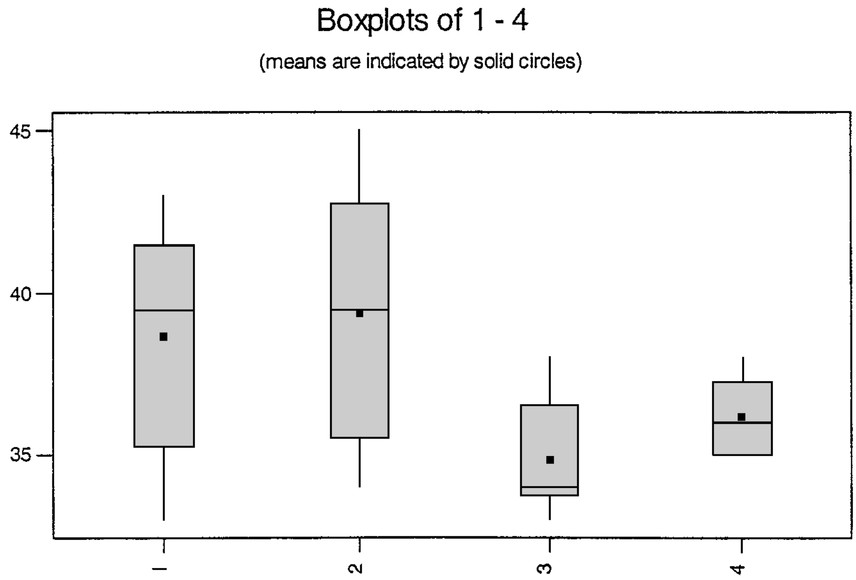
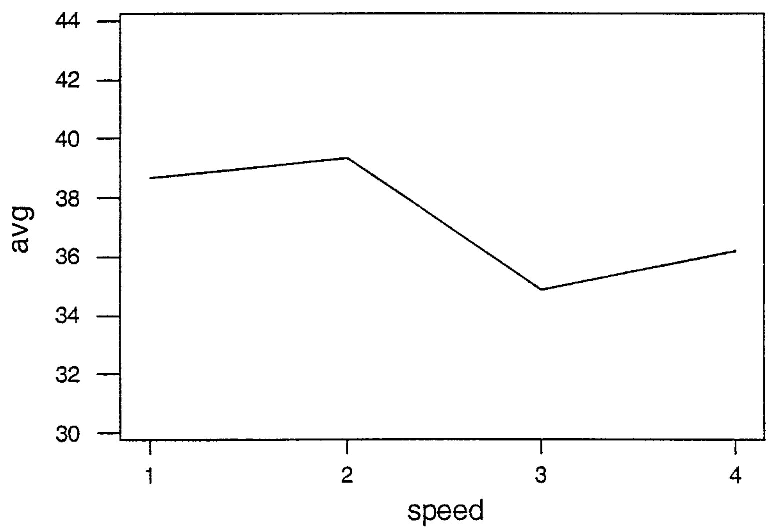
Chapter 12

12–1. (a) Analysis of Variance

Source	DF	SS	MS	F	P
Factor	3	80.17	26.72	3.17	0.047
Error	20	168.33	8.42		
Total	23	248.50			



(b)

