10.76	0.001
Error	11	27389	2490		
Total	15	134588			

Source	DF	Seq SS
A	1	1332
B	1	28392
C	1	20592
AC	1	56882

c)







The assumptions of normality and constant variance do not appear to be violated.

7-10.

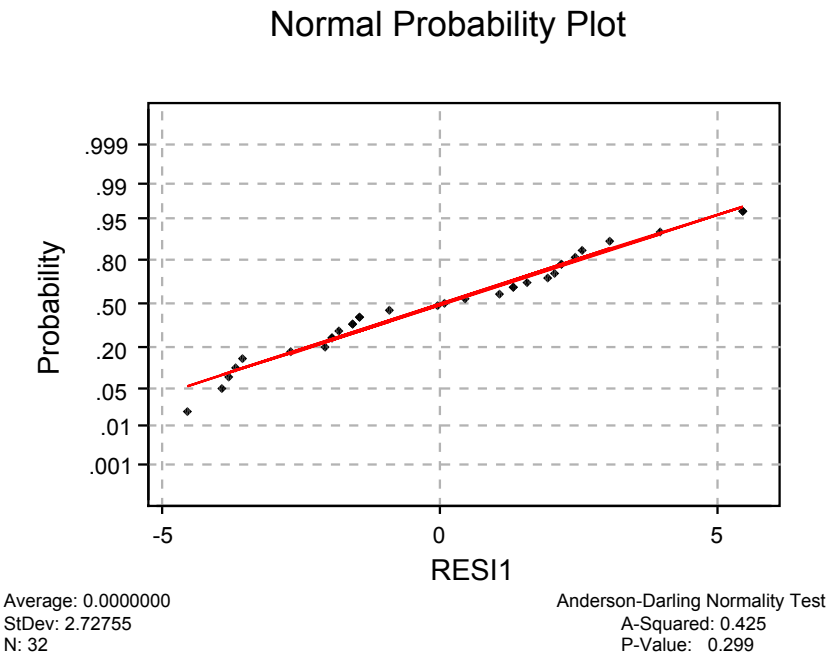
Coefficients for Score

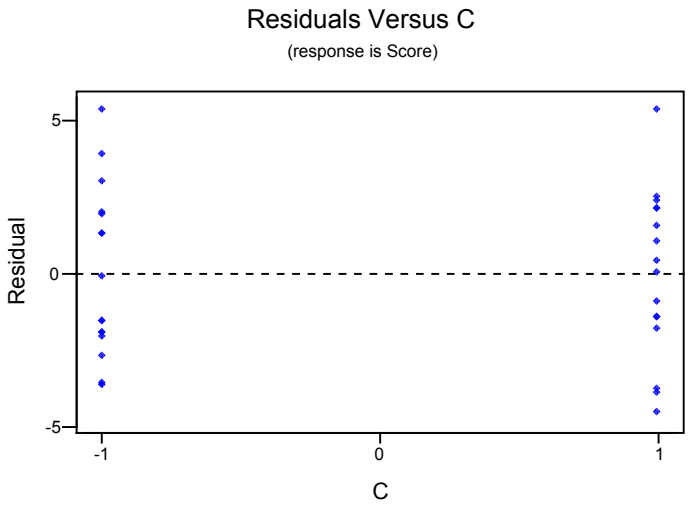
Term	Coef	StDev Coef	T	P
Constant	175.250	0.5467	320.59	0.000
A	8.500	0.5467	15.55	0.000
B	-0.813	0.5467	-1.49	0.157
C	5.437	0.5467	9.95	0.000
D	4.187	0.5467	7.66	0.000
A*B	-0.063	0.5467	-0.11	0.910
A*C	-0.312	0.5467	-0.57	0.575
A*D	4.562	0.5467	8.35	0.000
B*C	-0.125	0.5467	-0.23	0.822
B*D	0.625	0.5467	1.14	0.270
C*D	-0.625	0.5467	-1.14	0.270
A*B*C	0.375	0.5467	0.69	0.503
A*B*D	-0.250	0.5467	-0.46	0.654
A*C*D	-0.000	0.5467	-0.00	1.000
B*C*D	0.062	0.5467	0.11	0.910
A*B*C*D	-0.813	0.5467	-1.49	0.157

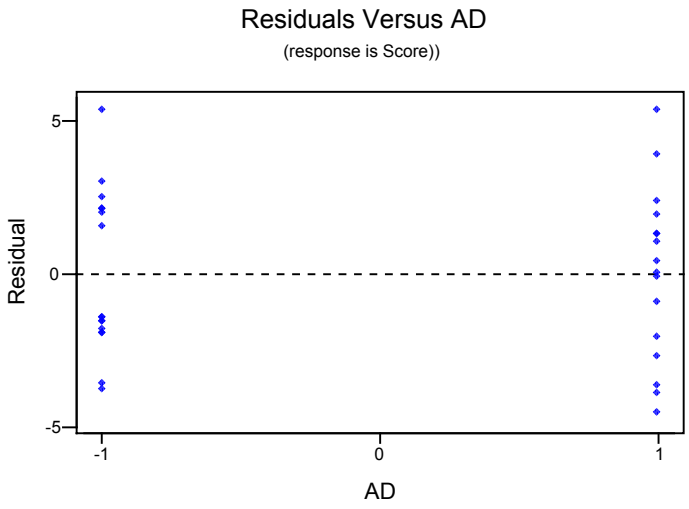
The table indicates that factors A, C, D, and the interaction of AD are significant at the 0.05 level of significance.

7-11. Using the results of 7-10, the regression analysis and final model are

```
The regression equation is
Score = 175 + 8.50 A + 5.44 C + 4.19 D + 4.56 AD

Predictor       Coef       StDev          T        P
Constant     175.250       0.517     339.20    0.000
A             8.5000      0.5166      16.45    0.000
C             5.4375      0.5166      10.52    0.000
D             4.1875      0.5166       8.11    0.000
AD            4.5625      0.5166       8.83    0.000

S = 2.923       R-Sq = 95.1%     R-Sq(adj) = 94.4%

Analysis of Variance

Source        DF          SS          MS         F        P
Regression     4      4485.4      1121.3    131.28    0.000
Error         27       230.6         8.5
Total         31      4716.0

Source        DF      Seq SS
A              1      2312.0
C              1       946.1
D              1       561.1
AD             1       666.1

Unusual Observations
Obs         A      Score        Fit  StDev Fit   Residual     St Resid
 18      1.00    175.000    169.562      1.155      5.438        2.03R
 21     -1.00    178.000    172.562      1.155      5.438        2.03R

R denotes an observation with a large standardized residual
```







The residual plots do not indicate any departure from normality or constant variance.

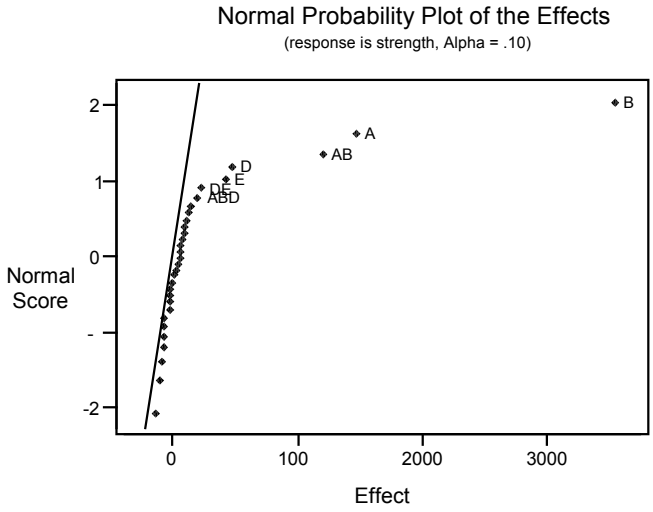
7-12. a)

`Estimated Effects for strength` (these can be found using the relationship: Estimated Effect = 2(coefficient))

Term	Effect
A	1462.13
B	3537.87
C	-137.12
D	474.62
E	425.38
A*B	1199.62
A*C	124.62
A*D	62.87
A*E	62.12
B*C	-99.63
B*D	-12.88
B*E	-12.12
C*D	112.13
C*E	-62.13

```
D*E           224.62
A*B*C         -62.88
A*B*D         200.38
A*B*E          49.63
A*C*D          75.38
A*C*E          99.63
A*D*E         -87.12
B*C*D          99.62
B*C*E         -74.63
B*D*E         -62.88
C*D*E          37.13
A*B*C*D       -12.12
A*B*C*E        12.13
A*B*D*E         0.37
A*C*D*E       150.37
B*C*D*E       -25.38
A*B*C*D*E      62.88
```

b)



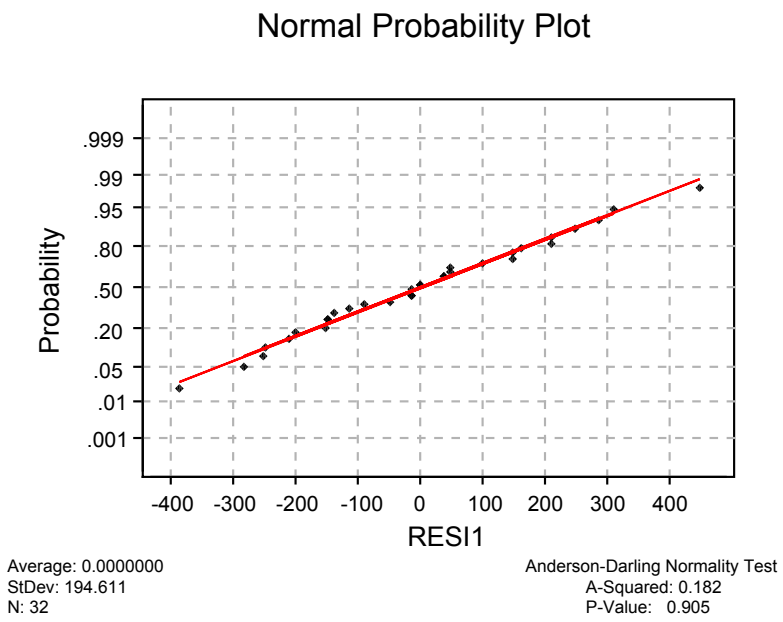
The effects that appear to be important are A, B, D, E, and the interactions AB, DE, and ABD.

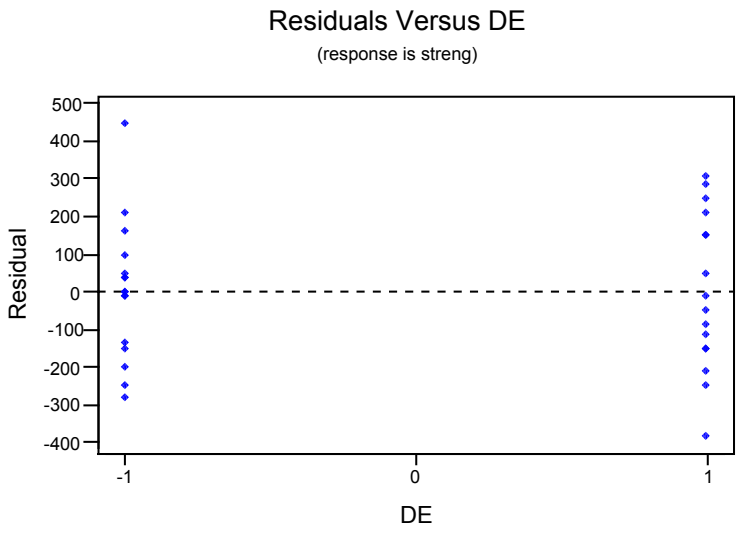
c) The regression analysis and final model are

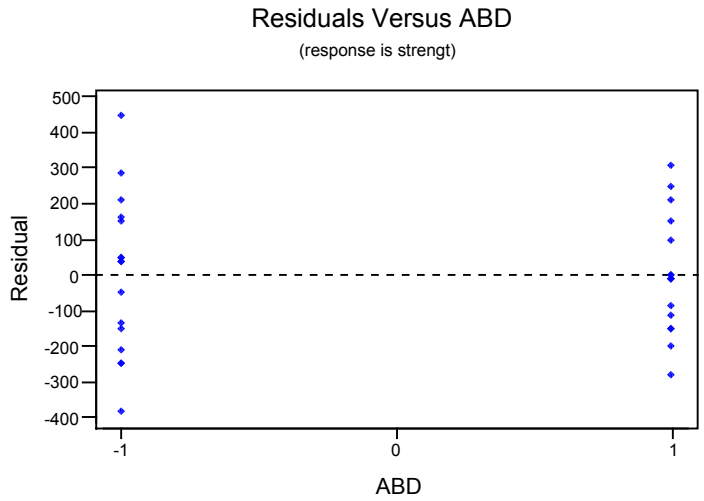
```
The regression equation is
strength = 2888 + 731 A + 1769 B + 237 D + 213 E + 600 AB + 112 DE +
100 ABD

Predictor       Coef       StDev          T        P
Constant     2887.69       39.10      73.86    0.000
A             731.06       39.10      18.70    0.000
B            1768.94       39.10      45.24    0.000
D             237.31       39.10       6.07    0.000
E             212.69       39.10       5.44    0.000
AB            599.81       39.10      15.34    0.000
DE            112.31       39.10       2.87    0.008
ABD           100.19       39.10       2.56    0.017

S = 221.2       R-Sq = 99.1%     R-Sq(adj) = 98.9%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    7   132722308    18960330    387.58    0.000
```

```
Error         24     1174077       48920
Total         31   133896385

Source        DF      Seq SS
A              1    17102476
B              1   100132476
D              1     1802151
E              1     1447551
AB             1    11512801
DE             1      403651
ABD            1      321201

Unusual Observations
Obs          A   strength        Fit   StDev Fit    Residual     St Resid
 20       1.00     6200.0     5750.4       110.6       449.6        2.35R
 31      -1.00     3400.0     3787.9       110.6      -387.9       -2.02R

R denotes an observation with a large standardized residual
```

Based on the analysis of variance, the model appears to be adequate.







The normal probability plot of the residuals indicates the assumption of normality is not violated. The model appears to be adequate.

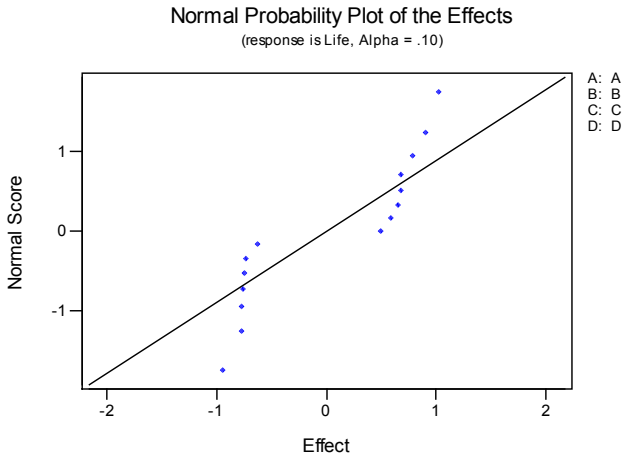
d) To maximize strength, the variables A, B, D, and E should be increased. Variable C is not significant thus any level of C would be acceptable.

7-13. a)

`Estimated Effects for inches` (the estimated effects are calculated using effect = 2(coefficient))

Term	Effect
A	-0.9412
B	-0.7712
C	0.6738
D	1.0237
A*B	-0.7512
A*C	0.7838
A*D	0.5888
B*C	0.6838
B*D	0.4987
C*D	-0.7763

```
A*B*C         0.8987
A*B*D         0.6487
A*C*D        -0.7562
B*C*D        -0.6262
A*B*C*D      -0.7313
```



Based on the normal probability plot, there do not appear to be any significant main factors or interactions. From these results, the t-ratios could be calculated for the main factors and two-factor interactions (since the higher-order interactions appear to be negligible).

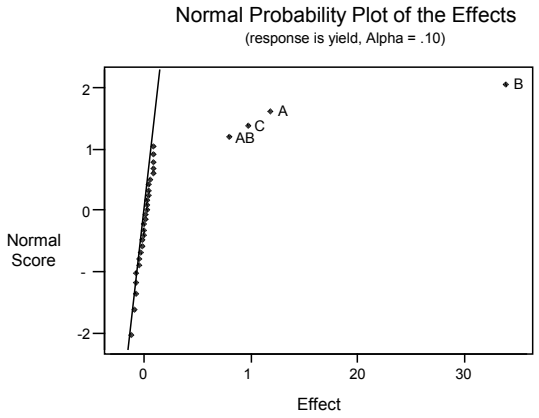
Estimated Effects and Coefficients for Life (coded units)

Term	Effect	Coef	StDev Coef	T	P
Constant		6.0069	0.3693	16.27	0.000
A	-0.9412	-0.4706	0.3693	-1.27	0.259
B	-0.7712	-0.3856	0.3693	-1.04	0.344
C	0.6738	0.3369	0.3693	0.91	0.403
D	1.0237	0.5119	0.3693	1.39	0.224
A*B	-0.7512	-0.3756	0.3693	-1.02	0.356
A*C	0.7837	0.3919	0.3693	1.06	0.337
A*D	0.5888	0.2944	0.3693	0.80	0.462
B*C	0.6837	0.3419	0.3693	0.93	0.397
B*D	0.4988	0.2494	0.3693	0.68	0.529
C*D	-0.7763	-0.3881	0.3693	-1.05	0.341

There are no significant factors or interactions.

b) There are no significant factors or interactions with which to build a model.

7-14. a)



Factors A, B, C, and the interaction AB appear to be significant.

b)

Estimated Effects and Coefficients for yield

Term	Effect	Coef	StDev Coef	T	P
Constant		30.5312	0.2786	109.57	0.000
A	11.8125	5.9063	0.2786	21.20	0.000
B	33.9375	16.9687	0.2786	60.90	0.000
C	9.6875	4.8437	0.2786	17.38	0.000
D	-0.8125	-0.4063	0.2786	-1.46	0.164
E	0.4375	0.2187	0.2786	0.79	0.444
A*B	7.9375	3.9687	0.2786	14.24	0.000
A*C	0.4375	0.2187	0.2786	0.79	0.444
A*D	-0.0625	-0.0313	0.2786	-0.11	0.912
A*E	0.9375	0.4687	0.2786	1.68	0.112
B*C	0.0625	0.0313	0.2786	0.11	0.912
B*D	-0.6875	-0.3437	0.2786	-1.23	0.235
B*E	0.5625	0.2813	0.2786	1.01	0.328
C*D	0.8125	0.4062	0.2786	1.46	0.164
C*E	0.3125	0.1563	0.2786	0.56	0.583
D*E	-1.1875	-0.5938	0.2786	-2.13	0.049

The t-ratios appear to support the findings of the normal probability plot.

The regression equation is
yield = 30.5 + 5.91 A + 17.0 B + 4.84 C + 3.97 AB

Predictor	Coef	StDev	T	P
Constant	30.5312	0.3021	101.07	0.000
A	5.9062	0.3021	19.55	0.000
B	16.9687	0.3021	56.17	0.000
C	4.8438	0.3021	16.03	0.000
AB	3.9688	0.3021	13.14	0.000

S = 1.709 R-Sq = 99.3% R-Sq(adj) = 99.2%

Analysis of Variance for yield

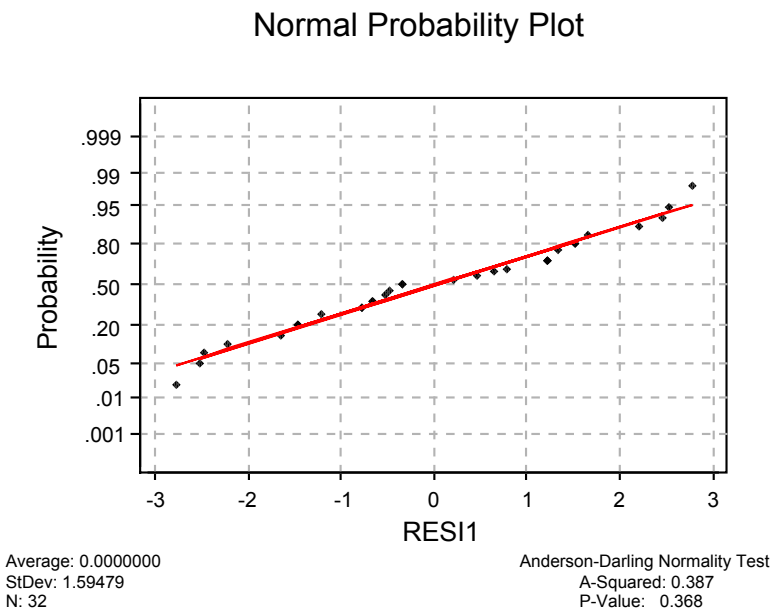
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	11081.1	11081.1	3693.70	1E+03	0.000
2-Way Interactions	1	504.0	504.0	504.03	172.61	0.000
Residual Error	27	78.8	78.8	2.92		
Lack of Fit	3	3.1	3.1	1.03	0.33	0.806
Pure Error	24	75.8	75.8	3.16		
Total	31	11664.0				

Estimate of the variance is: $\hat{\sigma}^2 = 2.92$

The t ratios in the table above were computed using s.e.(coefficient) = $\frac{1}{2}\sqrt{\frac{2(2.92)}{32/2}} = 0.3021$.

