

Chapter 13

13-1. Analysis of Variance for Wear

Source	DF	SS	MS	F	P
CS	2	0.0317805	0.0158903	15.94	0.000
DC	2	0.0271854	0.0135927	13.64	0.000
CS*DC	4	0.0006873	0.0001718	0.17	0.950
Error	18	0.0179413	0.0009967		
Total	26	0.0775945			

13-2. Analysis of Variance for Finish

Source	DF	SS	MS	F	P
Drying	2	27.4	13.7	0.01	0.986
Paint	1	355.6	355.6	0.38	0.601
Drying*Paint	2	1878.8	939.4	5.03	0.026
Error	12	2242.7	186.9		
Total	17	4504.4			

13-3. $-23.93 \leq \mu_1 - \mu_2 \leq 5.15$

13-4. Analysis of Variance for Strength

Source	DF	SS	MS	F	P
operator	2	2.250	1.125	0.29	0.759
machine	3	28.833	9.611	2.46	0.160
operator*machine	6	23.417	3.903	0.84	0.565
Error	12	56.000	4.667		
Total	23	110.500			

13-5. The results would be a mixed model. The test statistics would be:

Effect	F_0
Operator	0.241
Machine	2.46
Operator*Machine	0.84

There is no change in conclusions.

13-6. Analysis of Variance for time

Source	DF	SS	MS	F	P
operator	2	0.01005	0.00503	0.07	0.937
engineer	1	0.04688	0.04688	0.62	0.512
operator*engineer	2	0.15005	0.07503	1.26	0.350
Error	6	0.35785	0.05964		
Total	11	0.56483			

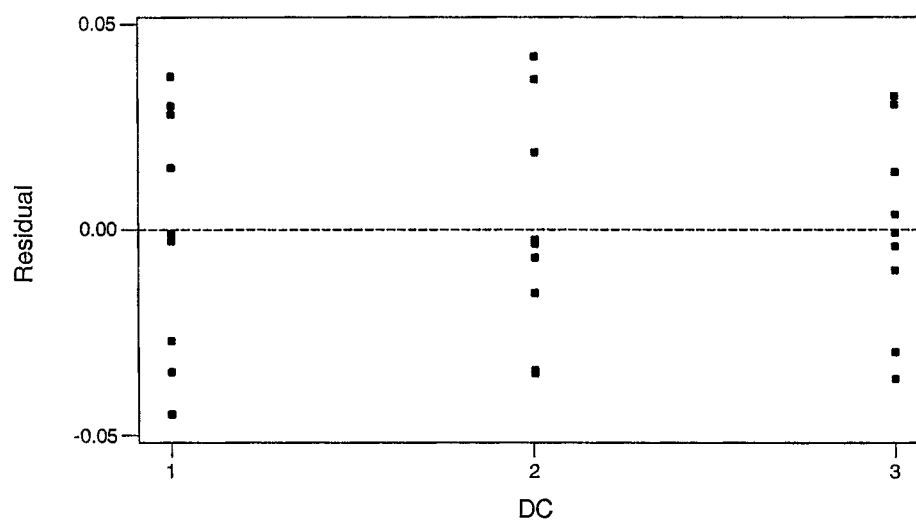
13-7. Analysis of Variance for current

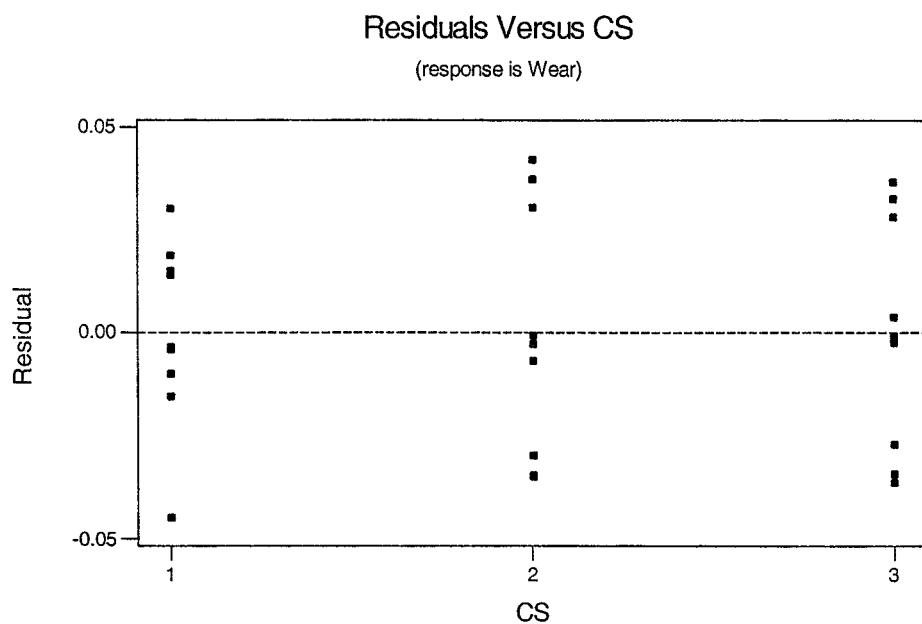
Source	DF	SS	MS	F	P
glass	1	14450.0	14450.0	273.79	0.000
phos	2	933.3	466.7	8.84	0.004
glass*phos	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

13-8.