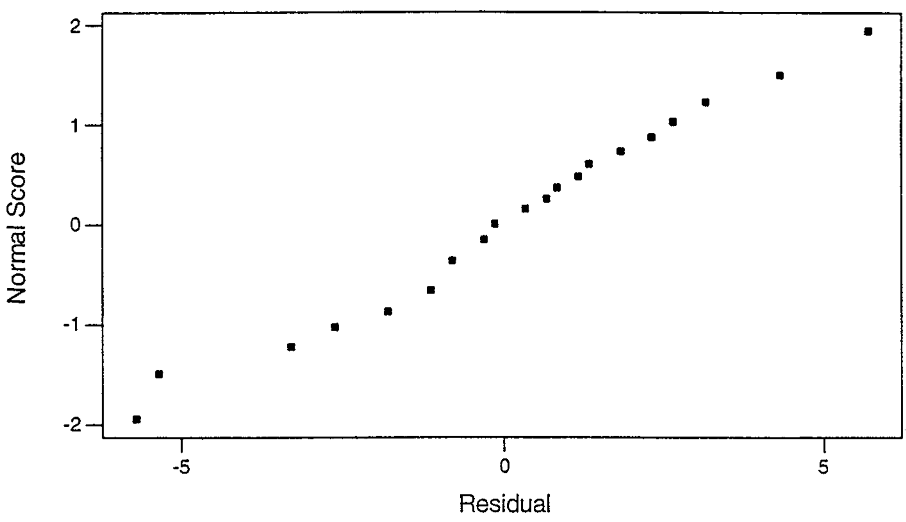


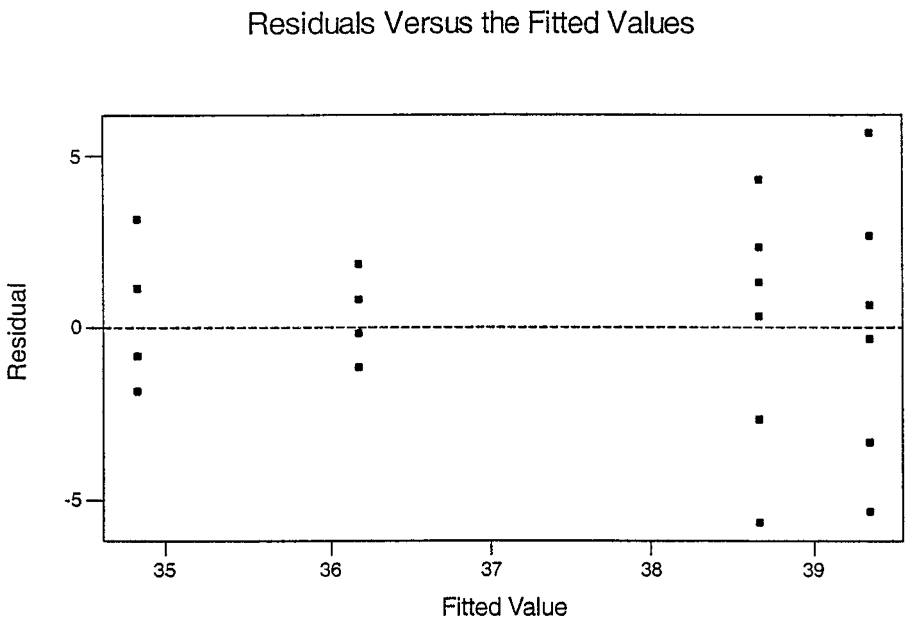
(c) Tukey's pairwise comparisons

```
    Family error rate = 0.0500
Individual error rate = 0.0111
Critical value = 3.96
Intervals for (column level mean) - (row level mean)
```

	1	2	3
2	-5.357 4.024		
3	-0.857 8.524	-0.190 9.190	
4	-2.190 7.190	-1.524 7.857	-6.024 3.357

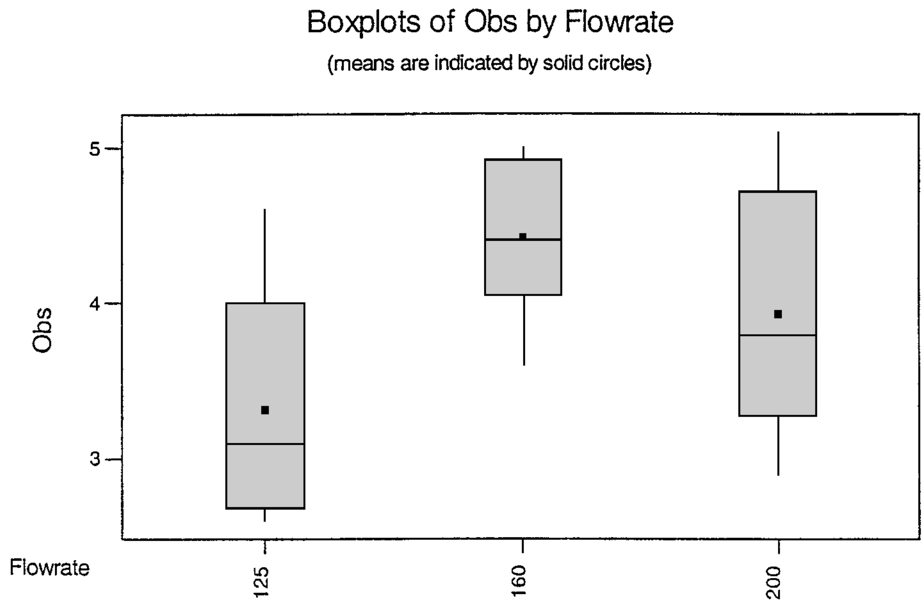
(d) Normal Probability Plot of the Residuals