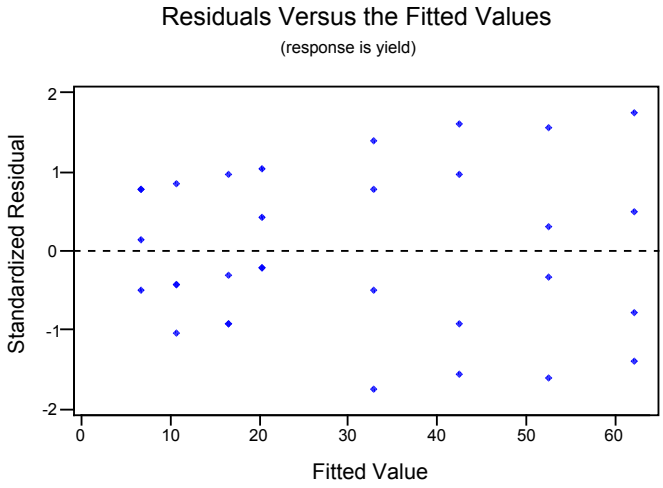
For example, for A, the ratio is 5.906/0.3021 = 19.55.

c)

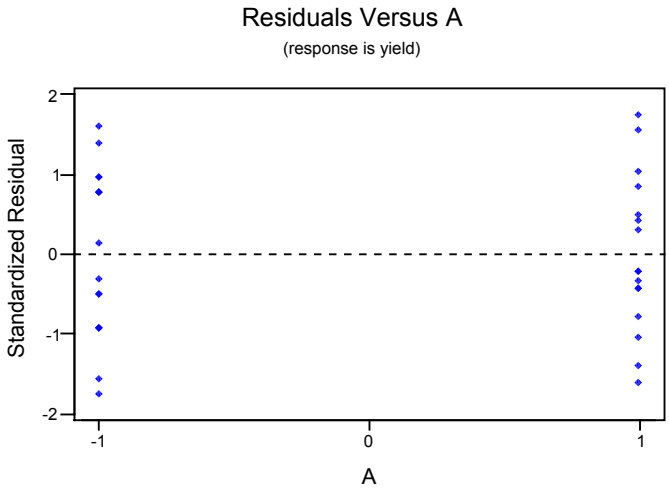


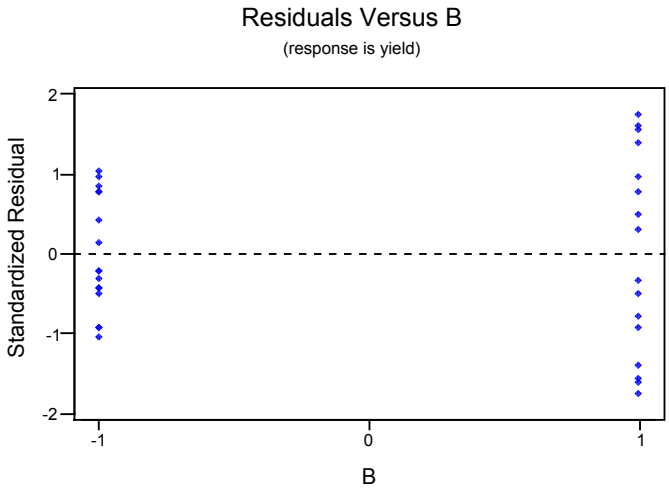
The residuals appear normally distributed.

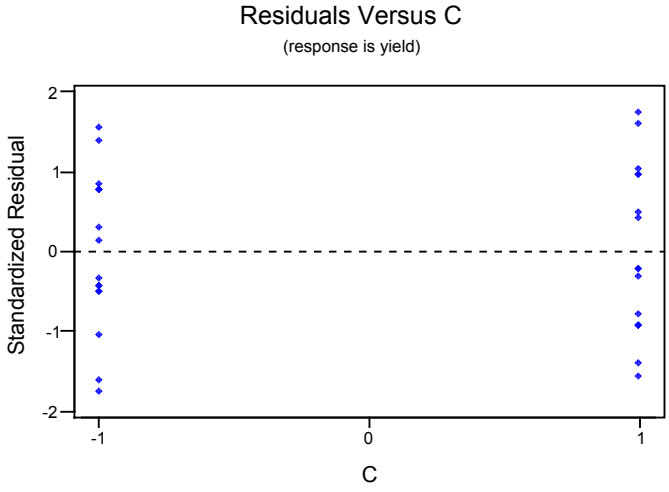
d)

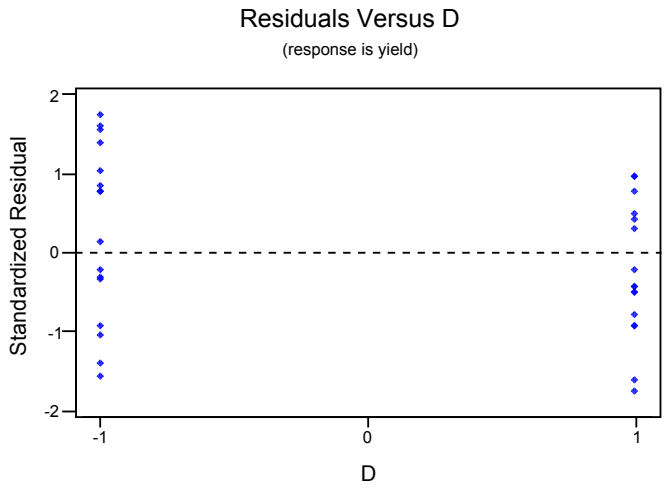


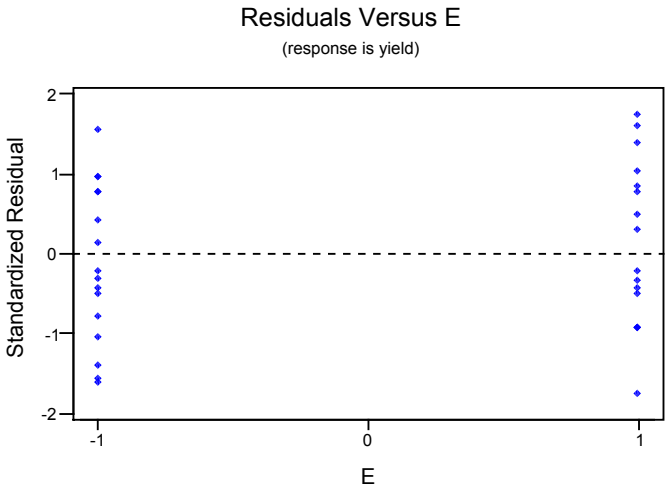
The variance appears to increase as the fitted value increases.


Residuals Versus A
(response is yield)
Standardized Residual
2
1
0
-1
-2
-1
0
1
A


Residuals Versus B
(response is yield)
Standardized Residual
2
1
0
-1
-2
-1
0
1
B

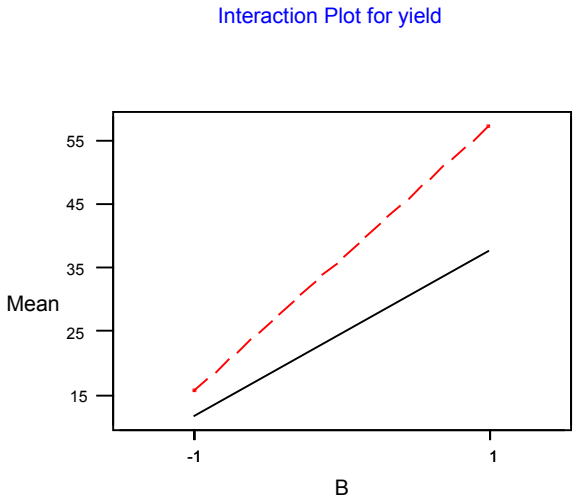

Residuals Versus C
(response is yield)
Standardized Residual
2
1
0
-1
-2
-1
0
1
C





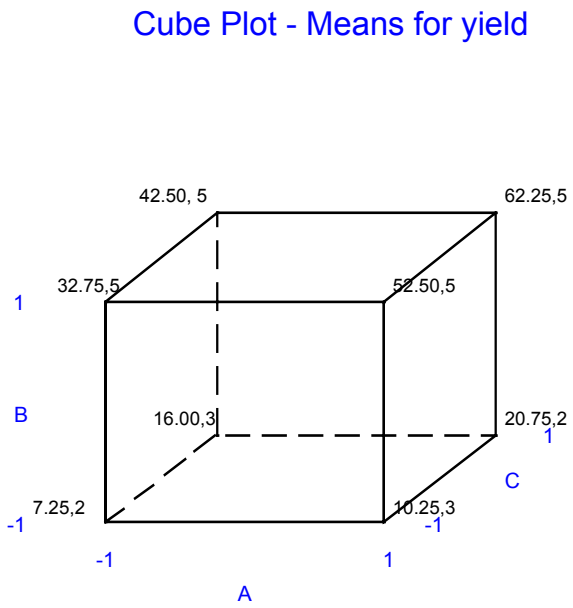
All plots support the constant variance assumption except B.

e) According to the t-ratios given in part b) above, the interaction AB appears to be significant. The interaction plot from MINITAB indicates that a high level of A and of B increases the mean yield. While low levels of each would lead to a reduction in the mean yield.



f) To increase yield, set A, B, and C at their high levels.

g)



It is evident from the plot that we should run the process with all factors set at their high level.

7-15. a) Note to Instructor: If insignificant terms are removed, Minitab will include a 'Lack of Fit' term in the analysis output; and, the $MSE_{curvature}$ computed by hand will not be equivalent to $MSE_{curvature}$ computed by Minitab.

```
Estimated Effects and Coefficients for inches

Term          Effect      Coef  StDev Coef       T      P
Constant                32.2432       1.982   16.27  0.000
A          11.8125     5.9062       2.131    2.77  0.039
```

B	33.9375	16.9687	2.131	7.96	0.001
C	9.6875	4.8437	2.131	2.27	0.072
D	-0.8125	-0.4063	2.131	-0.19	0.856
E	0.4375	0.2188	2.131	0.10	0.922
A*B	7.9375	3.9687	2.131	1.86	0.122
A*C	0.4375	0.2187	2.131	0.10	0.922
A*D	-0.0625	-0.0313	2.131	-0.01	0.989
A*E	0.9375	0.4688	2.131	0.22	0.835
B*C	0.0625	0.0313	2.131	0.01	0.989
B*D	-0.6875	-0.3437	2.131	-0.16	0.878
B*E	0.5625	0.2813	2.131	0.13	0.900
C*D	0.8125	0.4063	2.131	0.19	0.856
C*E	0.3125	0.1562	2.131	0.07	0.944
D*E	-1.1875	-0.5938	2.131	-0.28	0.792
A*B*C	-0.4375	-0.2188	2.131	-0.10	0.922
A*B*D	0.3125	0.1562	2.131	0.07	0.944
A*B*E	-0.1875	-0.0937	2.131	-0.04	0.967
A*C*D	-0.4375	-0.2188	2.131	-0.10	0.922
A*C*E	0.3125	0.1563	2.131	0.07	0.944
A*D*E	0.8125	0.4062	2.131	0.19	0.856
B*C*D	0.4375	0.2188	2.131	0.10	0.922
B*C*E	0.9375	0.4688	2.131	0.22	0.835
B*D*E	0.1875	0.0938	2.131	0.04	0.967
C*D*E	-0.8125	-0.4062	2.131	-0.19	0.856
A*B*C*D	-0.0625	-0.0313	2.131	-0.01	0.989
A*B*C*E	0.1875	0.0937	2.131	0.04	0.967
A*B*D*E	0.9375	0.4687	2.131	0.22	0.835
A*C*D*E	-0.3125	-0.1563	2.131	-0.07	0.944
B*C*D*E	-0.9375	-0.4688	2.131	-0.22	0.835
A*B*C*D*E	-0.1875	-0.0937	2.131	-0.04	0.967

Analysis of Variance for inches

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	11087.9	11087.9	2217.58	15.25	0.005
2-Way Interactions	10	536.3	536.3	53.63	0.37	0.916
3-Way Interactions	10	24.3	24.3	2.43	0.02	1.000
4-Way Interactions	5	15.2	15.2	3.03	0.02	1.000
5-Way Interactions	1	0.3	0.3	0.28	0.00	0.967
Residual Error	5	726.8	726.8	145.37		
Curvature	1	694.0	694.0	694.04	84.64	0.001
Pure Error	4	32.8	32.8	8.20		
Total	36	12390.8				

The hand calculation of $\hat{\sigma}_c^2$ is

$$\hat{\sigma}_c^2 = \sum_{\substack{\text{center} \\ \text{pts}}} \frac{(y_i - \bar{y}_c)^2}{(n_c - 1)} = \frac{32.8}{4} = 8.2 \qquad \text{where } \bar{y}_c = 43.2$$

The difference between this estimate and the estimate in Exercise 7-14 is due to the addition of centerpoints.

b) The t-ratio can be found from $|t_{curvature}| = \sqrt{F} = \sqrt{84.64} = 9.2$ or using the definition for $t_{curvature}$:

$$t_{curvature} = \frac{\bar{y}_F - \bar{y}_C}{\sqrt{\hat{\sigma}^2\left(\frac{1}{n_F} + \frac{1}{n_C}\right)}} = \frac{30.531 - 43.2}{\sqrt{8.20\left(\frac{1}{32} + \frac{1}{5}\right)}} = -9.20$$

The critical value for the t-test is $t_{0.025,4} = 2.776$. Thus since 9.2 > 2.776, we can conclude that curvature is significant.

7-16. a) Note to Instructor: If insignificant terms are removed, Minitab will include a 'Lack of Fit' term in the analysis output; and, the $MSE_{curvature}$ computed by hand will not be equivalent to $MSE_{curvature}$ computed by Minitab.

Estimated Effects and Coefficients for toolife

Term	Effect	Coef	StDev Coef	T	P
Constant		414.58	4.316	96.06	0.000
A	13.75	6.87	5.286	1.30	0.263
B	127.75	63.87	5.286	12.08	0.000
C	97.75	48.87	5.286	9.25	0.001
A*B	-21.25	-10.62	5.286	-2.01	0.115
A*C	-137.25	-68.62	5.286	-12.98	0.000
B*C	-52.25	-26.12	5.286	-4.94	0.008
A*B*C	-68.25	-34.12	5.286	-6.46	0.003

b)

Estimated Effects and Coefficients for toolife

Term	Effect	Coef	StDev Coef	T	P
Constant		414.58	4.316	96.06	0.000
A	13.75	6.87	5.286	1.30	0.263
B	127.75	63.87	5.286	12.08	0.000
C	97.75	48.87	5.286	9.25	0.001
A*B	-21.25	-10.62	5.286	-2.01	0.115
A*C	-137.25	-68.62	5.286	-12.98	0.000
B*C	-52.25	-26.12	5.286	-4.94	0.008
A*B*C	-68.25	-34.12	5.286	-6.46	0.003

Analysis of Variance for toolife

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	52128	52128.4	17376.1	77.74	0.001
2-Way Interactions	3	44038	44038.4	14679.5	65.68	0.001
3-Way Interactions	1	9316	9316.1	9316.1	41.68	0.003
Residual Error	4	894	894.0	223.5		
Curvature	1	176	176.0	176.0	0.74	0.454
Pure Error	3	718	718.0	239.3		
Total	11	106377				

The pure error effect using centerpoints would be 239.3

The hand calculation of $\hat{\sigma}_c^2$ is

$$\hat{\sigma}_c^2 = \sum_{\substack{\text{center} \\ \text{pts}}} \frac{(y_i - \bar{y}_c)^2}{(n_c - 1)} = \frac{718}{3} = 239.3 \quad \text{where } \bar{y}_c = 420$$

c) The t-ratio can be found from $|t_{curvature}| = \sqrt{F} = \sqrt{0.74} = 0.860$

$$t_{curvature} = \frac{\bar{y}_F - \bar{y}_C}{\sqrt{\hat{\sigma}^2\left(\frac{1}{n_F} + \frac{1}{n_C}\right)}} = \frac{411.875 - 420}{\sqrt{239.3\left(\frac{1}{8} + \frac{1}{4}\right)}} = -0.858$$

The critical value for the t-test is $t_{0.025,1} = 12.706$. Thus since $0.860 < 12.706$ we cannot conclude that curvature is significant.

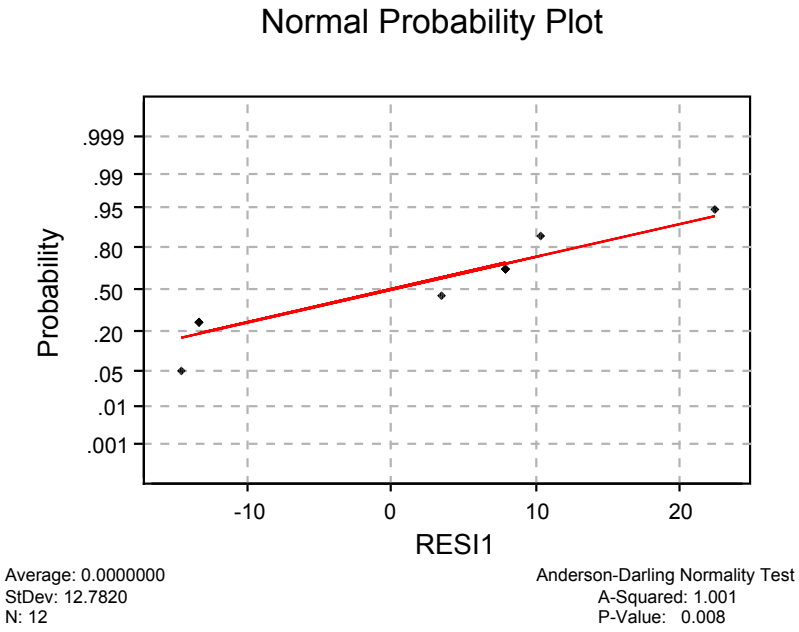
d) Testing for main effects and interaction effects yields B, C, AC, BC, and ABC as significant at the 0.05 level of significance. A is retained in the model for hierarchy.

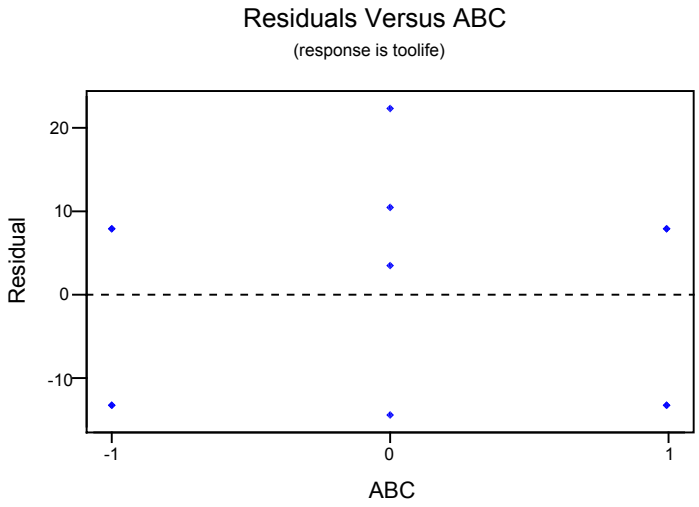
e) The appropriate regression model using 'Regression' in Minitab is

The regression equation is
toolife = 415 + 6.88 A + 63.9 B + 48.9 C - 68.6 AC - 26.1 BC - 34.1 ABC

Predictor	Coef	StDev	T	P
Constant	414.583	5.473	75.75	0.000
A	6.875	6.703	1.03	0.352

B	63.875	6.703	9.53	0.000
C	48.875	6.703	7.29	0.001
AC	-68.625	6.703	-10.24	0.000
BC	-26.125	6.703	-3.90	0.011
ABC	-34.125	6.703	-5.09	0.004



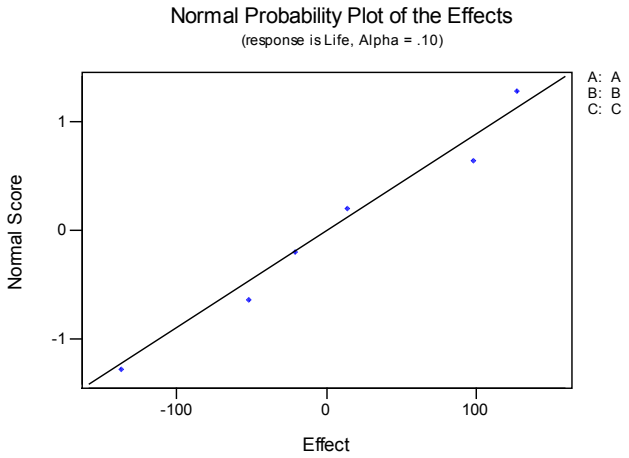


The residual plots do not indicate a serious departure from normality or constant variance.