Residuals Versus DC

(response is Wear)





There does not appear to be a problem with constant variance across levels of either factor.

13-9. Analysis of Variance for strength

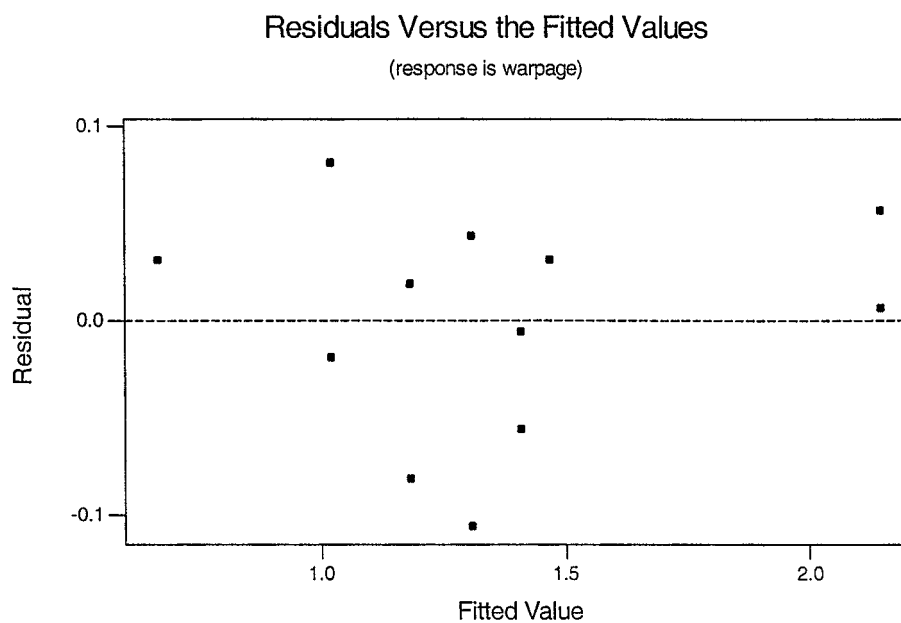
Source	DF	SS	MS	F	P
Conc	2	7.7639	3.8819	10.62	0.001
Freeness	2	19.3739	9.6869	26.50	0.000
Time	1	20.2500	20.2500	55.40	0.000
Conc*Freeness	4	6.0911	1.5228	4.17	0.015
Conc*Time	2	2.0817	1.0408	2.85	0.084
Freeness*Time	2	2.1950	1.0975	3.00	0.075
Conc*Freeness*Time	4	1.9733	0.4933	1.35	0.290
Error	18	6.5800	0.3656		
Total	35	66.3089			

13-10. Estimated Effects and Coefficients for warpage (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		1.3250	0.01398	94.81	0.000
A	0.2250	0.1125	0.01398	8.05	0.000
B	-0.1625	-0.0813	0.01398	-5.81	0.000
C	-0.4500	-0.2250	0.01398	-16.10	0.000
A*B	-0.5125	-0.2563	0.01398	-18.34	0.000
A*C	0.0000	0.0000	0.01398	0.00	1.000
B*C	0.2875	0.1438	0.01398	10.29	0.000
A*B*C	0.0625	0.0313	0.01398	2.24	0.056

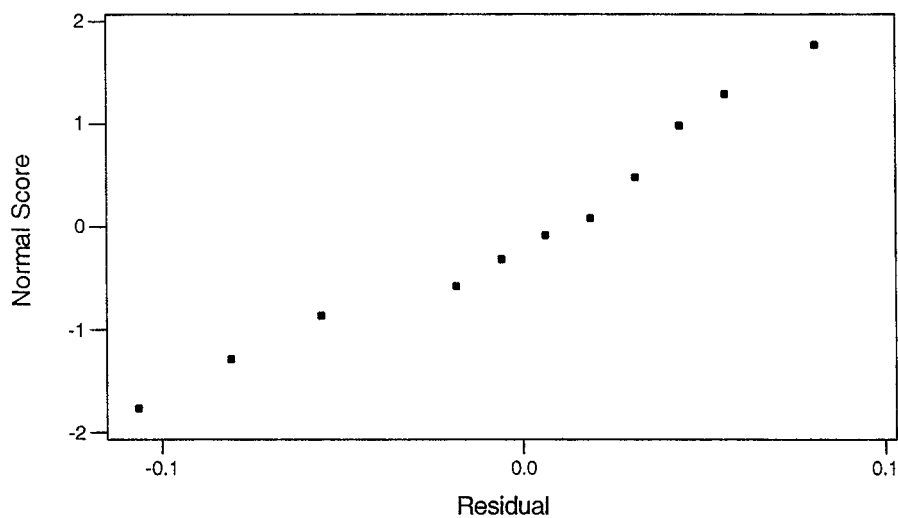
Based on the p -values, factors A , B , C , and the interactions AB and BC are significant at the 5% level of significance.

13-11. Using only the significant factors and interactions, the resulting residuals are as follows.



