Chapter 10

10–1. Both estimators are unbiased. Now, $V(\overline{X}_1) = \sigma^2/2n$ while $V(\overline{X}_2) = \sigma^2/n$. Since $V(\overline{X}_1) < V(\overline{X}_2)$, $\overline{X}_1$ is a more efficient estimator than $\overline{X}_2$.

10–2. $E(\hat{\theta}_1) = \mu$, $E(\hat{\theta}_2) = (1/2)E(2X_1 - X_6 + X_4) = (1/2)(2\mu - \mu + \mu) = \mu$. Both estimators are unbiased.

$$\begin{aligned} V(\hat{\theta}_1) &= \sigma^2/7, \\ V(\hat{\theta}_2) &= \left(\frac{1}{2}\right)^2 V(2X_1 - X_6 + X_4) \\ &= \left(\frac{1}{4}\right)[4V(X_1) + V(X_6) + V(X_4)] = \left(\frac{1}{4}\right) 6\sigma^2 = 3\sigma^2/2 \end{aligned}$$

$\hat{\theta}_1$ has a smaller variance than $\hat{\theta}_2$.

10–3. Since $\hat{\theta}_1$ is unbiased, $MSE(\hat{\theta}_1) = V(\hat{\theta}_1) = 10$.

$$MSE(\hat{\theta}_2) = V(\hat{\theta}_2) + (Bias)^2 = 4 + (\theta - \theta/2)^2 = 4 + \theta^2/4.$$

If $\theta < \sqrt{24} = 4.8990$, $\hat{\theta}_2$ is a better estimator of θ than $\hat{\theta}_1$, because it would have smaller MSE.

10–4. $MSE(\hat{\theta}_1) = V(\hat{\theta}_1) = 12$, $MSE(\hat{\theta}_2) = V(\hat{\theta}_2) = 10$,
$MSE(\hat{\theta}_3) = E(\hat{\theta}_3 - \theta)^2 = 6$. $\hat{\theta}_3$ is a better estimator because it has smaller MSE.

10–5. $E(S^2) = (1/24)E(10S_1^2 + 8S_2^2 + 6S_3^2) = (1/24)(10\sigma^2 + 8\sigma^2 + 6\sigma^2)$
$= (1/24)24\sigma^2 = \sigma^2$

10–6. Any linear estimator of μ is of the form $\hat{\theta} = \sum_{i=1}^{n} a_i X_i$ where a_i are constants. $\hat{\theta}$ is an unbiased estimator of μ only if $E(\hat{\theta}) = \mu$, which implies that $\sum_{i=1}^{n} a_i = 1$. Now $V(\hat{\theta}) = \sum_{i=1}^{n} a_i^2\sigma^2$. Thus we must choose the a_i to minimize $V(\hat{\theta})$ subject to the constraint $\sum a_i = 1$. Let λ be a Lagrange multiplier. Then

$$F(a_i, \lambda) = \sum_{i=1}^{n} a_i^2\sigma^2 - \lambda\left(\sum_{i=1}^{n} a_i - 1\right)$$

and $\partial F/\partial a_i = \partial F/\partial\lambda = 0$ gives

$$2a_i\sigma^2 - \lambda = 0; i = 1, 2, \ldots, a$$

$$\sum_{i=1}^{n} a_i = 1$$

The solution is $a_i = 1/n$. Thus $\hat{\theta} = \overline{X}$ is the best linear unbiased estimator of μ.

