Chapter 8

Note to the Instructor: Some of the revised control charts provided in the solutions retain the removed points only as place holders. Other revised control charts have been created by removing the out-of-control point from the worksheet and constructing the control charts again.

Section 8-4

8-1. a)

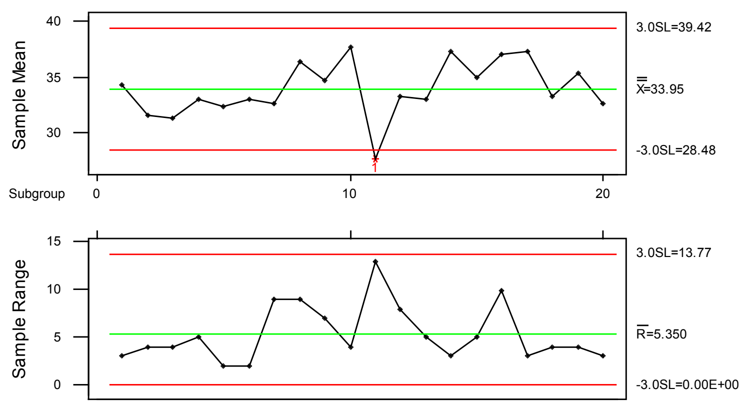
$\overline{x}$ chart

$UCL = CL + A_2\overline{r} = 39.42$
$CL = 33.95$
$LCL = CL - A_2\overline{r} = 28.48$

R chart

$UCL = D_4\overline{r} = 13.77$
$CL = 5.35$
$LCL = D_3\overline{r} = 0(5.35) = 0$

Xbar/R Chart for x1-x3



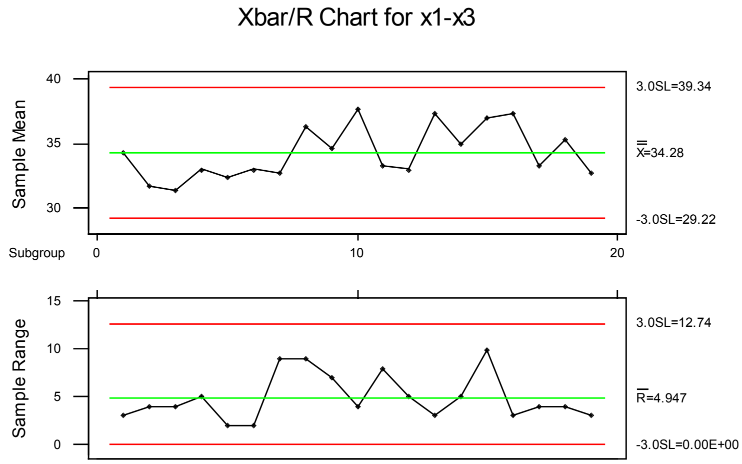
b)

$\overline{x}$ chart

$UCL = 39.34$
$CL = 34.28$
$LCL = 29.22$

R chart

$UCL = 12.74$
$CL = 4.947$
$LCL = D_3\overline{r} = 0(4.947) = 0$



8-2. a) $\bar{\bar{x}} = \dfrac{378.50}{20} = 18.925$ $\quad \bar{r} = \dfrac{7.80}{20} = 0.39$

$\bar{x}$ chart	R chart
$UCL = CL + A_2\bar{r} = 18.925 + 0.729(0.39) = 19.21$	$UCL = D_4\bar{r} = 2.282(0.39) = 0.89$
$CL = 18.925$	$CL = 0.39$
$LCL = CL - A_2\bar{r} = 18.925 - 0.729(0.39) = 18.64$	$LCL = D_3\bar{r} = 0(0.39) = 0$

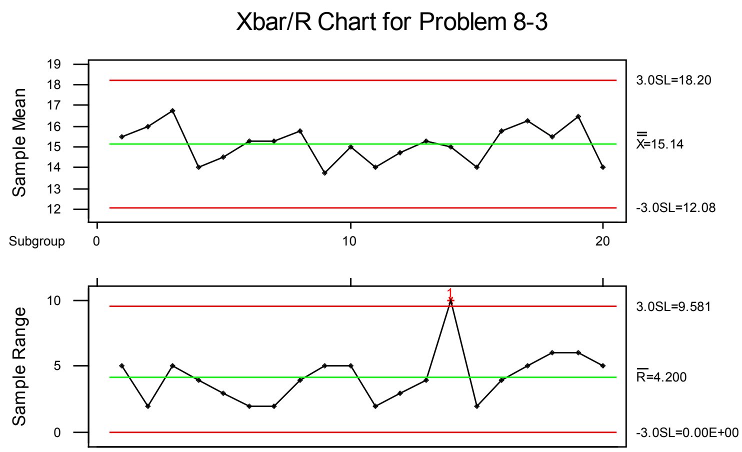
b)

$$\hat{\mu} = \bar{\bar{x}} = 18.925$$

$$\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{0.39}{2.059} = 0.1894$$

8-3. a) The trial control limits are as follows.

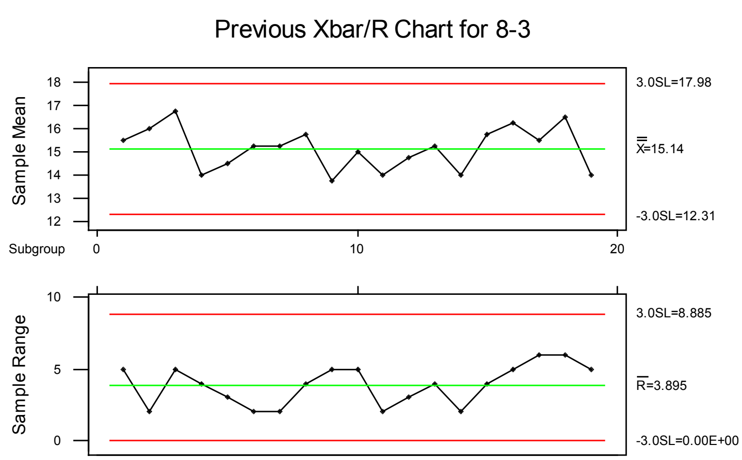
$\bar{x}$ chart	R chart
$UCL = 18.20$	$UCL = 9.581$
$CL = 15.14$	$CL = 4.2$
$LCL = 12.08$	$LCL = 0$



Based on the control charts, there is a single observation beyond the control limits. Observation 14 is above the upper control limit on the R chart.

b) With Observation 14 removed, the control limits and charts are as follows.

$\bar{x}$ chart	R chart
$UCL = 17.98$	$UCL = 8.885$
$CL = 15.14$	$CL = 3.895$
$LCL = 12.31$	$LCL = 0$



All points are within the control limits. The process is said to be in statistical control.

8-4. $\bar{\bar{x}} = 20.0 \quad \frac{\bar{r}}{d_2} = 1.4 \quad d_2 = 2.534$ giving $\bar{r} = 1.4(2.534) = 3.5476$

$\bar{x}$ chart	R chart
$UCL = CL + A_2\bar{r} = 20.0 + 0.483(3.5476) = 21.71$	$UCL = D_4\bar{r} = 2.004(3.53476) = 7.11$
$CL = 20.0$	$CL = 3.5476$
$LCL = CL - A_2\bar{r} = 20.0 - 0.483(3.5476) = 18.29$	$LCL = D_3\bar{r} = 0(3.5476) = 0$

8-5. a) $\bar{\bar{x}} = \frac{7657}{25} = 306.28$ $\quad \bar{r} = \frac{1180}{25} = 47.2$

$\bar{x}$ chart

$UCL = CL + A_2\bar{r} = 306.28 + 0.729(47.2) = 340.69$

$CL = 223$

$LCL = CL - A_2\bar{r} = 306.28 - 0.729(47.2) = 271.87$

R chart

$UCL = D_4\bar{r} = 2.282(47.2) = 107.71$

$CL = 47.2$

$LCL = D_3\bar{r} = 0(47.2) = 0$

b)

$\hat{\mu} = \bar{\bar{x}} = 306.28$

$\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{47.2}{2.059} = 22.92$

8-6. a)

$\bar{x}$ chart

$UCL = 0.06352$

$CL = 0.06294$

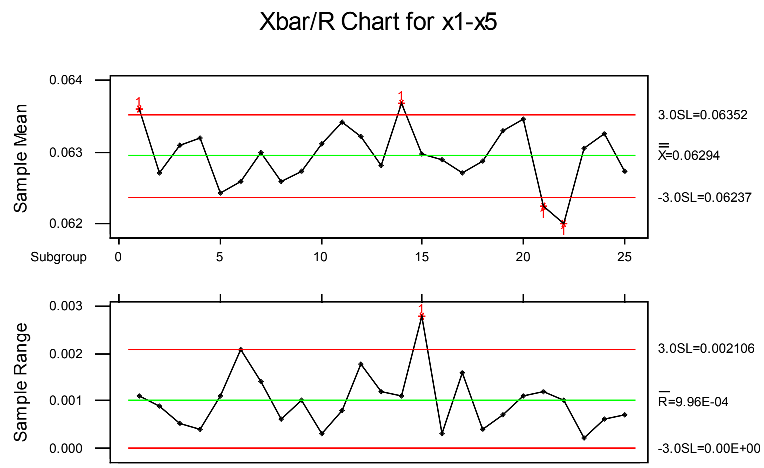
$LCL = 0.06237$

R chart

$UCL = 0.002106$

$CL = 0.000996$

$LCL = 0$



There are several points out of control. The control limits need to be revised. The points are 1, 14, 21, 22; or outside the control limits of the R chart: 15

b) Second revision, observations 5 and 6 are removed. Observation 12 is removed after the third revision. Observations 17 and 20 are removed after the fourth revision. Observations 8 and 11 are removed after the fifth revision. Observation 7 is removed after the sixth revision. After several revisions, the control chart is finally in control.