Normal Probability Plot of the Residuals

(response is warpage)



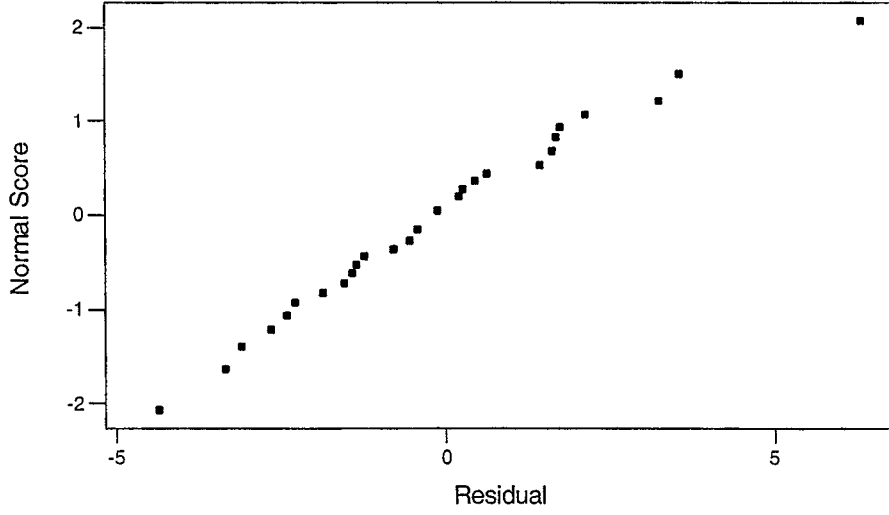
13-12. Estimated Effects and Coefficients for score (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		182.781	0.4891	373.68	0.000
A	-9.063	-4.531	0.4891	-9.26	0.000
B	-1.312	-0.656	0.4891	-1.34	0.198
C	-2.687	-1.344	0.4891	-2.75	0.014
D	3.937	1.969	0.4891	4.02	0.001
A*B	4.062	2.031	0.4891	4.15	0.001
A*C	0.688	0.344	0.4891	0.70	0.492
A*D	-2.187	-1.094	0.4891	-2.24	0.040
B*C	-0.563	-0.281	0.4891	-0.57	0.573
B*D	-0.188	-0.094	0.4891	-0.19	0.850
C*D	1.688	0.844	0.4891	1.72	0.104
A*B*C	-5.187	-2.594	0.4891	-5.30	0.000
A*B*D	4.687	2.344	0.4891	4.79	0.000
A*C*D	-0.938	-0.469	0.4891	-0.96	0.352
B*C*D	-0.938	-0.469	0.4891	-0.96	0.352
A*B*C*D	2.437	1.219	0.4891	2.49	0.024

13-13.

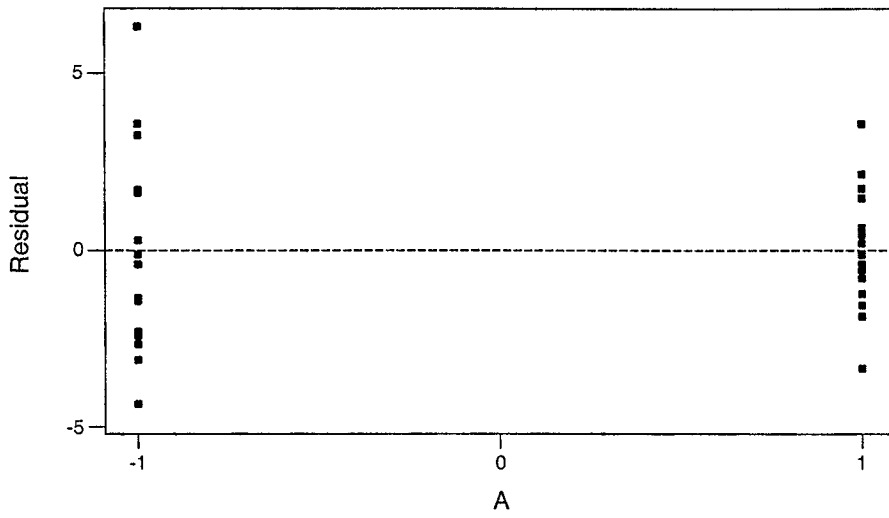
Normal Probability Plot of the Residuals

(response is score)