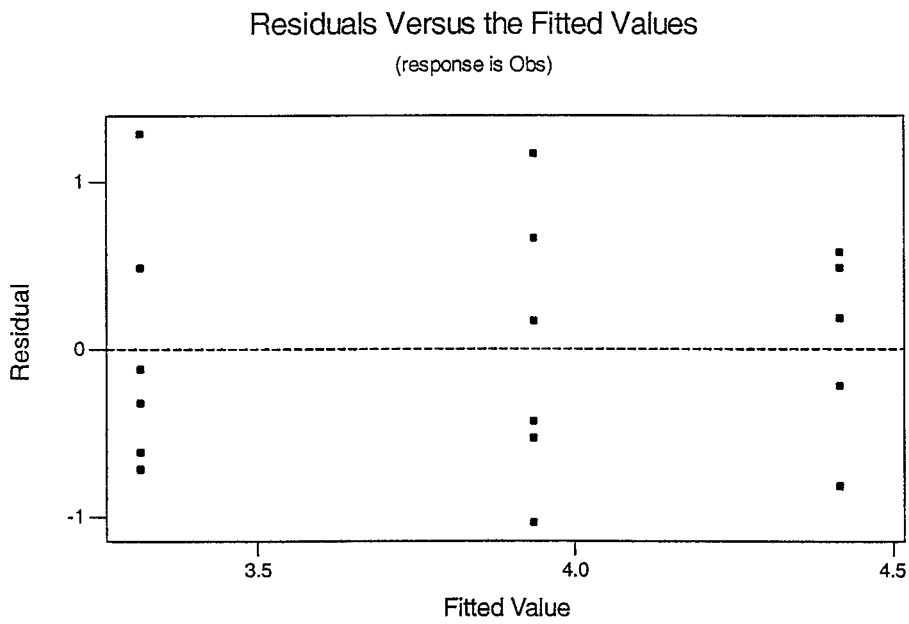


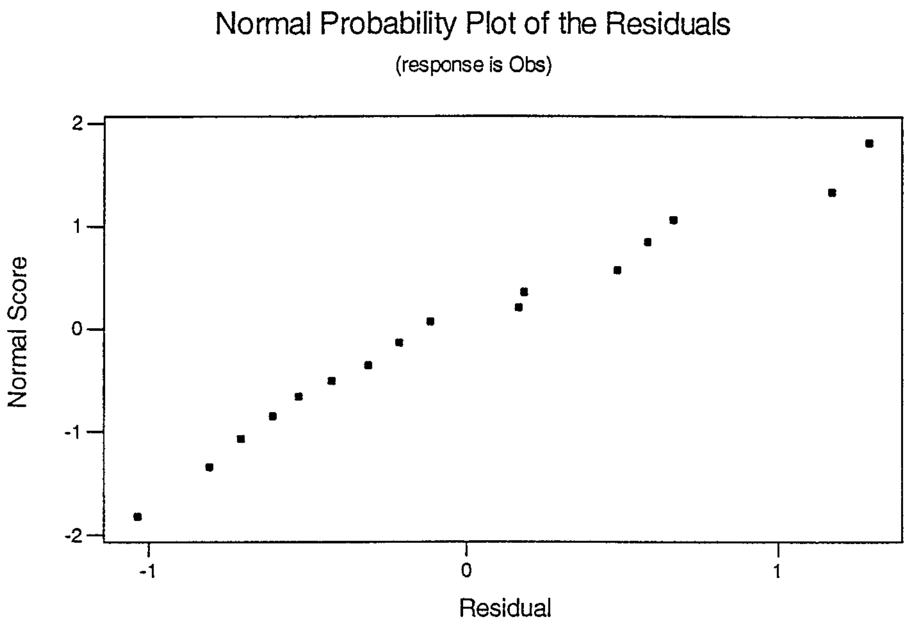
12–2. (a) Analysis of Variance for Obs

Source	DF	SS	MS	F	P
Flowrate	2	3.648	1.824	3.59	0.053
Error	15	7.630	0.509		
Total	17	11.278			


Boxplots of Obs by Flowrate
(means are indicated by solid circles)
Obs
5
4
3
Flowrate
125
160
200

(b)


Residuals Versus the Fitted Values
(response is Obs)
Residual
1
0
-1
3.5
4.0
4.5
Fitted Value



12–3. (a) Analysis of Variance for Strength

Source	DF	SS	MS	F	P
Technique	3	489740	163247	12.73	0.000
Error	12	153908	12826		
Total	15	643648			

(b) Tukey's pairwise comparisons

```
    Family error rate = 0.0500
Individual error rate = 0.0117
Critical value = 4.20
Intervals for (column level mean) - (row level mean)

                1            2            3
    2        -423
               53

    3        -201          -15
              275          460

    4          67          252           30
              543          728          505
```

12–4. (a) Random effects

Analysis of Variance for Output

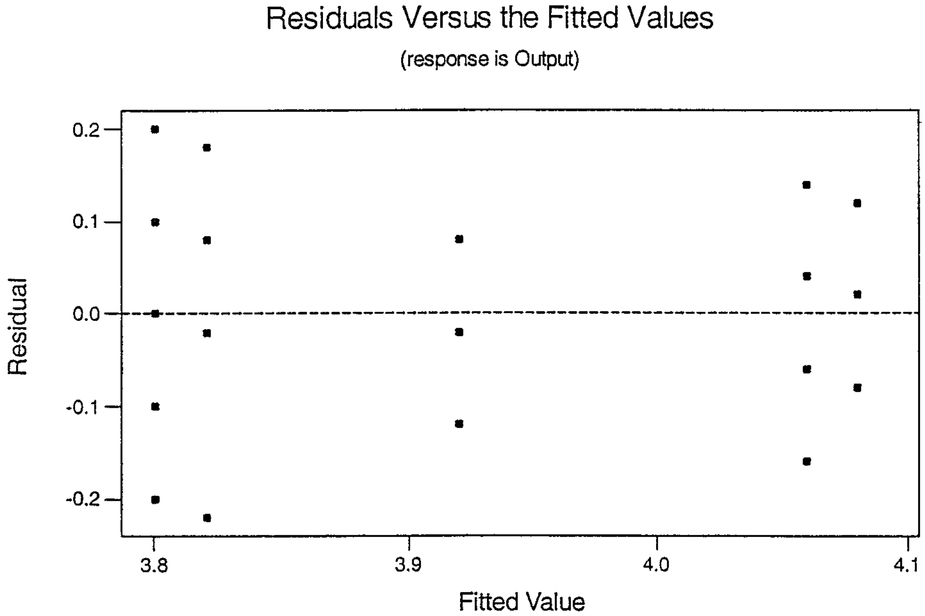
Source	DF	SS	MS	F	P
Loom	4	0.34160	0.08540	5.77	0.003
Error	20	0.29600	0.01480		
Total	24	0.63760			