7-17. Block 1: (1), ab, ac, bc Block 2: a, b, c, abc

Term	Effect	Coef
Constant		411.87
Block		34.12
A	13.75	6.87
B	127.75	63.87
C	97.75	48.88
A*B	-21.25	-10.63
A*C	-137.25	-68.62

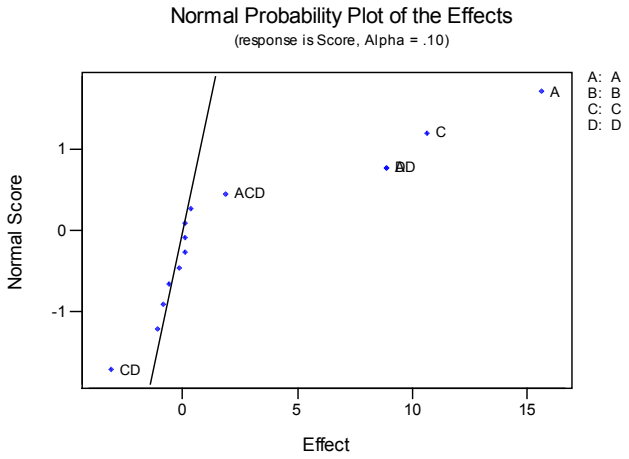
B*C	-52.25	-26.12



None of the factors or interactions appear to be significant using only Replicate 1.

7-18. Block 1: a, b, c, d, abc, abd, acd, bcd Block 2: (1), ab, ac ,bc, ad, bd, cd, abcd

Term	Effect	Coef
Constant		174.062
Block		0.438
A	15.625	7.812
B	-1.125	-0.562
C	10.625	5.312
D	8.875	4.437
A*B	-0.625	-0.312
A*C	0.125	0.062
A*D	8.875	4.437
B*C	0.375	0.188
B*D	0.125	0.062
C*D	-3.125	-1.563
A*B*C	-0.125	-0.062
A*B*D	-0.875	-0.438
A*C*D	1.875	0.937
B*C*D	0.125	0.063



The factors that appear to be significant are A, C, AD, CD, and ACD. The t-tests for these factors and interactions (and factor D and interactions AC and AD since they are involved in higher-order interactions) are given below.

Term	Effect	Coef	StDev Coef	T	P
Constant		174.062	0.3053	570.19	0.000
Block		0.438	0.3053	1.43	0.195
A	15.625	7.812	0.3053	25.59	0.000
C	10.625	5.312	0.3053	17.40	0.000
D	8.875	4.438	0.3053	14.54	0.000
A*C	0.125	0.063	0.3053	0.20	0.844
A*D	8.875	4.437	0.3053	14.54	0.000
C*D	-3.125	-1.562	0.3053	-5.12	0.001
A*C*D	1.875	0.937	0.3053	3.07	0.018

According to the t-tests, the AC interaction can be removed along with the blocking variable. The final t-tests and ANOVA are given below.

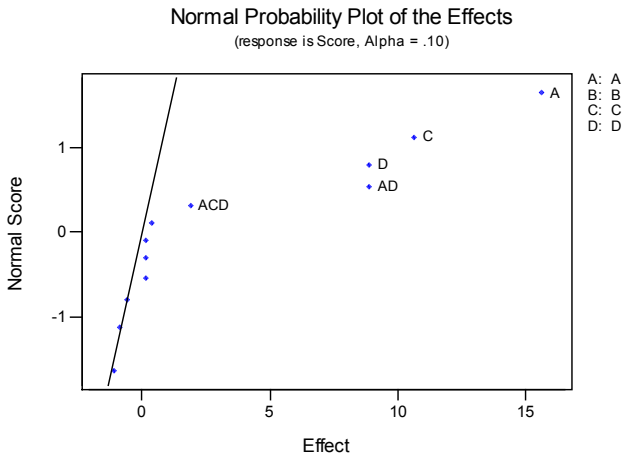
Term	Effect	Coef	StDev Coef	T	P
Constant		174.062	0.3069	567.17	0.000
A	15.625	7.812	0.3069	25.46	0.000
C	10.625	5.313	0.3069	17.31	0.000
D	8.875	4.437	0.3069	14.46	0.000
A*D	8.875	4.438	0.3069	14.46	0.000
C*D	-3.125	-1.563	0.3069	-5.09	0.001
A*C*D	1.875	0.937	0.3069	3.05	0.014

Analysis of Variance for Score (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	1743.19	1743.19	581.063	385.59	0.000
2-Way Interactions	2	354.13	354.13	177.063	117.50	0.000
3-Way Interactions	1	14.06	14.06	14.063	9.33	0.014
Residual Error	9	13.56	13.56	1.507		
Lack of Fit	1	0.06	0.06	0.063	0.04	0.852
Pure Error	8	13.50	13.50	1.688		
Total	15	2124.94				

7-25. Block 1: (1) acd bcd ab; Block 2: a, b, cd, abcd; Block 3: d, ac bc abd; Block 4: c ad bd abc

Term	Effect	Coef
Constant		174.062
Block 1		-1.062
2		-2.062
3		1.937
A	15.625	7.813
B	-1.125	-0.563
C	10.625	5.313
D	8.875	4.438
A*B	-0.625	-0.313
A*C	0.125	0.063
A*D	8.875	4.437
B*C	0.375	0.187
B*D	0.125	0.062
A*C*D	1.875	0.937
B*C*D	0.125	0.062
A*B*C*D	-0.875	-0.438



Based on the normal probability plot, factors A, C, D, and interactions AD and ACD appear to be significant. The t-tests for these factors and interactions (along with the CD interaction since it is involved in a higher-order interaction) are given below.

Term	Effect	Coef	StDev Coef	T	P
Constant		174.062	0.3053	570.19	0.000
Block 1		-1.062	0.5287	-2.01	0.084
2		-2.062	0.5287	-3.90	0.006
3		1.937	0.5287	3.66	0.008
A	15.625	7.813	0.3053	25.59	0.000
C	10.625	5.313	0.3053	17.40	0.000
D	8.875	4.437	0.3053	14.54	0.000
A*D	8.875	4.437	0.3053	14.54	0.000
A*C*D	1.875	0.938	0.3053	3.07	0.018

Analysis of Variance for Score (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Blocks	3	42.19	42.19	14.062	9.43	0.007
Main Effects	3	1743.19	1743.19	581.063	389.69	0.000
2-Way Interactions	1	315.06	315.06	315.062	211.30	0.000
3-Way Interactions	1	14.06	14.06	14.063	9.43	0.018
Residual Error	7	10.44	10.44	1.491		
Total	15	2124.94				

The factors A, C, and D and the interactions AD and ACD are significant.

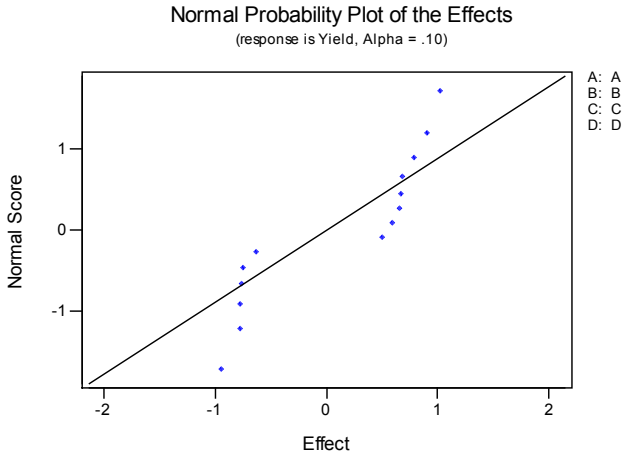
7-20. Block 1: (1), ab, bc, ad, bd, cd, abcd, ae, be, ce, abce, de, abde, acde, bcde
Block 2: a, b, c, d, e, abc, abd, acd, bcd, abe, ace, bce, ade, bde, cde, abcde

7-25. Using the block generators ABC and CDE, the blocks are:
Block 1: (1), ab, acd, bcd, ace, bce, de, abde
Block 2: ac, bc, d, abd, e, abe, acde, bcde
Block 3: a, b, cd, abcd, ce, abce, ade, bde
Block 4: c, abc, ad, bd, ae, be, cde, abcde

7-22.

Term	Effect	Coef
Constant		6.0069
Block		0.3656
A	-0.9412	-0.4706
B	-0.7713	-0.3856
C	0.6738	0.3369
D	1.0238	0.5119
A*B	-0.7513	-0.3756
A*C	0.7838	0.3919
A*D	0.5887	0.2944
B*C	0.6838	0.3419

B*D	0.4988	0.2494
C*D	-0.7762	-0.3881
A*B*C	0.8987	0.4494
A*B*D	0.6487	0.3244
A*C*D	-0.7563	-0.3781
B*C*D	-0.6263	-0.3131



According to the normal probability plot, there are no significant factors or interactions.

Section 7-7

7-23. a) The complete defining relation is I = ABCDE
The alias structure as given by Minitab (using 'Analyzing a Factorial Design' under "DOE")

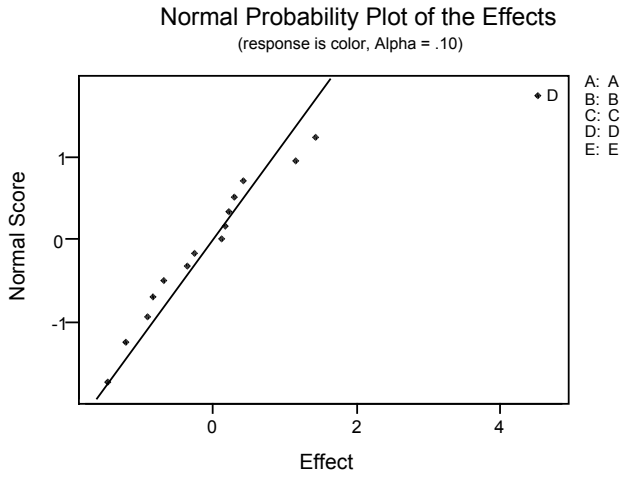
```
I + A*B*C*D*E
A + B*C*D*E
B + A*C*D*E
C + A*B*D*E
D + A*B*C*E
E + A*B*C*D
A*B + C*D*E
A*C + B*D*E
A*D + B*C*E
A*E + B*C*D
B*C + A*D*E
B*D + A*C*E
B*E + A*C*D
C*D + A*B*E
C*E + A*B*D
D*E + A*B*C
```

b) The estimated effects for this set of data is (found using 'Analyzing a Factorial Design' under "DOE" in Minitab)

Estimated Effects

Term	Effect
A	1.4350
B	-1.4650
C	-0.2725
D	4.5450
E	-0.7025
A*B	1.1500
A*C	-0.9125
A*D	-1.2300
A*E	0.4275
B*C	0.2925
B*D	0.1200

```
B*E           0.1625
```



It is not entirely obvious from the normal probability plots of the effects, which effects are the most significant besides D. Looking at the effects themselves, the largest are given by A, B, D, and the interaction of AD. Therefore, run an analysis using all main factors and some of the interaction terms with the largest effects.

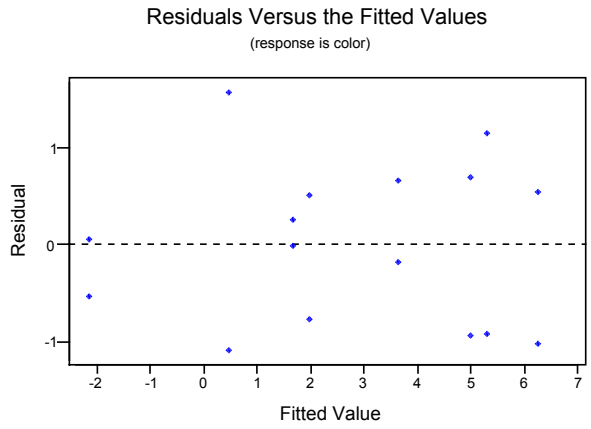
```
Estimated Effects and Coefficients for color

Term          Effect      Coef  StDev Coef       T      P
Constant                2.7700      0.2544   10.89  0.000
A             1.4350    0.7175      0.2544    2.82  0.018
B            -1.4650   -0.7325      0.2544   -2.88  0.016
D             4.5450    2.2725      0.2544    8.93  0.000
A*B           1.1500    0.5750      0.2544    2.26  0.047
A*D          -1.2300   -0.6150      0.2544   -2.42  0.036
```

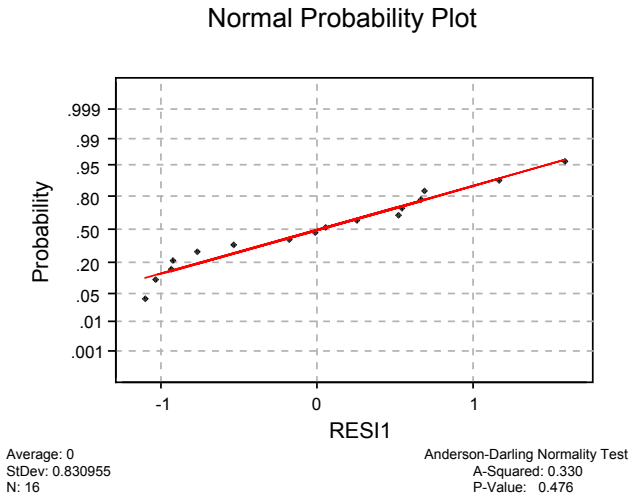
Using the main factors, A, B, and D were found to be significant. Of the interactions of these main factors, AB and AD are found to be significant at $\alpha = 0.05$.

c) Using the main factors A, B, and D, and the interaction terms AB and AD the appropriate model is (found using 'Regression' in Minitab)

```
The regression equation is
color = 2.77 + 0.718 A - 0.732 B + 2.27 D + 0.575 AB - 0.615 AD

Predictor       Coef       StDev          T        P
Constant      2.7700      0.2544      10.89    0.000
A             0.7175      0.2544       2.82    0.018
B            -0.7325      0.2544      -2.88    0.016
D             2.2725      0.2544       8.93    0.000
AB            0.5750      0.2544       2.26    0.047
AD           -0.6150      0.2544      -2.42    0.036

Source       DF      Seq SS
A             1       8.237
B             1       8.585
D             1      82.628
AB            1       5.290
AD            1       6.052
```

d) The residual plots are constructed using a model containing A, B, and D.





The residual plots appear adequate.

7-24. a) The complete defining relation is I = ABCD
The alias structure as given by Minitab is (using 'Analyzing a Factorial Design' under "DOE")

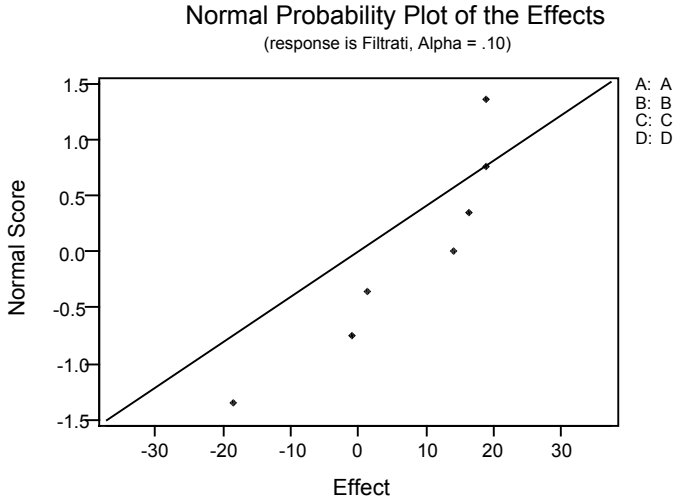
```
I + ABCD

A + BCD
B + ACD
C + ABD
D + ABC
AB + CD
AC + BD
AD + BC
```

b) The effect estimates as given by Minitab are

Alias Information for Terms in the Model.
Totally confounded terms were removed from the analysis

```
I + A*B*C*D
A + B*C*D
B + A*C*D
C + A*B*D
D + A*B*C
A*B + C*D
A*C + B*D
```

```
A*D + B*C

Estimated Effects for Filtration

Term          Effect
A             19.000
B              1.500
C             14.000
D             16.500
A*B           -1.000
A*C          -18.500
A*D           19.000
```



Although it is not entirely obvious from the normal probability plots which effects are active, investigating the effect estimates, it appears that factors A, C, and D may be significant. The interactions AC and AD also have large effect estimates.

c)

```
The regression equation is
Filtration Rate = 70.7 + 9.50 A + 0.750 B + 7.00 C + 8.25 D - 9.25 AC +
9.50 AD

Predictor       Coef       StDev          T        P
Constant     70.7500      0.5000     141.50    0.004
A             9.5000      0.5000      19.00    0.033
B             0.7500      0.5000       1.50    0.374
C             7.0000      0.5000      14.00    0.045
D             8.2500      0.5000      16.50    0.039
AC           -9.2500      0.5000     -18.50    0.034
AD            9.5000      0.5000      19.00    0.033
```

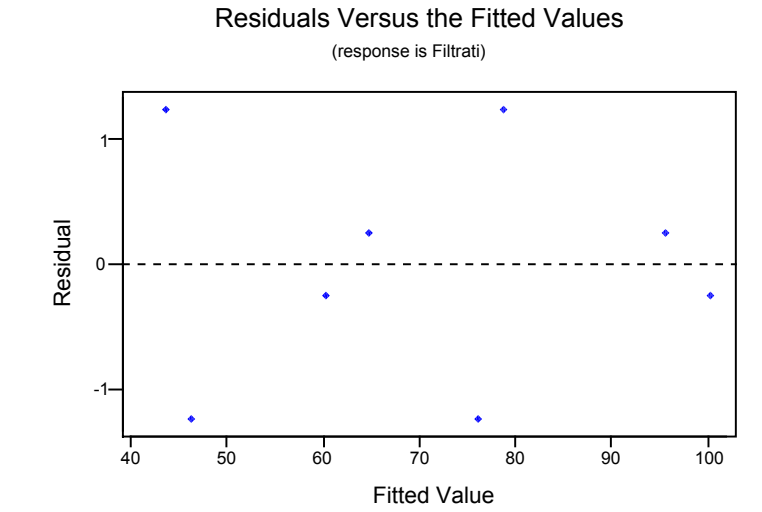
Based on the regression analysis, factors A, C, and D and the interactions AC and AD are significant.

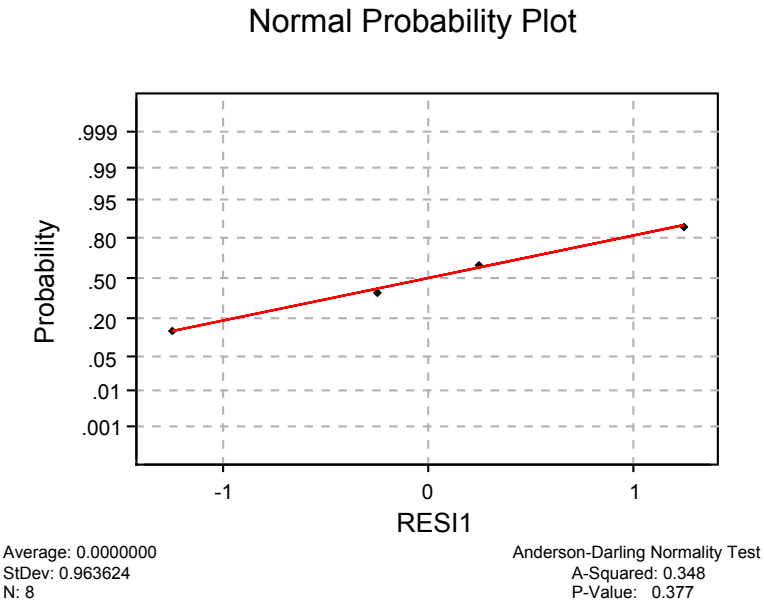
The regression analysis and final model are

```
The regression equation is
Filtration Rate = 70.7 + 9.50 A + 7.00 C + 8.25 D - 9.25 AC + 9.50 AD

Predictor       Coef       StDev          T        P
Constant     70.7500      0.6374     111.00    0.000
A             9.5000      0.6374      14.90    0.004
C             7.0000      0.6374      10.98    0.008
D             8.2500      0.6374      12.94    0.006
AC           -9.2500      0.6374     -14.51    0.005
AD            9.5000      0.6374      14.90    0.004
```

```
S = 1.803       R-Sq = 99.8%     R-Sq(adj) = 99.3%

Analysis of Variance

Source        DF          SS          MS          F         P
Regression     5     3065.00      613.00     188.62     0.005
Error          2        6.50        3.25
Total          7     3071.50

Source        DF       Seq SS
A              1       722.00
C              1       392.00
D              1       544.50
AC             1       684.50
AD             1       722.00
```

d)





The residual plots appear to be adequate.