$\bar{x}$ chart

$UCL = 0.06332$

$CL = 0.06298$

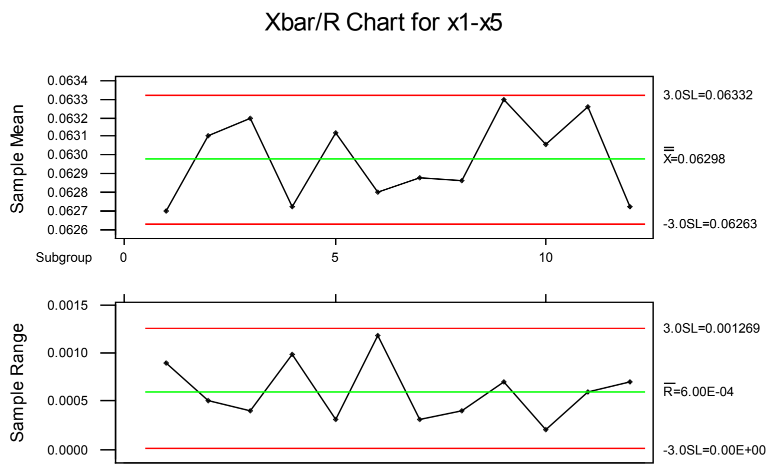
$LCL = 0.06263$

R chart

$UCL = 0.001269$

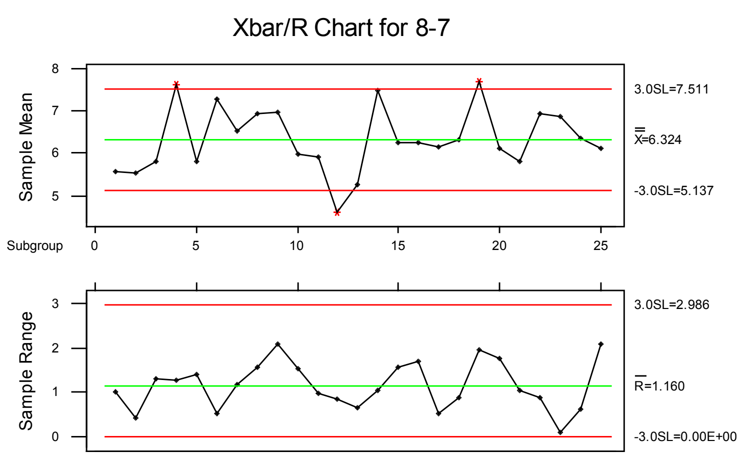
$CL = 0.0006$

$LCL = 0$



8-7. a)

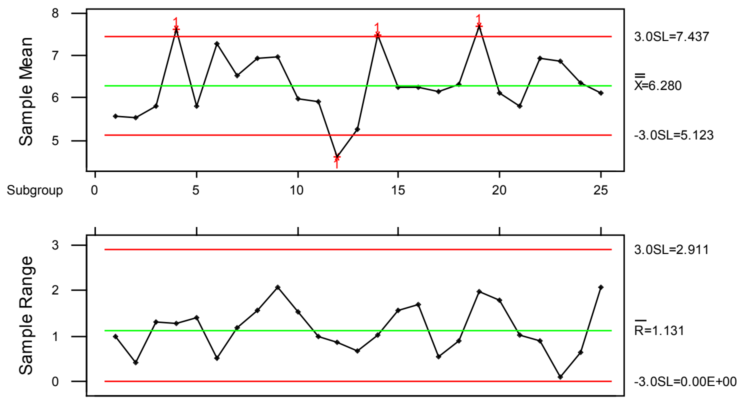
$\bar{x}$ chart	R chart
UCL = 7.511	UCL = 2.986
CL = 6.324	CL = 1.160
LCL = 5.137	LCL = 0



b)

$\bar{x}$ chart	R chart
UCL = 7.437	UCL = 2.911
CL = 6.280	CL = 1.131
LCL = 5.123	LCL = 0

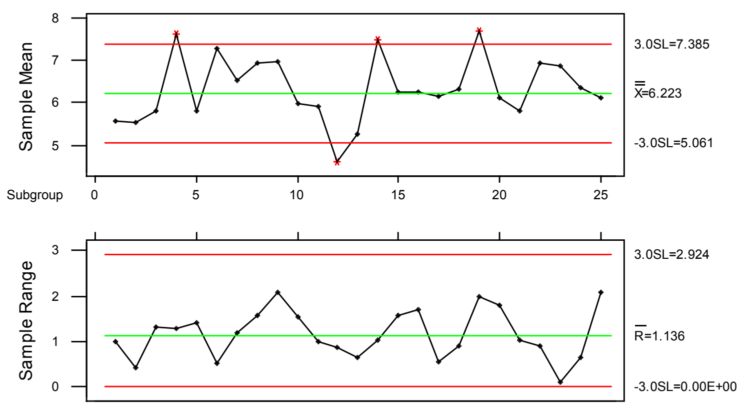
Revised Xbar/R Chart for 8-7



One point still beyond the control limits. Revise the limits again.

$\overline{x}$ chart	R chart
UCL = 7.385	UCL = 2.924
CL = 6.223	CL = 1.136
LCL = 5.061	LCL = 0

Xbar/R Chart for x1-x3

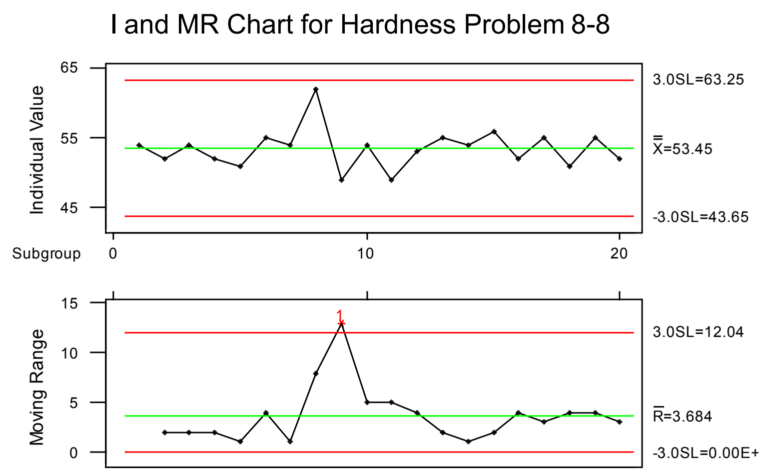


No point lies beyond the control limits, the process is now in control.

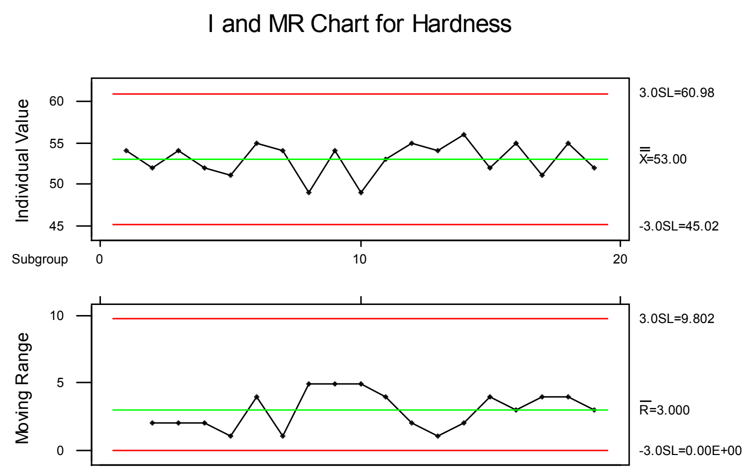
Section 8-5

8-8. a)

x chart	$\overline{MR}$ chart
UCL = 63.25	UCL = 12.04
CL = 53.45	CL = 3.684
LCL = 43.65	LCL = 0

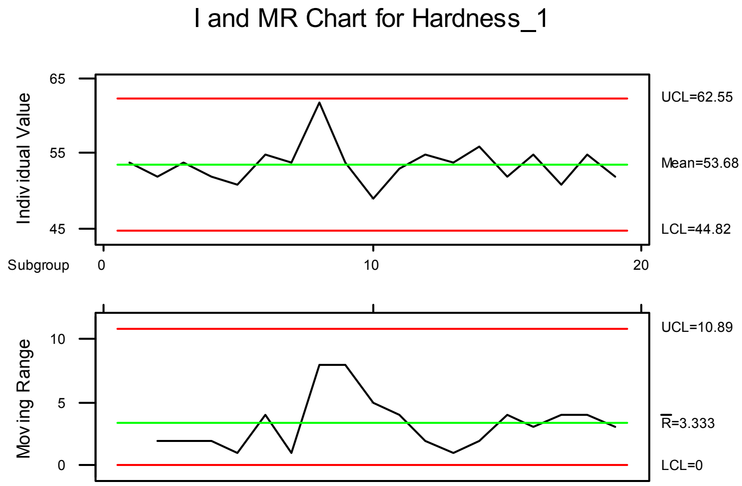


There is a single observation beyond the upper control limit on the R chart. Remove this observation (#8 in the dataset) and revise the control limits.



The process now appears to be in control.

There is a single observation beyond the upper control limit on the R chart. Remove this observation (#9 in the dataset) and revise the control limits.



b)

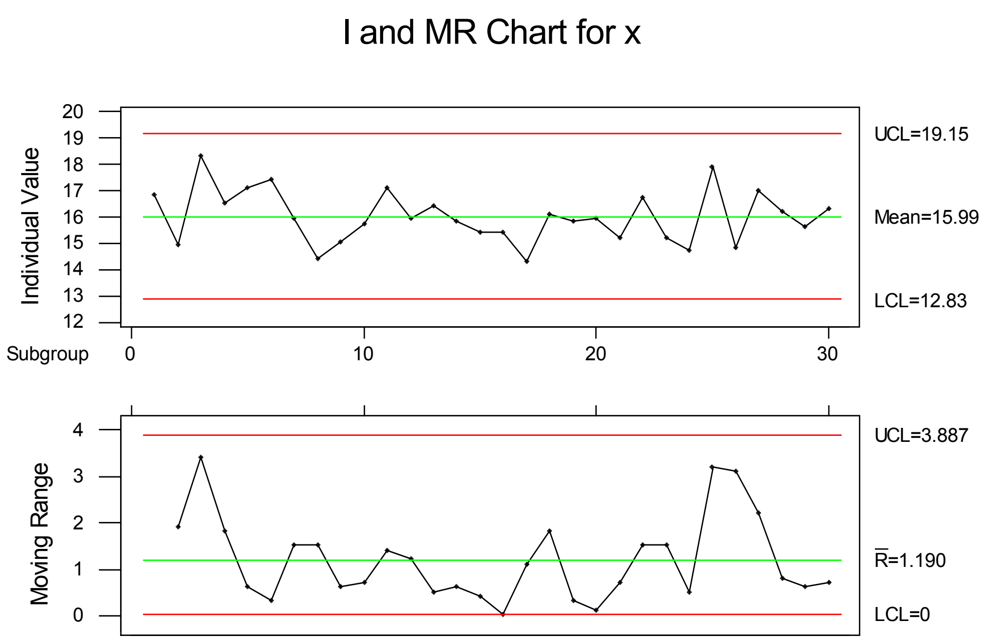
$\hat{\mu} = \bar{x} = 53$

$\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{3.00}{1.128} = 2.66$ if removing # 9 then $\hat{\mu} = \bar{x} = 53.68$

$\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{3.33}{1.128} = 2.952$

8-9. a) X chart: UCL = 19.15, LCL = 12.83, Mean = 15.99

$\overline{MR}$ chart: UCL = 3.887, LCL = 0, Mean = 1.190



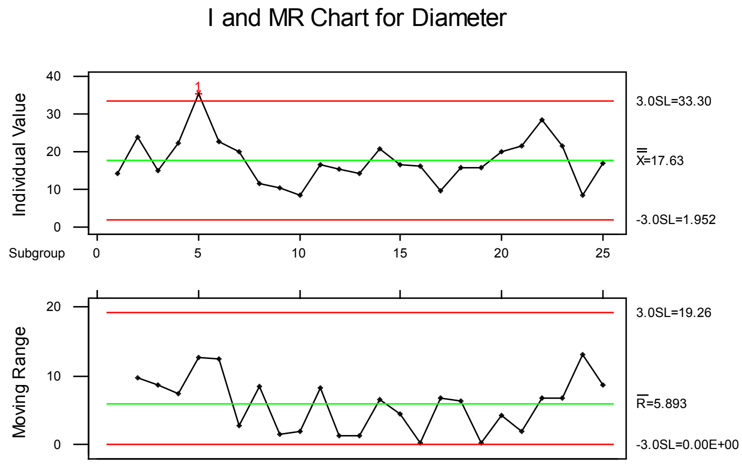
There are no points beyond the control limits. The process appears to be in control.

