Chapter 15

15–1. (a) $\hat{y} = 7.30 + 0.0183x_1 - 0.399x_4$

(b)

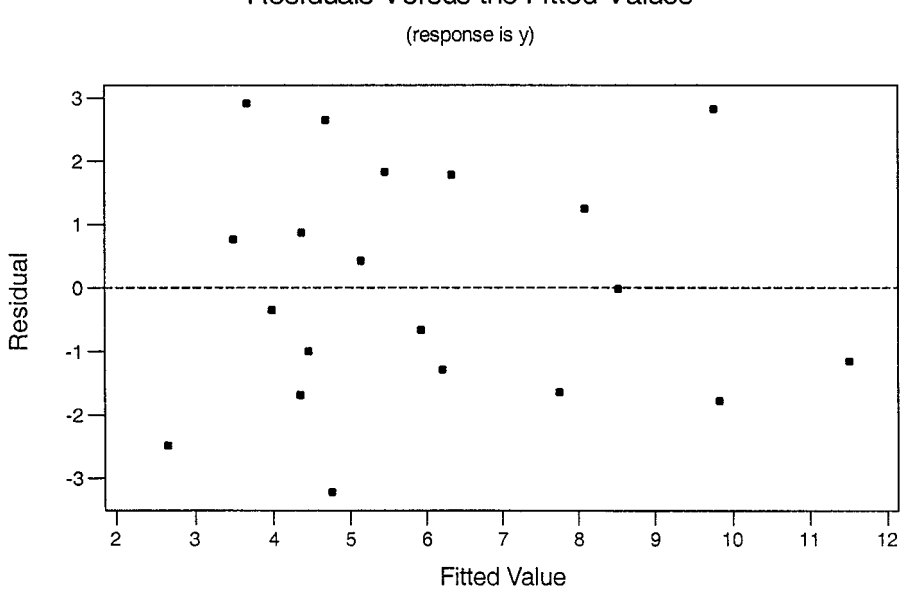
Predictor	Coef	SE Coef	T	P
Constant	7.304	5.179	1.41	0.176
x1	0.018299	0.004972	3.68	0.002
x4	-0.3986	0.1912	-2.08	0.053

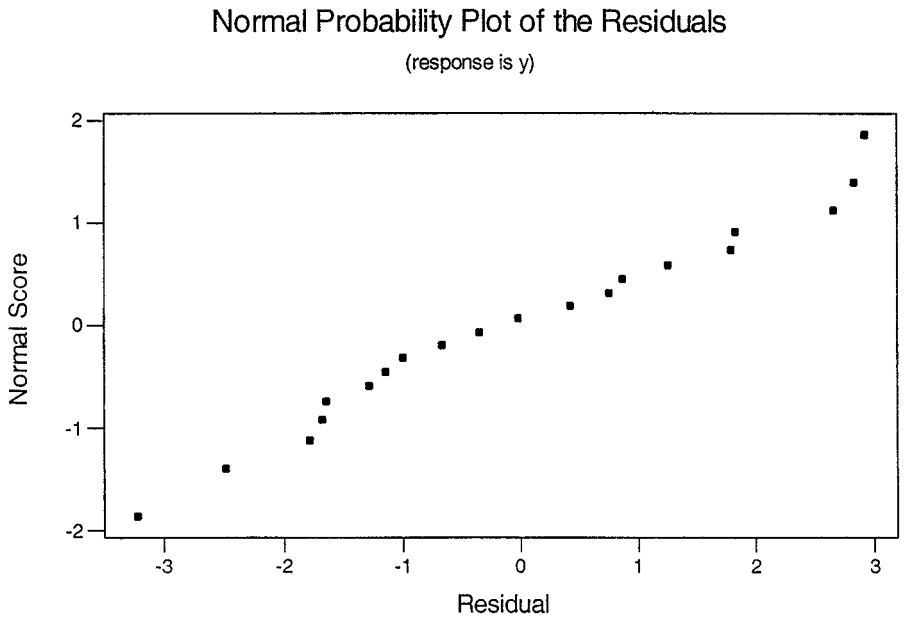
S = 1.922 R-Sq = 64.1% R-Sq(adj) = 59.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	112.263	56.131	15.19	0.000
Residual Error	17	62.824	3.696		
Total	19	175.086			

(c)





(d) The MS_E has improved with the x_1x_4 model, but the R^2 and adjusted R^2 have decreased.

15–2. (a) $\hat{y} = -27.9 + 0.0136x_1 + 30.7x_2 - 0.0670x_3$

(b)

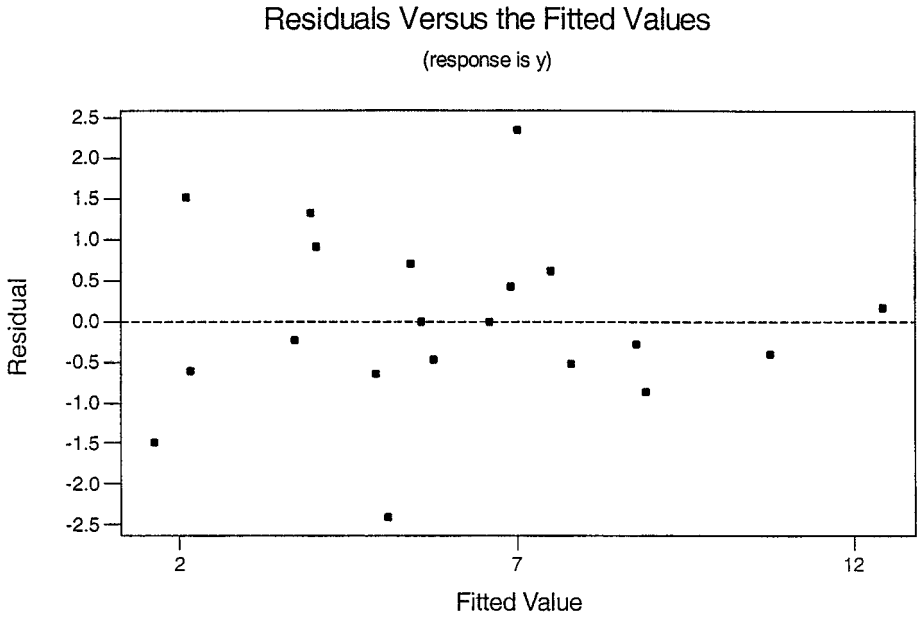
Predictor	Coef	SE Coef	T	P
Constant	-27.892	6.035	-4.62	0.000
x1	0.013597	0.003247	4.19	0.001
x2	30.685	6.513	4.71	0.000
x3	-0.06701	0.01916	-3.50	0.003