7-25. a) To verify I = ACE and I = BDE were the generators used in this problem, show that the column of −1 and 1 for D is equivalent to the column BE or the column of −1 and 1 for E is equivalent to the column AC.

The complete defining relation is I = ACE = BDE = ABCD

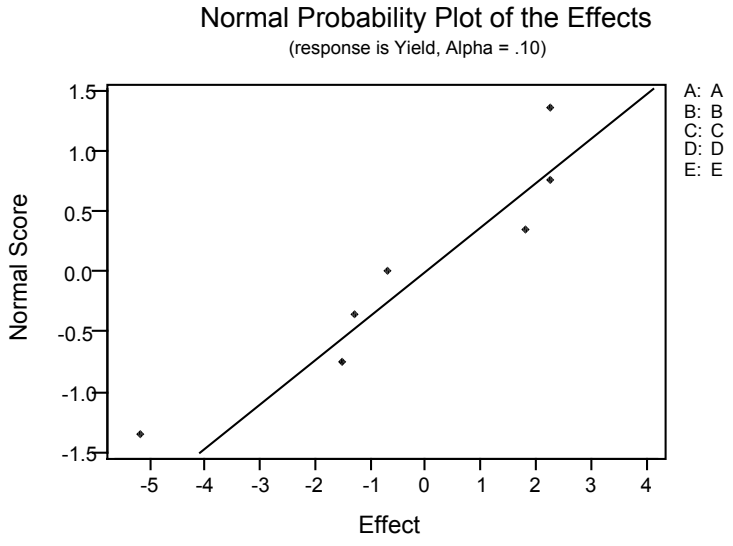
```
Design Generators:  D = BE  E = AC
```

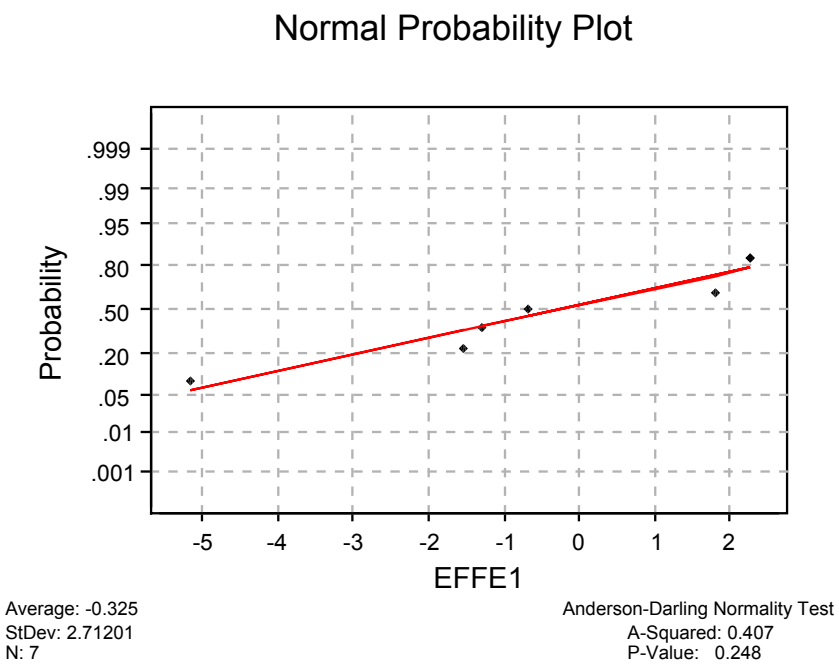
Alias Structure (up to order 3)
Alias Information for Terms in the Model.
Totally confounded terms were removed from the analysis

```
I + ACE + BDE

A + CE + BCD + ABDE
B + DE + ACD + ABCE
C + AE + ABD + BCDE
D + BE + ABC + ACDE
E + AC + BD + ABCDE
AB + CD + ADE + BCE
AD + BC + ABE + CDE
ABCD
```

b)

```
Estimated Effects

Term         Effect
A            -1.525
B            -5.175
C             2.275
D            -0.675
E             2.275
A*B           1.825
A*D          -1.275
```





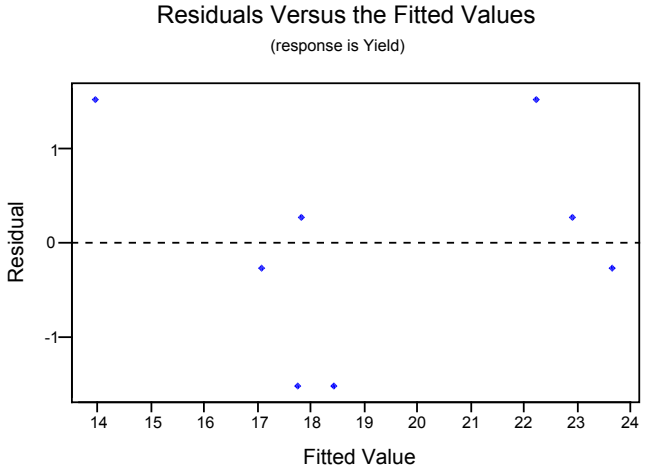
The effect estimates for the interactions AB and AD do not appear to be significant and therefore can possibly be used as error.

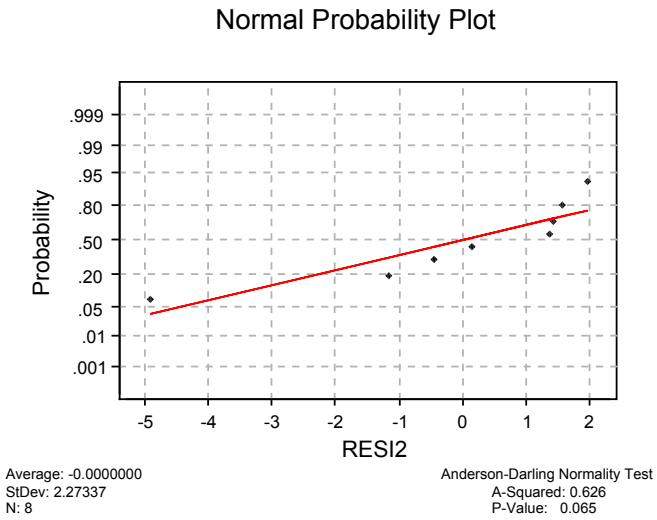
c) If we use AB and AD interactions for error the result is

Predictor	Coef	StDev	T	P
Constant	19.2375	0.7871	24.44	0.002
A	-0.7625	0.7871	-0.97	0.435
B	-2.5875	0.7871	-3.29	0.081
C	1.1375	0.7871	1.45	0.285
D	-0.3375	0.7871	-0.43	0.710
E	1.1375	0.7871	1.45	0.285

From the regression analysis, only factor B is significant at the 0.10 level of significance.

d) Using the only factor that may be significant, B, the residual plots are created.





The residual plots appear to be adequate.

7-26.

Alias Structure

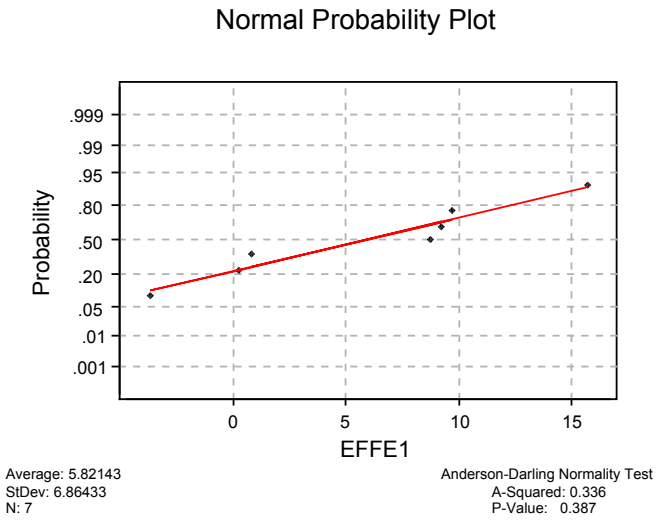
I + ABCD

A + BCD
B + ACD
C + ABD
D + ABC
AB + CD
AC + BD
AD + BC

A	B	C	D	taste
-1	-1	-1	-1	159
1	-1	-1	1	187
-1	1	-1	1	163
1	1	-1	-1	166
-1	-1	1	1	168
1	-1	1	-1	179
-1	1	1	-1	173
1	1	1	1	194

Estimated Effects

Term	Effect
A	15.750
B	0.750
C	9.750
D	8.750
A*B	-3.750
A*C	0.250
A*D	9.250



Although it is not obvious from the normal probability plot,from the effect estimates, it appears that factor B may be *insignificant* along with the interaction AC = BD.

A regression model with factor B only indicates that B is insignificant.

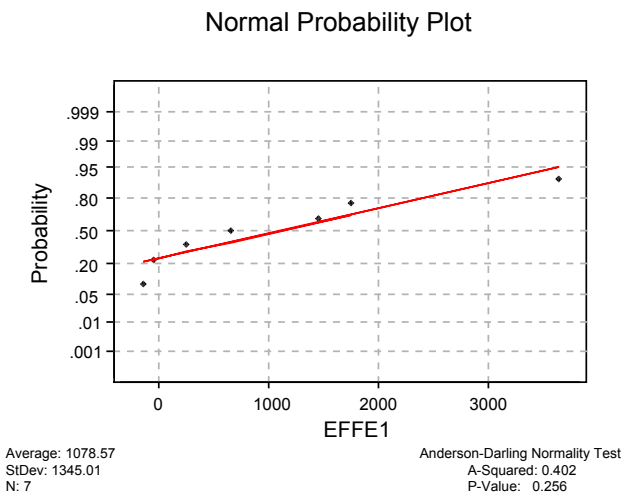
```
The regression equation is
taste = 174 + 0.37 B

Predictor       Coef       StDev          T        P
Constant     173.625       4.652      37.32    0.000
B              0.375       4.652       0.08    0.938
```

7-27.

```
Alias Structure

I + ABD + ACE + BCDE
A + BD + CE + ABCDE
B + AD + CDE + ABCE
C + AE + BDE + ABCD
D + AB + BCE + ACDE
E + AC + BCD + ABDE
BC + DE + ABE + ACD
BE + CD + ABC + ADE
```

A	B	C	D	E	Strength
-1	-1	-1	1	1	1900
1	-1	-1	-1	-1	900
-1	1	-1	-1	1	3500
1	1	-1	1	-1	6100
-1	-1	1	1	-1	800
1	-1	1	-1	1	1200
-1	1	1	-1	-1	3000
1	1	1	1	1	6800

```
Estimated Effects

Term         Effect
A           1450.00
B           3650.00
C           -150.00
D           1750.00
E            650.00
B*C          250.00
B*E          -50.00
```



Although it is not obvious from the normal probability plot,from the effect estimates it appears that A, B, D, and E (or their aliases) are significant.

A regression model with A, B, D, and E only indicates these main factors are significant.

```
The regression equation is
Strength = 3025 + 725 A + 1825 B + 875 D + 325 E

Predictor       Coef       StDev          T        P
Constant     3025.00       85.39      35.43    0.000
A             725.00       85.39       8.49    0.003
B            1825.00       85.39      21.37    0.000
D             875.00       85.39      10.25    0.002
E             325.00       85.39       3.81    0.032
```

7-28.

```
Alias Structure (up to order 4)

I + ABCE + ADEF + BCDF
A + BCE + DEF + ABCDF
B + ACE + CDF + ABDEF
C + ABE + BDF + ACDEF
D + AEF + BCF + ABCDE
E + ABC + ADF + BCDEF
F + ADE + BCD + ABCEF
AB + CE + ACDF + BDEF
AC + BE + ABDF + CDEF
AD + EF + ABCF + BCDE
AE + BC + DF + ABCDEF
AF + DE + ABCD + BCEF
BD + CF + ABEF + ACDE
BF + CD + ABDE + ACEF
ABD + ACF + BEF + CDE
ABF + ACD + BDE + CEF
```

A	B	C	D	E	F
1	1	-1	1	-1	-1
1	-1	-1	-1	1	-1
1	-1	-1	1	1	1
-1	1	1	-1	-1	-1
1	1	-1	-1	-1	1
-1	1	-1	-1	1	1
1	1	1	1	1	1
-1	-1	-1	1	-1	1
1	-1	1	-1	-1	1
-1	1	-1	1	1	-1
1	1	1	-1	1	-1
-1	-1	1	1	1	-1

```
-1   -1      1     -1      1      1
 1   -1      1      1     -1     -1
-1    1      1      1     -1      1
-1   -1     -1     -1     -1     -1
```

7-29.

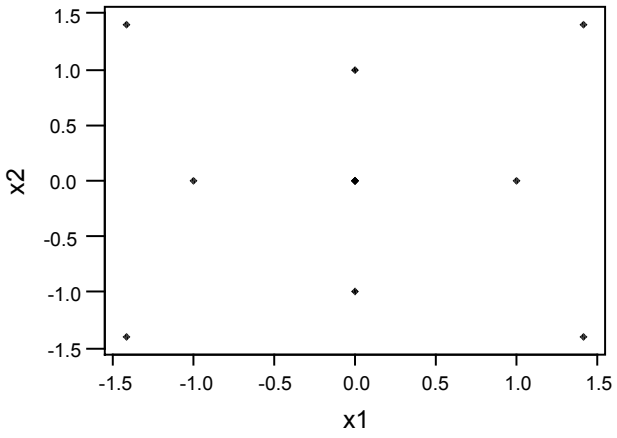
```
Design Generators:  D = AB  E = AC  F = BC

Alias Structure (up to order 2)

I
A + BD + CE + BEF + CDF + ABCF + ADEF + ABCDE
B + AD + CF + AEF + CDE + ABCE + BDEF + ABCDF
C + AE + BF + ADF + BDE + ABCD + CDEF + ABCEF
D + AB + EF + ACF + BCE + ACDE + BCDF + ABDEF
E + AC + DF + ABF + BCD + ABDE + BCEF + ACDEF
F + BC + DE + ABE + ACD + ABDF + ACEF + BCDEF
AF + BE + CD + ABC + ADE + BDF + CEF + ABCDEF
ABD + ACE + BCF + DEF + ABEF + ACDF + BCDE
```

A	B	C	D	E	F
-1	1	-1	-1	1	-1
1	1	-1	1	-1	-1
-1	-1	-1	1	1	1
-1	1	1	-1	-1	1
1	-1	1	-1	1	-1
1	-1	-1	-1	-1	1
-1	-1	1	1	-1	-1
1	1	1	1	1	1

Section 7-9

Note to the Instructor: The output in Minitab will name the sources differently than what is presented in the text. In the Analysis of Variance from regression analysis, the source, 'Regression' is equivalent to 'Model' used in the text. 'Residual Error' used by Minitab is equivalent to 'Residual' used the text.

7-30. a)



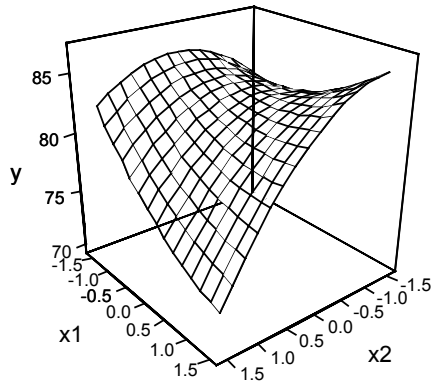
b)

```
Estimated Regression Coefficients for y
```

```
Term            Coef       StDev          T      P
Constant       82.024      0.5622    145.905  0.000
x1             -1.115      0.4397     -2.536  0.044
x2             -2.408      0.4397     -5.475  0.002
x1*x1           0.861      0.7343      1.172  0.286
x2*x2          -1.590      0.7342     -2.165  0.074
x1*x2          -1.801      0.3477     -5.178  0.002

S = 1.390       R-Sq = 92.0%     R-Sq(adj) = 85.3%

Analysis of Variance for y

Source          DF    Seq SS    Adj SS    Adj MS    F      P
Regression       5   132.837   132.837   26.5674  13.74  0.003
  Linear         2    70.393    70.391   35.1957  18.21  0.003
  Square         2    10.602    10.610    5.3048   2.74  0.142
  Interaction    1    51.842    51.842   51.8425  26.82  0.002
Residual Error   6    11.600    11.600    1.9333
  Lack-of-Fit    3    10.052    10.052    3.3507   6.50  0.079
  Pure Error     3     1.548     1.548    0.5158
Total           11   144.437

Unusual Observations for y

Obs          y        Fit  StDev Fit   Residual    St Resid
  4     75.900     78.027      0.929     -2.127      -2.06R
```

The second order model appears to be significant for the interaction term (p = 0.002). However, the squared terms are not significant (p = 0.142).

c)



There appears to be a saddle point in the experimental region. The yield increases as x_1 is decreased and x_2 is near the zero level.

7-31. a)

```
Estimated Regression Coefficients for y1

Term            Coef       StDev          T      P
Constant      500.37       4.527    110.542  0.000
x1             85.00       2.095     40.566  0.000
x2             35.00       2.095     16.703  0.000
x3            -71.67       2.095    -34.202  0.000
x1*x1           0.56       3.629      0.153  0.880
x2*x2          -2.78       3.629     -0.765  0.455
x3*x3           0.56       3.629      0.153  0.880
```

```
x1*x2          25.83       2.566     10.066  0.000
x1*x3          -1.67       2.566     -0.649  0.525
x2*x3           0.00       2.566      0.000  1.000

S = 8.890       R-Sq = 99.5%     R-Sq(adj) = 99.2%

Analysis of Variance for y1

Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         9      252642      252642     28071.3 355.20  0.000
  Linear           3      244550      244550     81516.7  1E+03  0.000
  Square           3          50          50        16.7   0.21  0.887
  Interaction      3        8042        8042      2680.6  33.92  0.000
Residual Error    17        1344        1344        79.0
Total             26      253985
```

With only the significant terms included, the analysis is

```
Estimated Regression Coefficients for y1

Term            Coef       StDev          T      P
Constant      499.26       1.550    322.129  0.000
x1             85.00       1.898     44.779  0.000
x2             35.00       1.898     18.439  0.000
x3            -71.67       1.898    -37.755  0.000
x1*x2          25.83       2.325     11.112  0.000

S = 8.053       R-Sq = 99.4%     R-Sq(adj) = 99.3%

Analysis of Variance for y1

Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         4      252558      252558     63139.6 973.52  0.000
  Linear           3      244550      244550     81516.7  1E+03  0.000
  Interaction      1        8008        8008      8008.3 123.48  0.000
Residual Error    22        1427        1427        64.9
Total             26      253985
```

The appropriate response surface model for viscosity is given by

$$y = 499.26 + 85.00x_1 + 35.00x_2 - 71.67x_3 + 25.83x_1x_2$$

b)

```
Estimated Regression Coefficients for y2

Term            Coef       StDev          T      P
Constant      299.00       2.596    115.190  0.000
x1             50.89       1.202     42.351  0.000
x2             75.78       1.202     63.065  0.000
x3             59.50       1.202     49.518  0.000
x1*x1         -17.33       2.081     -8.328  0.000
x2*x2         -34.00       2.081    -16.337  0.000
x3*x3         -17.17       2.081     -8.248  0.000
x1*x2          13.33       1.472      9.060  0.000
x1*x3          26.83       1.472     18.234  0.000
x2*x3         -17.92       1.472    -12.175  0.000

S = 5.098       R-Sq = 99.8%     R-Sq(adj) = 99.7%
Analysis of Variance for y2

Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         9      238832      238832     26536.9  1E+03  0.000
  Linear           3      213700      213700     71233.2  3E+03  0.000
  Square           3       10507       10507      3502.3 134.76  0.000
  Interaction      3       14626       14626      4875.2 187.59  0.000
Residual Error    17         442         442        26.0
```

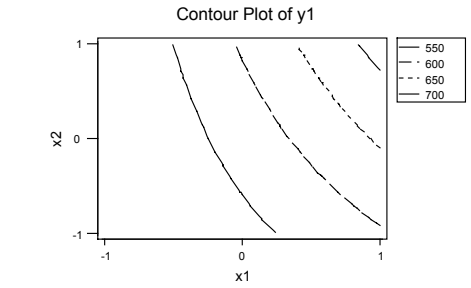
Total	26	239274

The appropriate response surface model for conversion is given by

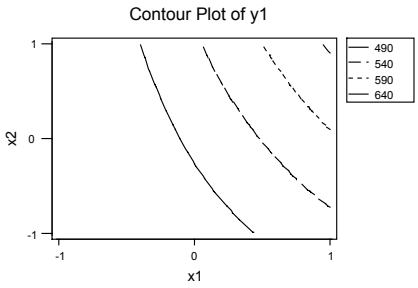
$$y_2 = 299 + 50.89x_1 + 75.78x_2 + 59.50x_3 - 17.33\,x_1^2 - 34\,x_2^2 - 17.17\,x_3^2 + 13.33x_1x_2 + 26.83x_1x_3 - 17.92x_2x_3$$

c) For the settings of x_1, x_2, and x_3 we can find the appropriate levels using contour plots. Say, for example, we set $x_3 = -1$, 0, and 1 and examine the contour plots.