10–7.
$$L(\alpha) = \prod_{i=1}^{n} \alpha^{X_i} e^{-\alpha}/X_i! = \alpha^{\Sigma X_i} e^{-n\alpha} \Bigg/ \prod_{i=1}^{n} X_i!$$
$$\ell\text{n}\, L(\alpha) = \sum_{i=1}^{n} X_i \,\ell\text{n}\,\alpha - n\alpha - \ell\text{n}\left(\prod_{i=1}^{n} X_i!\right)$$
$$\frac{d\,\ell\text{n}\, L(\alpha)}{d\alpha} = \sum_{i=1}^{n} X_i/\alpha - n = 0$$
$$\hat{\alpha} = \sum_{i=1}^{n} X_i/n = \overline{X}$$

10–8. For the Poisson distribution, $E(X) = \alpha = \mu_1'$. Also, $M_1' = \overline{X}$. Thus $\hat{\alpha} = \overline{X}$ is the moment estimator of α.

10–9.
$$L(\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda t_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} t_i}$$
$$\ell\text{n}\, L(\lambda) = n\,\ell\text{n}\,\lambda - \lambda \sum_{i=1}^{n} t_i$$
$$\frac{d\,\ell\text{n}\, L(\lambda)}{d\lambda} = (n/\lambda) - \sum_{i=1}^{n} t_i = 0$$
$$\hat{\lambda} = n \Bigg/ \sum_{i=1}^{n} t_i = (\bar{t})^{-1}$$

10–10. $E(t) = 1/\lambda = \mu_1'$, and $M_1' = \bar{t}$. Thus $1/\lambda = \bar{t}$ or $\hat{\lambda} = (\bar{t})^{-1}$.

10–11. If X is a gamma random variable, then $E(X) = r/\lambda$ and $V(X) = r/\lambda^2$. Thus $E(X^2) = (r + r^2)\lambda^2$. Now $M_1' = \overline{X}$ and $M_2' = (1/n)\sum_{i=1}^{n} X_i^2$. Equating moments, we obtain

$$r/\lambda = \overline{X}, \quad (r + r^2)\lambda^2 = (1/n)\sum_{i=1}^{n} X_i^2$$

or,

$$\hat{\lambda} = \overline{X} \Bigg/ \left[(1/n)\sum_{i=1}^{n} X_i^2 - \overline{X}^2\right]$$
$$\hat{r} = \overline{X}^2 \Bigg/ \left[(1/n)\sum_{i=1}^{n} X_i^2 - \overline{X}^2\right]$$

10–12. $E(X) = 1/p$, $M_1' = \overline{X}$. Thus $1/p = \overline{X}$ or $\hat{p} = 1/\overline{X}$.

10–13. $L(p) = \prod_{i=1}^{n}(1-p)^{X_i - 1}p = p^n(1-p)^{\Sigma X_i - n}$

$\ell\text{n}\, L(p) = n\,\ell\text{n}\, p + (\sum_{i=1}^{n} X_i - n)\ell\text{n}\,(1-p)$. From $d\,\ell\text{n}\, L(p)/d\rho = 0$, we obtain

$$(n/\hat{p}) - \left(\sum_{i=1}^{n} X_i - n\right) \bigg/ (1-\hat{\rho}) = 0$$

$$\hat{p} = n \bigg/ \sum_{i=1}^{n} X_i = 1/\overline{X}$$

10–14. $E(X) = p$, $M_1' = \overline{X}$. Thus $\hat{p} = \overline{X}$.

10–15. $E(X) = np$ (n is known), $M_1' = \overline{X}_N$ ($\overline{X}$ is based on a sample of N observations.) Thus $np = \overline{X}_N$ or $\hat{p} = \overline{X}_N/n$.

10–16. $E(X) = np$, $V(X) = np(1-p)$, $E(X^2) = np - np^2 + n^2p^2$
$M_1' = \overline{X}$, $M_2' = (1/N)\sum_{i=1}^{N} X_i^2$. Equating moments,

$$np = \overline{X}, np - np^2 + n^2p^2 = (1/N)\sum_{i=1}^{N} X_i^2$$

$$\hat{n} = \overline{X}^2 \bigg/ \left[\overline{X} - (1/N)\sum_{i=1}^{N}(X_i - \overline{X})^2\right],\ \hat{p} = \overline{X}/\hat{N}$$