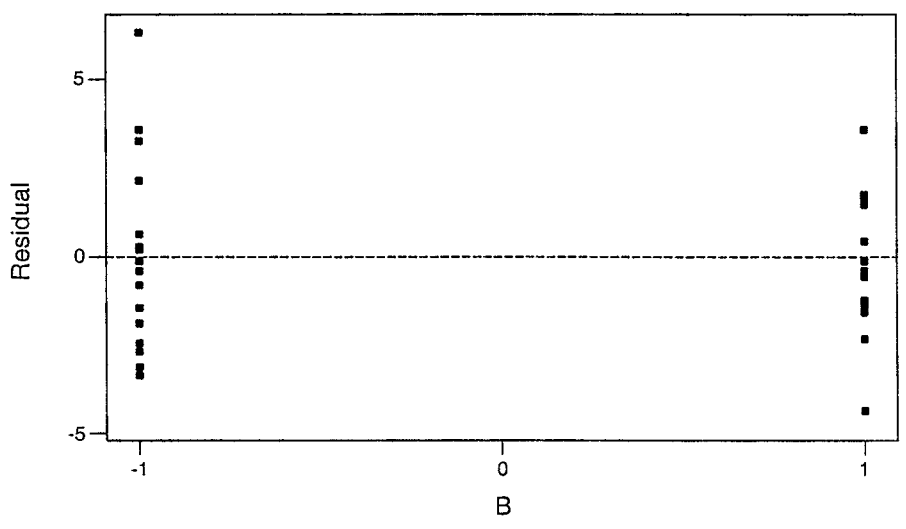


Residuals Versus A

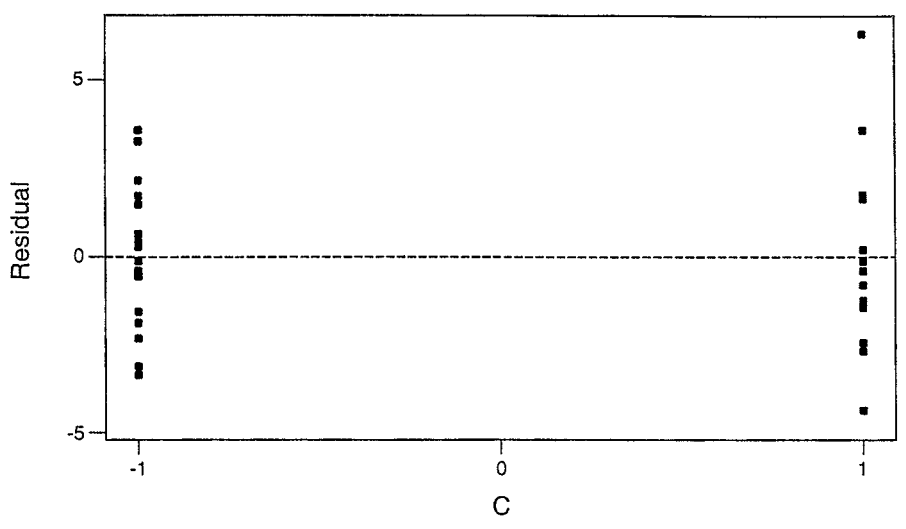
(response is score)

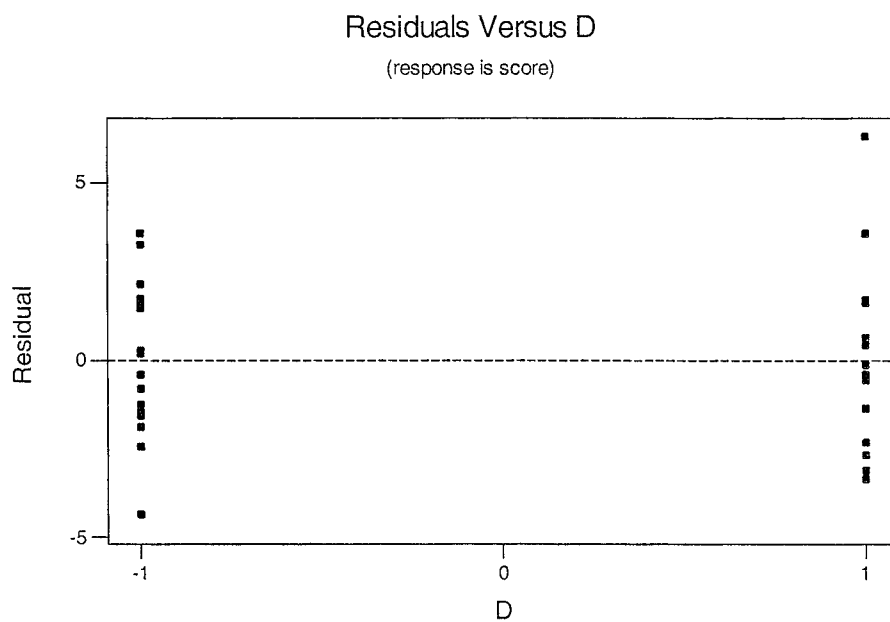


Residuals Versus B
(response is score)



Residuals Versus C
(response is score)





13–14. See solution for 13–12 for the standard errors.

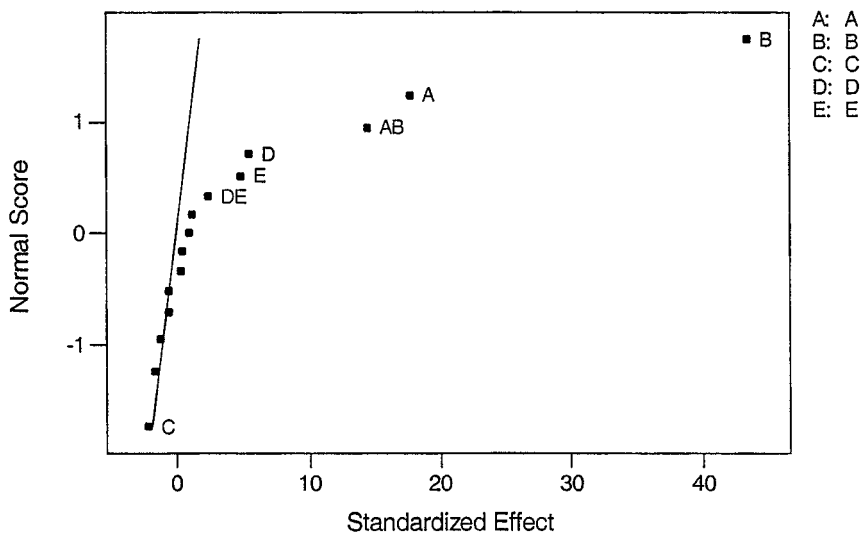
13–15. Estimated Effects and Coefficients for strength