Contour Plot for y_1 (viscosity) with $x_3 = -1$: Contour plot for y_1 with $x_3 = 0$

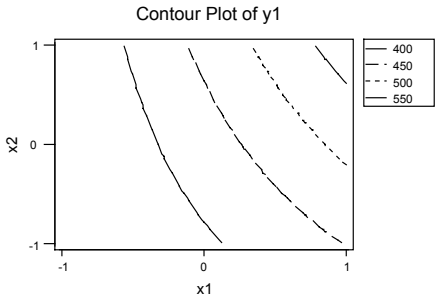


Hold values: x3:-1.0



Hold values: x3: 0.0

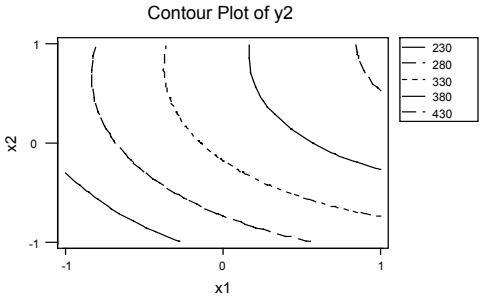
Contour Plot for y_1 with $x_3 = 1$



Hold values: x3: 1.0

From the third plot with $x_3 = 1$, we see a possible range for x_1 and x_2 to satisfy the constraint $450 < y_1 < 500$.

With $x_3 = 1$, set contour plots for various levels of x_1 and x_2 for y_2 (conversion).



Hold values: x3: 1.0

7-32. a) $10 + 5x_1 + 2x_2 > 12$ $\qquad$ $23 + 3x_1 + 4x_2 > 27.50$

$x_2 > -\frac{5}{2}x_1 + 1$ $\qquad$ $x_2 < -0.75x_1 + 1.125$

The feasible region can be shown graphically on the x_1-x_2 plane.

b) Operating the process with $x_1 = 1.5$ and $x_2 = -1.5$ results in y_1 and y_2 comfortably within the feasible region.

7-33. a) A central composite design has been used but it is not rotatable.

b)

Estimated Regression Coefficients for y

Term	Coef	StDev	T	P
Constant	150.04	7.821	19.184	0.000
x1	-58.47	5.384	-10.861	0.000
x2	3.35	5.384	0.623	0.556
x1*x1	-6.53	5.693	-1.147	0.295
x2*x2	10.58	5.693	1.859	0.112
x1*x2	0.50	7.848	0.064	0.951

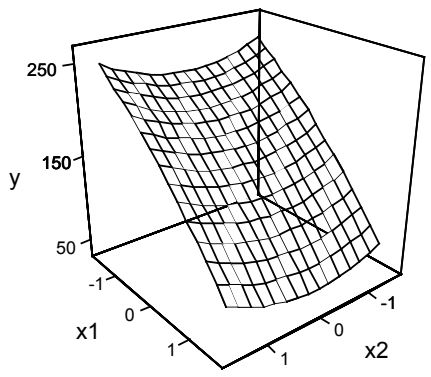
S = 15.70 R-Sq = 95.4% R-Sq(adj) = 91.6%

Analysis of Variance for y

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	5	30688.7	30688.7	6137.7	24.91	0.001
Linear	2	29155.4	29155.4	14577.7	59.17	0.000
Square	2	1532.3	1532.3	766.1	3.11	0.118
Interaction	1	1.0	1.0	1.0	0.00	0.951
Residual Error	6	1478.2	1478.2	246.4		
Lack-of-Fit	3	4.2	4.2	1.4	0.00	1.000
Pure Error	3	1474.0	1474.0	491.3		
Total	11	32166.9				

The linear terms appear to be significant ($p = 0.001$) while both the square terms and interaction terms are insignificant ($p = 0.118$ and $p = 0.951$, respectively).

c) Since x_1 is the only significant factor, to minimize ash, increase the value of x_1.



7-34. a)

Estimated Regression Coefficients for y1

Term	Coef	StDev	T	P
Constant	311.190	31.11	10.002	0.000
x1	158.983	14.35	11.083	0.000
x2	138.830	14.94	9.293	0.000
x3	160.870	14.94	10.768	0.000

```
x1*x1          27.252       26.21      1.040  0.314
x2*x2           7.020       25.19      0.279  0.784
x3*x3           0.346       25.19      0.014  0.989
x1*x2          39.001       17.57      2.220  0.041
x1*x3          48.443       17.57      2.757  0.014
x2*x3          87.690       18.65      4.702  0.000

S = 60.86       R-Sq = 95.5%     R-Sq(adj) = 93.0%

Analysis of Variance for y1

Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         9     1267693     1267693      140855  38.03  0.000
  Linear           3     1130334     1150835      383612 103.56  0.000
  Square           3        9052        4431        1477   0.40  0.756
  Interaction      3      128307      128307       42769  11.55  0.000
Residual Error    16       59267       59267        3704
Total             25     1326960

Unusual Observations for y1

Obs         y1        Fit  StDev Fit   Residual    St Resid
  9    271.670    385.619     43.572   -113.949      -2.68R
 19    220.670    111.733     43.572    108.937       2.56R

R denotes an observation with a large standardized residual
```

The quadratic model for y_1 is

$$y_1 = 331.190 + 158.983x_1 + 138.830x_2 + 160.870x_3 + 39.001x_1x_2 + 48.443x_1x_3 + 87.69x_2x_3$$

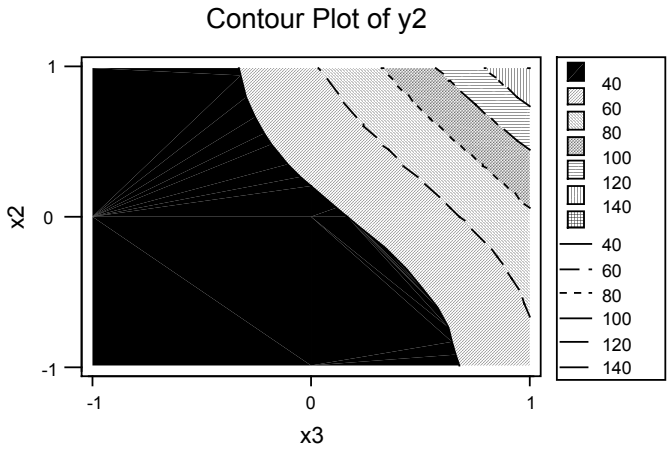
b)

```
Estimated Regression Coefficients for y2

Term             Coef       StDev          T      P
Constant       49.604       8.148      6.088  0.000
x1              6.893       9.769      0.706  0.488
x2             17.736      10.083      1.759  0.092
x3             31.605      10.083      3.135  0.005

S = 41.45       R-Sq = 36.8%     R-Sq(adj) = 28.1%

Analysis of Variance for y2

Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         3     21968.9     21968.9     7322.96   4.26  0.016
  Linear           3     21968.9     21968.9     7322.96   4.26  0.016
Residual Error    22     37793.3     37793.3     1717.88
Total             25     59762.2

Unusual Observations for y2

Obs         y2        Fit  StDev Fit   Residual    St Resid
 19    133.820     56.579     18.780     77.241       2.09R

R denotes an observation with a large standardized residual
```

The linear model for y_2 is given by

$$y_2 = 49.604 + 17.736x_2 + 31.605x_3$$

c)



The darkened section represents the smallest standard deviation for the combination of x_2 and x_3. If the level of 0 is chosen for both x_2 and x_3, then to attain the desired level of 500 for the mean, solve the quadratic equation found in part a using $y = 500$, $x_2 = 0$ and $x_3 = 0$ for x_1. The level of x_1 for this particular situation is approximately 1.

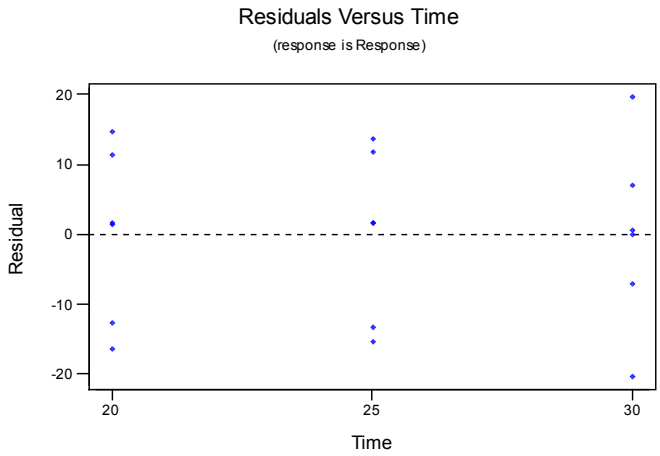
Section 7-11

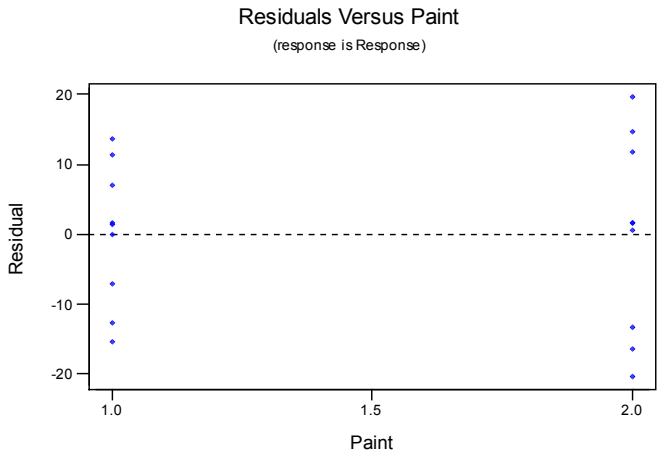
7-35. a)

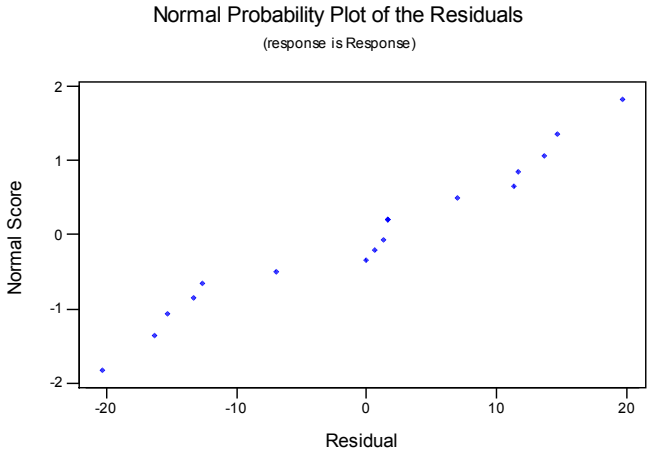
```
Factor     Type Levels Values
Paint     fixed      2      1     2
Time      fixed      3     20    25    30


Analysis of Variance for Response

Source        DF         SS         MS       F      P
Paint          1      420.5      420.5    2.09  0.174
Time           2       17.3        8.7    0.04  0.958
Paint*Time     2     1957.3      978.7    4.86  0.028
Error         12     2415.3      201.3
Total         17     4810.5
```

From the analysis of variance, paint type and drying time are not significant, but their interaction is significant at $\alpha = 0.05$

b)







The residual plots appear to be adequate. There is no serious departure from normality.

c) For smaller values, the paint type should be set at 1 and the drying time set at 25.

7-36. a)

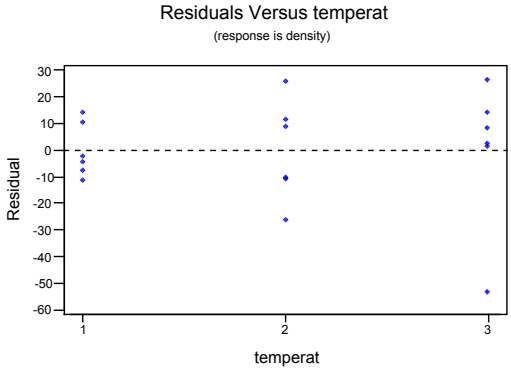
Factor	Type	Levels	Values		
position	fixed	2	1	2	
temperature	fixed	3	1	2	3

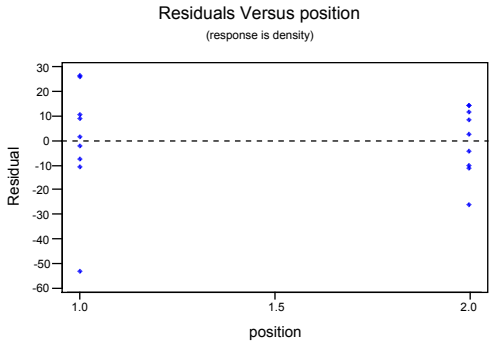
Analysis of Variance for density

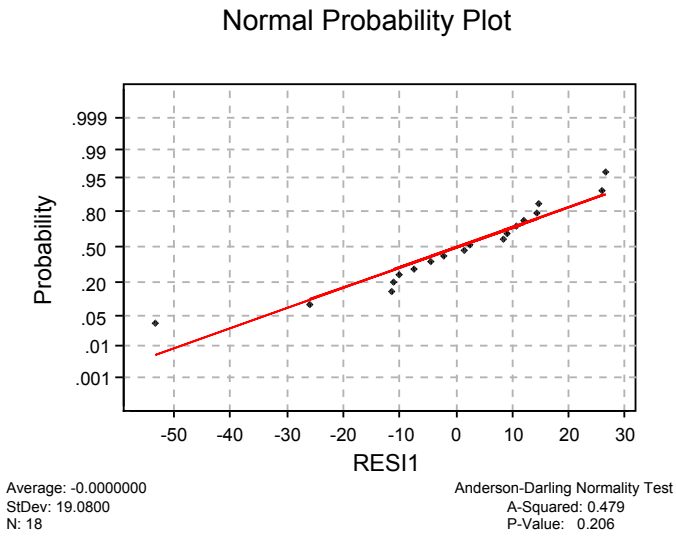
Source	DF	SS	MS	F	P
position	1	7160	7160	16.00	0.002
temperat	2	945342	472671	1056.12	0.000
position*temperat	2	818	409	0.91	0.427
Error	12	5371	448		
Total	17	958691			

The main effects of position and temperature are significant at the 0.05 level of significance, but the interaction is not significant at this level.

b)







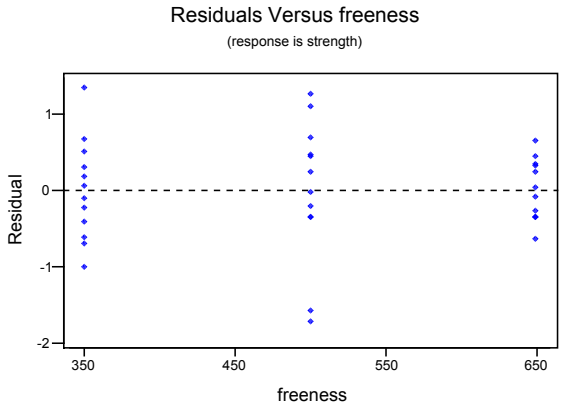
c) For higher density, set position to level 1 and temperature to level 2.

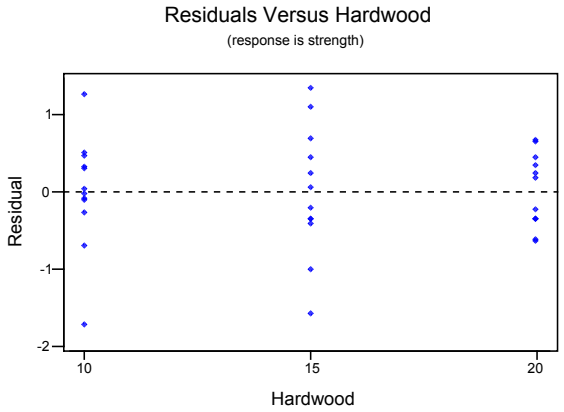
7-37. a)

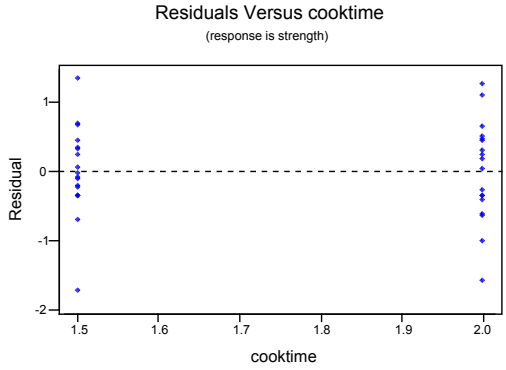
```
Factor     Type Levels Values
Hardwood   fixed      3     10     15     20
cooktime   fixed      2    1.5    2.0
freeness   fixed      3    350    500    650

Analysis of Variance for strength

Source                          DF         SS         MS        F       P
Hardwood                         2     8.3750     4.1875     7.55   0.004
cooktime                         1    17.3611    17.3611    31.31   0.000
freeness                         2    21.8517    10.9258    19.71   0.000
Hardwood*cooktime                2     3.2039     1.6019     2.89   0.082
Hardwood*freeness                4     6.5133     1.6283     2.94   0.050
cooktime*freeness                2     1.0506     0.5253     0.95   0.406
Hardwood*cooktime*freeness       4     2.0844     0.5211     0.94   0.463
Error                           18     9.9800     0.5544
Total                           35    70.4200
```

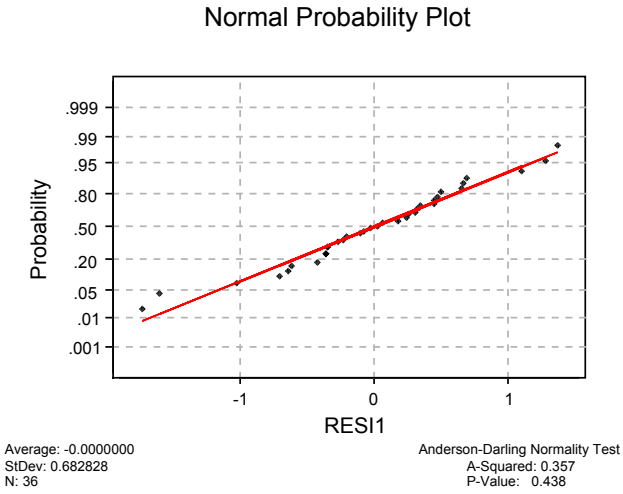
b) The F-ratios and p-values are given in part a. The main factors of hardwood, cook time, and freeness are all significant at the $\alpha = 0.05$ level. The interaction of hardness*freeness is slightly significant. It will be included in the final analysis.

c)









The residual plots appear to be adequate.