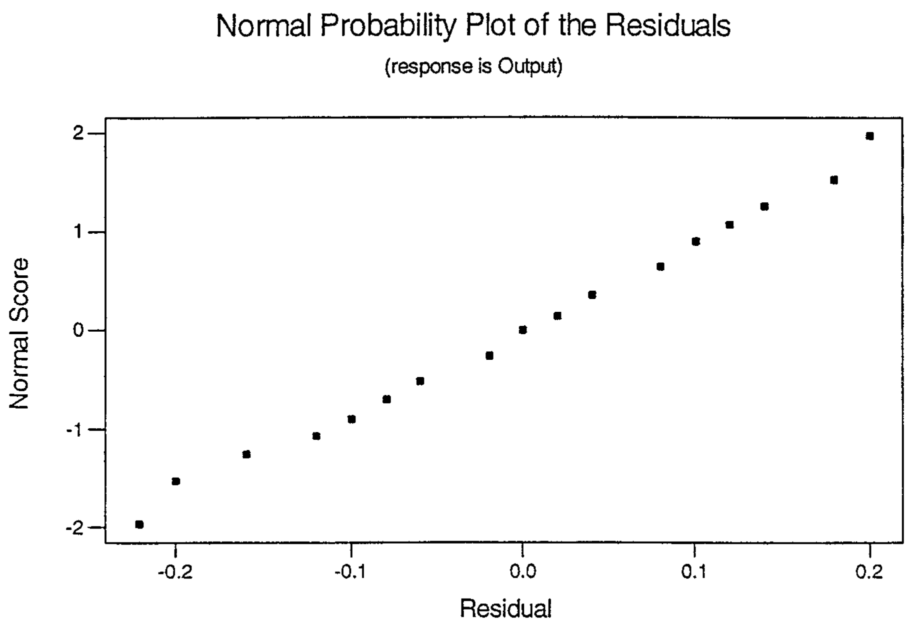
(b) $\sigma_\tau^2 = 0.01412$

(c) $\sigma^2 = 0.0148$

(d) 0.035

(e)



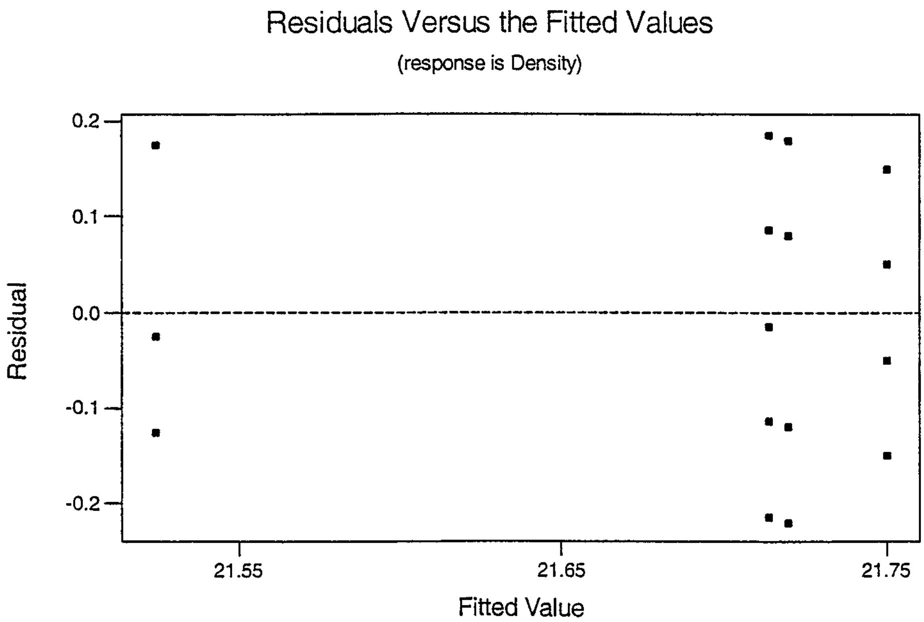


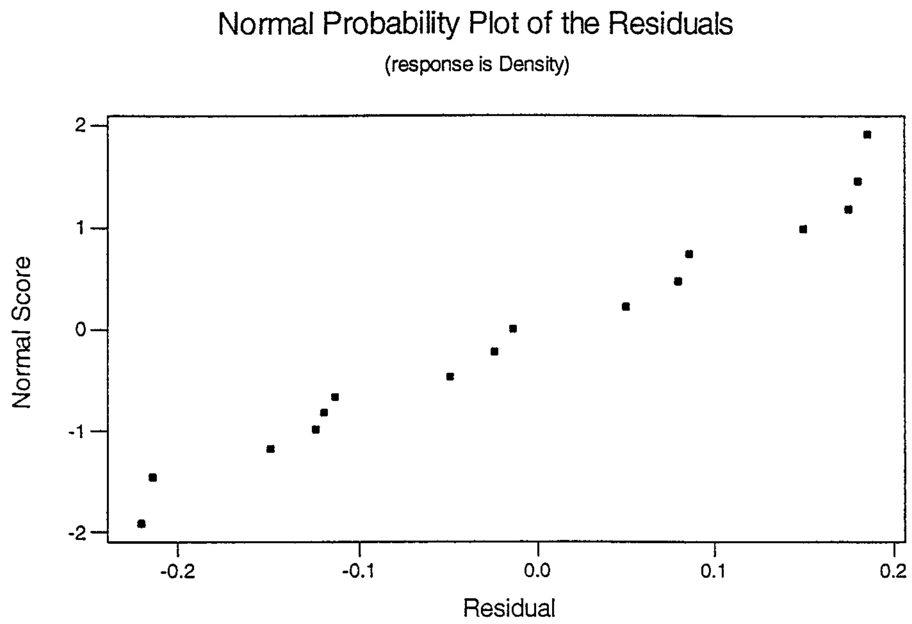
12–5. (a) Analysis of Variance for Density

Source	DF	SS	SS	MS	F	P
Temp	3	0.13911	0.13911	0.04637	2.62	0.083
Error	18	0.31907	0.31907	0.01773		
Total	21	0.45818				

(b) $\hat{\mu} = 21.70$, $\hat{\tau}_1 = 0.023$ $\hat{\tau}_2 = -0.166$ $\hat{\tau}_3 = 0.029$ $\hat{\tau}_4 = 0.059$

(c)





12–6. (a) Analysis of Variance for Conductivity

Source	DF	SS	MS	F	P
Coating	4	1060.50	265.13	16.35	0.000
Error	15	243.25	16.22		
Total	19	1303.75			