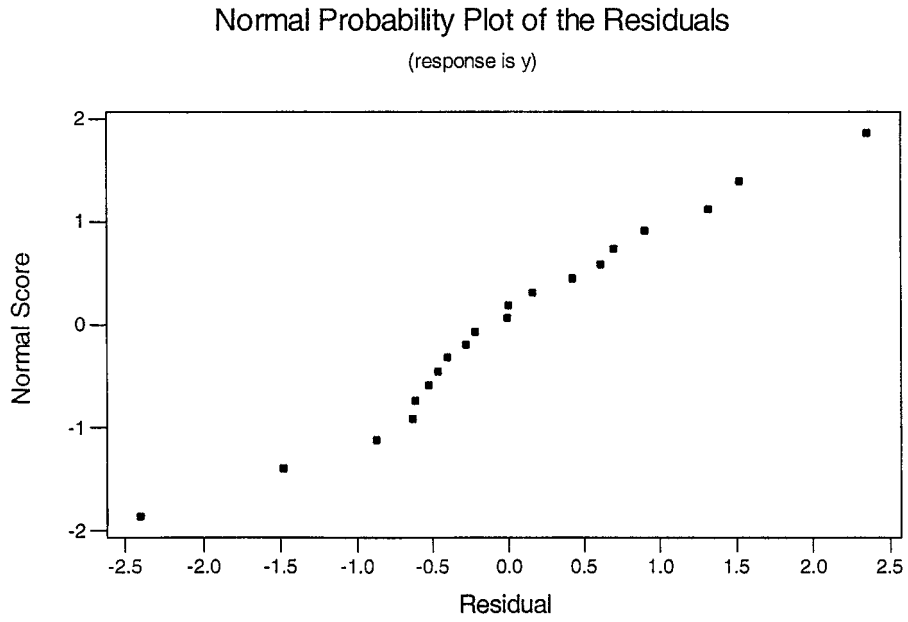
S = 1.167 R-Sq = 87.6% R-Sq(adj) = 85.2%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	153.305	51.102	37.54	0.000
Residual Error	16	21.782	1.361		
Total	19	175.086			

(c)



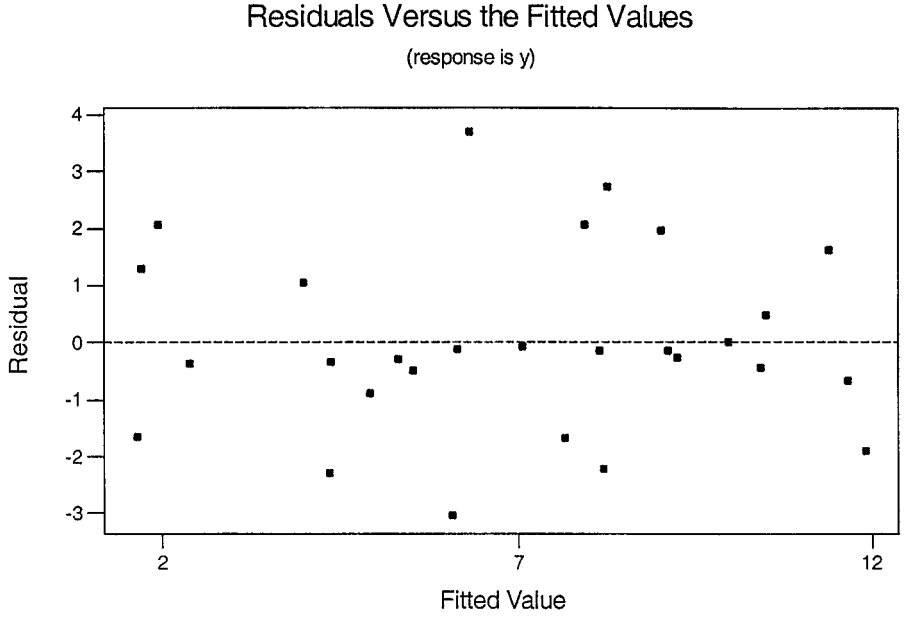


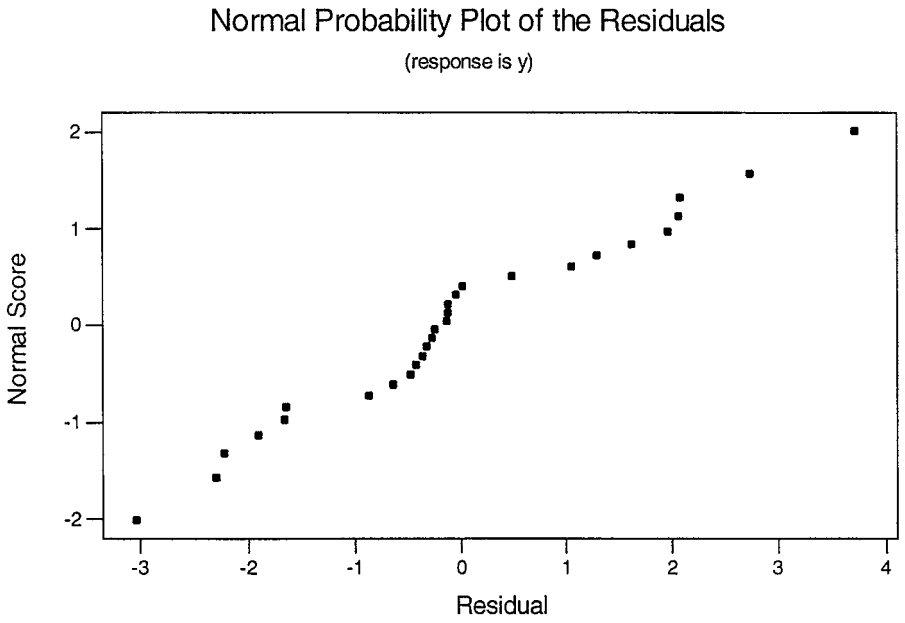
15–3. $(-0.8024, 0.0044)$

15–4. $(-0.1076, -0.0264)$

15–5. (a) $\hat{y} = -1.81 + 0.00360x_2 + 0.194x_7 - 0.00482x_8$

(b)