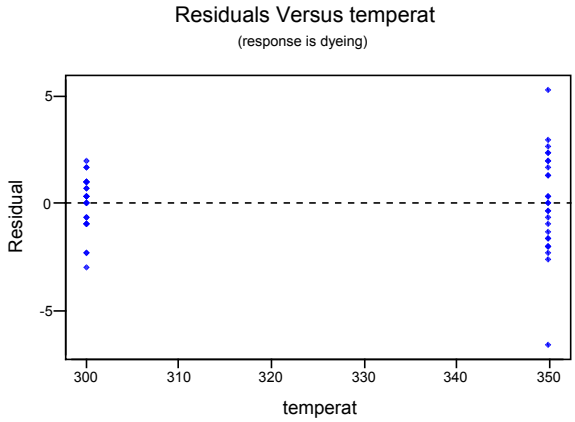
7-38. a)

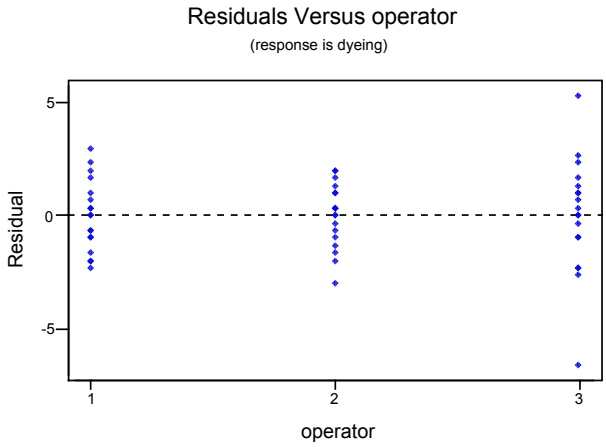
```
Factor     Type Levels Values
cycletim  fixed      3     40     50     60
operator  fixed      3      1      2      3
temperat  fixed      2    300    350

Analysis of Variance for dying

Source                          DF          SS          MS        F       P
```

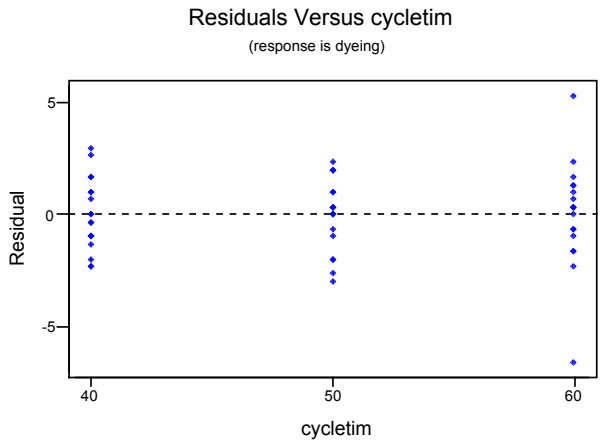
cycletim	2	361.593	180.796	34.50	0.000
operator	2	257.815	128.907	24.60	0.000
temperat	1	98.685	98.685	18.83	0.000
cycletim*operator	4	267.630	66.907	12.77	0.000
cycletim*temperat	2	66.704	33.352	6.36	0.004
operator*temperat	2	24.481	12.241	2.34	0.111
cycletim*operator*temperat	4	68.963	17.241	3.29	0.021
Error	36	188.667	5.241		
Total	53	1334.537			

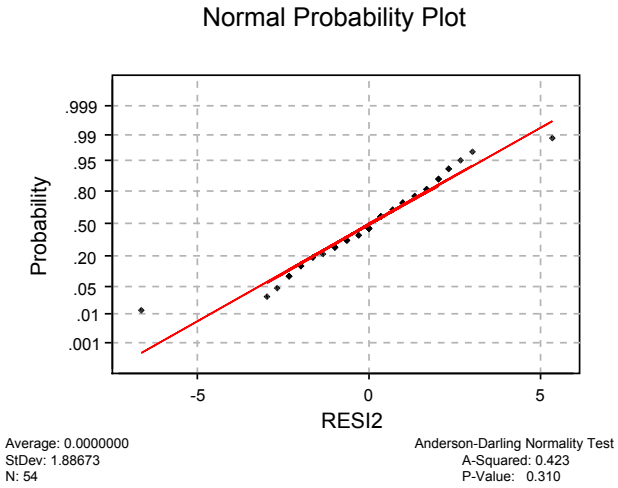
Only the interaction operator*temperature is insignificant at the 0.05 level of significance. All other main factors and interactions are significant.

b)









The residual plots appear to be adequate.

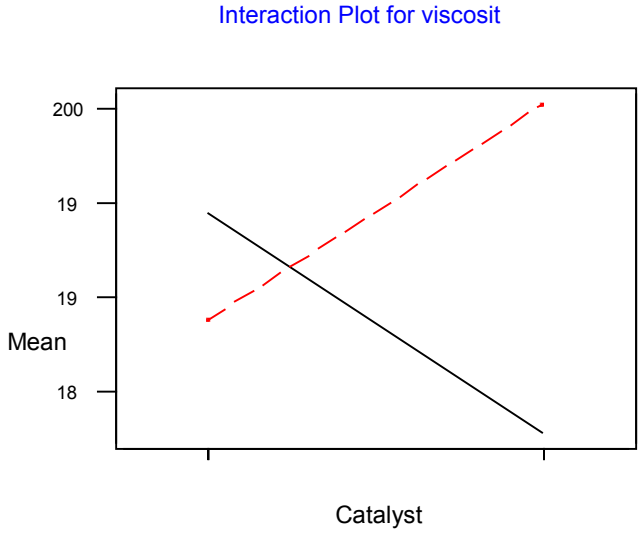
Supplemental Exercises

7-39. a)

```
Estimated Effects and Coefficients for viscosit
Term            Effect      Coef  StDev Coef       T      P
Constant                 191.563       1.158  165.49  0.000
pH               5.875     2.937       1.158    2.54  0.026
Catalyst        -0.125    -0.062       1.158   -0.05  0.958
pH*Catalyst     11.625     5.812       1.158    5.02  0.000
```

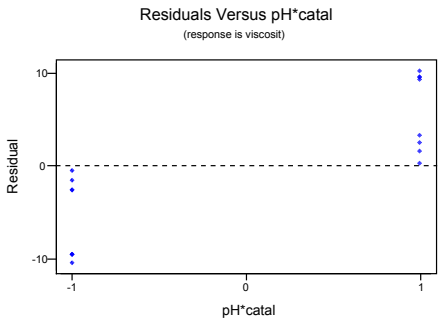
The main effect of pH and the interaction of pH and Catalyst are significant at the 0.05 level of significance.

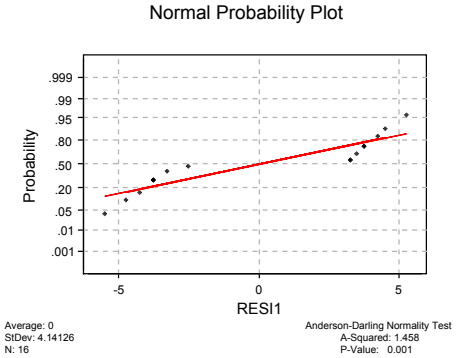
b)



The interaction plot supports the information found in the analysis of part a. There appears to be a significant interaction between pH and Catalyst.

c) The model used is viscosity = 191.563 + 2.937pH – 0.062 Catalyst + 5.812 pH*Catalyst





The residual plots are adequate. There does not appear to be any serious departure from normality or violation of the assumption of constant variance.

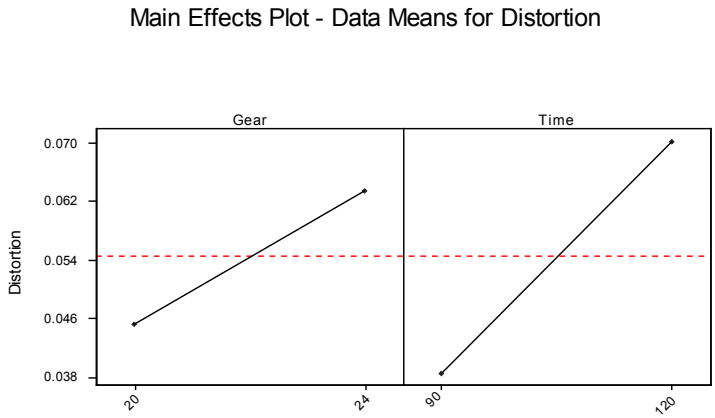
7-40. a)

```
Gear       fixed       2     20     24
Time       fixed       2     90    120

Analysis of Variance for Distortion
Source        DF          SS          MS        F      P
Gear           1 0.00065703 0.00065703    11.50  0.028
Time           1 0.00200028 0.00200028    35.00  0.004
```

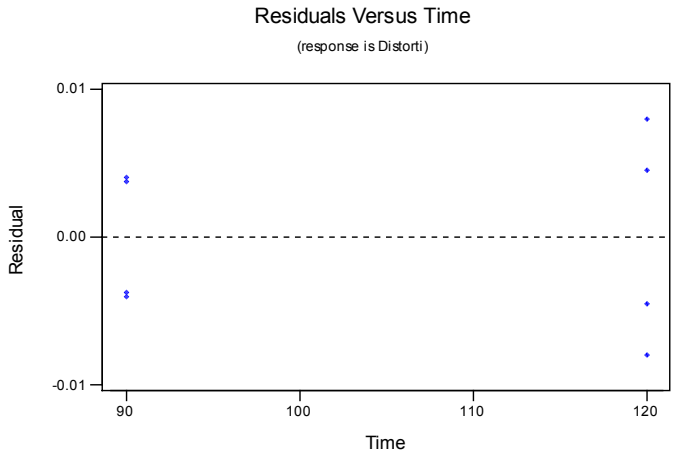
Gear*Time	1	0.00000378	0.00000378	0.07	0.810
Error	4	0.00022863	0.00005716		
Total	7	0.00288972			

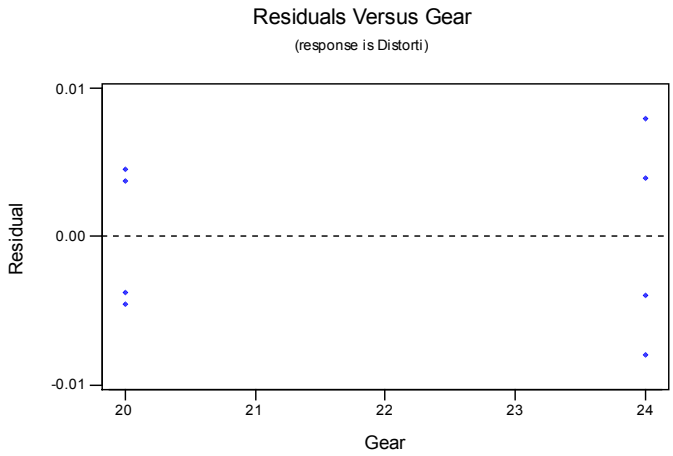
Both Gear Type and Time are significant factors. Their interaction is not significant.

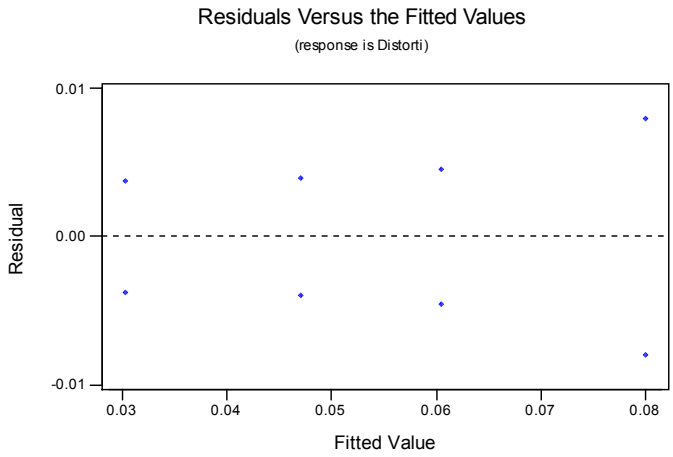
b)

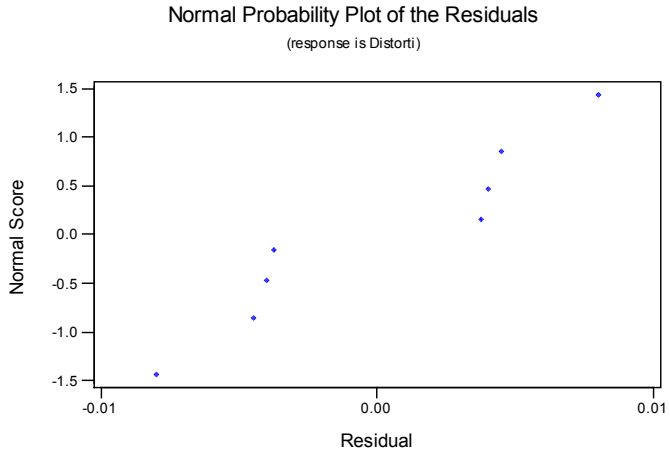


c)









There appears to be some problem with constant variance and possibility normality.

7-41. a)

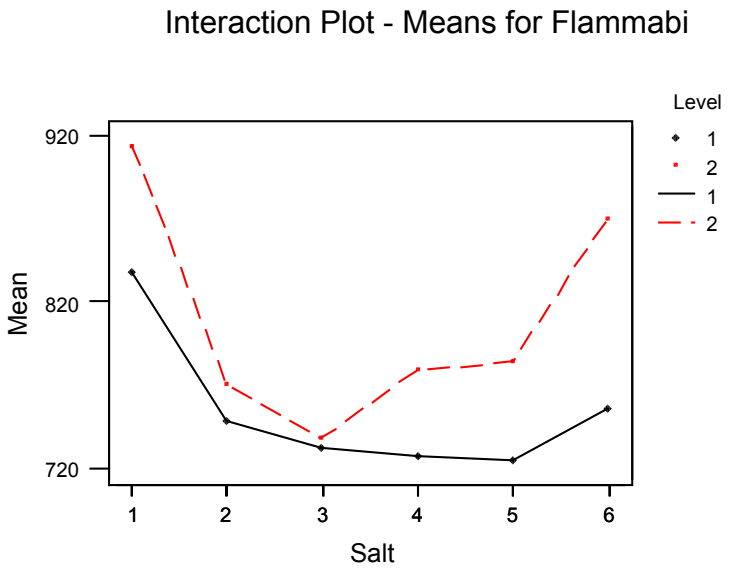
Factor	Type	Levels	Values					
Level	fixed	2	1	2				
Salt	fixed	6	1	2	3	4	5	6

Analysis of Variance for Flammability

Source	DF	SS	MS	F	P
Level	1	27390	27390	63.24	0.000
Salt	5	86087	17217	39.75	0.000
Level*Salt	5	11459	2292	5.29	0.002
Error	24	10395	433		
Total	35	135332			

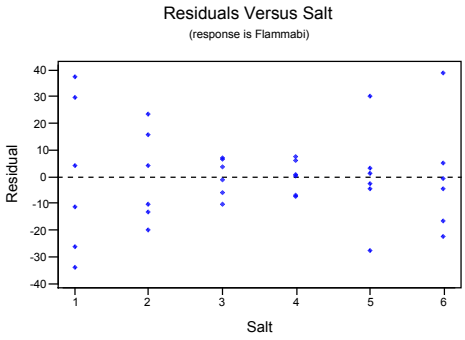
There is a significant difference between the application levels, the salts, and there is a significant difference between the levels of the interaction of the two.

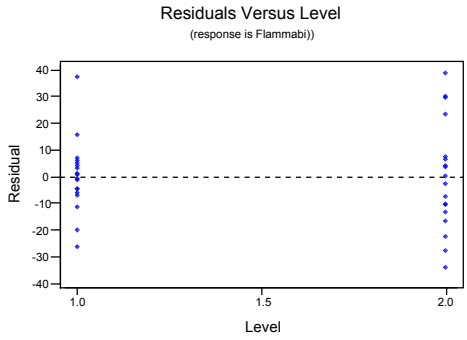
b)

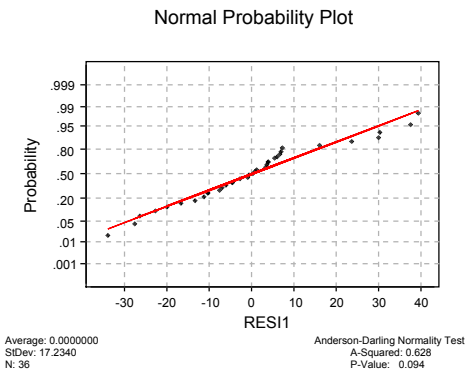


From the interaction plot, we see that the untreated salt has a higher flammability average than any of the other five levels. The remaining five levels ($MgCl_2$, NaCl, $CaCO_3$, $CaCl_2$, Na_2CO_3) appear to have similar flammability averages. Overall, application level 1 increases the flammability average.

c)







The residual plots indicate a significant difference among the salt types, while there does not appear to be a significant difference between residuals for the two application levels. There is no apparent violation of the normality assumption.

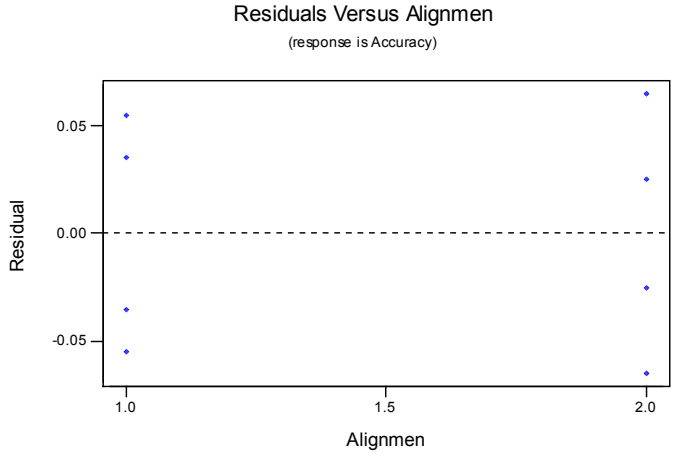
7-42. a) Analysis of Variance for Accuracy

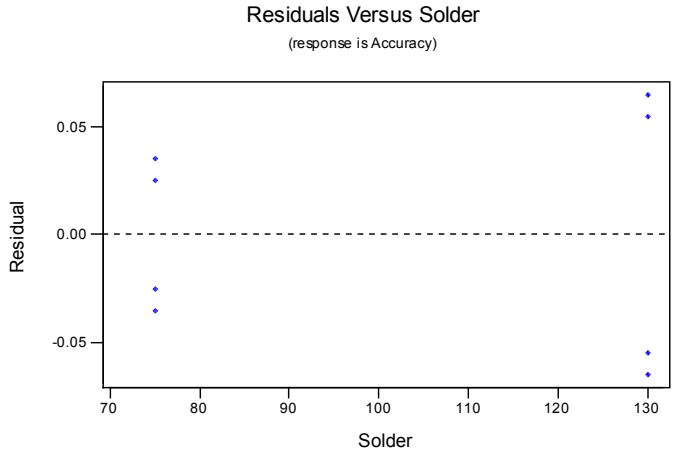
Source	DF	SS	MS	F	P
Solder	1	2.6912	2.6912	591.47	0.000
Alignmen	1	12.7008	12.7008	2791.38	0.000
Solder*Alignmen	1	2.0808	2.0808	457.32	0.000
Error	4	0.0182	0.0046		
Total	7	17.4910			

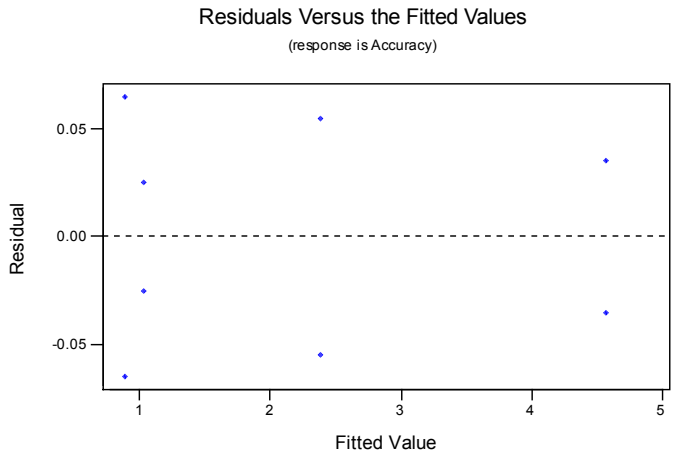
Solder bump size, alignment method, and their interaction are all significant.