b) Estimates are: $\hat{\mu} = 15.99$, $\hat{\sigma} = 1.19/1.128 = 1.055$

8-10. a)

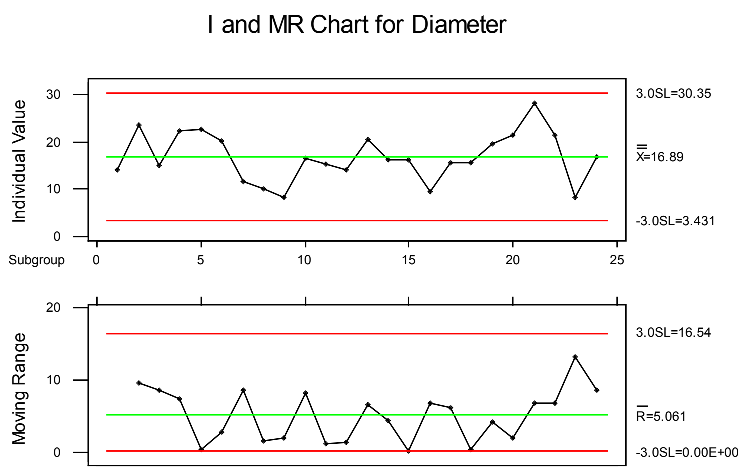
x chart $\quad\quad\quad\quad$ $\overline{MR}$ chart

x chart	$\overline{MR}$ chart
UCL = 34.63	UCL = 19.26
CL = 17.63	CL = 5.893
LCL = 1.952	LCL = 0



Observation 5 is beyond the upper control limit on the individual chart. Revise the control limits.

x chart	$\overline{MR}$ chart
UCL = 30.35	UCL = 16.54
CL = 16.89	CL = 5.061
LCL = 3.431	LCL = 0



All observations are within the control limits.

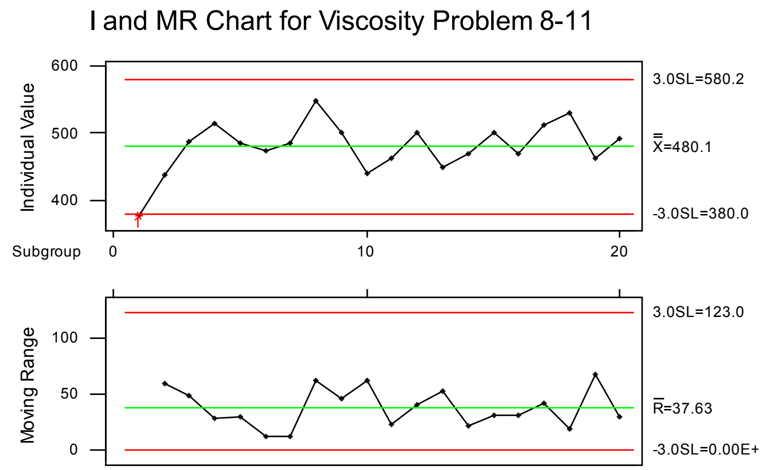
b)

$$\hat{\mu} = \overline{x} = 16.89$$

$$\hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{5.061}{1.128} = 4.487$$

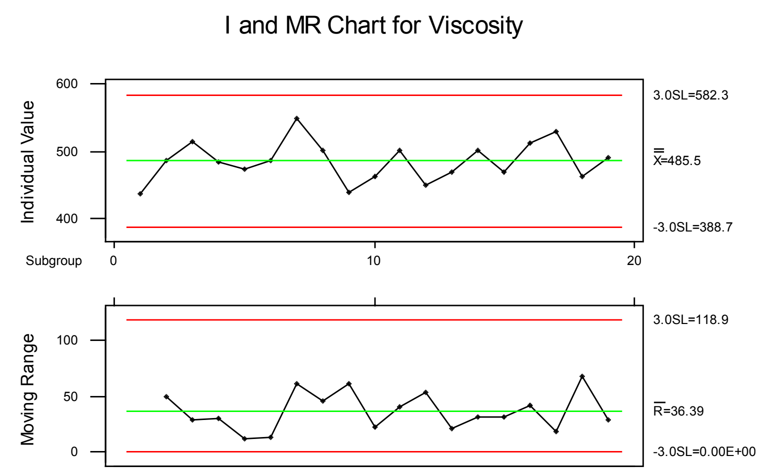
8-11. a)

x chart	$\overline{MR}$ chart
UCL = 580.2	UCL = 123
CL = 480.1	CL = 37.63
LCL = 380	LCL = 0



The first observation is below the lower control limit. Revise the control limits.

x chart	$\overline{MR}$ chart
UCL = 582.3	UCL = 118.9
CL = 485.5	CL = 36.39
LCL = 388.7	LCL = 0



The process is in control.

b) $\hat{\mu} = \overline{x} = 485.5 \qquad \hat{\sigma} = \frac{\overline{mr}}{d_2} = \frac{36.39}{1.128} = 32.26$

Section 8-6

8-12. a) If the process uses 66.7% of the specification band, then 6σ = 0.667(USL-LSL) then assume $\overline{\overline{x}} = \mu$ since the process is centered

$3\sigma = 0.667(USL - \overline{\overline{x}}) = 0.677(\overline{\overline{x}} - LSL) = 0.667(USL - \mu)$

$4.5\sigma = USL - \mu = LSU - \mu$

$$C_{pk} = \min\left[\frac{4.5\sigma}{3\sigma}, \frac{4.5\sigma}{3\sigma}\right] = 1.5$$

Since C_p and Cpk exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

b) Assuming a normal distribution with 6σ = 0.667(USL – LSL) and a centered process, then 3σ = 0.667(USL – μ). Consequently, USL – μ = 4.5σ and μ – LSL = 4.5σ

$$\begin{aligned} P(X > USL) &= P\left(Z > \frac{4.5\sigma}{\sigma}\right) \\ &= P(Z > 4.5) \\ &= 1 - P(Z < 4.5) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

By symmetry, the fraction defective is 2[P(X > USL)] = 0.

8-13. a) $\hat{\sigma} = \frac{\overline{r}}{d_2} = \frac{4.947}{1.693} = 2.922 \quad$ or $\quad \hat{\sigma} = 0.0002922$

$$C_p = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{0.4040 - 0.4020}{6(0.0002922)} = 1.141$$

$$\begin{aligned} C_{PK} &= \min\left[\frac{USL - \overline{\overline{x}}}{3\hat{\sigma}}, \frac{\overline{\overline{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{0.4040 - 0.40348}{3(0.0002922)}, \frac{0.40348 - 0.4020}{3(0.0002922)}\right] \\ &= \min[0.5932, 1.688] \\ &= 0.5932 \end{aligned}$$

Since Cp exceeds unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

Since Cpk ≠ Cp the process is off center.

b) Assuming a normal distribution with $\hat{\mu} = 0.40348$ and $\hat{\sigma} = 0.0002922$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P(Z < -5.07)$$
$$= 0$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P(Z > 1.78)$$
$$= 1 - P(Z < 1.78)$$
$$= 1 - 0.9624$$
$$= 0.0376$$

Therefore, the proportion nonconforming is given by
P(X<LSL) + P(X>USL) = 0.0376 + 0
= 0.0376

8-14. a) Assuming a normal distribution with $\hat{\mu}$ =18.925 and $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{0.39}{2.059} = 0.189$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z < \frac{18.00 - 18.925}{0.189}\right)$$
$$= P(Z < -4.89)$$
$$= 0$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z > \frac{19.00 - 18.925}{0.189}\right)$$
$$= P(Z > 0.40)$$
$$= 1 - P(Z < 0.40)$$
$$= 1 - 0.65542$$
$$= 0.34458$$

Therefore, the proportion nonconforming is given by
P(X<LSL) + P(X>USL) =0 + 0.3446
= 0.3446

b)

$$Cp = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{19.00 - 18.00}{6(0.189)} = 0.882$$

$$Cpk = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{19.00 - 18.925}{3(0.189)}, \frac{18.925 - 18.00}{3(0.189)}\right]$$
$$= \min[0.132, 1.63]$$
$$= 0.132$$

Since Cp less than unity, many defective units are being produced.

Cpk ≠ Cp the process is not centered.

8-15. $\hat{\sigma} = \frac{\bar{r}}{d_2} = \frac{3.895}{2.059} = 1.892$ or 0.01892 mm

$$\bar{x} = 15.14$$

$$P(X > 18) + P(X < 12)$$
$$= P\left(Z > \frac{18 - 15.14}{1.892}\right) + P\left(Z < \frac{12 - 15.14}{1.892}\right)$$
$$= P(Z > 1.51) + P(Z < -1.66)$$
$$= 0.065522 + 0.048457$$
$$= 0.113979$$

$$Cp = \frac{18 - 12}{6(1.892)} = 0.5285$$

With the Cp less than unity, the process capability appears to be poor.

8-16. a) If the process uses 85% of the spec band then 6σ = 0.85(USL – LSL) and

$$Cp = \frac{USL - LSL}{0.85(USL - LSL)} = \frac{1}{0.85} = 1.18$$

Assume $\bar{\bar{x}} = \mu$ and $3\sigma = 0.85(USL - \bar{\bar{x}}) = 0.85(\mu - LSL)$

Therefore,

$$Cpk = \min\left[\frac{3.53\hat{\sigma}}{3\hat{\sigma}}, \frac{3.53\hat{\sigma}}{3\hat{\sigma}}\right] = 1.18$$

Since Cp and Cpk exceed unity, the natural tolerance limits lie inside the specification limits and very few defective units will be produced.

b) Assuming a normal distribution with 6σ = 0.85(USL – LSL) and a centered process, then 3σ = 0.85(USL – μ). Consequently, USL – μ = 3.53σ and μ – LSL = 3.53σ

$$P(X > USL) = P\left(Z > \frac{3.53\sigma}{\sigma}\right)$$
$$= P(Z > 3.53)$$
$$= 1 - P(Z < 3.53)$$
$$= 1 - 0.9998$$
$$= 0.0002$$

By symmetry, the fraction defective is 2[P(X > USL)] = 0.0004.

8-17. Assuming a normal distribution with $\hat{\mu} = 306.28$ and $\hat{\sigma} = 22.923$

$$P(X < LSL) = P\left(Z < \frac{LSL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z < \frac{260 - 306.28}{22.923}\right)$$
$$= P(Z < -2.02)$$
$$= 0.0217$$

$$P(X > USL) = P\left(Z > \frac{USL - \hat{\mu}}{\hat{\sigma}}\right)$$
$$= P\left(Z > \frac{340 - 306.28}{22.923}\right)$$
$$= P(Z > 1.47)$$
$$= 1 - P(Z < 1.47)$$
$$= 1 - 0.9292$$
$$= 0.0708$$

Therefore, the proportion nonconforming is given by
P(X<LSL) + P(X>USL) = 0.0217 + 0.0708
= 0.0925

$$Cp = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{340 - 260}{6(22.923)} = 0.582$$
$$Cpk \quad = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{340 - 306.28}{3(22.923)}, \frac{306.28 - 260}{3(22.923)}\right]$$
$$= \min[0.490, 0.673]$$
$$= 0.490$$

The process capability is marginal.

