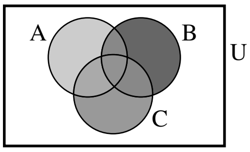
# Chapter 1

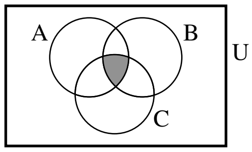
1–1. (a) $P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C) = 1 - 0.25 = 0.75$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.18$

1–2. (a)

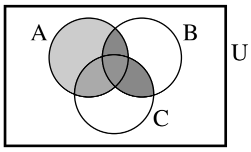



A∪(B∪C) = (A∪B)∪C




A∩(B∩C) = (A∩B)∩C

(b)

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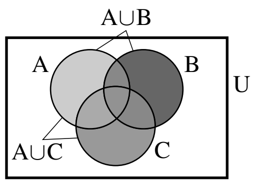
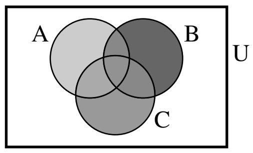



A∪(B∩C) = (A∪B)∩C(A∪C)

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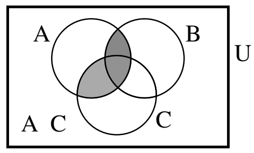
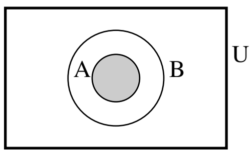


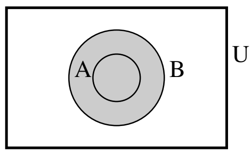

A∩(B∪C) = (A∩B)∪(A∩C)

(c)



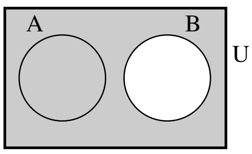

A∩B = A

(d)



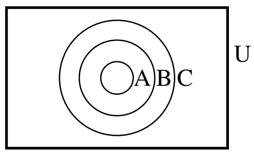

A∪B = B

(e)




A∩B = ∅ ⇒ A⊂$\overline{\text{B}}$

(f)




A⊂B and B⊂C ⇒ A⊂C

1–3. (a) $\overline{A} \cap B = \{5\}$,

(b) $\overline{A} \cup B = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$,

(c) $\overline{\overline{A} \cap \overline{B}} = \{2, 3, 4, 5\}$,

(d) $\overline{A \cap (B \cap C)} = U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

(e) $\overline{A \cap (B \cup C)} = \{1, 2, 5, 6, 7, 8, 9, 10\}$

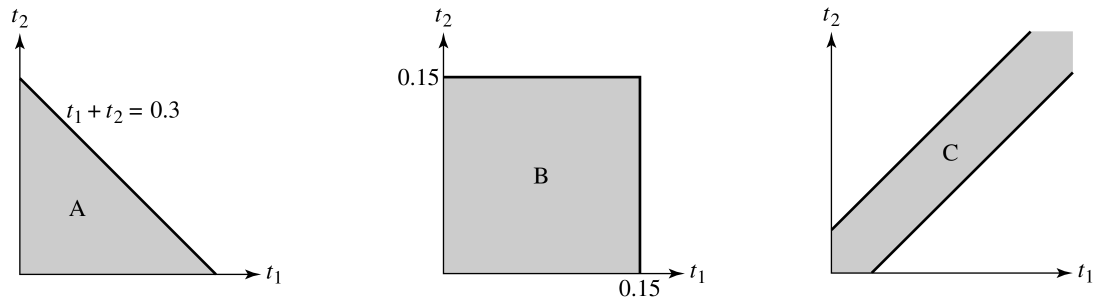
1–4. $P(A) = 0.02,\ P(B) = 0.01,\ P(C) = 0.015$

$P(A \cap B) = 0.005,\ P(A \cap C) = 0.006$

$P(B \cap C) = 0.004,\ P(A \cap B \cap C) = 0.002$

$P(A \cup B \cup C) = 0.02 + 0.01 + 0.015 - 0.005 - 0.006 - 0.004 + 0.002 = 0.032$

1–5. $S = \{(t_1, t_2)\colon\ t_1 \in R, t_2 \in R, t_1 \geq 0, t_2 \geq 0\}$



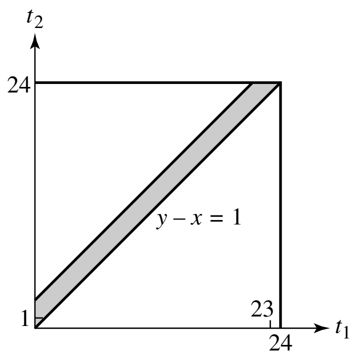

$A = \{(t_1, t_2)\colon\ t_1 \in R, t_2 \in R, 0 \leq t_1 \leq 0.3, 0 \leq t_2 \leq 0.3 - t_1\}$