(c)

Predictor	Coef	SE Coef	T	P
Constant	-1.808	7.901	-0.23	0.821
x2	0.0035981	0.0006950	5.18	0.000
x7	0.19396	0.08823	2.20	0.038
x8	-0.004815	0.001277	-3.77	0.001

S = 1.706 R-Sq = 78.6% R-Sq(adj) = 76.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	257.094	85.698	29.44	0.000
Residual Error	24	69.870	2.911		
Total	27	326.964			

15–6. (a) $\hat{y} = 33.45 - 0.05435x_1 + 1.0782x_6$

(b)

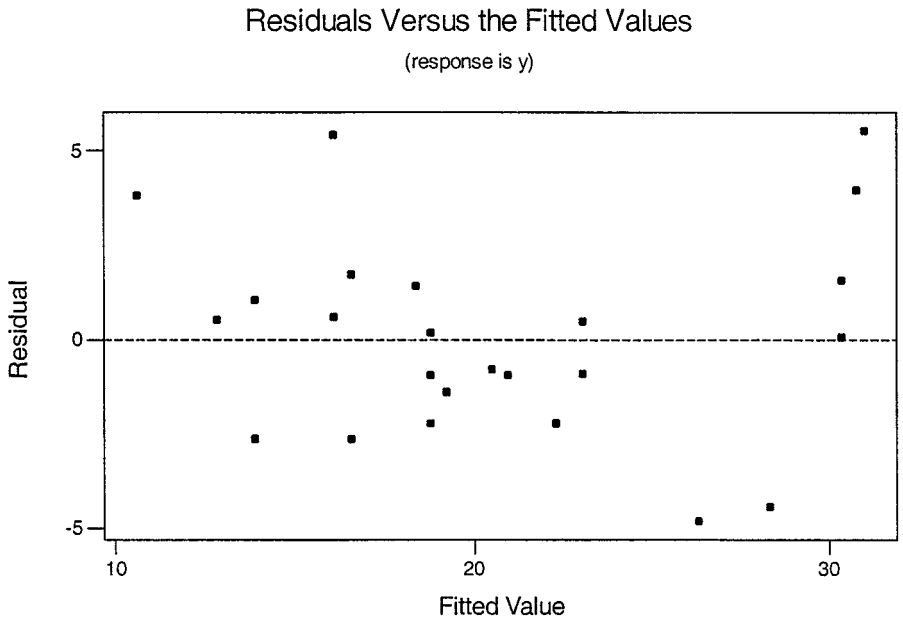
Predictor	Coef	SE Coef	T	P
Constant	33.449	1.576	21.22	0.000
x1	-0.054349	0.006329	-8.59	0.000
x6	1.0782	0.6997	1.54	0.138

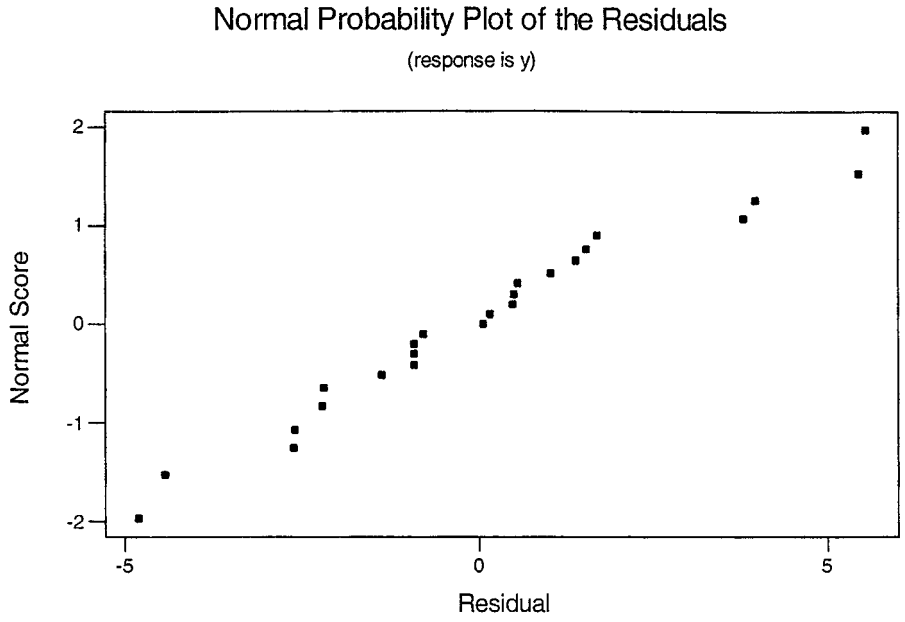
S = 2.834 R-Sq = 82.9% R-Sq(adj) = 81.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	856.24	428.12	53.32	0.000
Residual Error	22	176.66	8.03		
Total	24	1032.90			

(c)





(d) x_6 is not significant with x_1 included in the model.

15–7. (a) $\hat{y} = -103 + 0.605\,x_1 + 8.92\,x_2 + 1.44\,x_3 + 0.014\,x_4$

(b)

Predictor	Coef	SE Coef	T	P
Constant	-102.7	207.9	-0.49	0.636
x1	0.6054	0.3689	1.64	0.145
x2	8.924	5.301	1.68	0.136
x3	1.437	2.392	0.60	0.567
x4	0.0136	0.7338	0.02	0.986

