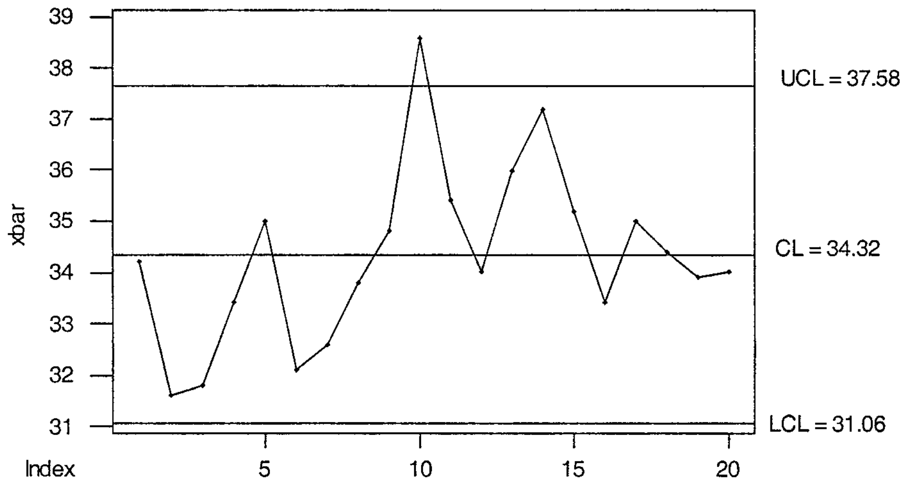
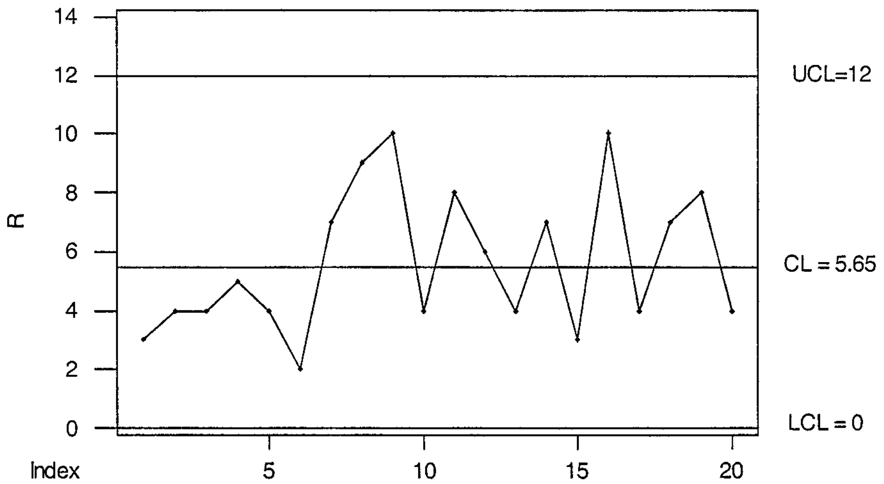
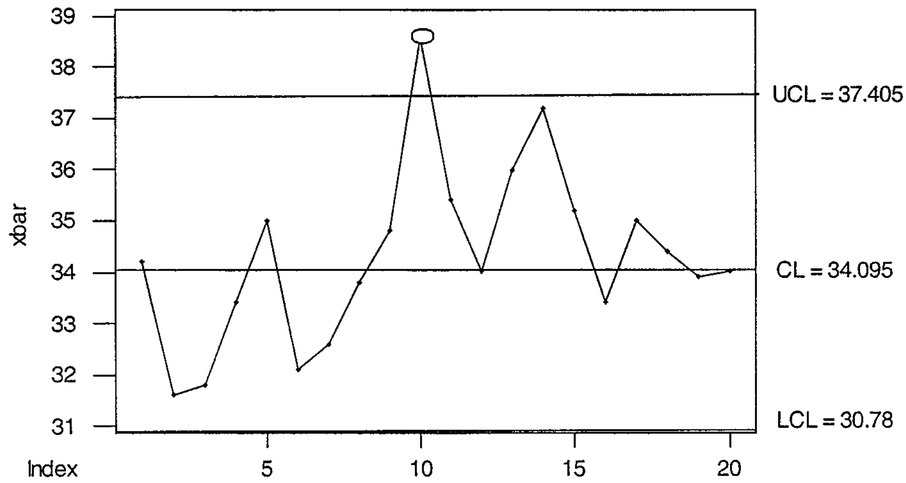
Chapter 17

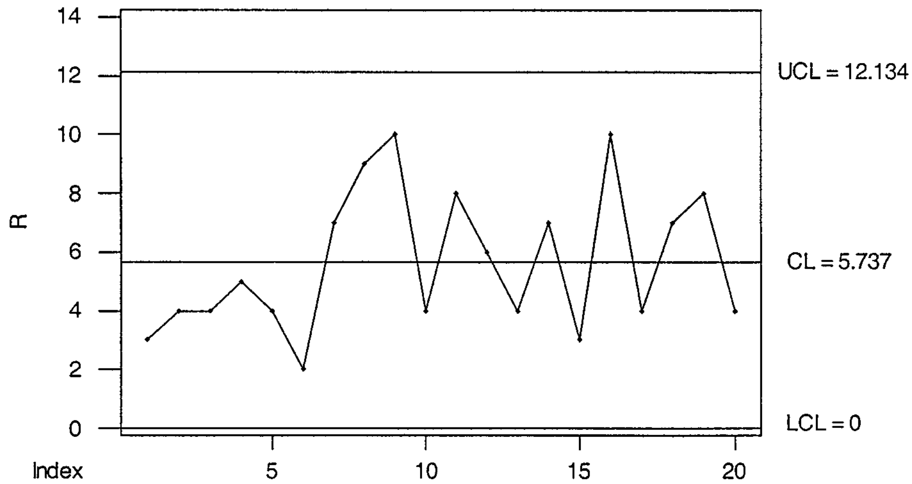
17–1. (a) X-bar Chart: $UCL = 34.32 + 0.577(5.65) = 37.58$
$LCL = 34.32 - 0.577(5.65) = 31.06$
R chart: $UCL = 2.115(5.65) = 12$
$LCL = 0$





There is one observation beyond the upper control limit. Removal of this point results in the following control charts:





The process now appears to be in control.