8-18. Assuming a normal distribution with $\hat{\mu}$ = 20.0 and $\hat{\sigma}$ = 1.4

$$Cp = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{25 - 15}{6(1.4)} = 1.19$$
$$Cpk \quad = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{25 - 20}{3(1.4)}, \frac{20 - 15}{3(1.4)}\right]$$
$$= \min[1.19, 1.19]$$
$$= 1.19$$

The process is capable.

8-19. Assuming a normal distribution with $\hat{\mu}$ = 6.223 and $\hat{\sigma} = \frac{1.136}{1.693} = 0.671$

$$Cp = \frac{USL - LSL}{6(\hat{\sigma})} = \frac{6.5 - 5.5}{6(0.671)} = 0.248$$

$$\begin{aligned} Cpk &= \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right] \\ &= \min\left[\frac{6.5 - 6.223}{3(0.671)}, \frac{6.223 - 5.5}{3(0.671)}\right] \\ &= \min[0.138, 0.359] \\ &= 0.138 \end{aligned}$$

The process capability is poor.

8-20. Assuming a normal distribution with $\hat{\mu} = 0.06298$ and $\hat{\sigma} = \frac{0.0006}{2.326} = 0.00026$

The natural tolerance limits are then

$$\begin{aligned} \hat{\mu} \pm 3\hat{\sigma} &= 0.06298 \pm 3(0.00026) \\ &= (0.0622, 0.0638) \end{aligned}$$

8-21. Assuming a normal distribution with $\hat{\mu} = 485.5$ and $\hat{\sigma} = 32.26$

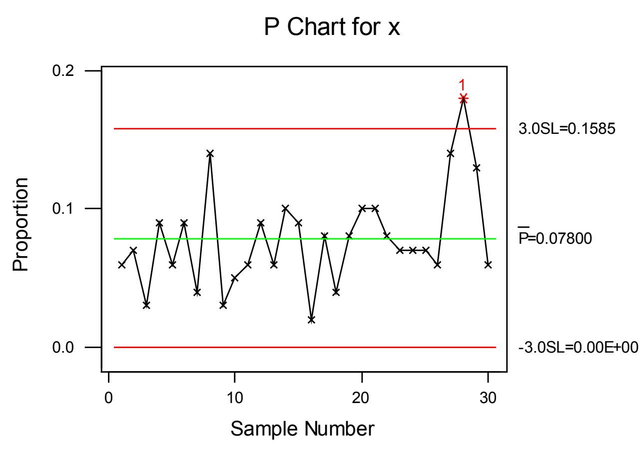
The natural tolerance limits are then

$$\begin{aligned} \hat{\mu} \pm 3\hat{\sigma} &= 485.5 \pm 3(32.26) \\ &= (388.72, 582.28) \end{aligned}$$

Section 8-7

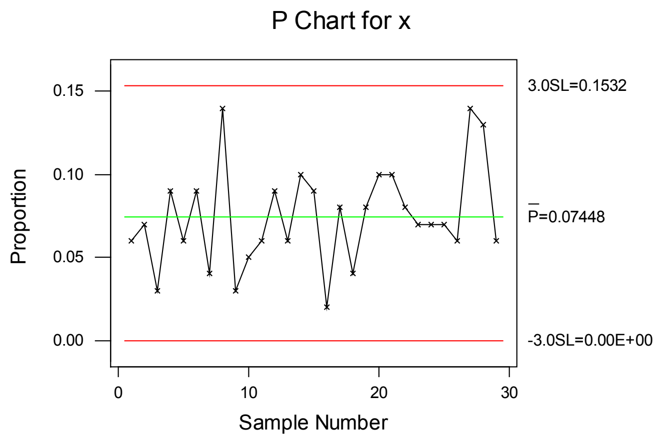
8-22. a) The control limits are

UCL = 0.1585
CL = 0.078
LCL = 0



b) The process is out of control, revised limits are

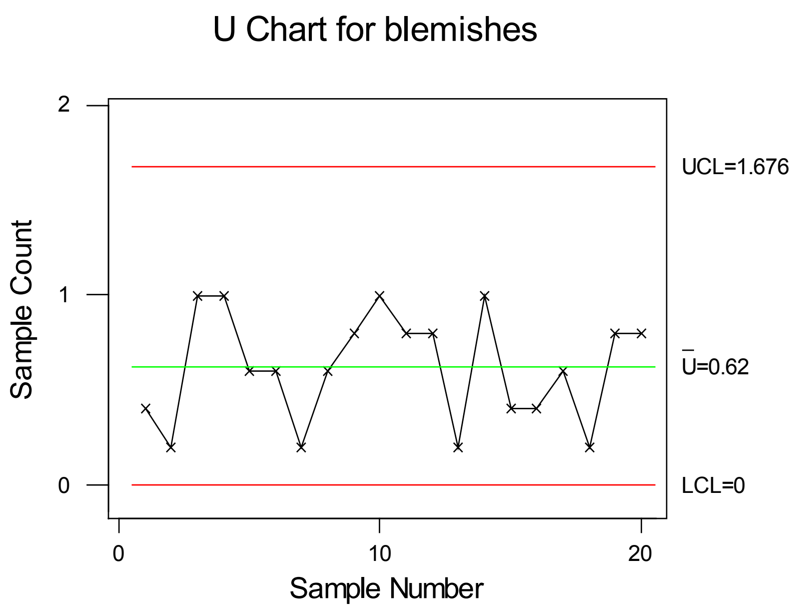
UCL = 0.1532
CL = 0.7448
LCL = 0



The process is now in control.

8-23. a) The control limits are

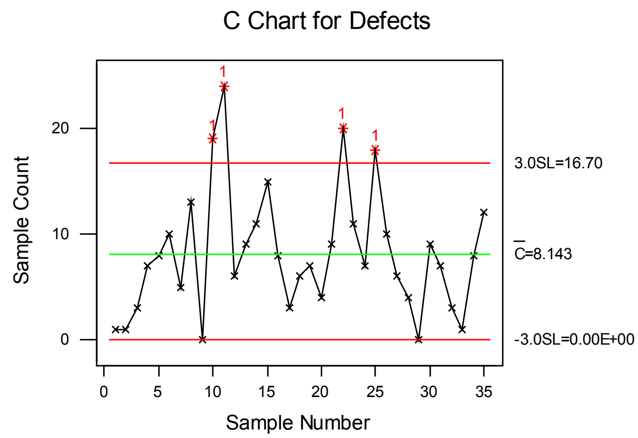
UCL = 1.676
CL = 0.62
LCL = 0



b) The process appears to be in control

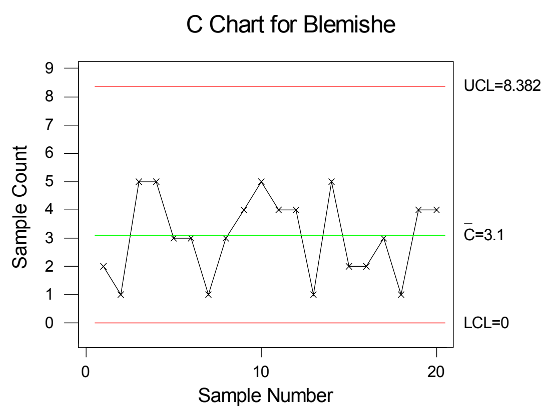
8-24. The control limits are

UCL = 16.70
CL = 8.143
LCL = 0



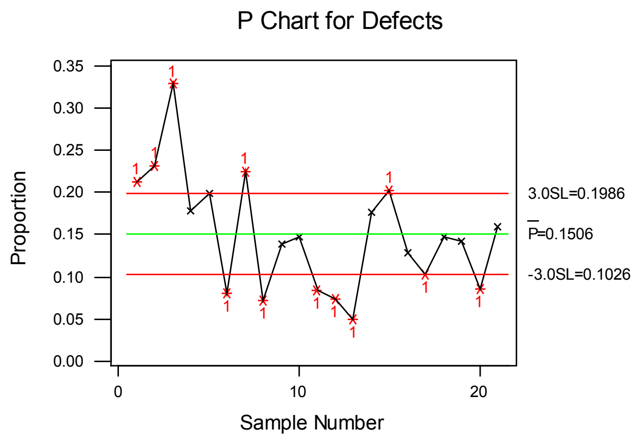
The process is not in control. This is evident by the fact that there are several points beyond the control limits.

8-25.



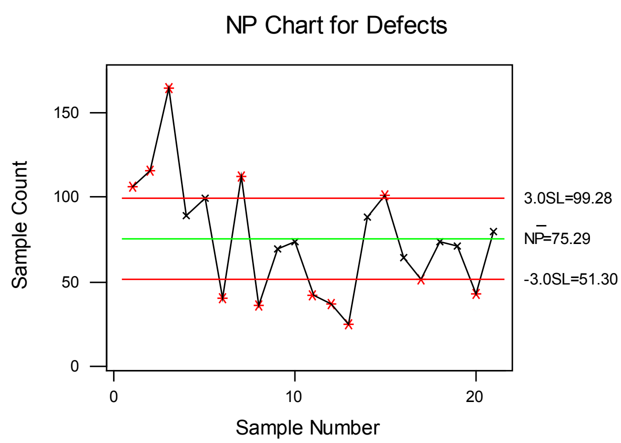
The U chart and the C chart both showed that the process is in control.

8-26. a) P-chart



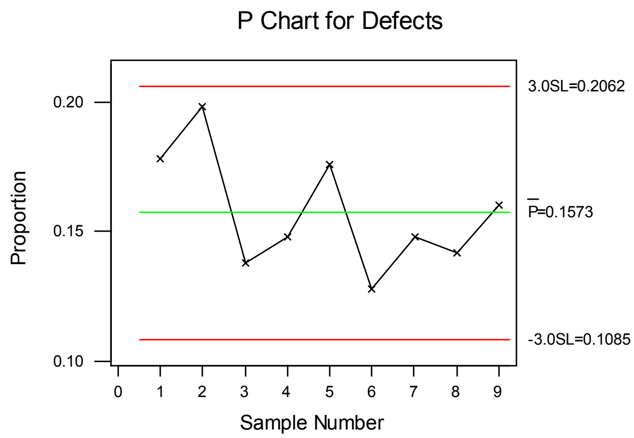
The samples with out-of-control points are 1, 2, 3, 6, 7, 8, 11, 12, 13, 15, 17, 20. There are several points are out of control. The control limits need to be revised.

nP chart



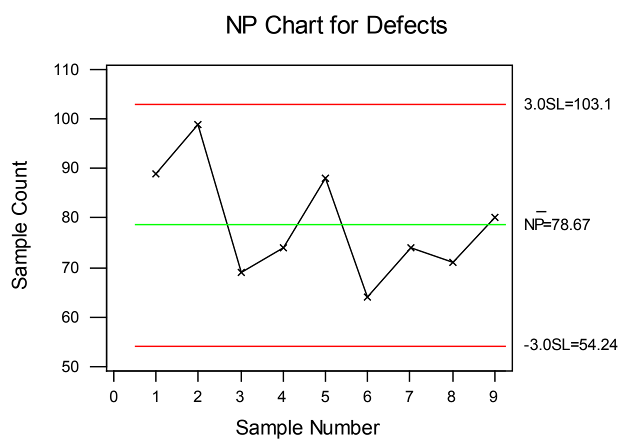
The samples with out-of-control points are 1, 2, 3, 6, 7, 8, 11, 12, 13, 15, 17, 20. There are several points are out of control. The control limits need to be revised.

b) P – chart revised



There are no points out of control for the revised limits.

nP chart - revised



The process is now in control.