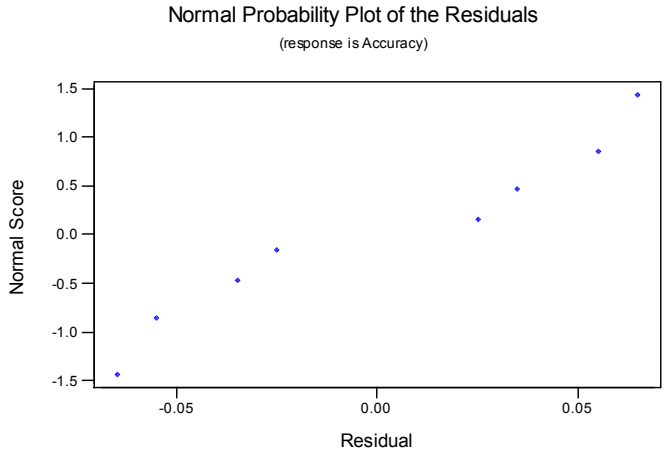
b) To decrease the response of interest, set Solder bump size at 130 and use alignment method 2.

c)









The only problem may be with constant variance as it appears in the residual plot versus the factor solder.

7-43. a) `Estimated Effects`

Term	Effect
A	-2.74
B	-6.66
C	3.49
A*B	-8.71
A*C	7.04
B*C	11.46
A*B*C	-6.49

b) The hand calculation of experimental error is found using:

$$\hat{\sigma}_c^2 = \sum_{\substack{\text{center} \\ \text{pts}}} \frac{(y_i - \bar{y}_c)^2}{(n_c - 1)} = 8.40$$

c) Estimated Effects and Coefficients for deltaline width

Term	Effect	Coef	StDev Coef	T	P
Constant		-11.82	1.328	-8.90	0.001
A	-2.74	-1.37	1.626	-0.84	0.447
B	-6.66	-3.33	1.626	-2.05	0.110
C	3.49	1.75	1.626	1.07	0.343
A*B	-8.71	-4.35	1.626	-2.68	0.055
A*C	7.04	3.52	1.626	2.17	0.096
B*C	11.46	5.73	1.626	3.52	0.024
A*B*C	-6.49	-3.25	1.626	-2.00	0.117

Analysis of Variance for deltaline width

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	128.082	128.082	42.694	2.02	0.254
2-Way Interactions	3	513.383	513.383	171.128	8.09	0.036
3-Way Interactions	1	84.305	84.305	84.305	3.99	0.117
Residual Error	4	84.608	84.608	21.152		
Curvature	1	59.441	59.441	59.441	7.09	0.076
Pure Error	3	25.167	25.167	8.389		
Total	11	810.379				

Using the output given in part b), it appears from the Minitab output that the 2-way interactions are significant (p = 0.036), while the main effects and 3-way interaction are not (p = 0.254 and 0.117, respectively). Curvature is also insignificant with p = 0.076 at 0.05. The interaction B*C is significant at 0.05.

d) Using 'Regression' in Minitab the resulting model should be

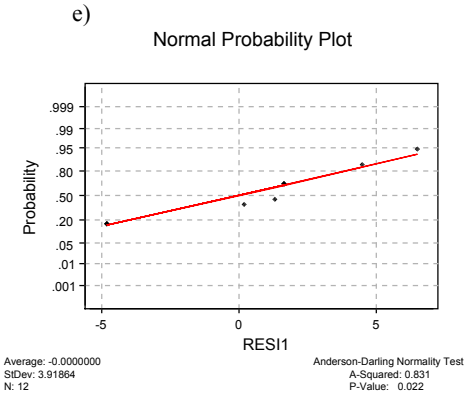
The regression equation is

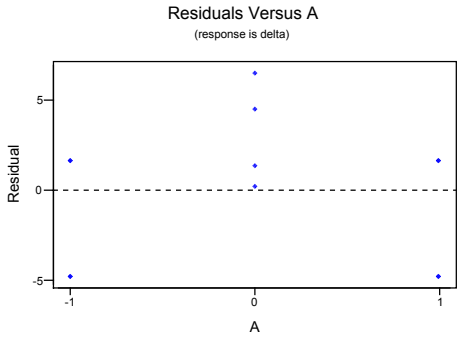
deltaline = - 11.8 - 1.37 A - 3.33 B + 1.75 C - 4.35 AB + 3.52 AC + 5.73 BC

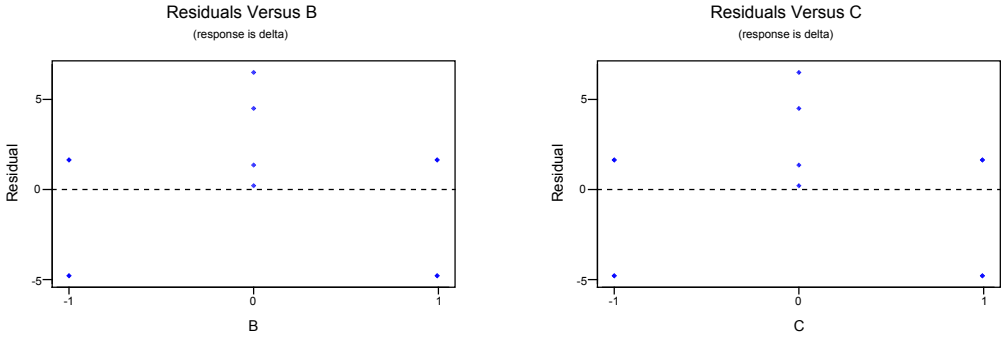
Predictor	Coef	StDev	T	P
Constant	-11.822	1.678	-7.05	0.001
A	-1.371	2.055	-0.67	0.534
B	-3.329	2.055	-1.62	0.166
C	1.746	2.055	0.85	0.434
ab	-4.354	2.055	-2.12	0.088
ac	3.521	2.055	1.71	0.147
bc	5.729	2.055	2.79	0.039

S = 5.812 R-Sq = 79.2% R-Sq(adj) = 54.1%

e)





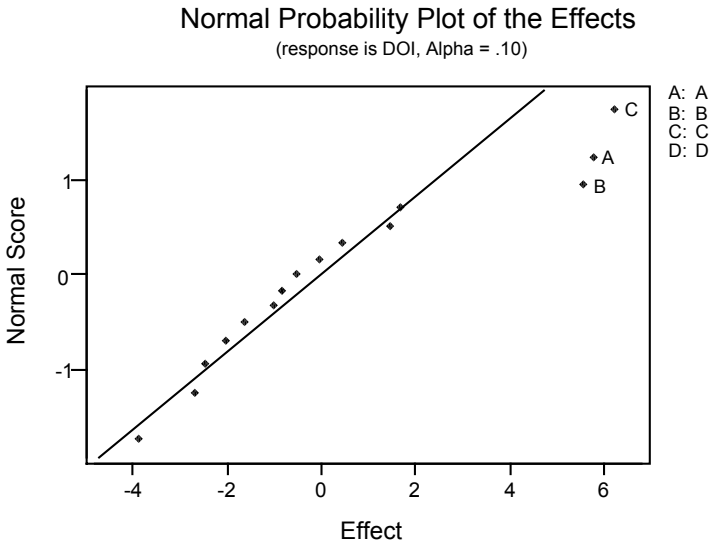


The residual plots appear to be adequate.

7-44. a) Estimated Effects and Coefficients for DOI

Term	Effect	Coef
Constant		72.894
A	5.763	2.881
B	5.563	2.781
C	6.238	3.119
D	-3.862	-1.931
A*B	-2.013	-1.006
A*C	-0.837	-0.419
A*D	-2.488	-1.244
B*C	-0.537	-0.269
B*D	1.462	0.731
C*D	-0.063	-0.031
A*B*C	0.437	0.219
A*B*D	-1.013	-0.506
A*C*D	1.663	0.831
B*C*D	-2.687	-1.344
A*B*C*D	-1.612	-0.806

b)



Based on the normal probability plot it appears that factors A, B, and C are significant.

c) Run an analysis using the main factors A, B, and C and interactions among these variables to see if any are significant.

Predictor	Coef	StDev	T	P
Constant	72.894	1.073	67.92	0.000
A	2.881	1.073	2.68	0.028
B	2.781	1.073	2.59	0.032
C	3.119	1.073	2.91	0.020

```
ab            -1.006       1.073      -0.94    0.376
ac            -0.419       1.073      -0.39    0.707
bc            -0.269       1.073      -0.25    0.809
abc            0.219       1.073       0.20    0.844

S = 4.293       R-Sq = 74.6%     R-Sq(adj) = 52.4%
```

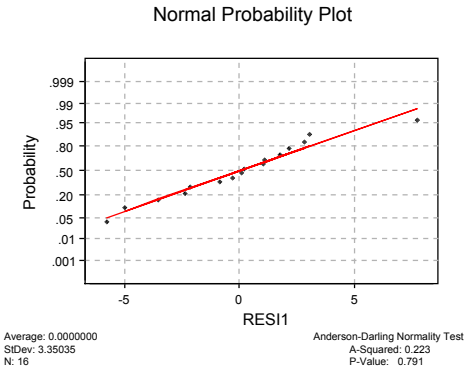
Based on this analysis, only the main factors A, B, and C are significant. Run a regression analysis using these important factors.

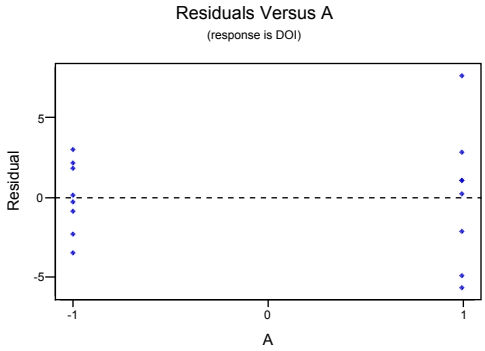
d)

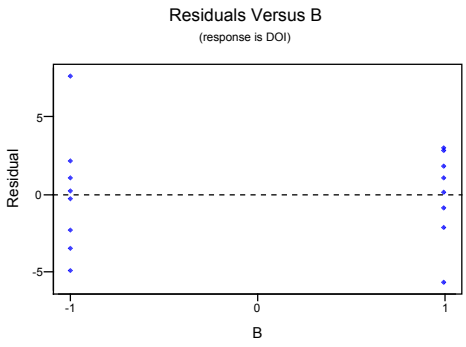
```
The regression equation is
DOI = 72.9 + 2.88 A + 2.78 B + 3.12 C
Predictor       Coef       StDev          T        P
Constant     72.8937      0.9365      77.84    0.000
A             2.8813      0.9365       3.08    0.010
B             2.7813      0.9365       2.97    0.012
C             3.1188      0.9365       3.33    0.006

S = 3.746       R-Sq = 71.0%     R-Sq(adj) = 63.7%

Analysis of Variance
Source       DF          SS          MS         F        P
Regression    3      412.22      137.41      9.79    0.002
Error        12      168.37       14.03
Total        15      580.59

Source       DF      Seq SS
A             1      132.83
B             1      123.77
C             1      155.63
```

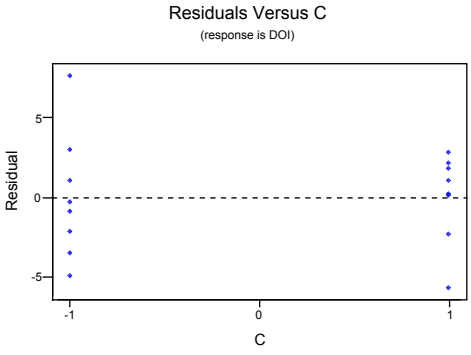
DOI = 72.9 + 2.88 A + 2.78 B + 3.12 C

e)







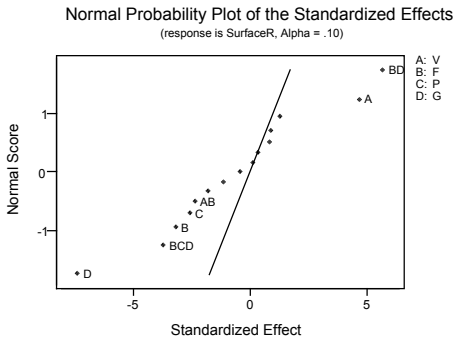


The residual plots appear to be adequate.

7-45. a)

```
    Estimated Effects
Term          Effect
V              15.75
F             -10.75
P              -8.75
G             -25.00
V*F            -8.00
V*P             3.00
V*G             2.75
F*P            -6.00
F*G            19.25
P*G            -3.75
V*F*P           1.25
V*F*G          -1.50
V*P*G           0.50
```

b)



According to the probability plot, factors V, F, P, and G along with the interactions VF and FPG. are possibly significant.

c) Running a regression analysis using these factors we find

```
SurfaceRough = 101 + 7.88 V - 5.37 F - 4.38 P - 12.5 G - 6.25 FPG - 4.00VF
Predictor       Coef       StDevCoeff    T        P
Constant     100.522       2.420      41.53    0.000
V              7.875       2.902       2.71    0.015
F             -5.375       2.902      -1.85    0.083
P             -4.375       2.902      -1.51    0.151
G            -12.500       2.902      -4.31    0.001
fpg           -6.250       2.902      -2.15    0.047
vf            -4.000       2.902      -1.38    0.187
S = 11.61       R-Sq = 70.5%     R-Sq(adj) = 59.4%

Analysis of Variance
Source       DF          SS          MS         F        P
Regression    6      5141.8       857.0      6.36    0.001
Error        16      2156.0       134.7
Total        22      7297.7

Source       DF      Seq SS
V             1       992.3
F             1       462.2
P             1       306.3
G             1      2500.0
fpg           1       625.0
vf            1       256.0
```

Based on this initial analysis we see that P, F, and the interaction VF are insignificant at the 0.05 level of significance.

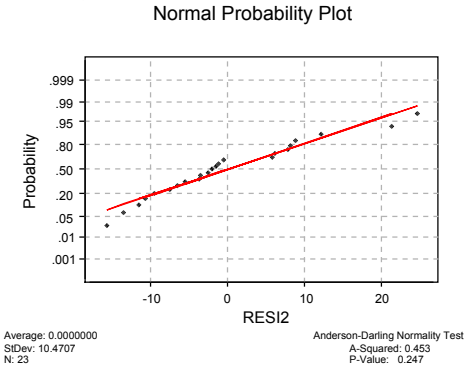
Thus, the regression analysis and final model are

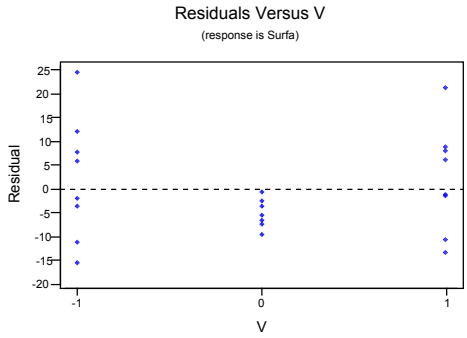
```
SurfaceRough = 101 + 7.88 V - 5.37 F - 4.38 P - 12.5 G - 6.25 fpg
Predictor       Coef       StDevCoef    T        P
Constant     100.522       2.484      40.47    0.000
V              7.875       2.978       2.64    0.017
F             -5.375       2.978      -1.80    0.089
P             -4.375       2.978      -1.47    0.160
G            -12.500       2.978      -4.20    0.001
```

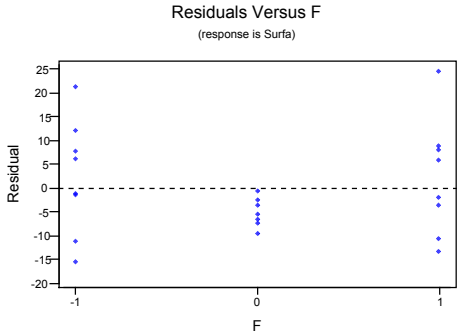
```
fpg           -6.250       2.978      -2.10    0.051

S = 11.91       R-Sq = 66.9%     R-Sq(adj) = 57.2%

Analysis of Variance
Source       DF          SS          MS         F        P
Regression    5      4885.8       977.2      6.89    0.001
Error        17      2412.0       141.9
Total        22      7297.7

Source       DF      Seq SS
V             1       992.3
F             1       462.2
P             1       306.3
G             1      2500.0
fpg           1       625.0
```

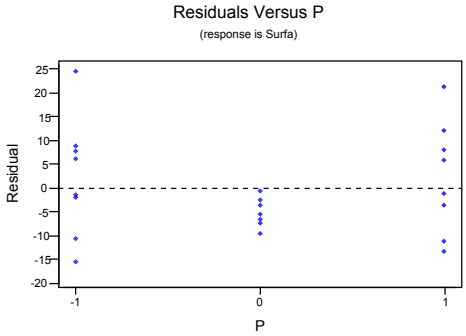
d) From the analysis, we see that jet traverse speed and abrasive grain size are significant along with the interaction of abrasive flow rate, abrasive grain size, and waterjet pressure. Since we have adopted a hierarchical modeling approach, flow rate and waterjet pressure are also included in the model.

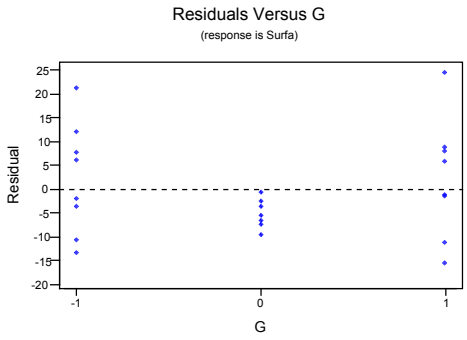
e)









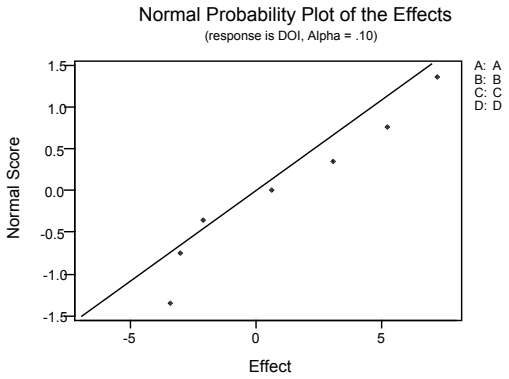


The residual plots appear to indicate the assumption of constant variance may not be met.

7-46. One possible design is

A	B	C	D	DOI	Estimated Effects

-1	-1	-1	-1	63.8	Term	Effect
1	-1	-1	1	64.9	A	3.075
-1	1	-1	1	72.7	B	7.225
1	1	-1	-1	76.5	C	5.225
-1	-1	1	1	68.0	D	-3.425
1	-1	1	-1	77.2	AB	-2.075
-1	1	1	-1	77.7	AC	0.625
1	1	1	1	75.9	AD	-3.025



It may be useful to run analysis with the main effects only to see which main effect is significant.

```
Term          Effect      Coef  StDev Coef       T      P
Constant                72.088       1.074   67.11  0.000
A              3.075     1.538       1.074    1.43  0.248
B              7.225     3.612       1.074    3.36  0.044
C              5.225     2.612       1.074    2.43  0.093
D             -3.425    -1.712       1.074   -1.59  0.209
Analysis of Variance for DOI
Source                DF      Seq SS     Adj SS     Adj MS      F      P
Main Effects           4      201.38     201.38     50.344   5.45  0.097
Residual Error         3       27.69      27.69      9.231
Total                  7      229.07
```

Based on this analysis, only factor B appears to be significant at the 0.05 level of significance. Comparing this result to that of problem 7-44, we see that the Factors A and C were not revealed as significant in the smaller design.

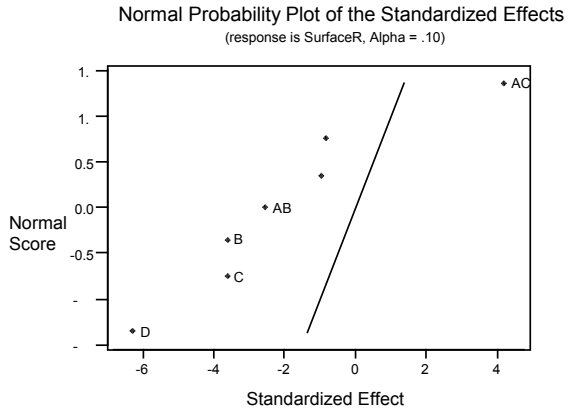
7-47. One possible design is

A	B	C	D	SurfaceRough
-1	-1	-1	-1	143
1	-1	-1	1	98
-1	1	-1	1	110
1	1	-1	-1	103
-1	-1	1	1	76
1	-1	1	-1	137
-1	1	1	-1	98
1	1	1	1	70
0	0	0	0	95
0	0	0	0	98
0	0	0	0	100
0	0	0	0	97
0	0	0	0	94
0	0	0	0	93
0	0	0	0	91

Where A = V, B = F, C = P, and D = G

Term	Effect
A	-4.75

B	-18.25
C	-18.25
D	-31.75
A*B	-12.75
A*C	21.25
A*D	-4.25



From the estimated effects, we could tentatively identify F, P, G, and the interactions VF and VP as significant.