(b) $\hat{\mu} = 139.25$, $\hat{\tau}_1 = 5.75$ $\hat{\tau}_2 = 6.00$ $\hat{\tau}_3 = -7.75$ $\hat{\tau}_4 = -10.00$ $\hat{\tau}_5 = 6.00$

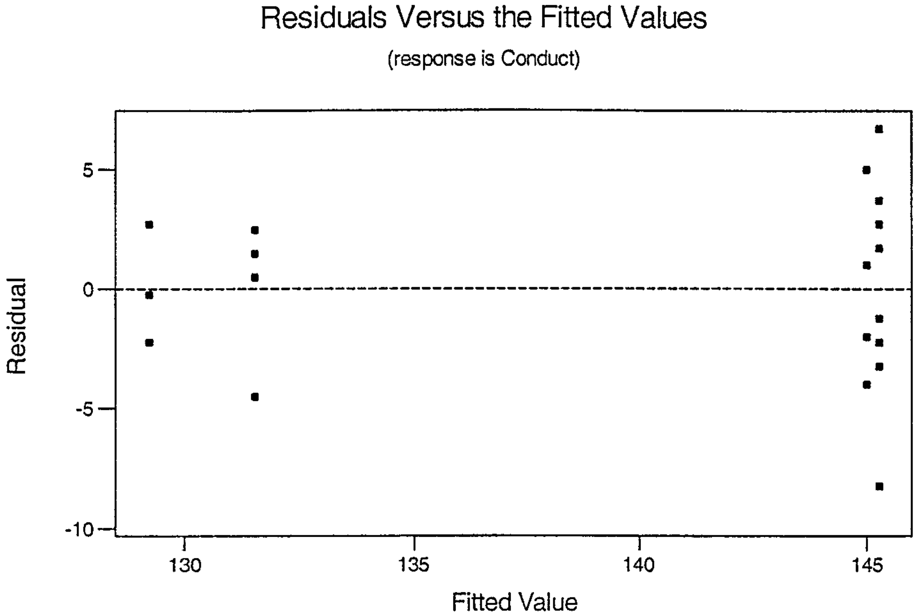
(c) $142.87 \leq \mu_1 \leq 147.13$, $7.363 \leq \mu_1 - \mu_2 \leq 24.137$

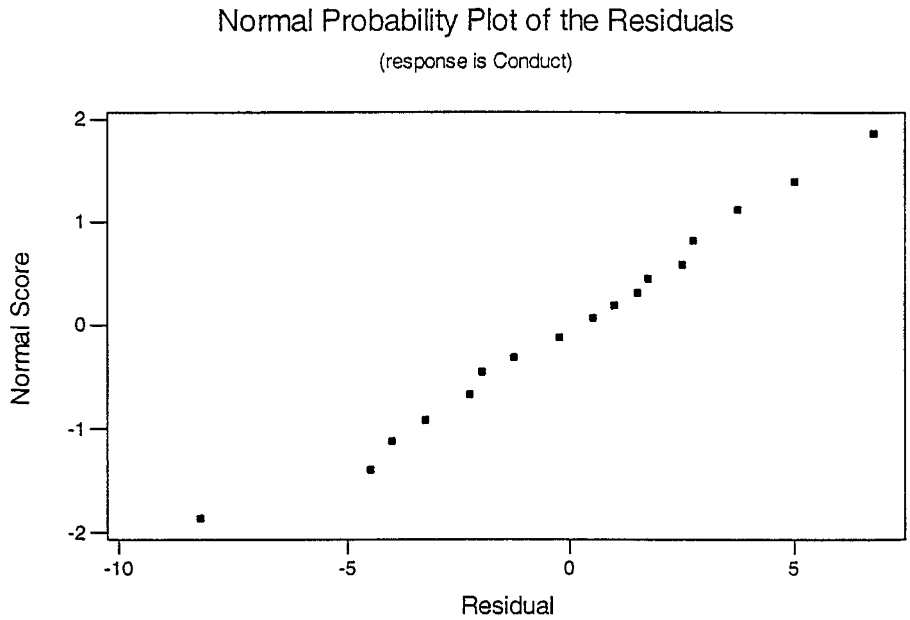
(d) Tukey's pairwise comparisons

```
    Family error rate = 0.0500
Individual error rate = 0.00747
Critical value = 4.37
Intervals for (column level mean) - (row level mean)
```

	1	2	3	4
2	-9.049			
	8.549			
3	4.701	4.951		
	22.299	22.549		
4	6.951	7.201	-6.549	
	24.549	24.799	11.049	
5	-9.049	-8.799	-22.549	-24.799
	8.549	8.799	-4.951	-7.201

(e)





12–7. (a) Analysis of Variance for Response Time

Source	DF	SS	MS	F	P
Circuit	2	260.9	130.5	4.01	0.046
Error	12	390.8	32.6		
Total	14	651.7			

(b) Tukey's pairwise comparisons

Family error rate = 0.0500
Individual error rate = 0.0206
Critical value = 3.77
Intervals for (column level mean) - (row level mean)