10–17. $$L(p) = \prod_{i=1}^{N}\binom{n}{X_i} p^{X_i}(1-p)^{n-X_i} = \left[\prod_{i=1}^{N}\binom{n}{X_i}\right] p^{\sum_{i=1}^{n} X_i}(1-p)^{nN - \sum_{i=1}^{N} X_i}$$

$$\ell\text{n}\, L(p) = \sum_{i=1}^{N}\ell\text{n}\binom{n}{X_i} + \left(\sum_{i=1}^{N} X_i\right)\ell\text{n}\, p + \left(nN - \sum_{i=1}^{N} X_i\right)\ell\text{n}(1-p)$$

$$\frac{d\,\ell\text{n}\, L(p)}{dp} = \sum_{i=1}^{N} X_i \bigg/ p - \left(nN - \sum_{i=1}^{N} X_i\right) \bigg/ (1-p) = 0$$

$$\hat{p} = \overline{X}/n$$

10–18. $L = \prod_{i=1}^{n} \left(\frac{\beta}{\delta}\right) \left(\frac{X_i - \gamma}{\delta}\right)^{\beta\tau} \exp\left[-\left(\frac{X_i - \gamma}{\delta}\right)^{\beta}\right]$

The system of partial derivatives $\partial L/\partial\delta = \partial L/\partial\beta = \partial L/\partial\gamma = 0$ yield simultaneous nonlinear equations that must be solved to produce the maximum likelihood estimators. In general, iterative methods must be used to find the maximum likelihood estimates. A number of special cases are of practical interest; for example, if $\gamma = 0$, the two-parameter Weibull distribution results. Both iterative and linear estimation techniques can be used for the two-parameter case.

10–19. Let X be a random variable and c be a constant. Then Chebychev's inequality is

$$P(|X - c| \geq \epsilon) \leq (1/\epsilon^2)E(X - c)^2$$

Thus,

$$P(|\hat{\theta} - \theta| \geq \epsilon) \leq (1/\epsilon^2)E(\hat{\theta} - \theta)^2$$

Now $E(\hat{\theta} - \theta)^2 = V(\hat{\theta}) + [E(\hat{\theta}) - \theta]^2$.
Then

$$P(|\hat{\theta} - \theta| \geq \epsilon) \leq (1/\epsilon^2)\{V(\hat{\theta}) + [E(\hat{\theta}) - \theta]^2\}$$

If $\hat{\theta}$ is unbiased then $E(\hat{\theta}) - \theta = 0$ and if $\lim_{n\to\infty} V(\hat{\theta}) = 0$ we see that $\lim_{n\to\infty} P(|\hat{\theta} - \theta| \geq \epsilon) \leq 0$, or $\lim_{n\to\infty} P(|\hat{\theta} - \theta| \geq \epsilon) = 0$.

10–20.
$$\begin{aligned} E(\overline{X}) &= E[a\overline{X}_1 + (1-a)\overline{X}_2] = aE(\overline{X}_1) + (1-a)E(\overline{X}_2) \\ &= a\mu + (1-a)\mu = \mu \end{aligned}$$

$$V(\overline{X}) = a^2V(\overline{X}_1) + (1-a)^2V(\overline{X}_2) = a^2(\sigma^2/n_1) + (1-a)^2(\sigma^2/n_2)$$

$$\frac{dV(\overline{X})}{da} = 2a\left(\frac{\sigma^2}{n_1}\right) - 2(1-a)\left(\frac{\sigma^2}{n_2}\right) = 0$$

$$a^* = \frac{\sigma^2/n_2}{\sigma^2/n_1 + \sigma^2/n_2} = \frac{n_1}{n_1 + n_2}$$