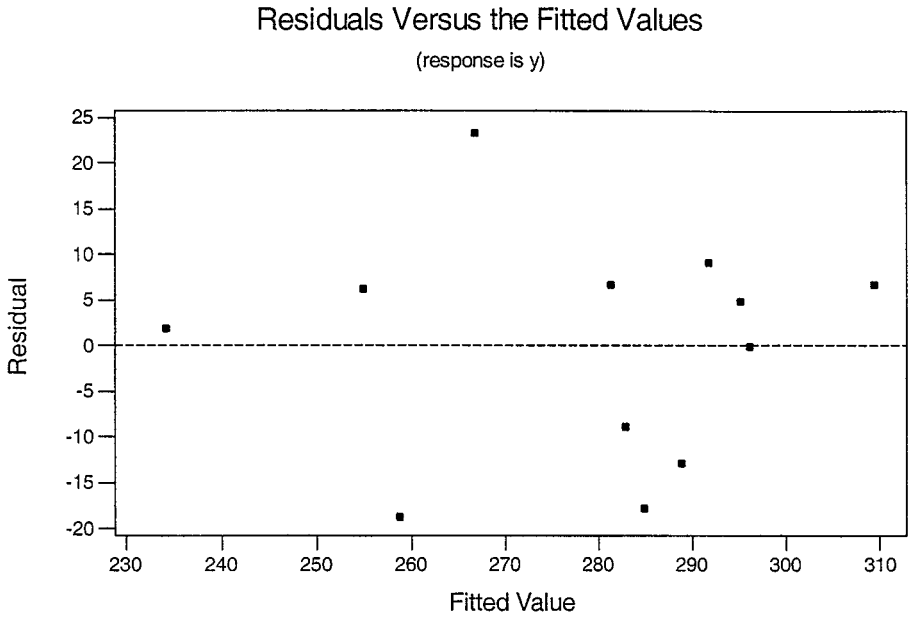
S = 15.58 R-Sq = 74.5% R-Sq(adj) = 59.9%

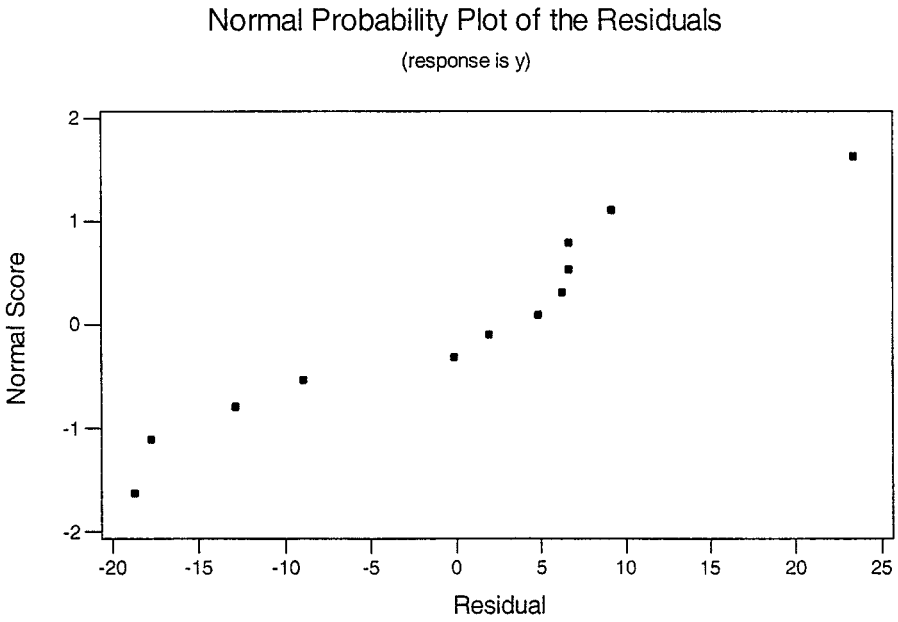
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	4	4957.2	1239.3	5.11	0.030
Residual Error	7	1699.0	242.7		
Total	11	6656.3			

(c) H_0: $\beta_3 = 0, F_0 = 0.361$ (not significant)
H_0: $\beta_4 = 0, F_0 = 0.0004$ (not significant)

(d)





15–8. (a) $y = 62.4 + 1.55\,x_1 + 0.510\,x_2 + 0.102\,x_3 - 0.144\,x_4$

(b)

Predictor	Coef	SE Coef	T	P
Constant	62.41	70.07	0.89	0.399
x1	1.5511	0.7448	2.08	0.071
x2	0.5102	0.7238	0.70	0.501
x3	0.1019	0.7547	0.14	0.896
x4	-0.1441	0.7091	-0.20	0.844

S = 2.446 R-Sq = 98.2% R-Sq(adj) = 97.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	4	2667.90	666.97	111.48	0.000
Residual Error	8	47.86	5.98		
Total	12	2715.76			

(c) H_0: $\beta_4 = 0$, $F_0 = 0.04$ (not significant)

(d) The t statistics are given in part (b)

(e) H_0: $\beta_2 = \beta_3 = \beta_4 = 0$,

$$\begin{aligned} SS_R(\beta_2, \beta_3, \beta_4 | \beta_1, \beta_0) &= SS_R(\beta_1, \beta_2, \beta_3, \beta_4 | \beta_0) - SS_R(\beta_1 | \beta_0) \\ &= 2667.90 - 1450.08 \\ &= 1217.82 \end{aligned}$$

$F_0 = (1217.82/3)/5.98 = 67.88$; at least one of the variables is significant.

(f) $(-1.1588, 2.1792)$

15–9. (a) $y = -26219 + 189x - 0.331x^2$

(b)

Predictor	Coef	SE Coef	T	P
Constant	-26219	11911	-2.20	0.079
x	189.20	80.24	2.36	0.065
x2	-0.3312	0.1350	-2.45	0.058