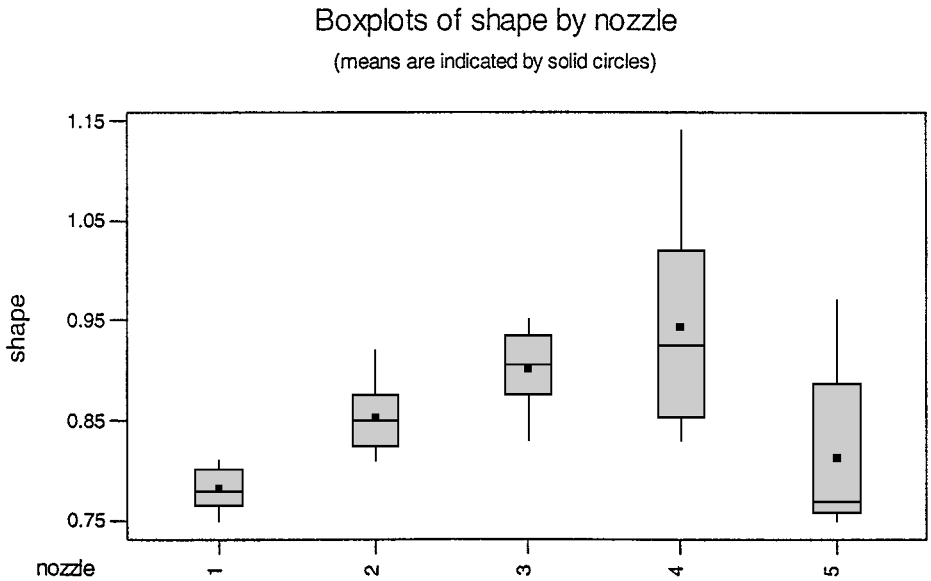
	1	2
2	-17.022 2.222	
3	-7.222 12.022	0.178 19.422

(c) $c_1 = y_{1\cdot} - 2y_{2\cdot} + y_{3\cdot}$, $SS_{c_1} = 14.4$
$c_2 = y_{1\cdot} - y_{3\cdot}$, $SS_{c_2} = 246.53$
Only c_2 is significant at $\alpha = 0.05$

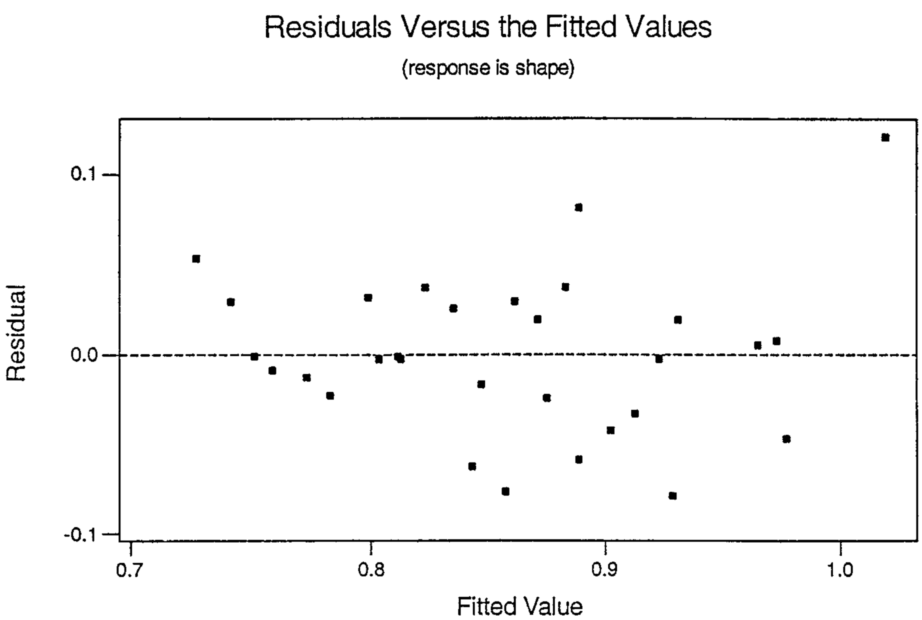
(d) 0.88

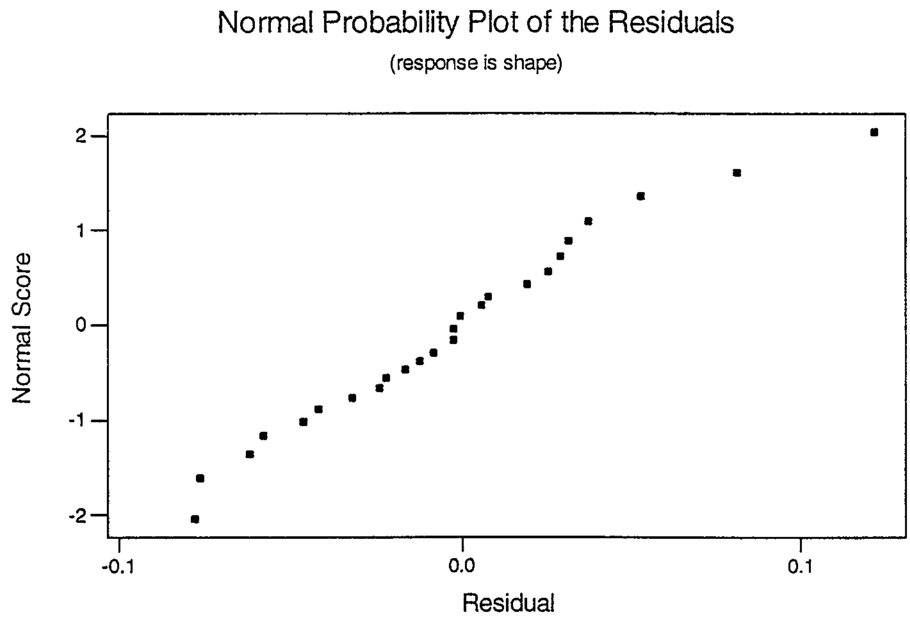
12–8. (a) Analysis of Variance for Shape

Source	DF	SS	MS	F	P
Nozzle	4	0.10218	0.02554	8.92	0.000
Efflux	5	0.06287	0.01257	4.39	0.007
Error	20	0.05730	0.00286		
Total	29	0.22235			