10–21.
$$L(\gamma) = \prod_{i=1}^{n}(\gamma + 1)X_i^{\gamma} = (\gamma + 1)^n \prod_{i=1}^{n} X_i^{\gamma}$$

$$\ell\text{n}\, L(\gamma) = n\,\ell\text{n}(\gamma + 1) + \sum_{i=1}^{n} \gamma\,\ell\text{n}\, X_i$$

$$\frac{d\,\ell\text{n}\,L(\gamma)}{d\gamma} = n \Bigg/ (\gamma+1) + \sum_{i=1}^{n} \ell\text{n}\,X_i = 0$$

$$\hat{\gamma} = -1 - \left(n \Bigg/ \sum_{i=1}^{n} \ell\text{n}\,X_i\right)$$

10–22. $$L(\gamma) = \prod_{i=1}^{n} \lambda e^{-\lambda(X_i - X_\ell)} = \lambda^n e^{-\lambda(\sum_{i=1}^{n} X_i - nX_\ell)}$$

$$\ell\text{n}\,L(\lambda) = n\,\ell\text{n}\,\lambda - \lambda\left(\sum_{i=1}^{n} X_i - nX_\ell\right)$$

$$\frac{d\,\ell\text{n}\,L(\lambda)}{d\lambda} = n \Bigg/ \lambda - \left(\sum_{i=1}^{n} X_i - nX_\ell\right) = 0$$

10–23. Assume X_ℓ unknown, and we want to maximize $n\,\ell\text{n}\,\lambda - \lambda\sum_{i=1}^{n}(X_i - X_\ell)$ with respect to X_ℓ, subject to $X_i \geq X_\ell$. Thus we want $\sum_{i=1}^{n}(X_i - X_\ell)$ to be a minimum, subject to $X_i \geq X_\ell$. Thus $\hat{X}_\ell = \min(X_1, X_2, \ldots, X_n) = X_{(1)}$.

10–24. $$E(G) = E\left[K\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2\right] = K\left[\sum_{i=1}^{n-1} E(X_{i+1} - X_i)^2\right]$$

$$= K\left[\sum_{i=1}^{n-1} E\left(X_{i+1}^2 - 2X_iX_{i+1} + X_i^2\right)\right]$$

$$= K\sum_{i=1}^{n-1}[E(X_{i+1}^2) - 2E(X_iX_{i+1}) + E(X_i^2)]$$

$$= K[(n-1)(\sigma^2 + \mu^2) - 2(n-1)\mu^2 + (n-1)(\mu^2 + \sigma^2)]$$

$$= K[2(n-1)\sigma^2]$$

For $K[2(n-1)\sigma^2]$ to equal σ^2, $K = [2(n-1)]^{-1}$. Thus

$$G = \frac{1}{2(n-1)}\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2$$

is an unbiased estimator of σ^2.

10–25. $f(x_1, x_2, \ldots, x_n|\mu) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2}\Sigma(x_i - \mu)^2/\sigma^2\right)$,

$$f(\mu) = (2\pi\sigma_0^2)^{-1/2} \exp\left(-\frac{1}{2}(x - \mu_0)^2/\sigma_0^2\right)$$

$$f(\mu|x_1, x_2, \ldots, x_n) = c^{-1/2}(2\pi)^{-1/2} \exp\left\{-\frac{c}{2}\left[\mu - \frac{1}{c}\left(\frac{n\overline{x}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}\right)\right]^2\right\}$$

where $c = \frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}$

10–26. $f(x_1, x_2, \ldots, x_n|1/\sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2}\Sigma(x_i - \mu)^2/\sigma^2\right)$

$$f(1/\sigma^2) = \frac{1}{\Gamma(m+1)}(m\sigma_0^2)^{m+1}(1/\sigma^2)^m e^{-m\sigma_0^2/\sigma^2}$$

The posterior density for $1/\sigma^2$ is gamma with parameters $m + (n/2) + 1$ and $m\sigma_0^2 + \Sigma(x_i - \mu)^2$.