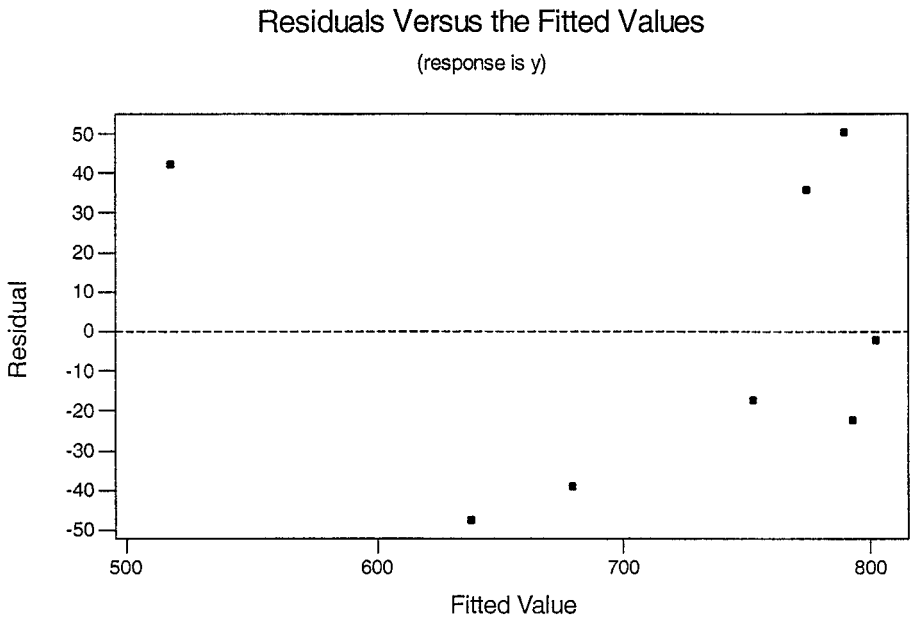
S = 45.20 R-Sq = 87.3% R-Sq(adj) = 82.2%

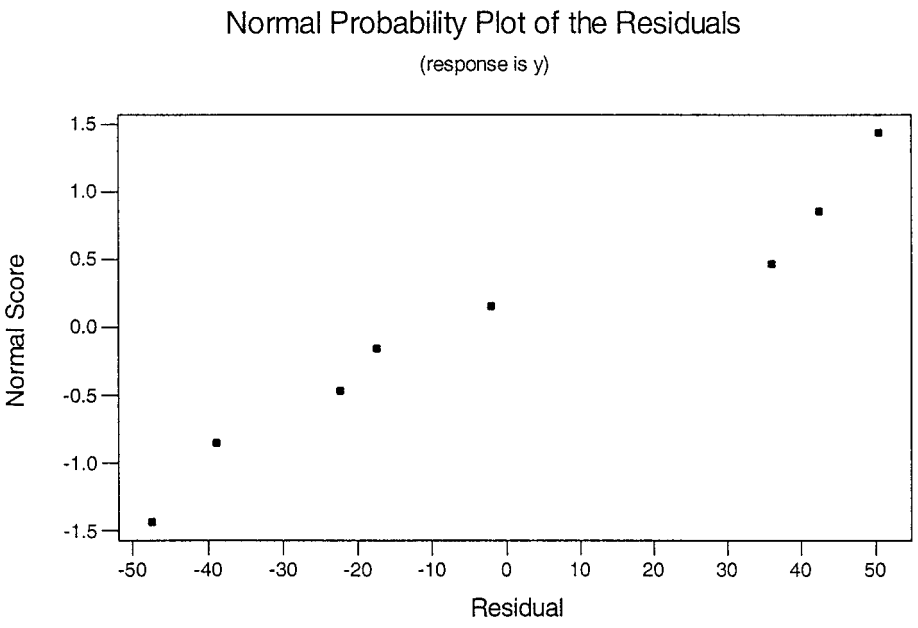
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	70284	35142	17.20	0.006
Residual Error	5	10213	2043		
Total	7	80497			

(c) See the t-test results in part (b)

(d)





15–10. (a) $y = -4.33 + 4.89x - 2.59x^2$

(b)

Predictor	Coef	SE Coef	T	P
Constant	-4.3330	0.8253	-5.25	0.001
x	4.887	1.379	3.54	0.009
x2	-2.5855	0.4886	-5.29	0.001

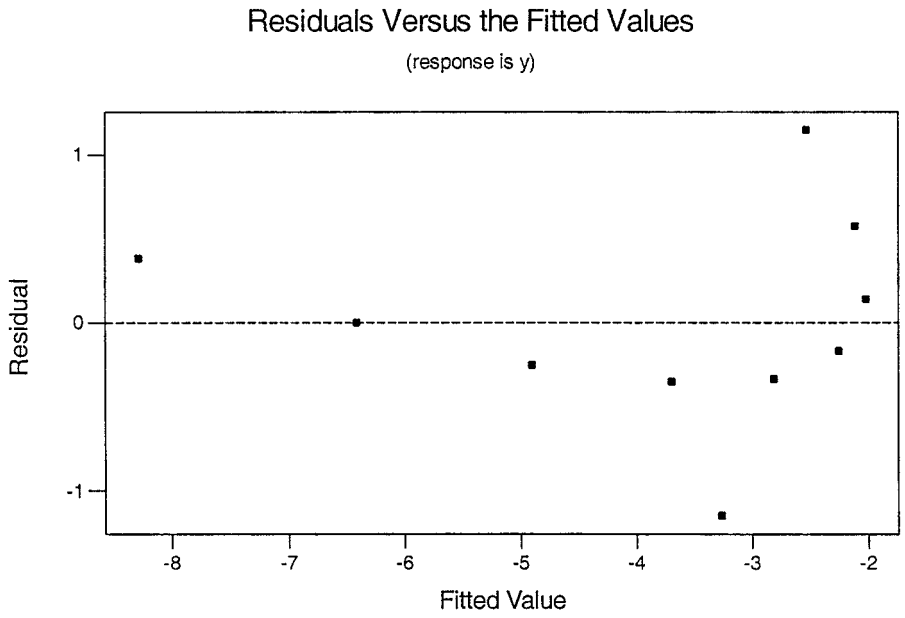
S = 0.7017 R-Sq = 91.9% R-Sq(adj) = 89.6%