$B = \{(t_1, t_2)\colon\ t_1 \in R, t_2 \in R, 0 \leq t_1 \leq 0.15, 0 \leq t_2 \leq 0.15\}$

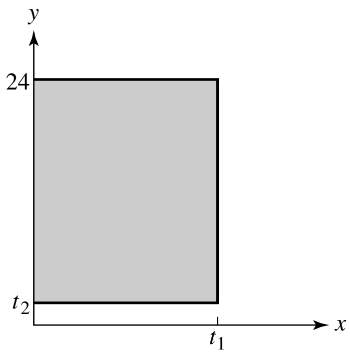
$C = \{(t_1, t_2)\colon\ t_1 \in R, t_2 \in R, t_1 \geq 0, t_2 \geq 0, t_1 - 0.06 \leq t_2 \leq t_1 + 0.06\}$

1–6. (a) $S = \{(x, y)\colon\ x \in R, y \in R, 0 \leq x \leq y \leq 24\}$

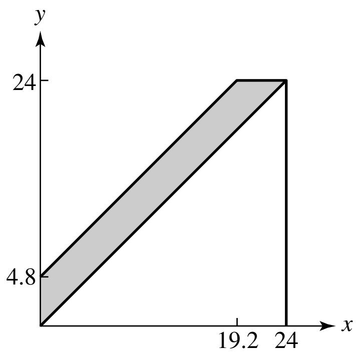
(b) i)




ii)




iii)



1–7. $S = \{NNNNN, NNNND, NNNDN, NNNDD, NNDNN,$
$NNDND, NNDD, NDNNN, NDNND, NDND,$
$NDD, DNNNN, DNNND, DNND, DND, DD\}$

1–8. $\{0,1\}^A = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\},$
$\{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$

1–9. $N$ = Not Defective, $D$ = Defective

(a) $S = \{NNN, NND, NDN, NDD, DNN, DND, DDN, DDD\}$

(b) $S = \{NNNN, NNND, NNDN, NDNN, DNNN\}$

1–10. $p'$ = Lot Fraction Defective
$50 \cdot p'$ = Lot No. of Defectives

$$P(\text{Scrap Lot}|n = 10, N = 50, p') = 1 - \frac{\binom{50p'}{0}\binom{50(1-p')}{10}}{\binom{50}{10}}$$

If $p' = 0.1$, $P(\text{scrap lot}) \cong 0.689$.
She might wish to increase sample size.

1–11. $6 \cdot 5 = 30$ routes

1–12. $26^3 \cdot 10^3 = 17{,}576{,}000$ possible plates $\Rightarrow$ scheme feasible

1–13. $\binom{15}{6}\binom{8}{2}\binom{4}{1} = 560{,}560$ ways

1–14. $$P(X \le 2) = \sum_{k=0}^{2} \frac{\binom{20}{k}\binom{80}{4-k}}{\binom{100}{0.4}} \cong 0.97$$

1–15. $$P(\text{Accept}|p') = \sum_{k=0}^{1} \frac{\binom{300p'}{k}\binom{300(1-p')}{10-k}}{\binom{300}{10}}$$

1–16. There are $5^{12}$ possibilities, so the probability of randomly selecting one is $5^{-12}$.

1–17. $\binom{8}{2} = 28$ comparisons

1–18. $\binom{40}{2} = 780$ tests

1–19. $P_2^{40} = \frac{40!}{38!} = 1560$ tests

1–20. $\binom{10}{5} = 252$

1–21. $\binom{5}{1}\binom{5}{1} = 25$

$\binom{5}{2}\binom{5}{2} = 100$

1–22. $[1 - (0.2)(0.1)(0.1)][1 - (0.2)(0.1)](0.99) = 0.968$

1–23. $[1 - (0.2)(0.1)(0.1)][1 - (0.2)(0.1)](0.9) = 0.880$

1–24. $R_S = R_1\{1 - [1 - (1 - (1 - R_2)(1 - R_4))(R_5)][1 - R_3]\}$

1–25. $S =$ Siberia $\quad U =$ Ural

$P(S) = 0.6, P(U) = 0.4, P(F|S) = P(\overline{F}|S) = 0.5$

$P(F|U) = 0.7, P(\overline{F}|U) = 0.3$

$$P(S|\overline{F}) = \frac{(0.6)\cdot(0.5)}{(0.6)(0.5) + (0.4)(0.3)} \doteq 0.714$$