Term	Effect	Coef	SE Coef	T	P
Constant		2872.06	40.47	70.97	0.000
A	1430.88	715.44	40.47	17.68	0.000
B	3506.62	1753.31	40.47	43.33	0.000
C	-168.37	-84.19	40.47	-2.08	0.054
D	443.37	221.69	40.47	5.48	0.000
E	394.13	197.06	40.47	4.87	0.000
A*B	1168.37	584.19	40.47	14.44	0.000
A*C	93.37	46.69	40.47	1.15	0.266
A*D	31.62	15.81	40.47	0.39	0.701
A*E	30.88	15.44	40.47	0.38	0.708
B*C	-130.87	-65.44	40.47	-1.62	0.125
B*D	-44.12	-22.06	40.47	-0.55	0.593
B*E	-43.37	-21.69	40.47	-0.54	0.599
C*D	80.88	40.44	40.47	1.00	0.333
C*E	-93.38	-46.69	40.47	-1.15	0.266
D*E	193.38	96.69	40.47	2.39	0.030

Main effects A , B , D , E and interactions AB and DE are significant.

Normal Probability Plot of the Standardized Effects

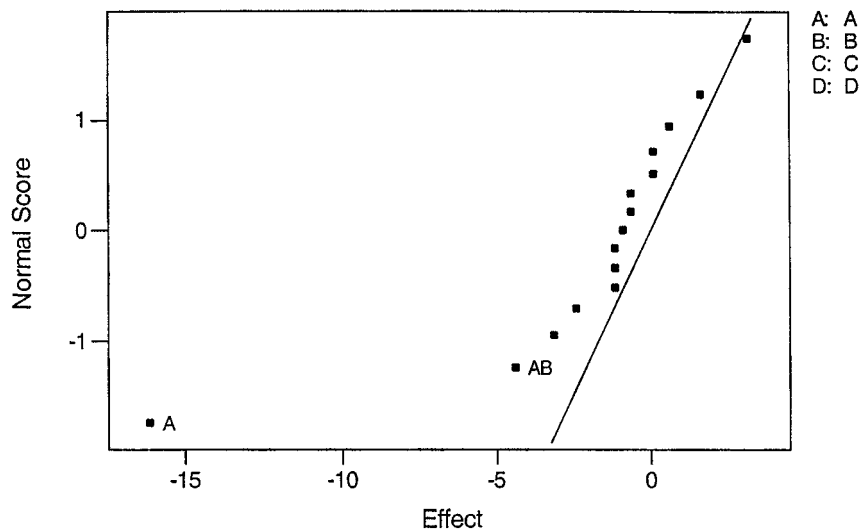
(response is strength, Alpha = .10)



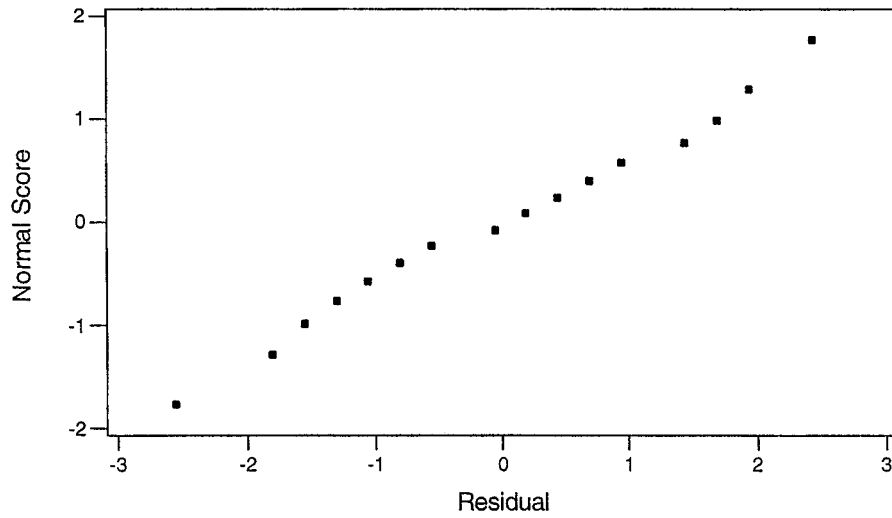
13-16. (a)

Normal Probability Plot of the Effects

(response is inches, Alpha = .10)



(b) Normal Probability Plot of the Residuals
(response is inches)



(c) Estimated Effects and Coefficients for inches

Term	Effect	Coef	SE Coef	T	P
Constant		35.938	0.6355	56.55	0.000
A	-16.125	-8.062	0.6355	-12.69	0.000
B	3.125	1.562	0.6355	2.46	0.057
C	-1.125	-0.562	0.6355	-0.89	0.417
D	-1.125	-0.562	0.6355	-0.89	0.417
A*B	-4.375	-2.188	0.6355	-3.44	0.018
A*C	-0.625	-0.313	0.6355	-0.49	0.644
A*D	-3.125	-1.563	0.6355	-2.46	0.057
B*C	1.625	0.812	0.6355	1.28	0.257
B*D	0.125	0.063	0.6355	0.10	0.925
C*D	-0.625	-0.312	0.6355	-0.49	0.644