Section 8-8

8-27. a) $\bar{x} = 74.01$ $\quad \sigma_{\bar{x}} = 0.0045$ $\quad \mu = 74.01$

$$P(73.9865 < \overline{X} < 74.0135)$$
$$= P\left(\frac{73.9865 - 74.01}{0.0045} < \frac{X - \mu}{\hat{\sigma}_{\bar{x}}} < \frac{74.0135 - 74.01}{0.0045}\right)$$
$$= P(-5.22 < Z < 0.78)$$
$$= P(Z < 0.78) - P(Z < -5.22)$$
$$= P(Z < 0.78) - [1 - P(Z < 5.22)]$$
$$= 0.7823 - (1 - 1)$$
$$= 0.7823$$

The probability that this shift will be detected on the next sample is p = 1–0.7823 = 0.2177.

b) $\text{ARL} = \frac{1}{p} = \frac{1}{0.2177} = 4.6$

8-28. a) $\mu + 3\frac{\sigma}{\sqrt{n}} = \text{UCL}$

$$100 + 3\frac{\sigma}{\sqrt{6}} = 106$$

$$\sigma = \frac{\sqrt{6}}{3}(106 - 100) = 4.9$$

b) $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{4.9}{2.45} = 2, \quad \mu = 105$

$$\begin{aligned}
&P(94 < \bar{X} < 106) \\
&= P\left(\frac{94 - 105}{2} < \frac{\bar{X} - \mu}{\sigma_{\bar{x}}} < \frac{106 - 105}{2}\right) \\
&= P(-5.5 < Z < 0.5) \\
&= P(Z < 0.5) - P(Z < -5.5) \\
&= P(Z < 0.5) - [1 - P(Z < 5.5)] \\
&= 0.69146 - 0 \\
&= 0.69146
\end{aligned}$$

The probability that this shift will be detected on the next sample is p = 1–0.69146 = 0.30854.

c) $\text{ARL} = \frac{1}{p} = \frac{1}{0.30854} = 3.24$

8-29. a) $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{2.922}{\sqrt{3}} = 1.69$, $\mu = 38$

$$\begin{aligned}
&P(29.22 < \bar{X} < 39.34) \\
&= P\left(\frac{29.22 - 39}{1.69} < \frac{\bar{X} - \mu}{\hat{\sigma}_{\bar{x}}} < \frac{39.34 - 39}{1.69}\right) \\
&= P(-5.78 < Z < 0.201) \\
&= P(Z < 0.201) - P(Z < -5.78) \\
&= 0.5797 - 0 \\
&= 0.5797
\end{aligned}$$

The probability that this shift will be detected on the next sample is p = 1–0.5797 = 0.4203.

b) $ARL = \frac{1}{p} = \frac{1}{0.4203} = 2.38$

8-30. a) $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.39}{2.059} = 0.189$ $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{0.189}{\sqrt{4}} = 0.0945$, $\mu = 18.7$

$$P(18.641 < X < 19.209)$$
$$= P\left(\frac{18.641 - 19.1}{0.0945} < \frac{X - \mu}{\sigma_x} < \frac{19.209 - 19.1}{0.0945}\right)$$
$$= P(-4.86 < Z < 1.15)$$
$$= P(Z < 1.15) - P(Z < -4.86)$$
$$= 0.874928 - 0$$
$$= 0.874928$$

The probability that this shift will be detected on the next sample is p = 1–0.874928 = 0.125072

b) $ARL = \frac{1}{p} = \frac{1}{0.125072} = 7.995$

8-31. a) $\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{3.895}{2.059} = 1.89$ $\hat{\sigma}_{\overline{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{1.89}{\sqrt{4}} = 0.945$, $\mu = 14.5$

$$P(12.31 < \overline{X} < 17.98)$$
$$= P\left(\frac{12.31 - 12.8}{0.945} < \frac{\overline{X} - \mu}{\sigma_{\overline{x}}} < \frac{17.89 - 12.8}{0.945}\right)$$
$$= P(-0.519 < Z < 5.39)$$
$$= P(Z < 5.39) - P(Z < -0.518)$$
$$= 1 - 0.3022$$
$$= 0.6978$$

The probability that this shift will be detected on the next sample is p = 1–0.6978 = 0.3022.

b) $ARL = \frac{1}{p} = \frac{1}{0.3022} = 3.31$

8-32. a) $\hat{\sigma} = \frac{\overline{R}}{d_2} = 1.4$ $\hat{\sigma}_{\overline{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{1.4}{\sqrt{6}} = 0.572$, $\mu = 19$

$$P(18.12 < X < 21.88)$$
$$= P\left(\frac{18.29 - 18.5}{0.572} < \frac{\overline{X} - \mu}{\sigma_{\overline{x}}} < \frac{21.71 - 18.5}{0.572}\right)$$
$$= P(-0.37 < Z < 5.61)$$
$$= P(Z < 5.61) - P(Z < -0.37)$$
$$= 1 - 0.355691$$
$$= 0.644309$$

The probability that this shift will be detected on the next sample is p = 1– 0.644309= 0.355691.

b) $ARL = \frac{1}{p} = \frac{1}{0.355691} = 2.811$

8-33. a) $\hat{\sigma} = \frac{47.2}{2.059} = 22.92$ $\hat{\sigma}_{\overline{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{22.92}{\sqrt{4}} = 11.46$

$$P(271.87 < \overline{X} < 340.69)$$
$$= P\left(\frac{271.87-310}{11.46} < \frac{\overline{X}-\mu}{\sigma_{\overline{X}}} < \frac{340.69-310}{11.46}\right)$$
$$= P(-3.33 < Z < 2.68)$$
$$= P(Z < 2.68) - P(Z < -3.33)$$
$$= 0.9963 - 0.00043$$
$$= 0.99587$$

The probability that this shift will be detected on the next sample is = 1–0.99587 = 0.00413.

b) $ARL = \frac{1}{p} = \frac{1}{0.00413} = 242.13$

8-34. a) $\hat{\sigma} = 0.00024$ $\hat{\sigma}_{\overline{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{0.00024}{\sqrt{5}} = 0.000107$, $\mu = 0.0628$

$$P(0.06266 < \overline{X} < 0.06331)$$
$$= P\left(\frac{0.06266-0.063}{0.000107} < \frac{\overline{X}-\mu}{\sigma_{\overline{x}}} < \frac{0.06331-0.063}{0.000107}\right)$$
$$= P(-3.18 < Z < 2.90)$$
$$= P(Z < 2.90) - P(Z < -3.18)$$
$$= 0.998134 - 0.000736$$
$$= 0.997398$$

The probability that this shift will be detected on the next sample is p = 1 – 0.997398 = 0.002602

b) $ARL = \frac{1}{p} = \frac{1}{0.002602} = 384.32$

8-35. a) $\hat{\sigma} = \frac{1.136}{1.693} = 0.671$ $\hat{\sigma}_{\overline{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{0.671}{\sqrt{3}} = 0.387$, $\mu = 5.5$

$$P(5.061 < \overline{X} < 7.385)$$
$$= P\left(\frac{5.061-6.8}{0387} < \frac{\overline{X}-\mu}{\sigma_{\overline{x}}} < \frac{7.385-6.8}{0.387}\right)$$
$$= P(-4.49 < Z < 1.51)$$
$$= P(Z < 1.51) - P(Z < -4.49)$$
$$= 0.93448 - 0$$
$$= 0.93448$$

The probability that this shift will be detected on the next sample is p = 1–0.93448 = 0.06552.

b) $ARL = \frac{1}{p} = \frac{1}{0.06552} = 15.26$

<u>Section 8-9</u>

8-36. a) One-Way ANOVA Table

Source	DF	SS	MS	F	P
Wafer	19	7209.4	379.442	5.01024	0.00001

Repeatability	40	3029.3	75.733
Total	59	10238.7	

There is significant difference in the parts used in the study based on these three measurements.

b) Gage R&R

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	75.73	42.79
Repeatability	75.73	42.79
Wafer -to- Wafer	101.24	57.21
Total Variation	176.97	100.00

c)

Source	StdDev (SD)	Study Var (5.15*SD)	%Study Var (%SV)
Total Gage R&R	8.7025	44.8178	65.42
Repeatability	8.7025	44.8178	65.42
Part-to-Part	10.0616	51.8174	75.63
Total Variation	13.3030	68.5104	100.00

The gauge contributes about 65.42% of total variability.

8-37. a) Analysis of Variance for purity

Source	DF	SS	MS	F	P
Measure	9	0.32050	0.03561	0.43	0.889
Error	10	0.82500	0.08250		
Total	19	1.14550			

There is no significant difference in the measuring device.

b)

Source	Variance component	Error term	Expected Mean Square for Each Term (using unrestricted model)
1 measure	-0.02344	2	(2) + 2(1)
2 Error	0.08250		(2)

total = 0 + 0.08250 = 0.08250

c) 100%. You have a desirable situation.

8-38. a) One-Way ANOVA Table

Source	DF	SS	MS	F	P
Unit	7	306.167	43.7382	1.93137	0.18804
Repeatability	8	181.170	22.6463		
Total	15	487.337			

There is no significant difference in the units used in the study.

b) Gage R&R

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	22.646	68.23
Repeatability	22.646	68.23
Unit-to-Unit	10.546	31.77
Total Variation	33.192	100.00

c)

Source	StdDev (SD)	Study Var (5.15*SD)	%Study Var (%SV)
Total Gage R&R	4.75881	24.5079	82.60
Repeatability	4.75881	24.5079	82.60
Part-to-Part	3.24746	16.7244	56.37
Total Variation	5.76127	29.6705	100.00

The gauge contributes about 82.6% of total variability.

8-39. a) Analysis of Variance for Strength

Source	DF	SS	MS	F	P
Part	5	457.8	91.6	1.61	0.307
Operator	1	121.4	121.4	2.14	0.204
Part*Operator	5	284.1	56.8	0.46	0.799
Error	12	1485.8	123.8		
Total	23	2349.1			

None of the terms is significant.

b)

Source	Variance component	Error term	Expected Mean Square for Each Term (using unrestricted model)
1 Part	8.683	3	(4) + 2(3) + 4(1)
2 Operator	5.378	3	(4) + 2(3) + 12(2)
3 Part*Operator	-33.497	4	(4) + 2(3)
4 Error	123.820		(4)

Using MINITAB13>Stat>Quality Tools> Gauge R&R Study(Crossed), we obtain

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	105.55	100.00
Repeatability	104.12	98.64
Reproducibility	1.44	1.36
Operator	1.44	1.36
Part-To-Part	0.00	0.00
Total Variation	105.55	100.00

c)

Source	StdDev (SD)	Study Var (5.15*SD)	%Study Var (%SV)
Total Gage R&R	10.2739	52.9106	100.00
Repeatability	10.2037	52.5491	99.32
Reproducibility	1.1989	6.1745	11.67
Operator	1.1989	6.1745	11.67
Part-To-Part	0.0000	0.0000	0.00
Total Variation	10.2739	52.9106	100.00

Overall most of the variability in the gauge comes from the measurement tool (repeatability).