1–26. $R_S = (0.995)(0.993)(0.994) = 0.9821$

1–27. $A$: 1st ball numbered 1
$B$: 2nd ball numbered 2

$$\begin{aligned} P(B) &= P(A)\cdot P(B|A) + P(\overline{A})\cdot P(B|\overline{A}) \\ &= \frac{1}{m}\cdot\frac{1}{m-1} + \frac{m-1}{m}\cdot\frac{1}{m} \\ &= \frac{m^2 - m + 1}{m^2(m-1)} \end{aligned}$$

1–28. $9 \times 9 - 9 = 72$ possible numbers

$D_1 + D_2$ even: 32 possibilities

$P(D_1$ odd <u>and</u> $D_2$ odd$|D_1 + D_2$ even$) = \frac{20}{32}$.

1–29. $A$: over $6'$
$M$: male
$F$: female

$$P(M) = 0.6, P(F) = 0.4, P(A|M) = 0.2, P(A|F) = 0.01$$

$$\begin{aligned} P(F|A) &= \frac{P(F) \cdot P(A|F)}{P(F) \cdot P(A|F) + P(M) \cdot P(A|M)} \\ &= \frac{(0.04)(0.01)}{(0.4)(0.01) + (0.6)(0.2)} = \frac{0.004}{0.124} \\ &\cong 0.0323 \end{aligned}$$

1–30. $A$: defective
$B_i$: production on machine $i$

(a) $$\begin{aligned} P(A) &= P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2) + P(B_3) \cdot P(A|B_3) \\ &\quad + P(B_4) \cdot P(A|B_4) \\ &= (0.15)(0.04) + (0.30)(0.03) + (0.20)(0.05) + (0.35)(0.02) \\ &= 0.032 \end{aligned}$$

(b) $$P(B_3|A) = \frac{(0.2)(0.05)}{0.032} = 0.3125$$

1–31. $r$ = radius

$$P(\text{closer to center}) = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

1–32. $$\begin{aligned} P(A \cup B \cup C) &= P((A \cup B) \cup C) \qquad \text{(associative law)} \\ &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\ &= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C)) \\ &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ &\quad - P(B \cap C) + P(A \cap B \cap C) \end{aligned}$$

1–33. For $k = 2$, $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$; Thm. 1–3.
Using induction we show that if true for $k - 1$, then true for $k$, i.e.,

If

$$P(A_2 \cup A_3 \cup \cdots \cup A_k) = \sum_{i=2}^{k} P(A_i) - \sum_{2\le i<j\le k} P(A_i \cap A_j) + \sum_{2\le i<j<r\le k} P(A_i \cap A_j \cap A_r) - \sum_{2\le i<j<r<\ell\le k} P(A_i \cap A_j \cap A_r \cap A_\ell) + \cdots \qquad \text{(Eq. 1)}$$

Then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = \sum_{i=1}^{k} P(A_i) - \sum_{1\le i<j\le k} P(A_i \cap A_j) + \sum_{1\le i<j<r\le k} P(A_i \cap A_j \cap A_r) - \sum_{1\le i<j<r<\ell\le k} P(A_i \cap A_j \cap A_r \cap A_\ell) + \cdots \qquad \text{(Eq. 2)}$$

By hypothesis, and letting $A_1 \cap A_i$ replace $A_i$ in Eq. 1,

$$P((A_1\cap A_2)\cup(A_1\cap A_3)\cup\cdots\cup(A_1\cap A_k)) = \sum_{i=2}^{k} P(A_1\cap A_i) - \sum_{2\le i<j\le k} P(A_1\cap A_i\cap A_j) + \sum_{2\le i<j<r\le k} P(A_1\cap A_i\cap A_j\cap A_r) - \sum_{2\le i<j<r\le k} P(A_1\cap A_i\cap A_j\cap A_r\cap A_\ell) + \cdots \qquad \text{(Eq. 3)}$$

By Thm. 1–3,

$$P(A_1 \cup (A_2 \cup A_3 \cup \cdots \cup A_k)) = P(A_1) + P(A_2 \cup A_3 \cup \cdots \cup A_k) - P((A_1 \cap A_2) \cup \cdots \cup (A_1 \cap A_k))$$