10–27. $f(x_1, x_2, \ldots, x_n|p_0) = p^n(1 - p)^{\Sigma x_i - n}$,

$$f(p) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p^{a-1}(1-p)^{b-1}$$

The posterior density for p is a beta distribution with parameters $a + n$ and $b + \Sigma x_i - n$.

10–28. $f(x_1, x_2, \ldots, x_n|p) = p^{\Sigma x_i}(1 - p)^{n - \Sigma x_i}$,

$$f(p) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p^{a-1}(1-p)^{b-1},$$

The posterior density for p is a beta distribution with parameters $a + \Sigma x_i$ and $b + n - \Sigma x_i$

10–29. $f(x_1, x_2, \ldots, x_n|\lambda) = \lambda^{\Sigma x_i} e^{-n\lambda}/\Pi x_i!$

$$f(\lambda) = \frac{1}{\Gamma(m+1)}\left(\frac{m+1}{\lambda_0}\right)^{m+1} \lambda^m e^{-(m+1)\lambda/\lambda_0}$$

The posterior density for λ is gamma with parameters $r = m + \Sigma x_i + 1$ and $\delta = n + (m+1)/\lambda_0$.

10–30. From Exercise 10–25 and using the relationship that the Bayes' estimator for μ using a squared-error loss function is given by $\hat{\mu} = \frac{1}{c}[\frac{n\overline{x}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}]$, we have

$$\hat{\mu} - \left[\frac{25}{40} + \frac{1}{8}\right]^{-1} \left[\frac{25(4.85)}{40} + \frac{4}{8}\right] = 4.708$$

10–31. $$\hat{\mu} = 1.05 - \frac{\frac{0.1}{2}}{\Phi\left(\frac{1.20 - 1.05}{\frac{0.1}{2}}\right) - \Phi\left(\frac{0.98 - 1.05}{\frac{0.1}{2}}\right)}(2\pi)^{-1/2}$$

$$\times \left[e^{-1/2(\frac{1.20-1.05}{0.1/2})} - e^{-1/2(\frac{0.98-1.05}{0.1/2})}\right]$$

$$= 1.05 - (0.0545)(0.399)(0.223 - 4.055) = \underline{0.967}$$

10–32. $\Sigma\chi_i = 6270$, $\hat{\lambda} = 0.000323$

10–33. $$\hat{\mu} = \left[\frac{25}{0.1} + \frac{1}{0.04}\right]^{-1} \left[\frac{25(10.05)}{0.1} + \frac{10}{0.04}\right] = 10.045$$

weight $\sim N(10.045, 0.1)$

$$P(\text{weight} < 9.95) = P\left(Z < \frac{9.95 - 10.045}{0.316}\right) = \Phi(-0.301) \approx 0.3783$$

10–34. From a previous exercise, the posterior is gamma with parameters $r = \Sigma x_i + 1$ and $\delta = n + (1/\lambda_0)$. Since $n = 10$ and $\Sigma x_i = 45$,

$$f(\lambda|x_1, \ldots, x_{10}) = \frac{1}{\Gamma(46)}(14)^{46}\lambda^{45}e^{-46\lambda}$$

The Bayes interval requires us to find L and U so that

$$\frac{(14)^{46}}{\Gamma(46)} \int_L^U \lambda^{45}e^{-46\lambda}\, d\lambda = 0.95$$

Since r is integer, tables of the Poisson distribution could be used to find L and U.

10–35. (a) $f(x_1|\theta) = \frac{2x}{\theta^2}$, $\quad f(\theta) = 1, 0 < \theta < 1$, $\quad f(x_1, \theta) = \frac{2x}{\theta^2}$, and

$$f(x_1) = \int_x^1 \frac{2x}{\theta^2} d\theta = -2x\frac{1}{\theta}\int_x^1 = 2 - 2x; 0 < x < 1$$