13-17. Block 1 Block 2

(1)	<i>a</i>
<i>ab</i>	<i>b</i>
<i>ac</i>	<i>c</i>
<i>bc</i>	<i>abc</i>

13-18.	Block 1	Block 2
(1)	<i>ad</i>	<i>a abc</i>
	<i>ab bd</i>	<i>b bcd</i>
	<i>ac cd</i>	<i>c acd</i>
	<i>bc abcd</i>	<i>d abd</i>

13-19.	Block 1	Block 2	Block 3	Block 4
(1)	<i>a</i>	<i>c</i>	<i>d</i>	
	<i>ab</i>	<i>b</i>	<i>abc</i>	<i>abd</i>
	<i>bcd</i>	<i>cd</i>	<i>bd</i>	<i>bc</i>
	<i>acd</i>	<i>abcd</i>	<i>ad</i>	<i>ac</i>

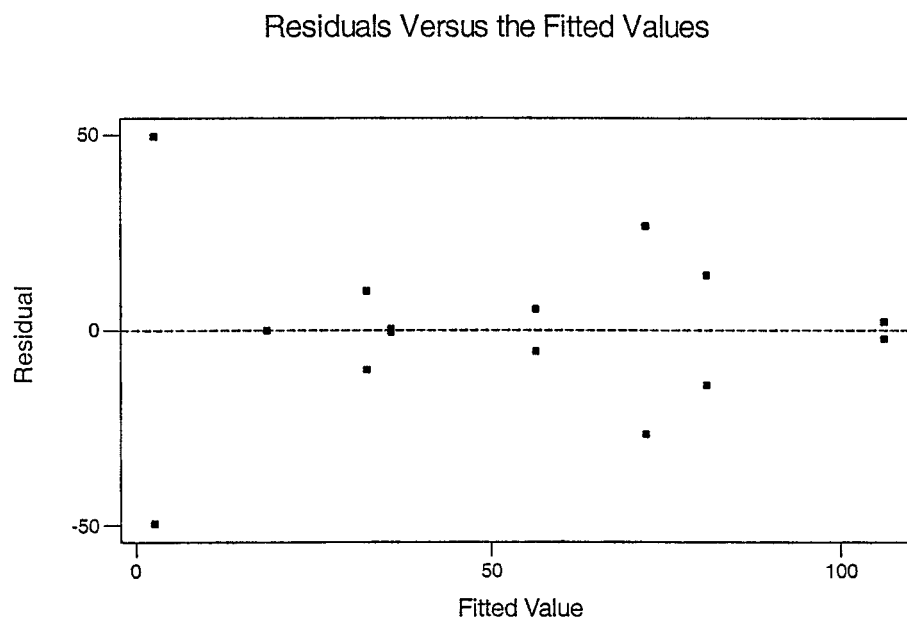
13-20. *AC* and *BDE* confounded: *ABCDE* generalized interaction

	Block 1	Block 2	Block 3	Block 4
(1)	<i>a</i>	<i>b</i>	<i>ab</i>	
	<i>ac</i>	<i>c</i>	<i>abc</i>	<i>bc</i>
	<i>bd</i>	<i>abd</i>	<i>d</i>	<i>ad</i>
	<i>abcd</i>	<i>bcd</i>	<i>acd</i>	<i>cd</i>
	<i>be</i>	<i>abe</i>	<i>e</i>	<i>ae</i>
	<i>abce</i>	<i>bce</i>	<i>ace</i>	<i>ce</i>
	<i>de</i>	<i>ade</i>	<i>bde</i>	<i>abde</i>
	<i>acde</i>	<i>cde</i>	<i>abcde</i>	<i>bcde</i>

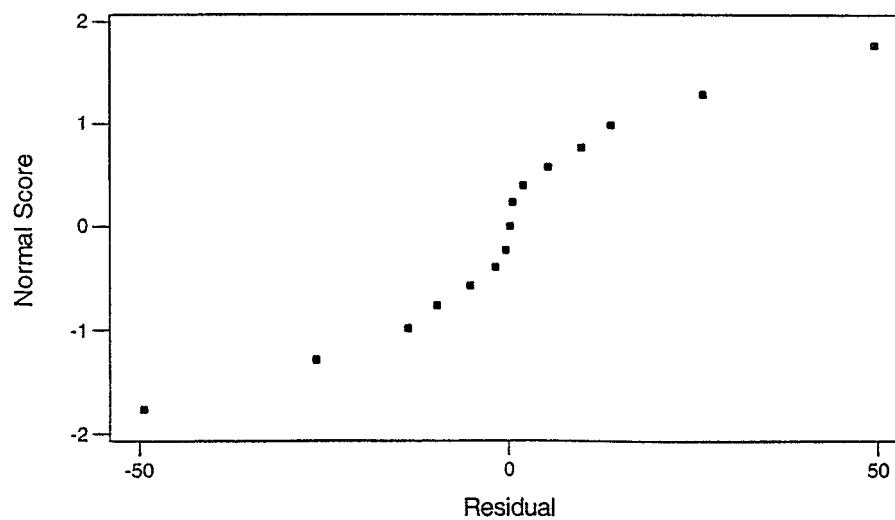
13-21. (a) Estimated Effects and Coefficients for strength