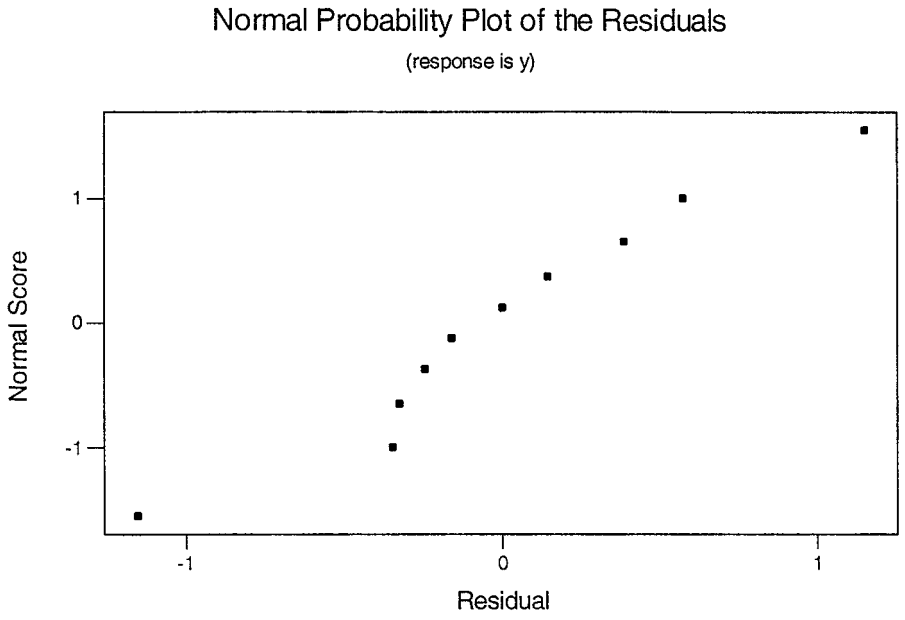
Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	39.274	19.637	39.89	0.000
Residual Error	7	3.446	0.492		
Total	9	42.720			

(c) see part (b)

(d)


Residuals Versus the Fitted Values
(response is y)
Residual
Fitted Value
1
0
-1
-8
-7
-6
-5
-4
-3
-2


Normal Probability Plot of the Residuals
(response is y)
Normal Score
Residual
1
0
-1
-1
0
1

15–11. $\hat{y} = 759.39 - 7.607x' - 0.331(x')^2$

15–13. (a) $y = -4.5 + 1.38x + 1.47x^2$

(b)

Predictor	Coef	SE Coef	T	P
Constant	-4.46	14.63	-0.30	0.768
x	1.384	5.497	0.25	0.807
x2	1.4670	0.4936	2.97	0.016

S = 1.657 R-Sq = 99.6% R-Sq(adj) = 99.5%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	5740.6	2870.3	1044.99	0.000
Residual Error	9	24.7	2.7		
Total	11	5765.3			

(c) see part (b)

15–15. The fitted model is $\hat{y} = 11.503 + 0.153x_1 - 0.094x_2 - 0.0306x_1x_2$
The t-statistic for H_0: $\beta_3 = 0$ is $t_0 = 1.79$. We conclude that the slopes are the same.

15–16. $y = \beta_0 + \beta_1 x_1 + \beta_2(x_1 - x^*)x_2 + \varepsilon$, where x_2 is an indicator variable with $x_2 = 0$ if $x_1 \leq x^*$ and $x_2 = 1$ if $x_1 > x^*$.

15–17. $y = \beta_0 + \beta_1 x_1 + \beta_2(x_1 - x^*)x_2 + \beta_3 + \varepsilon$, where x_2 and x_3 are indicator variables with $x_2 = x_3 = 0$ if $x_1 \leq x^*$ and $x_2 = x_3 = 1$ if $x_1 > x^*$. β_3 estimates the effect of the discontinuity.

15–18. The model is as in Exercise 15–16, except now X^* is unknown and must be estimated. This is a nonlinear regression problem. It could be solved by using one-dimensional or line search methods, which could be used to obtain the trial values of x^*.

15–19. $\hat{b}_1 = 0.594 \qquad \hat{b}_4 = -0.336$

15–20. $\hat{b}_1 = 0.441 \qquad \hat{b}_2 = 0.505 \qquad \hat{b}_3 = -0.315$

15–21. $VIF_1 = 1.2$, $VIF_4 = 1.2$

15–22. (a) All possible regressions from Minitab displaying the best two models for each combination of variables.

Vars	R-Sq	R-Sq(adj)	C-p	S	x 1	x 2	x 3	x 4	x 5	x 6	x 7	x 8	x 9
1	54.5	52.7	20.4	2.3929								X	
1	35.2	32.7	39.3	2.8548	X								
2	74.3	72.3	3.1	1.8324		X						X	
2	66.0	63.2	11.2	2.1097		X					X		
3	78.6	76.0	0.9	1.7062		X					X	X	
3	77.8	75.0	1.7	1.7410	X	X						X	
4	80.1	76.7	1.4	1.6812		X					X	X	X
4	79.5	75.9	2.0	1.7073	X	X						X	X
5	80.7	76.3	2.8	1.6941	X	X					X	X	X
5	80.7	76.3	2.9	1.6957		X		X			X	X	X
6	81.2	75.8	4.4	1.7118		X	X	X			X	X	X
6	81.1	75.6	4.5	1.7174	X	X		X			X	X	X
7	81.4	74.9	6.2	1.7442		X	X	X		X	X	X	X
7	81.3	74.8	6.2	1.7470	X	X		X		X	X	X	X
8	81.6	73.8	8.0	1.7814	X	X	X	X		X	X	X	X
8	81.4	73.6	8.2	1.7895		X	X	X	X	X	X	X	X
9	81.6	72.3	10.0	1.8302	X	X	X	X	X	X	X	X	X

(b) Stepwise regression from Minitab:

```
 Alpha-to-Enter: 0.15  Alpha-to-Remove: 0.15
Response is    y     on  9 predictors, with N =   28

    Step         1        2        3
Constant    21.788   14.713   -1.808
x8        -0.00703 -0.00681 -0.00482
T-Value      -5.58    -7.05    -3.77
P-Value      0.000    0.000    0.001

x2                  0.00311  0.00360
T-Value                4.40     5.18
P-Value                0.000    0.000

x7                              0.194
T-Value                          2.20
P-Value                         0.038
```

```
S             2.39     1.83     1.71
R-Sq         54.47    74.33    78.63
R-Sq(adj)    52.72    72.27    75.96
C-p           20.4      3.1      0.9
```

The stepwise procedure found variables x_2, x_7, and x_8 significant.