The posterior density is

$$f(\theta|x_1) = \frac{f(x_1,\theta)}{f(x_1)} = \frac{2x}{\theta^2(2-2x)}$$

(b) The estimator must minimize

$$Z = \int \ell(\hat{\theta};\theta) f(\theta|x_1)\, d\theta = \int_0^1 \theta^2(\hat{\theta}-\theta)^2 \frac{2x}{\theta^2(2-2x)} d\theta$$
$$= \frac{2x}{2-2x}\left[\hat{\theta}^2 - \hat{\theta} + \frac{1}{3}\right]$$

From $\frac{dZ}{d\hat{\theta}} = 0$ we get $\hat{\theta} = \frac{1}{2}$

10–36. $f(x_1|p) = p^x(1-p)^{1-x}, f(x_1,p) = 6p^{x+1}(1-p)^{2-x}$, and

$$f(x_1) = \int_0^1 6p^{x+1}(1-p)^{2-x}\, dp = \frac{\Gamma(x+2)\Gamma(3-x)}{4}$$

The posterior density for p is

$$f(p|x_1) = \frac{24p^{x+1}(1-p)^{2-x}}{\Gamma(x+2)\Gamma(3-x)}$$

For a squared-error loss, the Bayes estimator is

$$\hat{p} = E(p|x_1) = \int_0^1 \frac{24p^{x+2}(1-p)^{2-x}}{\Gamma(x+2)\Gamma(3-x)} = \frac{\Gamma(x+3)}{5\Gamma(x+2)} = \frac{x+3}{5}$$

If $\ell(\hat{p};p) = 2(\hat{p}-p)^2$, the Bayes estimator must minimize

$$Z = \frac{24}{\Gamma(x+2)\Gamma(3-x)} \int_0^1 2(\hat{p}-p)^2 p^{x+1}(1-p)^{2-x}\, dp$$
$$= \frac{2}{\Gamma(x+2)}\left[\hat{p}^2\Gamma(x+2) - \frac{2\hat{p}\Gamma(x+3)}{5} + \frac{\Gamma(x+4)}{30}\right]$$

From $dz/d\hat{p} = 0$, $\hat{p} = \frac{x+2}{5}$

10–37. For $\alpha_1 = \alpha_2 = \alpha/2$, $\alpha = 0.05$; $\overline{x} \pm 1.96(\frac{\sigma}{\sqrt{n}})$

For $\alpha_1 = 0.01$, $\alpha_2 = 0.04$; $\overline{x} - 1.751(\frac{\sigma}{\sqrt{n}}) \leq \mu \leq \overline{x} + 2.323(\frac{\sigma}{\sqrt{n}})$

$\alpha_1 = \alpha_2 = \alpha/2$ is shorter.

10–38. (a) $N(0,1)$

(b) $\overline{X} - Z_{\alpha/2}\left(\sqrt{\frac{\overline{X}}{n}}\right) \leq \lambda \leq \overline{X} + Z_{\alpha/2}\left(\sqrt{\frac{\overline{X}}{n}}\right)$

10–39. (a) $\overline{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$74.03533 \leq \mu \leq 74.03666$

(b) $\overline{x} - Z_{\alpha}\frac{\sigma}{\sqrt{n}} \leq \mu$

$74.0356 \leq \mu$

10–40. (a) $1003.04 \leq \mu \leq 1024.96$

(b) $1004.80 \leq \mu$

10–41. (a) $3232.11 \leq \mu \leq 3267.89$

(b) $3226.49 \leq \mu \leq 3273.51$

The width of the confidence interval in (a) is 35.78, and the width of the interval in (b) is 47.01. The wider confidence interval in (b) reflects the higher confidence coefficient.

10–42. $n = (Z_{\alpha/2}\sigma/E)^2 = [(1.96)25/5]^2 = 96.04 \simeq 97$

10–43. For the total width to be 8, the half-width must be 4, therefore $n = (Z_{\alpha/2}\sigma/E)^2 = [(1.96)25/4]^2 = 150.06 \simeq 150$ or 151.