Term	Effect	Coef	SE Coef	T	P
Constant		50.50	7.377	6.85	0.000
Block		-3.50	7.377	-0.47	0.648
A	-57.00	-28.50	7.377	-3.86	0.005
B	-13.25	-6.62	7.377	-0.90	0.395
C	26.25	13.12	7.377	1.78	0.113
A*B	7.25	3.62	7.377	0.49	0.636
A*C	-2.75	-1.38	7.377	-0.19	0.857
B*C	2.50	1.25	7.377	0.17	0.870

(b)



Normal Probability Plot of the Residuals



(c) & (d) This design is not as efficient as possible. If we were to confound a different interaction in each replicate this would provide some information on all interactions.

13–22. Please refer to the original reference for an analysis of the data from this experiment.

13–23. (a) $I = ABCD$. Alias Structure:

$$\begin{array}{lll}
 \ell_A = A + BCD & \ell_{AB} = AB + CD & \ell_{CE} = CE + ABD \\
 \ell_B = B + ACD & \ell_{AC} = AC + BD & \ell_{AD} = AD + CB \\
 \ell_C = C + ABD & \ell_{AE} = AE + BCD & \ell_{BD} = BD + AC \\
 \ell_D = D + ABC & \ell_{BC} = BC + AD & \ell_{CD} = CD + AB \\
 \ell_E = E + ABCD & \ell_{BE} = BE + ACD & \ell_{DE} = DE + ABC
 \end{array}$$

(b) design: $2^{5-1}D = ABC$

$$\begin{array}{lll}
 \ell_A = 0.238 & \ell_{AB} = -0.024 & \ell_{BD} = 0.042 \\
 \ell_B = -0.16 & \ell_{AC} = 0.0042 & \ell_{CD} = -0.024 \\
 \ell_C = -0.043 & \ell_{BC} = -0.026 & \ell_{BE} = 0.1575 \\
 \ell_D = 0.0867 & \ell_{AD} = -0.026 & \ell_{CE} = -0.029 \\
 \ell_E = -0.242 & \ell_{AE} = 0.059 & \ell_{DE} = 0.036
 \end{array}$$

Conclusion: assuming 3 and 4 factor interactions insignificant, factors A & E are important; possible also B and BE .

13–24. (a) The generators used were $I = ACE$ and $I = BDE$.

(b) $I = ACE = BDE = ABCD$

13–25. (a) $D = ABC$

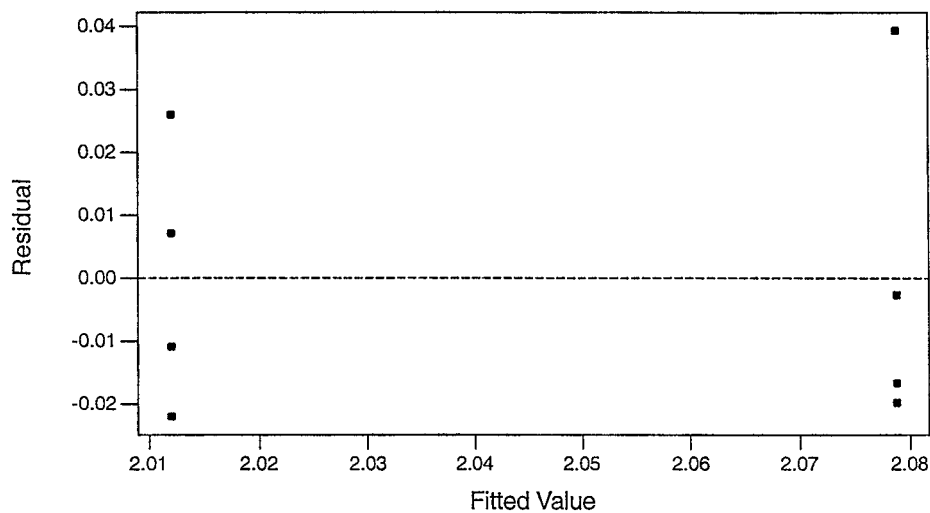
(b) Term	Effect	Coef	SE Coef	T	P
Constant		2.04538	0.007423	275.55	0.000
A	0.06675	0.03338	0.007423	4.50	0.021
B	0.02625	0.01313	0.007423	1.77	0.175
C	0.02025	0.01012	0.007423	1.36	0.266
D	-0.00375	-0.00187	0.007423	-0.25	0.817

Only factor A appears to be significant.