(c)

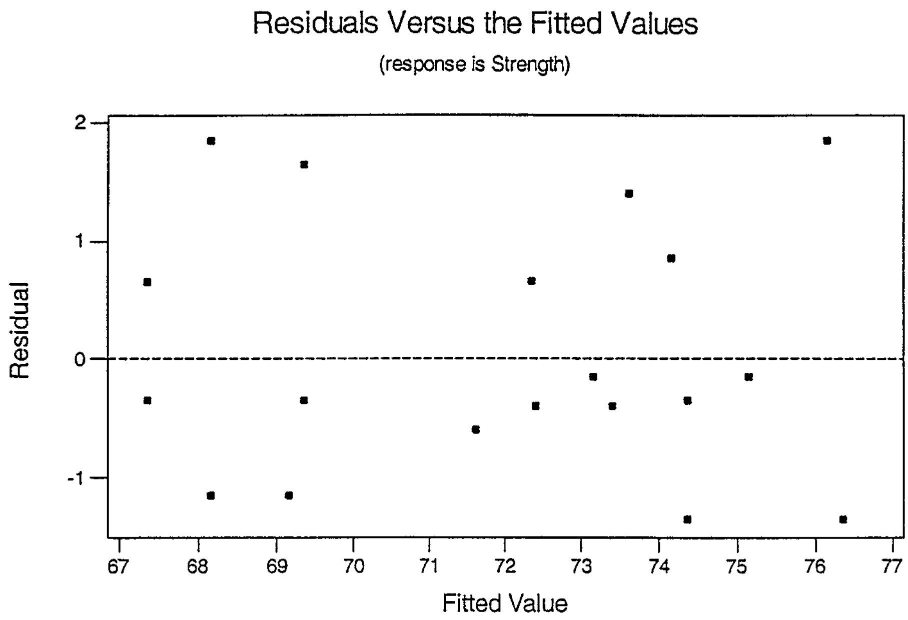


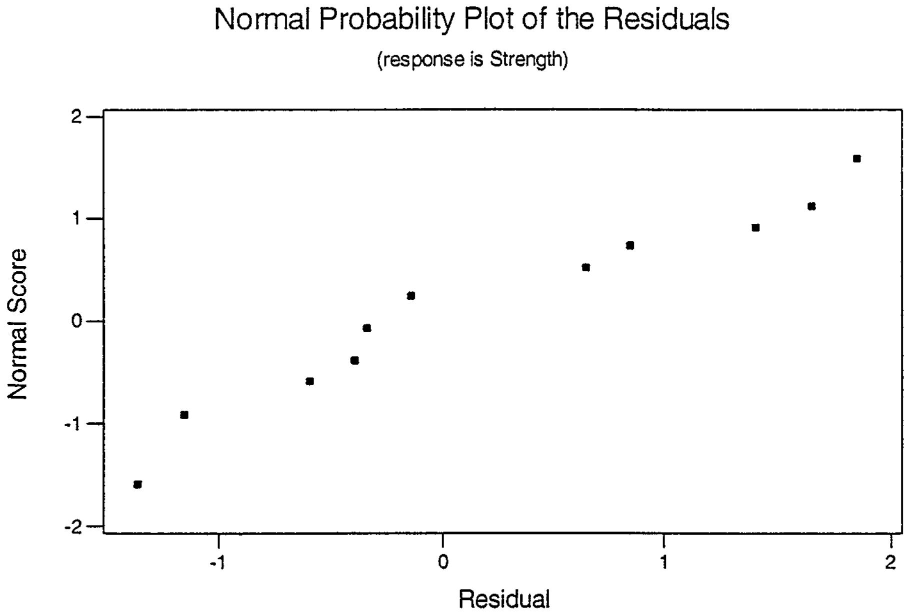


12–9. (a) Analysis of Variance for Strength

Source	DF	SS	MS	F	P
Chemical	3	12.95	4.32	2.38	0.121
Bolt	4	157.00	39.25	21.61	0.000
Error	12	21.80	1.82		
Total	19	191.75			

(c)





12–10. $\mu = \frac{1}{4}\Sigma\mu_1 = \frac{220}{4} = 55$, $\tau_1 = 50 - 55 = -5$, $\tau_2 = 60 - 55 = 5$, $\tau_3 = 50 - 55 = -5$, $\tau_4 = 60 - 55 = 5$. Then $\Phi^2 = \frac{2}{a}\frac{n\Sigma\tau_i^2}{2\sigma^2} = \frac{2}{4}\frac{n(100)}{2(25)} = n$, $\Phi = \sqrt{n}$. $\beta < 0.1$, $\alpha = 0.05$, OC curves give $n = 7$.

12–11. $\mu = \dfrac{1}{5}\Sigma\mu_i = \dfrac{940}{5} = 188, \tau_i = \mu_i - 188, i = 1, 2, \ldots, 5$

$$\tau_1 = -13, \tau_2 = 2, \tau_3 = -28, \tau_4 = 12, \tau_5 = 27$$

$$\Phi^2 = \frac{n}{a}\frac{\Sigma\tau_i^2}{\sigma^2} = \frac{n}{5}\left(\frac{1830}{100}\right) = 3.66n, \Phi = 1.91\sqrt{n}$$

If $\beta \leq 0.05$, $\alpha = 0.01$, OC curves give $n = 3$. If $n = 3$, then $\beta \cong 0.03$.