10–44. $n = (Z_{\alpha/2}\sigma/E)^2 = 1000(1.96/15)^2 = 17.07 \simeq 18$.

10–45. (a) $0.0723 \leq \mu_1 - \mu_2 \leq 0.3076$

(b) $0.0499 \leq \mu_1 - \mu_2 \leq 0.33$

(c) $\mu_1 - \mu_2 \leq 0.3076$

10–46. $3.553 \leq \mu_1 - \mu_2 \leq 8.447$

10–47. $-3.68 \leq \mu_1 - \mu_2 \leq -2.12$

10–48. (a) $2238.6 \leq \mu \leq 2275.4$

(b) $2242.63 \leq \mu$

(c) $2240.11 \leq \mu \leq 2275.39$

10–49. $183.0 \le \mu \le 256.6$

10–50. $4.05 - t_{0.10,24}(0.08/\sqrt{25}) \le \mu \Rightarrow 4.029 \le \mu$

10–51. 13

10–52. (a) $546.12 \le \mu \le 553.88$
(b) $546.82 \le m$
(c) $\mu \le 553.18$

10–53. $94.282 \le \mu \le 111.518$

10–54. (a) $7.65 \le \mu_1 - \mu_2 \le 12.346$
(b) $8.03 \le \mu_1 - \mu_2$
(c) $\mu_1 - \mu_2 \le 11.97$

10–55. $-0.839 \le \mu_1 - \mu_2 \le -0.679$

10–56. (a) $-0.561 \le \mu_1 - \mu_2 \le 1.561$
(b) $\mu_1 - \mu_2 \le 1.384$
(c) $-0.384 \le \mu_1 - \mu_2$

10–57. $0.355 \le \mu_1 - \mu_2 \le 0.455$

10–58. $-30.24 \le \mu_1 - \mu_2 \le -19.76$

10–59. From 10–48, $s = 34.51$, $n = 16$, and $(n-1)s^2 = 17864.1$
(a) $649.60 \le \sigma^2 \le 2853.69$
(b) $714.56 \le \sigma^2$
(c) $\sigma^2 \le 2460.62$

10–60. (a) $1606.18 \le \sigma^2 \le 26322.15$
(b) $1755.68 \le \sigma^2$
(c) $\sigma^2 \le 21376.78$

10–61. $0.0039 \le \sigma^2 \le 0.0157$

10–62. $\sigma^2 \le 193.09$

10–63. $0.574 \le \sigma_1^2/\sigma_2^2 \le 3.614$

10–64. $s_1^2 = 0.29$, $s_2^2 = 0.34$, $s_1^2/s_2^2 = 1.208$, $n_1 = 12$, $n_2 = 18$

(a) $0.502 \le \sigma_1^2/\sigma_2^2 \le 2.924$

(b) $0.423 \le \sigma_1^2/\sigma_2^2 \le 3.468$

(c) $0.613 \le \sigma_1^2/\sigma_2^2$

(d) $\sigma_1^2/\sigma_2^2 \le 2.598$

10–65. $0.11 \le \sigma_1^2/\sigma_2^2 \le 0.86$

10–66. $0.089818 \le p \le 0.155939$

10–67. $n = 4057$

10–68. $p \le 0.00348$

10–69. $n = (Z_{\alpha/2}/E)^2 p(1-p) = (2.575/0.01)^2 p(1-p) = 66306.25p(1-p)$. The most conservative choice of p is $p = 0.5$, giving $n = 16576.56$ or $n = 16577$ homeowners.

10–70. $0.0282410 \le p_1 - p_2 \le 0.0677590$

10–71. $-0.0244 \le p_1 - p_2 \le 0.0024$

10–72. $-8.50 \le \mu_1 - \mu_2 \le 1.94$

10–73. $-2038 \le \mu_1 - \mu_2 \le 3774.8$

10–74. Since $\overline{X}$ and S^2 are independent, we can construct confidence intervals for μ and σ^2 such that we are 90 percent confident that both intervals provide correct conclusions by constructing a $100(0.90)^{1/2}$ percent confidence interval for each parameter. That is, we need a 95 percent confidence interval on μ and σ^2. Thus, $3.938 \le \mu \le 4.057$ and $0.0049 \le \sigma^2 \le 0.0157$ provides the desired simultaneous confidence intervals.