(c) Forward selection

```
Forward selection.  Alpha-to-Enter: 0.25
 Response is    y     on  9 predictors, with N =  28
    Step          1        2        3        4
Constant     21.788   14.713   -1.808   -1.822

x8         -0.00703 -0.00681 -0.00482 -0.00401
T-Value       -5.58    -7.05    -3.77    -2.87
P-Value       0.000    0.000    0.001    0.009

x2                   0.00311  0.00360  0.00382
T-Value                 4.40     5.18     5.42
P-Value                 0.000    0.000    0.000

x7                               0.194    0.217
T-Value                          2.20     2.45
P-Value                          0.038    0.023

x9                                       -0.0016
T-Value                                    -1.31
P-Value                                    0.202

S              2.39     1.83     1.71     1.68
R-Sq          54.47    74.33    78.63    80.12
R-Sq(adj)     52.72    72.27    75.96    76.66
C-p            20.4      3.1      0.9      1.4
```

Forward selection found variables x_2, x_7, x_8, and x_9 significant.

(d) Backward elimination

```
Backward elimination.  Alpha-to-Remove: 0.1
 Response is    y     on  9 predictors, with N =  28
    Step          1        2        3        4        5        6        7
Constant     -7.292   -7.294   -9.130   -7.695   -4.627   -1.822   -1.808

x1           0.0008   0.0008
T-Value        0.40     0.42
P-Value       0.690    0.681

x2          0.00363  0.00363  0.00363  0.00358  0.00371  0.00382  0.00360
T-Value        4.32     4.59     4.69     4.76     5.13     5.42     5.18
P-Value       0.000    0.000    0.000    0.000    0.000    0.000    0.000

x3             0.12     0.12     0.17     0.17
T-Value        0.47     0.49     0.75     0.77
P-Value       0.643    0.632    0.461    0.451

x4             0.032    0.032    0.037    0.035    0.026
T-Value         0.77     0.80     1.00     0.97     0.78
P-Value        0.453    0.431    0.329    0.342    0.445

x5             0.000
T-Value         0.00
P-Value        1.000

x6           0.0016   0.0016   0.0015
T-Value        0.49     0.51     0.48
P-Value       0.630    0.618    0.639

x7             0.154    0.154    0.189    0.193    0.235    0.217    0.194
T-Value         1.02     1.10     1.72     1.79     2.54     2.45     2.20
P-Value        0.324    0.284    0.102    0.088    0.019    0.023    0.038

x8          -0.0039  -0.0039  -0.0042  -0.0044  -0.0037  -0.0040  -0.0048
T-Value       -1.90    -1.95    -2.34    -2.50    -2.48    -2.87    -3.77
P-Value       0.074    0.066    0.030    0.021    0.021    0.009    0.001

x9          -0.0018  -0.0018  -0.0017  -0.0017  -0.0018  -0.0016
T-Value       -1.26    -1.30    -1.26    -1.28    -1.40    -1.31
P-Value       0.222    0.210    0.221    0.213    0.176    0.202

S              1.83     1.78     1.74     1.71     1.70     1.68     1.71
R-Sq          81.56    81.56    81.39    81.18    80.65    80.12    78.63
R-Sq(adj)     72.34    73.80    74.88    75.80    76.25    76.66    75.96
C-p            10.0      8.0      6.2      4.4      2.9      1.4      0.9
```

Backward elimination found variables x_2, x_7, and x_8 significant.

15–23. (a) All possible regressions from Minitab displaying the best two models for each combination of variables.

```
24 cases used; 1 case contains missing values.
```

Vars	R-Sq	R-Sq(adj)	C-p	S	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
1	80.9	80.1	5.6	2.9679	X										
1	77.1	76.1	10.7	3.2514		X									
2	82.6	81.0	5.2	2.8974	X					X					
2	82.5	80.9	5.4	2.9082	X			X							
3	84.3	82.0	5.0	2.8231					X			X		X	
3	84.0	81.5	5.5	2.8552	X		X			X					
4	85.0	81.9	6.0	2.8283			X		X			X		X	
4	84.9	81.7	6.2	2.8411	X	X	X			X					
5	86.7	83.0	5.8	2.7400	X		X		X			X		X	
5	85.8	81.8	7.1	2.8347	X		X	X	X		X				
6	88.6	84.5	5.3	2.6141	X		X		X		X	X		X	
6	87.6	83.2	6.6	2.7244	X		X		X			X		X	X
7	89.6	85.1	5.9	2.5649	X		X	X	X		X	X		X	
7	89.0	84.1	6.8	2.6465	X		X		X		X	X		X	X
8	90.2	84.9	7.2	2.5786	X		X	X	X		X	X		X	X
8	90.0	84.6	7.4	2.6068	X	X	X		X	X	X	X		X	
9	90.5	84.4	8.8	2.6285	X	X	X		X	X	X	X		X	X

(b) Stepwise regression

```
 Alpha-to-Enter: 0.15   Alpha-to-Remove: 0.15
 Response is    y     on 11 predictors, with N =   24
 N(cases with missing observations) =   1 N(all cases) =   25
    Step          1
Constant      34.43

x1          -0.0482
T-Value       -9.66
P-Value       0.000

S              2.97
R-Sq          80.93
R-Sq(adj)     80.06
C-p             5.6
```

The stepwise procedure found x_1 significant.

(c) Forward selection

```
Forward selection.  Alpha-to-Enter: 0.25
 Response is    y     on 11 predictors, with N =   24
 N(cases with missing observations) =   1 N(all cases) =   25
    Step          1        2        3
Constant      34.43    33.50    32.36

x1          -0.0482  -0.0544  -0.1034
T-Value       -9.66    -8.40    -2.65
P-Value       0.000    0.000    0.015

x6                      1.05     1.02
T-Value                 1.44     1.43
P-Value                0.164    0.169

x3                              0.070
T-Value                          1.28
P-Value                         0.217

S              2.97     2.90     2.86
R-Sq          80.93    82.65    83.95
R-Sq(adj)     80.06    80.99    81.55
C-p             5.6      5.2      5.5
```

The forward selection procedure found x_1, x_3, and x_6 significant.