The regression analysis gives

SurfaceRough = 100 - 2.37V - 9.13F - 9.13P - 15.9G - 6.37VF + 10.6VP- 2.13VG

Predictor	Coef	StDev	T	P
Constant	100.200	1.843	54.38	0.000
V	-2.375	2.523	-0.94	0.378
F	-9.125	2.523	-3.62	0.009
P	-9.125	2.523	-3.62	0.009
G	-15.875	2.523	-6.29	0.000
VF	-6.375	2.523	-2.53	0.039
VP	10.625	2.523	4.21	0.004
VG	-2.125	2.523	-0.84	0.428

From the analysis we see that factors F, P, G, VF, and VP are significant. In the analysis of 7-39, we found only V, G, and FPG significant.

7-48.

Fractional Factorial Design
Factors: 8 Base Design: 8, 16 Resolution: IV
Runs: 16 Replicates: 1 Fraction: 1/16
Design Generators: E = BCD F = ACD G = ABC H = ABD
Defining Relation: I = BCDE = ACDF = ABCG = ABDH = ABEF = ADEG = ACEH = BDFG = BCFH = CDGH = CEFG = DEFH = BEGH = AFGH = ABCDEFGH
Alias Structure (up to order 4)
I + ABCG + ABDH + ABEF + ACDF + ACEH + ADEG + AFGH + BCDE + BCFH + BDFG + BEGH + CDGH + CEFG + DEFH

A + BCG + BDH + BEF + CDF + CEH + DEG + FGH
B + ACG + ADH + AEF + CDE + CFH + DFG + EGH
C + ABG + ADF + AEH + BDE + BFH + DGH + EFG
D + ABH + ACF + AEG + BCE + BFG + CGH + EFH
E + ABF + ACH + ADG + BCD + BGH + CFG + DFH
F + ABE + ACD + AGH + BCH + BDG + CEG + DEH
G + ABC + ADE + AFH + BDF + BEH + CDH + CEF
H + ABD + ACE + AFG + BCF + BEG + CDG + DEF
AB + CG + DH + EF + ACDE + ACFH + ADFG + AEGH + BCDF + BCEH + BDEG + BFGH
AC + BG + DF + EH + ABDE + ABFH + ADGH + AEFG + BCDH + BCEF + CDEG + CFGH
AD + BH + CF + EG + ABCE + ABFG + ACGH + AEFH + BCDG + BDEF + CDEH + DFGH
AE + BF + CH + DG + ABCD + ABGH + ACFG + ADFH + BCEG + BDEH + CDEF + EFGH
AF + BE + CD + GH + ABCH + ABDG + ACEG + ADEH + BCFG + BDFH + CEFH + DEFG
AG + BC + DE + FH + ABDF + ABEH + ACDH + ACEF + BDGH + BEFG + CDFG + CEGH
AH + BD + CE + FG + ABCF + ABEG + ACDG + ADEF + BCGH + BEFH + CDFH + DEGH

Run	A	B	C	D	E	F	G	H
1	-	-	-	-	-	-	-	-
2	+	-	-	-	-	+	+	+

3	−	+	−	−	+	−	+	+
4	+	+	−	−	+	+	−	−
5	−	−	+	−	+	+	+	−
6	+	−	+	−	+	−	−	+
7	−	+	+	−	−	+	−	+
8	+	+	+	−	−	−	+	−
9	−	−	−	+	+	+	−	+
10	+	−	−	+	+	−	+	−
11	−	+	−	+	−	+	+	−
12	+	+	−	+	−	−	−	+
13	−	−	+	+	−	−	+	+
14	+	−	+	+	−	+	−	−
15	−	+	+	+	+	−	−	−
16	+	+	+	+	+	+	+	+

7-49.

Fractional Factorial Design

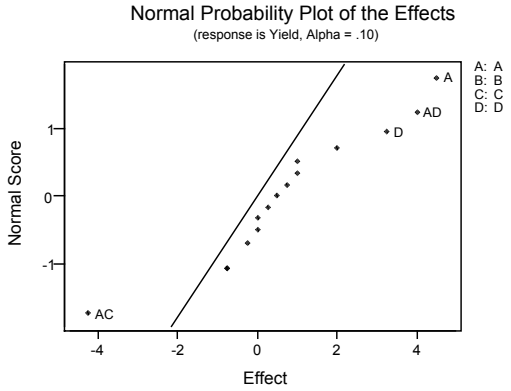
Factors: 5 Base Design: 5, 8 Resolution: III
Runs: 8 Replicates: 1 Fraction: 1/4
Blocks: none Center pts (total): 0

*** NOTE *** Some main effects are confounded with two-way interactions

Design Generators: D = AB E = AC
Defining Relation: I = ABD = ACE = BCDE
Alias Structure I + ABD + ACE + BCDE
A + BD + CE + ABCDE
B + AD + CDE + ABCE
C + AE + BDE + ABCD
D + AB + BCE + ACDE
E + AC + BCD + ABDE
BC + DE + ABE + ACD
BE + CD + ABC + ADE

Run	A	B	C	D	E
1	−	−	−	+	+
2	+	−	−	−	−
3	−	+	−	−	+
4	+	+	−	+	−
5	−	−	+	+	−
6	+	−	+	−	+
7	−	+	+	−	−
8	+	+	+	+	+

7-50. a)



The factors that appear to have large effects are A, D, AC, and AD.

b) The regression analysis is

Estimated Effects and Coefficients for Yield

Term	Effect	Coef	StDev Coef	T	P
Constant		17.375	0.3187	54.52	0.000
A	4.500	2.250	0.3187	7.06	0.000
C	2.000	1.000	0.3187	3.14	0.011
D	3.250	1.625	0.3187	5.10	0.000
A*C	-4.250	-2.125	0.3187	-6.67	0.000
A*D	4.000	2.000	0.3187	6.28	0.000

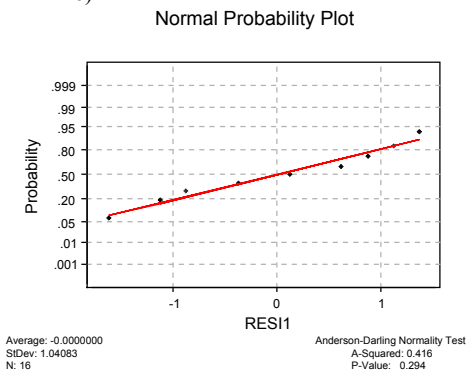
Analysis of Variance for Yield

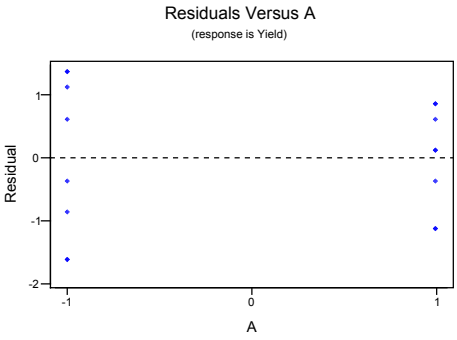
Source	DF	Seq SS	Adj SS	Adj MS	F	P

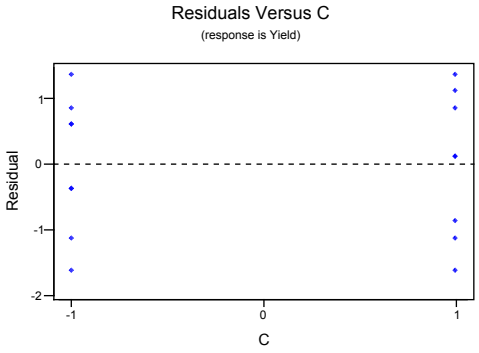
Main Effects	3	139.250	139.250	46.4167	28.56	0.000
2-Way Interactions	2	136.250	136.250	68.1250	41.92	0.000
Residual Error	10	16.250	16.250	1.6250		
Lack of Fit	2	0.250	0.250	0.1250	0.06	0.940
Pure Error	8	16.000	16.000	2.0000		
Total	15	291.750				

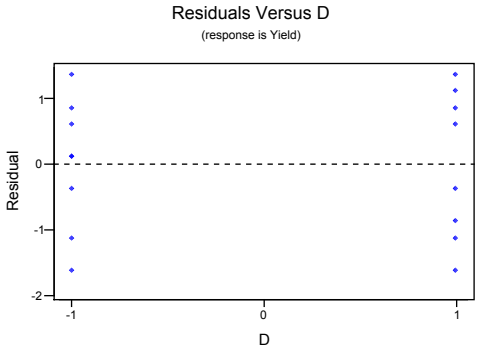
The conclusions are that factors A, C, and D are significant along with the interactions AC and AD.

c)





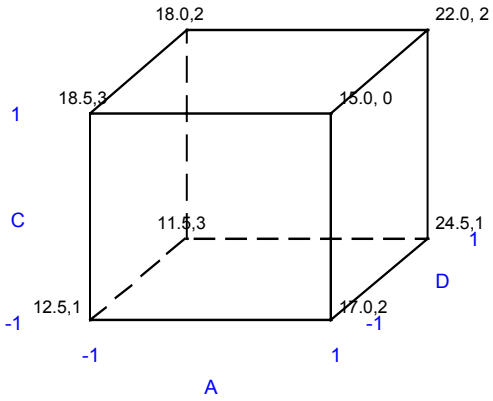




The residual plots appear to be adequate.

d) The design can be collapsed into a 2^3 design as shown

Cube Plot - Means for Yield



The data are Mean, Range for each location. It is evident that the mean yield is largest for high levels of A and D and low level of C.

7-51. If we consider the main effects A, C, and D as significant, we may want to investigate the interactions containing these factors.

Term	Effect	Coef	StDev Coef	T	P
A	4.500	2.250	0.3536	6.36	0.000
C	2.000	1.000	0.3536	2.83	0.022
D	3.250	1.625	0.3536	4.60	0.000
A*C	-4.250	-2.125	0.3536	-6.01	0.000
A*D	4.000	2.000	0.3536	5.66	0.000

Using the relationship 2(st.dev coef) = st. err. we have
2(0.3536) = 0.7072
For a 95% confidence interval, $t_{8,0.025} = 2.307$
Therefore the 95% confidence intervals for the effects are given by: effect ± 2.307(0.072)
or effect ± 1.6315.

7-52. a) The generator for this fraction was I=ABCD
Totally confounded terms were removed from the analysis

```
I + a*b*c*d
a + b*c*d
b + a*c*d
c + a*b*d
d + a*b*c
e + a*b*c*d*e
a*b + c*d
a*c + b*d
a*d + b*c
a*e + b*c*d*e
b*e + a*c*d*e
c*e + a*b*d*e
d*e + a*b*c*e
a*b*e + c*d*e
a*c*e + b*d*e
a*d*e + b*c*e
```

b) Estimated Effects and Coefficients for freeheig

Term	Effect	Coef	StDev Coef	T	P
Constant		7.6400	0.01901	401.97	0.000
a	0.2133	0.1067	0.01901	5.61	0.000
b	-0.1925	-0.0963	0.01901	-5.06	0.000

c	-0.0783	-0.0392	0.01901	-2.06	0.048
d	0.0625	0.0313	0.01901	1.64	0.110
e	-0.2100	-0.1050	0.01901	-5.52	0.000
a*b	-0.0008	-0.0004	0.01901	-0.02	0.983
a*c	0.0300	0.0150	0.01901	0.79	0.436
a*d	0.0058	0.0029	0.01901	0.15	0.879
a*e	0.0350	0.0175	0.01901	0.92	0.364
b*e	0.1242	0.0621	0.01901	3.27	0.003
c*e	-0.0617	-0.0308	0.01901	-1.62	0.115
d*e	0.0108	0.0054	0.01901	0.28	0.777
a*b*e	0.0308	0.0154	0.01901	0.81	0.423
a*c*e	0.0483	0.0242	0.01901	1.27	0.213
a*d*e	-0.0308	-0.0154	0.01901	-0.81	0.423

Analysis of Variance for freeheig

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	1.64052	1.64052	0.32810	18.92	0.000
2-Way Interactions	7	0.25797	0.25797	0.03685	2.13	0.069
3-Way Interactions	3	0.05085	0.05085	0.01695	0.98	0.416
Residual Error	32	0.55487	0.55487	0.01734		
Pure Error	32	0.55487	0.55487	0.01734		
Total	47	2.50420				

Based on the analysis, factors A, B, C, and E are significant. The interaction of BE is also significant.

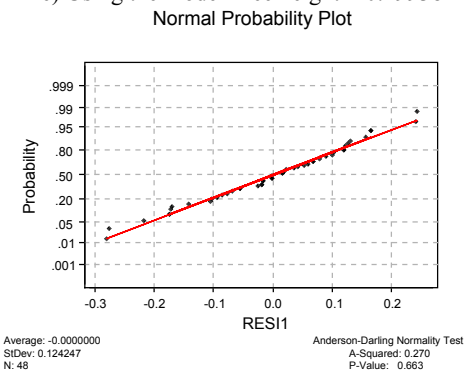
c)

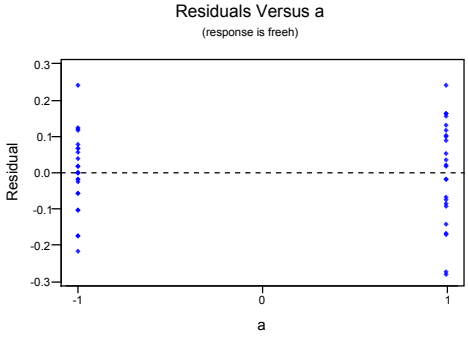
a	b	c	d	e	Range
-1	-1	-1	-1	-1	0.03
1	-1	-1	1	-1	0.30
-1	1	-1	1	-1	0.06
1	1	-1	-1	-1	0.19
-1	-1	1	1	-1	0.46
1	-1	1	-1	-1	0.40
-1	1	1	-1	-1	0.12
1	1	1	1	-1	0.25
-1	-1	-1	-1	1	0.06
1	-1	-1	1	1	0.44
-1	1	-1	1	1	0.06
1	1	-1	-1	1	0.19
-1	-1	1	1	1	0.12
1	-1	1	-1	1	0.13
-1	1	1	-1	1	0.07
1	1	1	1	1	0.31

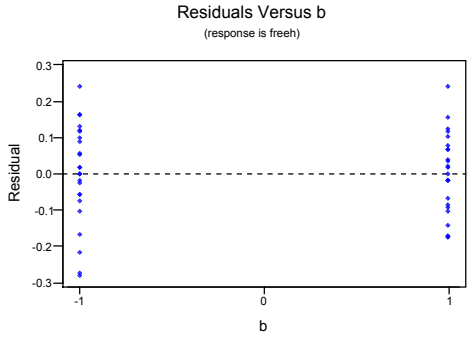
```
Estimated Effects and Coefficients for Range
Term         Effect      Coef  StDev Coef       T      P
Constant                0.19938     0.02714    7.35  0.000
a           0.15375   0.07688     0.02714    2.83  0.018
b          -0.08625  -0.04313     0.02714   -1.59  0.143
c           0.06625   0.03312     0.02714    1.22  0.250
d           0.10125   0.05062     0.02714    1.87  0.092
e          -0.05375  -0.02687     0.02714   -0.99  0.345

Analysis of Variance for Range
Source                DF      Seq SS      Adj SS      Adj MS      F      P
Main Effects           5      0.1944      0.1944     0.03889   3.30  0.051
Residual Error        10      0.1179      0.1179     0.01179
Total                 15      0.3123
```

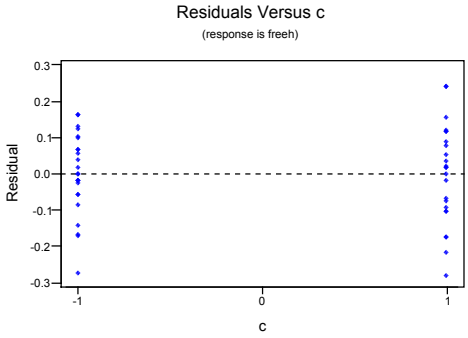
From the analysis we see that factor A is significant in affecting variability in free height.

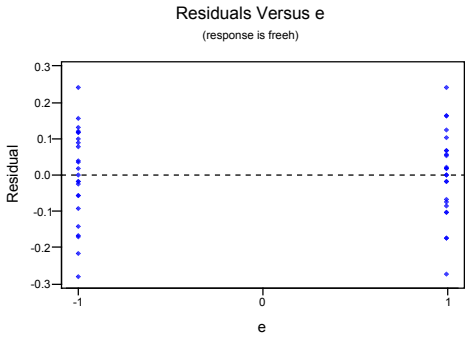
e) Using the model Free height = 0.19938 + 0.07688 A











The residual plots appear to be adequate.

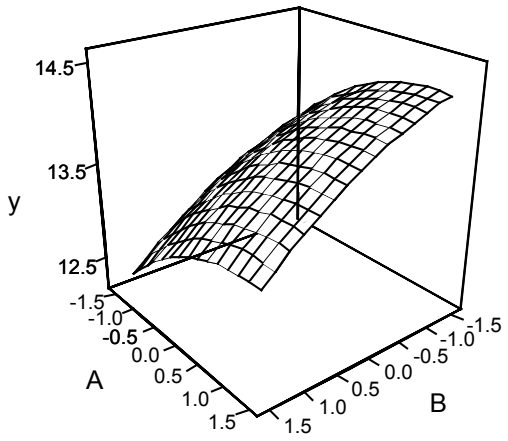
7-53. The analysis was done using coded units.

```
Estimated Regression Coefficients for y
Term             Coef       StDev          T      P
Constant      13.7280     0.04201    326.790  0.000
A              0.2966     0.03321      8.931  0.000
```

```
B             -0.4052     0.03321    -12.202  0.000
A*A           -0.1240     0.03561     -3.482  0.010
B*B           -0.0790     0.03561     -2.218  0.062
A*B            0.0550     0.04697      1.171  0.280

S = 0.09393     R-Sq = 97.2%     R-Sq(adj) = 95.2%
Analysis of Variance for y
Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         5     2.16451     2.16451     0.43290  49.06  0.000
  Linear           2     2.01751     2.01751     1.00876 114.32  0.000
  Square           2     0.13490     0.13490     0.06745   7.64  0.017
  Interaction      1     0.01210     0.01210     0.01210   1.37  0.280
Residual Error     7     0.06177     0.06177     0.00882
  Lack-of-Fit      3     0.02949     0.02949     0.00983   1.22  0.412
  Pure Error       4     0.03228     0.03228     0.00807
Total             12     2.22628
```

From the analysis, we see that the main effects and the square terms are significant, but the interactions are not. Running an analysis using only the significant values we have

```
Term             Coef       StDev          T      P
Constant      13.7280     0.04297    319.460  0.000
A              0.2966     0.03397      8.731  0.000
B             -0.4052     0.03397    -11.928  0.000
A*A           -0.1240     0.03643     -3.404  0.009
B*B           -0.0790     0.03643     -2.168  0.062

S = 0.09609     R-Sq = 96.7%     R-Sq(adj) = 95.0%

Analysis of Variance for y
Source            DF      Seq SS      Adj SS      Adj MS      F      P
Regression         4     2.15241     2.15241     0.53810  58.28  0.000
  Linear           2     2.01751     2.01751     1.00876 109.25  0.000
  Square           2     0.13490     0.13490     0.06745   7.31  0.016
Residual Error     8     0.07387     0.07387     0.00923
  Lack-of-Fit      4     0.04159     0.04159     0.01040   1.29  0.406
  Pure Error       4     0.03228     0.03228     0.00807
Total             12     2.22628
```



Maximum viscosity is found by computing the derivatives with respect to A (x_1) and B (x_2) of the model. Then setting these equations equal to 0 and solving:

Model: $13.728 + 0.2966\ A - 0.4052\ B - 0.1240\ A^2 - 0.079\ B^2 = y$

$$\frac{\partial y}{\partial A} = 0.2966 - 0.248A = 0$$

$$A = 1.19$$

$$\frac{\partial y}{\partial B} = -0.4052 - 0.158B = 0$$

$$B = -2.56$$

The maximum viscosity is given by:

$$13.728 + 0.2966\,(1.19) - 0.4052\,(-2.56) - 0.1240\,(1.19)^2 - 0.079\,(-2.56)^2 = 14.425$$