10–75. Assume that all three variances are equal. A 95 percent simultaneous confidence interval on $\mu_1 - \mu_2$, $\mu_1 - \mu_3$, and $\mu_2 - \mu_3$ will require that the individual intervals use $\alpha/3 = 0.05/3 = 0.0167$.

For $\mu_1 - \mu_2, s_p^2 = 1.97, t_{0.0167/2,18} \simeq 2.64; -3.1529 \le \mu_1 - \mu_2 \le 0.1529$
For $\mu_1 - \mu_3, s_p^2 = 1.76, t_{0.0167/2,23} \simeq 2.59; -1.9015 \le \mu_1 - \mu_3 \le 0.9015$
For $\mu_2 - \mu_3, s_p^2 = 1.24, t_{0.0167/2,23} \simeq 2.59; -0.1775 \le \mu_2 - \mu_3 \le 2.1775$

10–76. The posterior density for μ is truncated normal:

$$f(\mu|x_1, \ldots, x_{16}) = \sqrt{\frac{16}{2\pi 10}} \left[\Phi\left(\frac{12-8}{\sqrt{10/16}}\right) - \Phi\left(\frac{6-8}{\sqrt{10/16}}\right) \right]^{-1} e^{(-1/2)(\frac{\mu-8}{10/16})^2}$$

for $6 < \mu \leq 12$. From the normal tables, the 90% interval estimate for μ is centered at 8 and is from $8 - (1.795)(1.054) = 6.108$ to 9.892. Since $6.108 < 9 < 9.892$, we have no evidence to reject H_0.

10–77. The posterior density for $1/\sigma^2$ is gamma w/parameters $r + (n/2)$ and $\lambda + \Sigma(x_i - \mu)^2$. For $r = 3$, $\lambda = 1$, $n = 10$, $\mu = 5$, $\Sigma(x_i - 5)^2 = 4.92$, the Bayes estimate of $1/\sigma^2$ is $(1/\sigma^2) = \frac{3+5}{1+4.92} = 1.35$. The integral:

$$0.90 = \int_L^U \frac{1}{8}(5.92)^8 (1/\sigma^2)^7 e^{-5.92/\sigma^2}$$

10–78. $Z = \int_{-\infty}^{\infty} (\hat{\theta} - \theta)^2 f(\theta|x_1, x_2, \ldots, x_n)\, d\theta$

$$= \hat{\theta}^2 \int_{-\infty}^{\infty} f(\theta|x_1, x_2, \ldots, x_n)\, d\theta - 2\hat{\theta} \int_{-\infty}^{\infty} \theta f(\theta|x_1, x_2, \ldots, x_n)\, d\theta$$

$$+ \int_{-\infty}^{\infty} \theta^2 f(\theta|x_1, x_2, \ldots, x_n)\, d\theta$$

Let

$$E(\hat{\theta}) = \int_{-\infty}^{\infty} \theta f(\theta|x_1, x_2, \ldots, x_n)\, d\theta = \mu_\theta$$

$$E(\hat{\theta}^2) = \int_{-\infty}^{\infty} \theta^2 f(\theta|x_1, x_2, \ldots, x_n)\, d\theta = \tau_\theta$$

Then

$$Z = \hat{\theta}^2 - 2\hat{\theta}\mu_\theta + \tau_\theta$$

$$\frac{dZ}{d\hat{\theta}} = 2\hat{\theta} - 2\mu_\theta = 0 \quad \text{so} \quad \hat{\theta} = \mu_\theta.$$