(d) Backward elimination

```
Backward elimination.  Alpha-to-Remove: 0.1
 Response is    y     on 11 predictors, with N =   24
 N(cases with missing observations) =   1 N(all cases) =   25

    Step          1        2        3        4        5        6        7        8        9
Constant   -17.6442 -18.5202  -3.7497  -0.9652  -0.6957  -4.4555  -1.9112  -8.9148   0.3409

x1           -0.142   -0.142   -0.139   -0.127   -0.102   -0.089   -0.061
T-Value       -2.57    -2.68    -2.66    -2.54    -2.18    -2.11    -1.50
P-Value       0.024    0.019    0.019    0.023    0.044    0.050    0.152

x2           -0.076   -0.075   -0.096   -0.092
T-Value       -0.93    -0.97    -1.30    -1.26
P-Value       0.369    0.348    0.215    0.227

x3            0.231    0.230    0.240    0.224    0.140    0.146    0.099    0.035
T-Value        2.34     2.44     2.58     2.48     2.26     2.44     1.79     0.96
P-Value       0.037    0.030    0.022    0.026    0.038    0.026    0.090    0.348

x4              2.4      2.4
T-Value        0.87     0.90
P-Value       0.402    0.383

x5              6.8      6.7      6.8      6.4      6.1      6.6      3.0      3.6      2.9
T-Value        2.06     2.34     2.40     2.31     2.15     2.47     1.83     2.23     2.02
P-Value       0.062    0.036    0.031    0.036    0.047    0.024    0.083    0.038    0.057

x6             1.11     1.14     1.42     1.37     0.66
T-Value        0.88     0.99     1.30     1.26     0.70
P-Value       0.398    0.338    0.215    0.226    0.494

x7             -4.2     -4.1     -3.4     -3.8     -3.9     -3.6
T-Value       -1.47    -1.67    -1.46    -1.72    -1.74    -1.67
P-Value       0.167    0.118    0.165    0.106    0.100    0.114

x8             0.28     0.28     0.30     0.29     0.28     0.31     0.26    0.324    0.246
T-Value        2.11     2.20     2.40     2.31     2.26     2.54     2.12     2.72     2.81
P-Value       0.056    0.046    0.031    0.035    0.038    0.021    0.048    0.014    0.011

x9           -0.02
T-Value      -0.05
P-Value      0.959

x10        -0.0119  -0.0121  -0.0126  -0.0121  -0.0115  -0.0133   0.0113  -0.0142  -0.0099
T-Value      -1.82    -2.09    -2.21    -2.15    -2.01    -2.65    -2.21    -2.90    -5.00
P-Value      0.093    0.057    0.044    0.049    0.061    0.017    0.040    0.009    0.000

x11            -2.4     -2.5     -2.3
T-Value       -0.89    -0.93    -0.87
P-Value       0.393    0.370    0.400

S              2.75     2.65     2.63     2.61     2.65     2.61     2.74     2.83     2.82
R-Sq          91.04    91.04    90.48    89.97    88.90    88.57    86.70    85.04    84.31
R-Sq(adj)     82.83    84.15    84.36    84.62    84.05    84.53    83.00    81.89    81.96
C-p            12.0     10.0      8.8      7.4      6.9      5.3      5.8      6.0      5.0
```

Backward elimination found x_5, x_8, and x_{10} significant.

15–24. (a) All possible regressions from Minitab displaying the best two models for each combination of variables.

Vars	R-Sq	R-Sq(adj)	C-p	S	x1	x2	x3	x4
1	67.5	64.5	138.7	8.9639				X
1	66.6	63.6	142.5	9.0771		X		
2	97.9	97.4	2.7	2.4063	X	X		
2	97.2	96.7	5.5	2.7343	X			X
3	98.2	97.6	3.0	2.3087	X	X		X
3	98.2	97.6	3.0	2.3121	X	X	X	
4	98.2	97.4	5.0	2.4460	X	X	X	X

(b) Stepwise regression

Alpha-to-Enter: 0.15 Alpha-to-Remove: 0.15
Response is y on 4 predictors, with N = 13

Step	1	2	3	4
Constant	117.57	103.10	71.65	52.58
x4	-0.738	-0.614	-0.237	
T-Value	-4.77	-12.62	-1.37	
P-Value	0.001	0.000	0.205	
x1		1.44	1.45	1.47
T-Value		10.40	12.41	12.10
P-Value		0.000	0.000	0.000
x2			0.416	0.662
T-Value			2.24	14.44
P-Value			0.052	0.000
S	8.96	2.73	2.31	2.41
R-Sq	67.45	97.25	98.23	97.87
R-Sq(adj)	64.50	96.70	97.64	97.44
C-p	138.7	5.5	3.0	2.7

Stepwise procedure found x_1 and x_2 significant.

(c) Forward selection

```
Forward selection.   Alpha-to-Enter: 0.25
 Response is    y      on  4 predictors, with N =   13

    Step           1        2        3
Constant      117.57   103.10    71.65

x4            -0.738   -0.614   -0.237
T-Value        -4.77   -12.62    -1.37
P-Value        0.001    0.000    0.205

x1                      1.44     1.45
T-Value                10.40    12.41
P-Value                0.000    0.000

x2                              0.42
T-Value                         2.24
P-Value                        0.052

S              8.96     2.73     2.31
R-Sq          67.45    97.25    98.23
R-Sq(adj)     64.50    96.70    97.64
C-p           138.7      5.5      3.0
```

Forward selection found x_1, x_2, and x_4 significant.

(d) Backward elimination

```
Backward elimination.   Alpha-to-Remove: 0.1
 Response is    y      on  4 predictors, with N =   13
    Step          1         2         3
Constant      62.41     71.65     52.58

x1             1.55      1.45      1.47
T-Value        2.08     12.41     12.10
P-Value       0.071     0.000     0.000

x2            0.510     0.416     0.662
T-Value        0.70      2.24     14.44
P-Value       0.501     0.052     0.000

x3             0.10
T-Value        0.14
P-Value       0.896

x4            -0.14     -0.24
T-Value       -0.20     -1.37
P-Value       0.844     0.205

S              2.45      2.31      2.41
R-Sq          98.24     98.23     97.87
R-Sq(adj)     97.36     97.64     97.44
C-p             5.0       3.0       2.7
```

Backward elimination found x_1 and x_2 significant.

15–25. $VIF_1 = 38.5$, $VIF_2 = 254.4$, $VIF_3 = 46.9$, $VIF_4 = 282.5$. The variance inflation factors indicate a problem with multicollinearity.