8-40. a) Analysis of Variance for diameter

Source	DF	SS	MS	F	P
Part	4	25.269	6.317	1.73	0.237
Operator	2	2.601	1.300	0.36	0.712
Part*Operator	8	29.290	3.661	0.61	0.759
Error	30	179.050	5.968		
Total	44	236.209			

None of the terms is significant.

b)

Source	Variance component	Error term	Expected Mean Square for Each Term (using unrestricted model)
1 Part	0.2951	3	(4) + 3(3) + 9(1)
2 Operator	-0.1574	3	(4) + 3(3) + 15(2)
3 Part*Operator	-0.7690	4	(4) + 3(3)
4 Error	5.9683		(4)

Using MINITAB13>Stat>Quality Tools> Gauge R&R Study(Crossed), we obtain

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	5.4826	98.34
Repeatability	5.4826	98.34
Reproducibility	0.0000	0.00
Operator	0.0000	0.00
Part-To-Part	0.0927	1.66
Total Variation	5.5754	100.00

c)

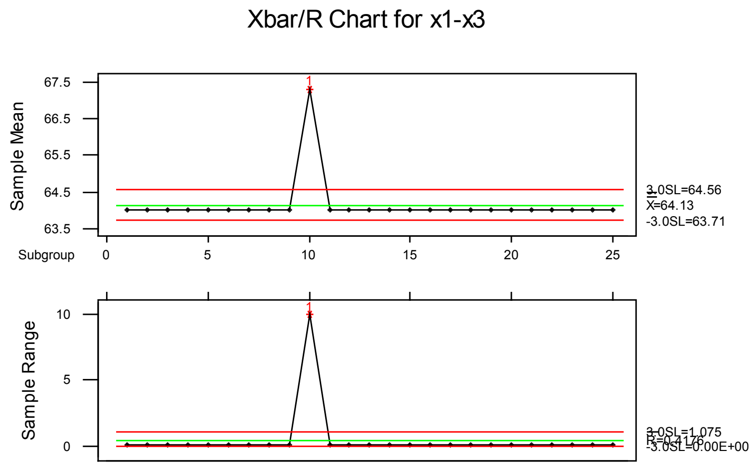
Source	StdDev (SD)	Study Var (5.15*SD)	%Study Var (%SV)
Total Gage R&R	2.34150	12.0587	99.16
Repeatability	2.34150	12.0587	99.16
Reproducibility	0.00000	0.0000	0.00
Operator	0.00000	0.0000	0.00
Part-To-Part	0.30451	1.5682	12.90
Total Variation	2.36122	12.1603	100.00

Overall most of the variability in the gauge comes from the measurement tool (repeatability).

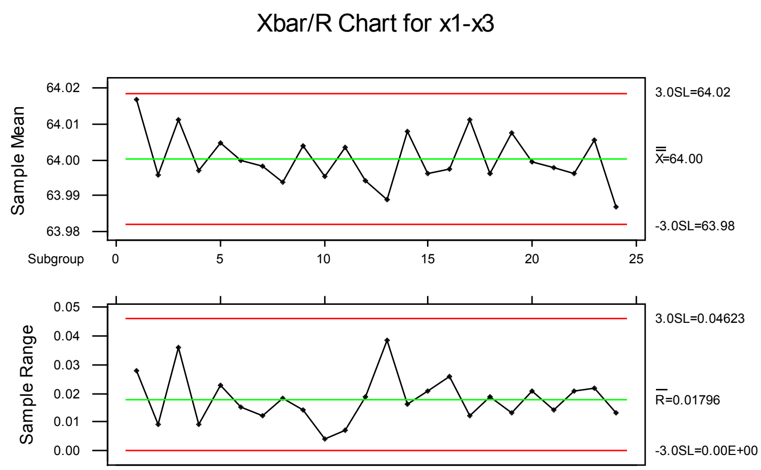
Supplementary Exercises

8-41. a)

x chart	$\overline{R}$ chart
$UCL = 64.56$	$UCL = 1.075$
$CL = 64.13$	$CL = 0.4176$
$LCL = 63.71$	$LCL = 0$



Observation # 10 is out of control.



The process is now in control.

b) $\hat{\mu} = \bar{\bar{x}} = 64$ $\qquad \hat{\sigma} = \dfrac{\bar{R}}{d_2} = \dfrac{0.01769}{1.693} = 0.0104$

c) $$Cp = \frac{USL - LSL}{6\hat{\sigma}} = \frac{64.02 - 63.98}{6(0.0104)} = 0.641$$

The process does not meet the minimum capability level of $Cp \geq 1.33$.

d)

$$Cpk = \min\left[\frac{USL - \bar{\bar{x}}}{3\hat{\sigma}}, \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{64.02 - 64}{3(0.0104)}, \frac{64 - 63.98}{3(0.0104)}\right]$$
$$= \min[0.641, 0.641]$$
$$= 0.641$$

e) In order to make this process a "six-sigma process", the variance σ^2 would have to be decreased such that Cpk = 2.0. The value of the variance is found by solving $Cpk = \dfrac{\bar{\bar{x}} - LSL}{3\sigma} = 2.0$ for σ:

$$\frac{64-61}{3\sigma} = 2.0$$
$$6\sigma = 64.-61$$
$$\sigma = \frac{64.-61}{6}$$
$$\sigma = 0.50$$

Therefore, the process variance would have to be decreased to $\sigma^2 = (0.50)^2 = 0.025$.

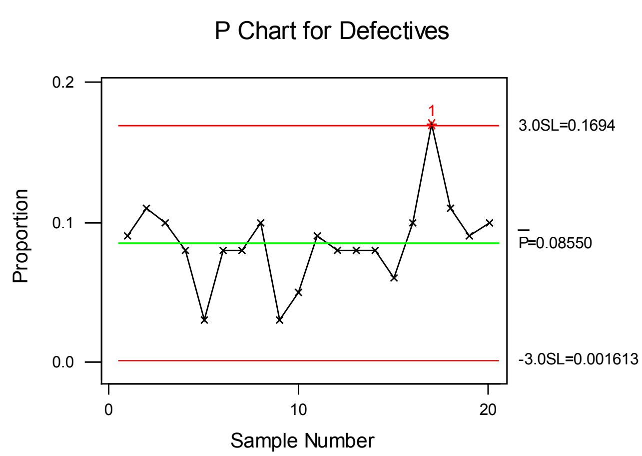
f) $\hat{\sigma}_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{0.0104}{\sqrt{3}} = 0.006$

$$P(63.98 < \bar{X} < 64.02)$$
$$= P\left(\frac{63.98 - 64.005}{0.006} < \frac{\bar{X}-\mu}{\sigma_x} < \frac{64.02 - 64.005}{0.006}\right)$$
$$= P(-4.17 < Z < 2.5)$$
$$= P(Z < 2.5) - P(Z < -4.17)$$
$$= 0.99379 - 0$$
$$= 0.99379$$

The probability that this shift will be detected on the next sample is p = 1–0.99379 = 0.00621

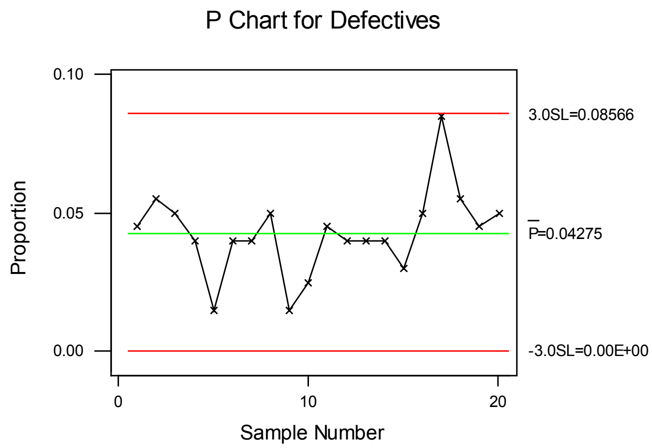
$$ARL = \frac{1}{p} = \frac{1}{0.00621} = 161$$

8-42. a)



The process is not in statistical control. Observation number 17 is beyond the UCL.

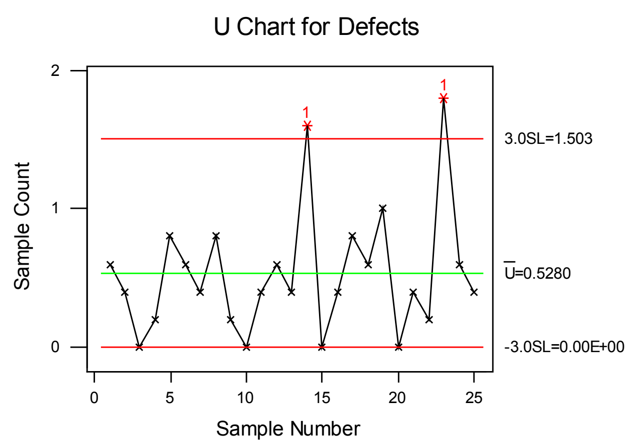
b)



The process appears to be in control.

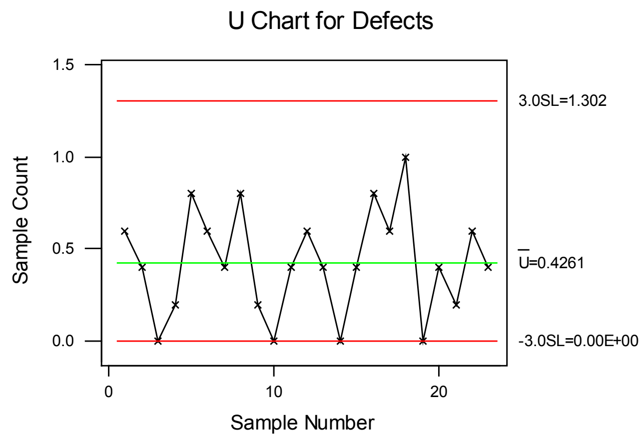
c) A larger sample size with the same number of defective items will result in more narrow control limits. The control limits corresponding to the larger sample size are more sensitive.

8-43. a)



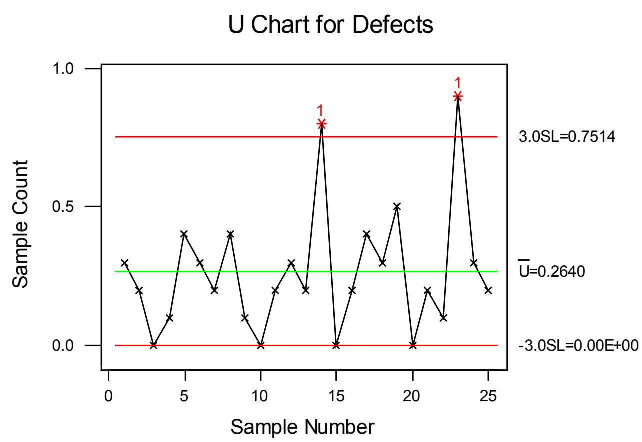
The process is out of control. Observation # 14 and 23 are beyond the control limits.

b)

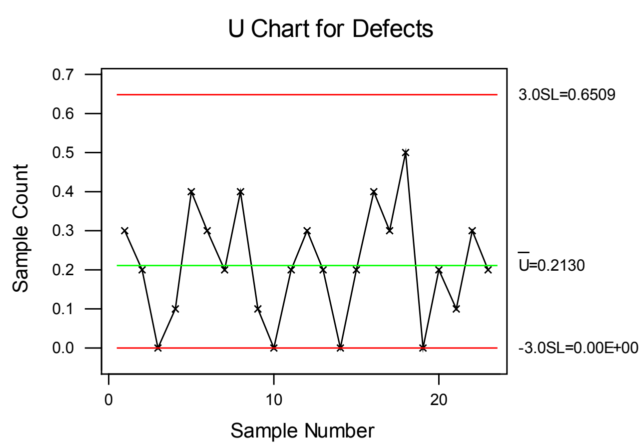


There are no points beyond the limits. The process is now in control.

c)

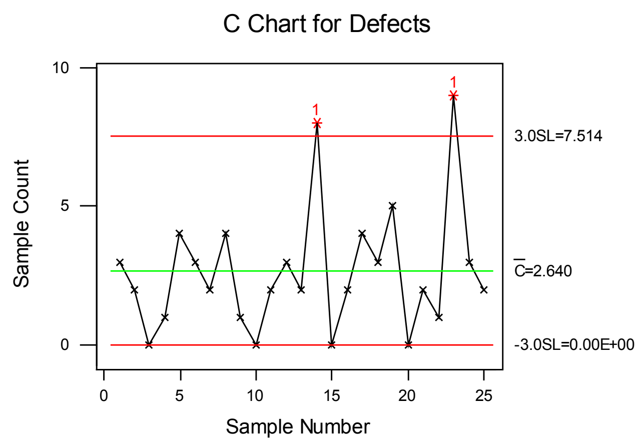


Revised U – Chart after removing point # 14 and 23.