So from using Eq. 1 through 3,

$$P(A_1 \cup A_2 \cup \cdots \cup A_k)$$

$$= P(A_1) + \left[\sum_{i=2}^{k} P(A_i) - \sum_{2\le i<j\le k} P(A_i \cap A_j) + \sum_{2\le i<j<r\le k} P(A_i \cap A_j \cap A_r) - \cdots\right]$$

$$- \left[\sum_{i=2}^{k} P(A_1 \cap A_i) - \sum_{2\le i<j\le k} P(A_1 \cap A_i \cap A_j) + \sum_{2\le i<j<r\le k} P(A_1 \cap A_i \cap A_j \cap A_r) - \cdots\right]$$

$$= \sum_{i=1}^{k} P(A_i) - \sum_{1\le i<j\le k} P(A_i \cap A_j) + \sum_{1\le i<j<r\le k} P(A_i \cap A_j \cap A_r)$$

$$+ \cdots + (-1)^{k-1} \cdot P(A_1 \cap A_2 \cap \cdots \cap A_j)$$

1–35. $P(\overline{B}) = \dfrac{(365)(364)\cdots(365-n+1)}{365^n}$

| $n$ | 10 | 20 | 21 | 22 | 23 | 24 | 25 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P(B)$ | 0.117 | 0.411 | 0.444 | 0.476 | 0.507 | 0.538 | 0.569 | 0.706 | 0.891 | 0.970 | 0.994 |

1–36. $P(\text{win on 1st throw}) = \dfrac{6}{36} + \dfrac{2}{36} = \dfrac{8}{36}$

$$P(\text{win after 1st throw}) = P(\text{win on 4}) + P(\text{win on 5}) + P(\text{win on 6}) + P(\text{win on 8}) + P(\text{win on 9}) + P(\text{win on 10})$$

$$P(\text{win on 4}) = \frac{3}{36}\cdot\left[\frac{3}{36} + \left(\frac{27}{36}\right)\cdot\frac{3}{36} + \left(\frac{27}{36}\right)^2\cdot\frac{3}{36} + \cdots\right] = \frac{1}{36}$$

$$P(\text{win on 5}) = \frac{4}{36}\cdot\left[\frac{4}{36} + \left(\frac{26}{36}\right)\left(\frac{4}{36}\right) + \left(\frac{26}{36}\right)^2\cdot\left(\frac{4}{36}\right) + \cdots\right] = \frac{2}{45}$$

$$P(\text{win on 6}) = \frac{5}{36}\left[\frac{5}{36} + \left(\frac{25}{36}\right)\left(\frac{5}{36}\right) + \left(\frac{25}{36}\right)^2\cdot\left(\frac{5}{36}\right) + \cdots\right] = \frac{25}{396}$$

$$P(\text{win on 8}) = P(\text{win on 6}) = \frac{25}{396}$$

$$P(\text{win on 9}) = P(\text{win on 5}) = \frac{2}{45} \qquad P(\text{win on 10}) = P(\text{win on 4}) = \frac{1}{36}$$

$$P(\text{win}) = \frac{8}{36} + \left[2\cdot\frac{1}{36} + 2\cdot\frac{2}{45} + 2\cdot\frac{25}{396}\right] = 0.4929$$

1–37. $P_8^8 = 8! = 40{,}320$

1–38. Let $B$, $C$, $D$, $E$, $X$ represent the events of arriving at points so labeled.

$$P(B) = P(C) = P(D) = P(E) = \frac{1}{4}$$

$$P(X|B) = \frac{1}{3}, \qquad P(X|C) = 1, \qquad P(X|D) = 1, \qquad P(X|E) = \frac{2}{5}$$

$$P(X) = P(B)\cdot P(X|B) + P(C)\cdot P(X|C) + P(D)\cdot P(X|D) + P(E)\cdot P(X|E)$$
$$= \left(\frac{1}{4}\cdot\frac{1}{3}\right) + \left(\frac{1}{4}\cdot 1\right) + \left(\frac{1}{4}\cdot 1\right) + \left(\frac{1}{4}\cdot\frac{2}{5}\right) = \frac{41}{60}$$

1–39. $$P(B_3|A) = \frac{P(B_3)\cdot P(A|B_3)}{\sum_{i=1}^{3} P(B_i)\cdot P(A|B_i)} = \left[\frac{(0.5)(0.3)}{(0.2)(0.2)+(0.3)(0.5)+(0.5)(0.3)}\right]$$
$$= 0.441$$

1–40. $F$: Structural Failure
$D_S$: Diagnosis as Structural Failure

$$P(F|D_S) = \frac{P(F)\cdot P(D_S|F)}{P(F)\cdot P(D_S|F) + P(\overline{F})\cdot P(D_S|\overline{F})}$$
$$= \frac{(0.25)(0.9)}{(0.25)(0.9)+(0.75)(0.2)} = \frac{0.225}{0.225+0.150} = 0.6$$