(c)

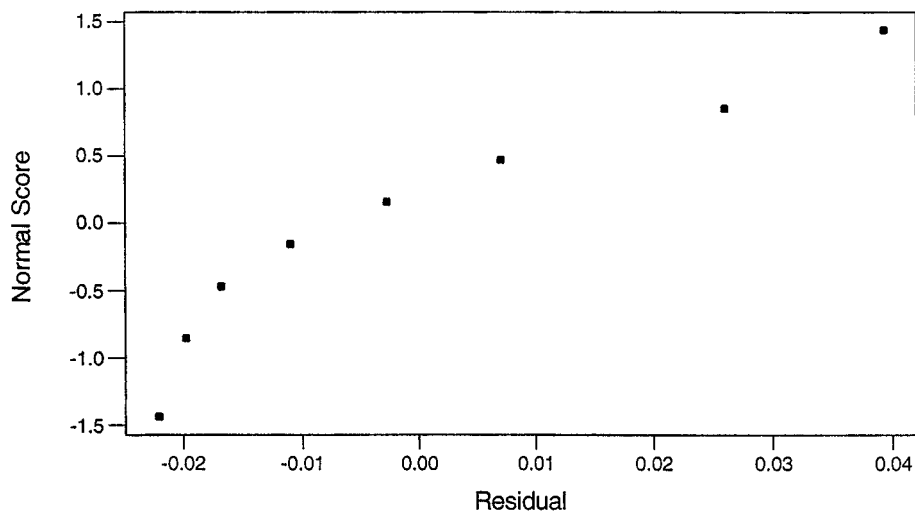
Residuals Versus the Fitted Values

(response is density)



Normal Probability Plot of the Residuals

(response is density)

13-26. 2^{3-1} with 2 replicates.

13-27. 2^{4-1} $I = ABCD$ Aliases:

$l_A = A + BCD$	$l_{AB} = AB + CD$
$l_B = B + ACD$	$l_{AC} = AC + BD$
$l_C = C + ABD$	$l_{AD} = AD + BC$
$l_D = D = ABC$	

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D = ABC</i>	
(1)	-	-	-	-	190
<i>a</i>	+	-	-	+	174
<i>b</i>	-	+	-	+	181
<i>ab</i>	+	+	-	-	183
<i>c</i>	-	-	+	+	177
<i>ac</i>	+	-	+	-	181
<i>bc</i>	-	+	+	-	188
<i>abc</i>	+	+	+	+	173

Sweetener (*A*) & Temperature (*D*)
influence taste.

$$\begin{aligned}
 A &= -6.25 & \ell_{AB} &= -0.25 \\
 B &= 0.75 & \ell_{AC} &= 0.75 \\
 C &= -2.25 & \ell_{AD} &= 0.75 \\
 D &= -9.25
 \end{aligned}$$

13–28. 2^{5-1} $I = ABCDE$

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E = ABCD</i>	Treatment Combination	Strength
-	-	-	-	+	<i>e</i>	800
+	-	-	-	-	<i>a</i>	900
-	+	-	-	-	<i>b</i>	3400
+	+	-	-	+	<i>abe</i>	6200
-	-	+	-	-	<i>c</i>	600
+	-	+	-	+	<i>ace</i>	1200
-	+	+	-	+	<i>bce</i>	3006
+	+	+	-	-	<i>abc</i>	5300
-	-	-	+	-	<i>d</i>	1000
+	-	-	+	+	<i>ade</i>	1500
-	+	-	+	+	<i>bde</i>	4000
+	+	-	+	-	<i>abd</i>	6100
-	-	+	+	+	<i>cde</i>	1500
+	-	+	+	-	<i>acd</i>	1100
-	+	+	+	-	<i>bcd</i>	3300
+	+	+	+	+	<i>abcde</i>	6300

$\ell_A = 10,994$ $\ell_{AB} = 9394$ The estimates for *A*, *B*, and *AB* are large

$$\ell_B = 29,006$$

$$\ell_C = -1594$$

$$\ell_D = 3394$$

$$\ell_E = 2806$$

13-29. 2^{5-2}

$$I = ABCD, I = ACE$$

					Treatment	
<i>A</i>	<i>B</i>	<i>C</i>	<i>D = ABC</i>	<i>E = AC</i>	Combination	Strength
-	-	-	-	+	<i>e</i>	800
+	-	-	+	-	<i>ade</i>	1500
-	+	-	+	+	<i>bde</i>	4000
+	+	-	-	-	<i>abe</i>	6200
-	-	+	+	-	<i>cde</i>	1500
+	-	+	-	+	<i>ace</i>	1200
-	+	+	-	-	<i>bce</i>	3006
+	+	+	+	+	<i>abcde</i>	6300
$\ell_A = 5894$			$\ell_C = -494$		$\ell_{AB} = 8094$	
$\ell_B = 14506$			$\ell_D = 2094$			
			$\ell_E = 94$			

13-30. 2_{III}^{6-3}

$$I = ABD = ACE = BCF = BCDE = ABEF = ACDF = DEF$$

$$\begin{aligned}
 A &= BD = CE = ABCF = ABCDE = BEF = CDF = ADEF \\
 B &= AD = CE = ABCF = CDE = AEF = ABCEF = BDEF \\
 C &= ABCD = AE = BD = BDE = ABCEF = ADF = CDEF \\
 D &= AB = ACDE = BCDF = BCE = ABDEF = ACF = EF \\
 E &= ABDE = AC = BCEF = BCD = ABF = ACDEF = DF \\
 F &= ABDF = ACEF = BC = BCDEF = ABE = ACD = DE
 \end{aligned}$$