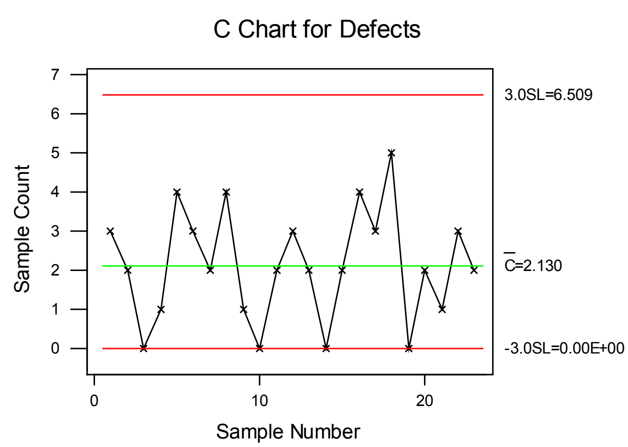
There are no points beyond the control limits. The process is now in control. Larger sample size narrows the control limits and causes more sample observations to be deemed out-of-control.

8-44. a)



There are points beyond the control limits. The process is out of control. The points are 14 and 23.

b)



There are no points beyond the control limits. The process is in control.

c) The control charts will not change since the sample size is not used in the c-chart.

8-45. a) Let p denote the probability that a point plots outside of the control limits when the mean has shifted from μ_0 to $\mu = \mu_0 + 1.75\sigma$. Then,

$$
\begin{aligned}
&P(LCL < \overline{X} < UCL) \\
&\quad = P\left(\mu_0 - \frac{3\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + \frac{3\sigma}{\sqrt{n}}\right) \\
&\quad = P\left(\frac{-1.75\sigma}{\sigma/\sqrt{4}} - 3 < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{-1.75\sigma}{\sigma/\sqrt{4}} + 3\right) \\
&\quad = P(-6.5 < Z < -0.5) \\
&\quad = P(Z < -0.5) - P(Z < -6.5) \\
&\quad = 0.30854 - 0 \\
&\quad = 0.30854
\end{aligned}
$$

Therefore, the probability the shift is undetected for three consecutive samples is $(1-p)^3 = (0.30854)^3 = 0.0294$.

b) If 2-sigma control limits were used, then

$$
\begin{aligned}
&1 - p = P(LCL < \overline{X} < UCL) \\
&\quad = P\left(\mu_0 - \frac{2\sigma}{\sqrt{n}} < \overline{X} < \mu_0 + \frac{2\sigma}{\sqrt{n}}\right) \\
&\quad = P\left(\frac{-1.75\sigma}{\sigma/\sqrt{n}} - 2 < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{-1.75\sigma}{\sigma/\sqrt{n}} + 2\right) \\
&\quad = P(-5.5 < Z < -1.5) \\
&\quad = P(Z < -1.5) - P(Z < -5.5) \\
&\quad = 0.06681
\end{aligned}
$$

Therefore, the probability the shift is undetected for three consecutive samples is $(1-p)^3 = (0.06681)^3 = 0.0003$ 0.004.

c) The 2-sigma limits are more narrow than the 3-sigma limits. Since the 2-sigma limits have a smaller probability of a shift being undetected, it would be better than the 3-sigma limits for a mean shift of 1.5σ.

8-46. ARL = 1/p where p is the probability a point falls outside the control limits.

a) $\mu = \mu_0 + \sigma$ and $n = 1$

$$
\begin{aligned}
p &= P(\overline{X} > UCL) + P(\overline{X} < LCL) \\
&= P\left(Z > \frac{\mu_0 + \frac{3\sigma}{\sqrt{n}} - \mu_0 - \sigma}{\sigma/\sqrt{n}}\right) + P\left(Z < \frac{\mu_0 - \frac{3\sigma}{\sqrt{n}} - \mu_0 - \sigma}{\sigma/\sqrt{n}}\right) \\
&= P(Z > 3 - \sqrt{n}) + P(Z < -3 - \sqrt{n}) \\
&= P(Z > 2) + P(Z < -4) \qquad \text{when } n = 1 \\
&= 1 - P(Z < 2) + [1 - P(Z < 4)] \\
&= 1 - 0.97725 + [1 - 1] \\
&= 0.02275
\end{aligned}
$$

Therefore, ARL = 1/p = 1/0.02275 = 43.9.

b) $\mu = \mu_0 + 2\sigma$

$P(\overline{X} > UCL) + P(\overline{X} < LCL)$

$$= P\left(Z > \frac{\mu_0 + \frac{3\sigma}{\sqrt{n}} - \mu_0 - 2\sigma}{\sigma/\sqrt{n}} \right) + P\left(Z < \frac{\mu_0 - \frac{3\sigma}{\sqrt{n}} - \mu_0 - 2\sigma}{\sigma/\sqrt{n}} \right)$$

$= P(Z > 3 - 2\sqrt{n}) + P(Z < -3 - 2\sqrt{n})$

$= P(Z > 1) + P(Z < -5)$ when $n = 1$

$= 1 - P(Z < 1) + [1 - P(Z < 5)]$

$= 1 - 0.84134 + [1 - 1]$

$= 0.15866$

Therefore, ARL = 1/p = 1/0.15866 = 6.30.

c) $\mu = \mu_0 + 3\sigma$

$P(\overline{X} > UCL) + P(\overline{X} < LCL)$

$$= P\left(Z > \frac{\mu_0 + \frac{3\sigma}{\sqrt{n}} - \mu_0 - 3\sigma}{\sigma/\sqrt{n}} \right) + P\left(Z < \frac{\mu_0 - \frac{3\sigma}{\sqrt{n}} - \mu_0 - 3\sigma}{\sigma/\sqrt{n}} \right)$$

$= P(Z > 3 - 3\sqrt{n}) + P(Z < -3 - 3\sqrt{n})$

$= P(Z > 0) + P(Z < -6)$ when $n = 1$

$= 1 - P(Z < 0) + [1 - P(Z < 6)]$

$= 1 - 0.50 + [1 - 1]$

$= 0.50$

Therefore, ARL = 1/p = 1/0.50 = 2.00.

d) The ARL is decreasing as the magnitude of the shift increases from σ to 2σ to 3σ. The ARL will decrease as the magnitude of the shift increases since a larger shift is more likely to be detected earlier than a smaller shift.

8-47. a) Because ARL = 370, on the average we expect there to be one false alarm every 370 hours. Each 30-day month contains 30 × 24 = 720 hours of operation. Consequently, we expect 720/370 = 1.9 false alarms each month.

b) The 2-sigma limits do reduce the ARL for detecting a shift in the mean of magnitude σ since the limits are narrower. The number of false alarms has increased using 2-sigma limits.

c) With 2-sigma limits the probability of a point plotting out of control is determined as follows, when $\mu = \mu_0 + \sigma$

$P(X > UCL) + P(X < LCL)$

$$= P\left(\frac{X - \mu_0 - \sigma}{\sigma} > \frac{\mu_0 + 2\sigma - \mu_0 - \sigma}{\sigma} \right) + P\left(\frac{X - \mu_0 - \sigma}{\sigma} < \frac{\mu_0 - 2\sigma - \mu_0 - \sigma}{\sigma} \right)$$

$= P(Z > 1) + P(Z < -3)$

$= 1 - P(Z < 1) + [1 - P(Z < 3)]$

$= 1 - 0.84134 + 1 - 0.99865$

$= 0.160$

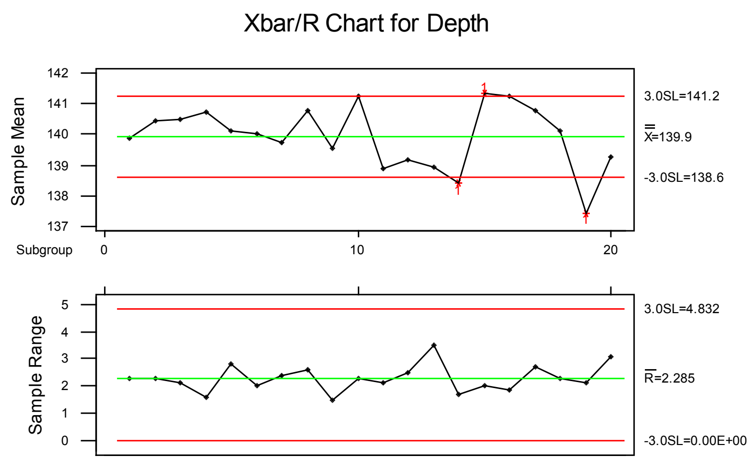
Therefore, ARL=1/p = 1/0.160 = 6.25. The 2-sigma limits do reduce the ARL for detecting a shift in the mean of magnitude σ. The number of false alarms has increased using 2-sigma limits.

d) The in-control ARL = 1/p, where

$$
\begin{aligned}
p &= P(X > UCL|\mu = \mu_0) + P(X < LCL|\mu = \mu_0) \\
&= P\left(\frac{X-\mu_0}{\sigma} > \frac{\mu_0 + 2\sigma - \mu_0}{\sigma}\right) + P\left(\frac{X-\mu_0}{\sigma} < \frac{\mu_0 - 2\sigma - \mu_0}{\sigma}\right) \\
&= P(Z > 2) + P(Z < -2) \\
&= 1 - P(Z < 2) + [1 - P(Z < 2)] \\
&= 2(1 - 0.97725) \\
&= 0.0455
\end{aligned}
$$

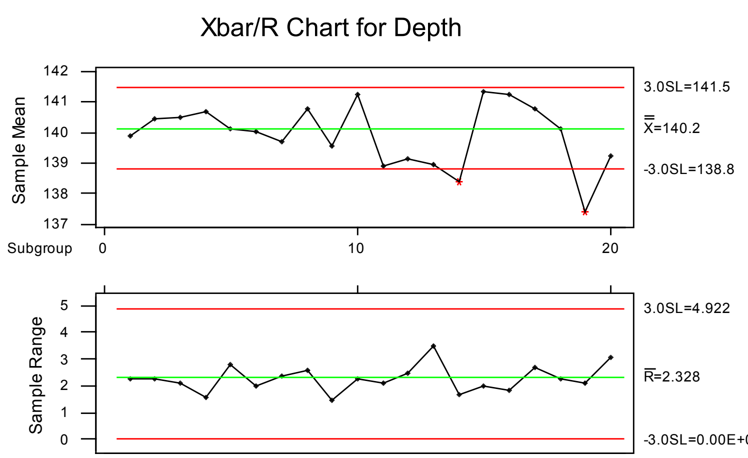
Therefore, ARL = 1/0.0455 = 21.98. The number of false alarms per month is 720/21.98 = 32.76. This is an excessive number of false alarms (more than one per day) and 2-sigma limits are not recommended for routine production. Thus, this in-control ARL performance is probably not satisfactory.