12–12. The test statistic for the two-sample t-test (with $n_1 = n_2 = n$) is

$$t_0 = \frac{\overline{y}_{1\cdot} - \overline{y}_{2\cdot}}{S_p\sqrt{2/n}} - t_{2n-2}$$

$$t_0^2 = \frac{(\overline{y}_{1\cdot} - \overline{y}_{2\cdot})^2(n/2)}{S_p^2}$$

$$= \frac{\left(\frac{y_{1\cdot}}{n} + \frac{y_{2\cdot}}{n}\right)^2 \cdot \left(\frac{n}{2}\right)}{S_p^2} = \frac{\frac{y_{1\cdot}^2}{2n} + \frac{y_{2\cdot}^2}{2n} - \frac{y_{1\cdot}y_{2\cdot}}{n}}{S_p^2}$$

But since $y_{1\cdot}y_{2\cdot} = \frac{y_{1\cdot}^2}{2} + \frac{y_{2\cdot}^2}{2} - \frac{(y_{1\cdot}+y_{2\cdot})^2}{2}$, the last equation becomes

$$t_0^2 = \frac{\dfrac{y_{1\cdot}^2}{n} + \dfrac{y_{2\cdot}^2}{n} - \dfrac{(y_{1\cdot} + y_{2\cdot})^2}{2n}}{S_p^2} = \frac{SS_{\text{Treatments}}}{S_p^2}$$

Note that $S_p^2 = \frac{\sum_{i=1}^2 \sum_{j=1}^n (y_{ij} - \overline{y}_{i\cdot})^2}{2n-2} = MS_E$. Therefore, $t_0^2 = \frac{SS_{\text{Treatments}}}{MS_E}$, and since the square of t_u is $F_{1,u}$ (in general), we see that the two tests are equivalent.

12–13. $$V\left(\sum_{i=1}^a c_i y_{i\cdot}\right) = \sum_{i=1}^a V(c_i y_{i\cdot}) = \sum_{i=1}^a c_i^2 V(y_{i\cdot})$$
$$= \sum_{i=1}^a c_i^2 V\left(\sum_{j=1}^{n_i} y_{ij}\right) = \sum_{i=1}^a c_i^2 \sum_{j=1}^{n_i} V(y)$$
$$= \sigma^2 \sum_{i=1}^a n_i c_i^2$$

12–14. For 4 treatments, a set of orthogonal contrasts is

$$\begin{aligned} 3y_{1\cdot} - y_{2\cdot} - y_{3\cdot} - y_{4\cdot} \\ 2y_{2\cdot} - y_{3\cdot} - y_{4\cdot} \\ y_{3\cdot} - y_{4\cdot} \end{aligned}$$

Assuming equal n, the contrast sums of squares are

$$Q_1^2 = \frac{(3y_{1\cdot} - y_{2\cdot} - y_{3\cdot} - y_{4\cdot})^2}{12n}, \quad Q_2^2 = \frac{(2y_{2\cdot} - y_{3\cdot} - y_{4\cdot})^2}{6n}$$
$$Q_3^2 = \frac{(y_{3\cdot} - y_{4\cdot})^2}{2n}$$

Now

$$Q_1^2 + Q_2^2 + Q_3^2 = \frac{9\sum_{i=1}^4 y_{i\cdot}^2 - 6\sum\sum_{i<j} y_{i\cdot}y_{j\cdot}}{12n}$$

and since

$$\sum\sum_{i<j} y_{i\cdot}y_{j\cdot} = \frac{1}{2}\left[y_{\cdot\cdot}^2 - \sum_{i=1}^4 y_{i\cdot}^2\right],$$

$$Q_1^2 + Q_2^2 + Q_3^2 = \frac{12\sum_{i=1}^4 y_{i\cdot}^2 - 3y_{\cdot\cdot}^2}{12n} = \frac{\sum_{i=1}^4 y_{i\cdot}^2}{n} - \frac{y_{\cdot\cdot}^2}{4n}$$
$$= SS_{\text{Treatments}}$$

12–15. (a)

$$\begin{aligned} 15\hat{\mu} + 5\hat{\tau}_1 + 5\hat{\tau}_2 + 5\hat{\tau}_3 &= 307 \\ 5\hat{\mu} + 5\hat{\tau}_1 \qquad\qquad\quad &= 104 \\ 5\hat{\mu} \qquad\quad + 5\hat{\tau}_2 \qquad\quad &= 111 \\ 5\hat{\mu} \qquad\qquad\qquad + 5\hat{\tau}_3 &= 92 \end{aligned}$$

If $\sum_{i=1}^{3} \hat{\tau}_i = 0$, then $\hat{\mu} = 20.47$, $\hat{\tau}_1 = 0.33$, $\hat{\tau}_2 = 1.73$, and $\hat{\tau}_3 = 2.07$.

$$\widehat{\tau_1 - \tau_2} = \hat{\tau}_1 - \hat{\tau}_2 = -1.40$$

(b) If $\hat{\tau}_3 = 0$, then $\hat{\mu} = 18.40$, $\hat{\tau}_1 = 2.40$, $\hat{\tau}_2 = 3.80$, and $\hat{\tau}_3 = 0$. These estimators differ from those found in part (a). However, note that

$$\widehat{\tau_1 - \tau_2} = \hat{\tau}_1 - \hat{\tau}_2 = 2.40 - 3.80 = -1.40$$

which agrees with part (a), because *contrasts* in the τ_i are uniquely estimated.

(c)

	Estimate Using	
Function	Part (a) Solution	Part (b) Solution
$\mu + \tau_1$	20.80	20.80
$2\tau_1 - \tau_2 - \tau_3$	1.00	1.00
$\mu + \tau_1 + \tau_2$	22.53	24.60