8-48. a)



There are two points beyond the control limits. The process is out of control.

b)



The process is now in control.

The process standard deviation estimate is given by $\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{2.328}{2.326} = 1$

c) $Cp = \frac{USL - LSL}{6\hat{\sigma}} = \frac{142 - 138}{6(1)} = 0.67$

$$Cpk = \min\left[\frac{USL - \overline{\overline{x}}}{3\hat{\sigma}}, \frac{\overline{\overline{x}} - LSL}{3\hat{\sigma}}\right]$$
$$= \min\left[\frac{142 - 140.2}{3(1)}, \frac{140.2 - 138}{3(1)}\right]$$
$$= \min[0.60, 0.73]$$
$$= 0.60$$

Since Cp is less than unity, the natural tolerance limits lie outside the specification limits and many defective units will be produced.

Cp is slightly larger than Cpk indicating that the process is somewhat off center.

d) In order to make this process a "six-sigma process", the variance σ^2 would have to be decreased such that Cpk = 2.0. The value of the variance is found by solving $Cpk = \frac{\overline{\overline{x}} - LSL}{3\sigma} = 2.0$ for σ:

$$\frac{140.2 - 138}{3\sigma} = 2.0$$
$$6\sigma = 140.2 - 138$$
$$\sigma = \frac{140.2 - 138}{6}$$
$$\sigma = 0.367$$

Therefore, the process variance would have to be decreased to $\sigma^2 = (0.367)^2 = 0.135$.

e) $\hat{\sigma}_{\overline{x}} = \frac{1}{\sqrt{5}} = 0.45$

$$p = P(138.8 < \overline{x} < 141.5 \mid \mu = 139.7)$$
$$= P\left(\frac{138.8 - 139.7}{0.45} < \frac{\overline{x} - \mu}{\sigma_{\overline{x}}} < \frac{141.5 - 139.7}{0.45}\right)$$
$$= P(-2 < Z < 4)$$
$$= P(Z < 4) - P(Z < -2)$$
$$= 1 - 0.02275$$
$$= 0.97725$$

The probability that this shift will be detected on the next sample is $1 - p = 1 - 0.97725 = 0.02275$

$$ARL = \frac{1}{1-p} = \frac{1}{0.02275} = 43.96$$

8-49. a) The $P(LCL < \hat{P} < UCL)$, when $p = 0.06$, is needed.

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.05 - 3\sqrt{\frac{0.05(1-0.05)}{100}} = -0.015 \rightarrow 0$$
$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.05 + 3\sqrt{\frac{0.05(1-0.05)}{100}} = 0.115$$

Therefore, when p = 0.06

$$P(0 \le \hat{P} \le 0.115) = P(\hat{P} \le 0.115)$$

$$= P\left(\frac{\hat{P} - 0.06}{\sqrt{\frac{0.06(0.94)}{100}}} \le \frac{0.115 - 0.06}{\sqrt{\frac{0.06(0.94)}{100}}} \right)$$

$$= P(Z \le 2.32)$$

$$= 0.989830$$

using the normal approximation to the distribution of $\hat{P}$. Therefore, the probability of detecting the shift on the first sample following the shift is 1 – 0.98983 = 0.01017.

b) The probability that the control chart detects a shift to 0.06 on the second shift is 0.01017(0.98983) = 0.0101.

c) p = 0.08

$$P(0 \le \hat{P} \le 0.115) = P(\hat{P} \le 0.115)$$

$$= P\left(\frac{\hat{P} - 0.10}{\sqrt{\frac{0.08(0.92)}{100}}} \le \frac{0.115 - 0.08}{\sqrt{\frac{0.08(0.92)}{100}}} \right)$$

$$= P(Z \le 1.29)$$

$$= 0.901475$$

using the normal approximation to the distribution of $\hat{P}$. Therefore, the probability of detecting the shift on the first sample following the shift is 1 – 0.901475 = 0.098525.

The probability that the control chart detects a shift to 0.10 on the second shift is 0.901475(0.098525) = 0.0888.

d) A larger shift is generally easier to detect. Therefore, we should expect a shift to 0.08 to be detected quicker than a shift to 0.06.

8-50. $\bar{u} = 8$

a) n = 5

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 8 + 3\sqrt{\frac{8}{5}} = 11.79$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 8 - 3\sqrt{\frac{8}{5}} = 4.21$$

$$P(\bar{U} > 11.79 \text{ when } \lambda = 16) = P\left(Z > \frac{11.79 - 16}{\sqrt{16/5}} \right)$$

= P(Z > –2.35)
= 1 – P(Z < –2.35)
= 1 – 0.009387
= 0.990613

$$P(\bar{U} < 4.21 \text{ when } \lambda = 16) = P\left(Z < \frac{4.21 - 16}{\sqrt{16/5}} \right)$$

= P(Z < –6.59)
= 0

The probability is then 0.990613

b) n = 8

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}} = 8 + 3\sqrt{\frac{8}{8}} = 11$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} = 8 - 3\sqrt{\frac{8}{8}} = 5$$

P($\overline{U}$ > 11 when λ = 16) = $P\left(Z > \frac{11-16}{\sqrt{\frac{16}{8}}}\right)$

= P(Z > −3.54)
= 0.9998

P($\overline{U}$ < 5 when λ = 16) = $P\left(Z < \frac{5-16}{\sqrt{16/8}}\right)$

= P(Z < −7.78)
= 0

The probability is then 0.9998

8-51. $\overline{u} = 10$

a) n = 3

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}} = 10 + 3\sqrt{\frac{10}{3}} = 15.48$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} = 10 - 3\sqrt{\frac{10}{3}} = 4.52$$

P($\overline{U}$ > 15.48 when λ = 14) = $P\left(Z > \frac{15.48-14}{\sqrt{14/3}}\right)$

= P(Z > 0.685)
= 1 – P(Z < 0.685)
= 1 – 0.7533
= 0.2467

and

P($\overline{U}$ < 4.52) = $P\left(Z < \frac{4.52-14}{\sqrt{14/3}}\right) = 0$

The probability is then 0.2467

b) n = 6

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}} = 10 + 3\sqrt{\frac{10}{6}} = 13.87$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} = 10 - 3\sqrt{\frac{10}{6}} = 6.13$$

P($\overline{U}$ > 13.87 when λ = 14) = $P\left(Z > \frac{13.87-14}{\sqrt{\frac{14}{6}}}\right)$

= P(Z >-0.085)
= 1 – 0.4661

$= 0.5339$

$$P(\overline{U} < 6.13 \text{ when } \lambda = 14) = P\left(Z < \frac{6.13-14}{\sqrt{\frac{14}{6}}}\right)$$
$= P(Z < -5.15)$
$= 0$

The probability is then 0.5339

8-52.

$CL = \mu$

$UCL = \mu + 2\frac{\sigma}{\sqrt{n}}$

$LCL = \mu - 2\frac{\sigma}{\sqrt{n}}$

$$P\left(\overline{X} > \mu + 2\frac{\sigma}{\sqrt{n}}\right) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > 2\right)$$
$= P(Z > 2)$
$= 1 - P(Z < 2)$
$= 1 - 0.97725$
$= 0.02275$

and

$$P\left(\overline{X} < \mu - 2\frac{\sigma}{\sqrt{n}}\right) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < -2\right)$$
$= P(Z < -2)$
$= 1 - P(Z < 2)$
$= 1 - 0.97725$
$= 0.02275$

The answer is 0.02275 + 0.02275 = 0.0455.
The answer for 3-sigma control limits is 0.0027. The 3-sigma control limits result in much fewer false alarms.

8-53.

$CL = \mu$

$UCL = \mu + k\frac{\sigma}{\sqrt{n}}$

$LCL = \mu - k\frac{\sigma}{\sqrt{n}}$

$$P\left(\overline{X} > \mu + k\frac{\sigma}{\sqrt{n}}\right) = P\left(\frac{\overline{X} - \mu - \delta}{\sigma/\sqrt{n}} > k - \frac{\delta}{\sigma/\sqrt{n}}\right)$$

$$= P\left(Z > k - \frac{\delta}{\sigma/\sqrt{n}}\right)$$

$$= 1 - \Phi\left(k - \frac{\delta}{\sigma/\sqrt{n}}\right)$$

$$P\left(\overline{X} < \mu + k\frac{\sigma}{\sqrt{n}}\right) = P\left(\frac{\overline{X} - \mu - \delta}{\sigma/\sqrt{n}} < -k - \frac{\delta}{\sigma/\sqrt{n}}\right)$$

$$= P\left(Z < -k - \frac{\delta}{\sigma/\sqrt{n}}\right)$$

$$= \Phi\left(-k - \frac{\delta}{\sigma/\sqrt{n}}\right)$$

The answer is

$$1 - \Phi\left(k - \frac{\delta\sqrt{n}}{\sigma}\right) + \Phi\left(-k - \frac{\delta\sqrt{n}}{\sigma}\right).$$

8-54. From Exercise 8-52, p = 0.0455.

a) The probability of a false alarm is the probability that $\overline{X}$ is outside the control limits when there is no shift, i.e., 0.0455. The probability there is not a false alarm on the first sample is 1 – 0.0455 = 0.9545. Hence, the probability of a false alarm on the second sample but not on the first (assuming the two samples are independent) is (0.9545)(0.0455) = 0.04343.

b) Assuming all the samples are independent, the probability that there is not a false alarm in the first three samples is $(0.9545)^3 = 0.86962$.

8-55. Cp = 2 but $\mu = USL + 3\sigma$

$$P(X < USL) = P\left(Z < \frac{(\mu - 3\sigma) - \mu}{\sigma}\right) = P(Z < -3) = 0.00135$$

8-56. a) From Prob. 8-37, we have $\hat{\sigma}_{gauge} = 0.2872$. Thus the P/T ratio is

$$\text{P/T ratio} = \frac{6\hat{\sigma}_{gauge}}{USL - LSL} = \frac{6(0.2872)}{2.8 - 1.2} = 1.077$$

b) Because P/T ratio is 1.077 higher than 0.1, the gauge is not adequate.

8-57. a) From Prob. 8-38, we have $\hat{\sigma}_{gauge} = 4.7588$. Thus the P/T ratio is

$$\text{P/T ratio} = \frac{6\hat{\sigma}_{gauge}}{USL - LSL} = \frac{6(4.7588)}{665 - 635} = 0.9518$$

b) Because P/T ratio is 0.9518 higher than 0.1, the gauge is not adequate.