(b) $\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{5.74}{2.326} = 2.468, PCR = \frac{USL - LSL}{6\sigma} = \frac{20}{6(2.468)} = 1.35$

$$PCR_K = \min\left[\frac{45 - 34.09}{3(2.468)}, \frac{34.09 - 25}{3(2.468)}\right] = \min[1.474, 1.2277] = 1.2277$$

(c) 0.205%

17–2. $P(\overline{X} < \mu + 3\sigma/\sqrt{n} | \mu_{\overline{X}} = \mu + 1.5\sigma)$

$$= P(Z < \frac{\mu + 3\sigma/\sqrt{n} - (\mu + 1.5\sigma)}{\sigma/\sqrt{n}}$$

$= P(Z < 3 - 1.5\sqrt{n})$
$=$ probability of failing to detect shift on 1st sample following the shift.

$[P(Z < 3 - 1.5\sqrt{n})]^3 =$ prob of failing to detect shift for 3 consecutive samples following the shift.

For $n = 4$, $[P(Z < 0)]^3 = (0.5)^3 = 0.125$

For $n = 4$ with 2-sigma limits,

$$[P(Z < 2 - 3)]^3 = [P(Z < -1)]^3 \;=\; (0.1587)^3 \;=\; 0.003997$$

17–3. (a) $ARL = 1/\alpha$

(b) $ARL = 1/(1 - \beta)$

(c) If k changes from 3 to 2, the in-control ARL will get much shorter (from about 370 to 20). This is not desirable.

(d) For a 1-sigma shift, $\beta \simeq 0.8$, so the ARL is approximately $ARL = 1/(1 - 0.8) = 5$.

17–4. $\overline{\overline{X}} = \dfrac{362.75}{25} = 14.51$, $\overline{R} = \dfrac{8.60}{25} = 0.34$

(a) $\overline{X}$ Chart: $UCL = 14.706$, $CL = 14.31$, $LCL = 14.314$

$\overline{R}$ Chart: $UCL = 0.719$, $CL = 0.34$, $LCL = 0$

(b) $\hat{\sigma} = \overline{R}/d_2 = 0.34/2.326 = 0.14617$, $\hat{\mu} = 14.51$

6σ natural tolerance limits $= 14.51 \pm 3(0.14617) \;=\; 14.51 \pm 0.4385$

$P(X > 14.90) = 0.00379$, $P(X < 14.10) = 0.00252$

Fraction defective $= 0.00631$

(c) $PCR = \dfrac{15 - 14}{6(0.146)} = 1.141 \qquad PCR_K = \min[1.119, 1.164] = 1.119$

17–5. $D/2$

17–6. (a) $\overline{\overline{X}} = \dfrac{214.25}{20} = 10.7125 \qquad \overline{R} = \dfrac{133}{20} = 6.65$

$\overline{X}$ Chart: $UCL = 10.7125 + 0.729(6.65) = 15.56$

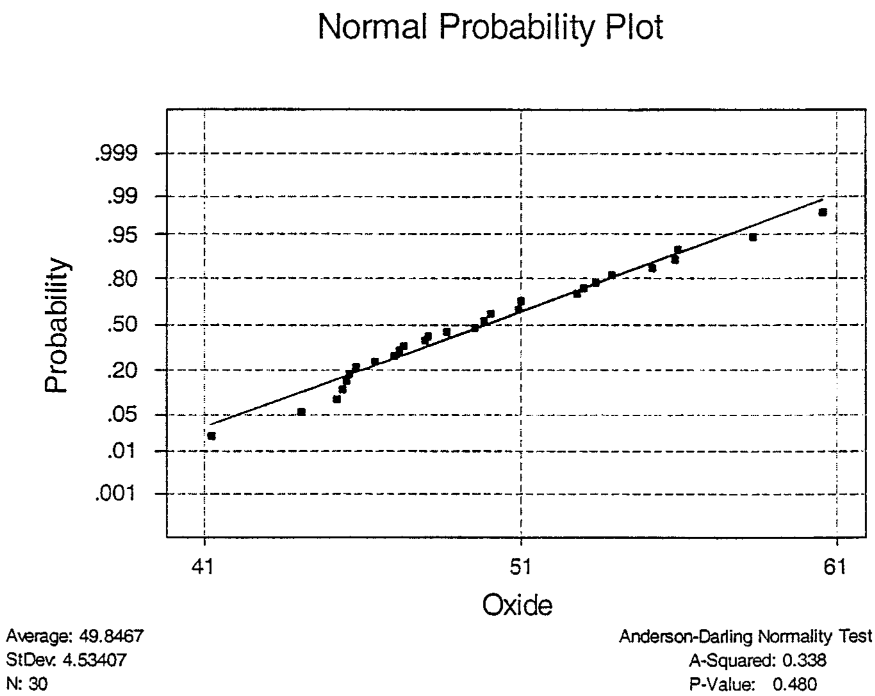
$LCL = 5.86$

R Chart: $UCL = 2.282(6.65) = 15.175$, $LCL = 0$; process in control

(b) $\hat{\sigma} = \overline{R}d_2 = 6.65/2.059 = 3.23$

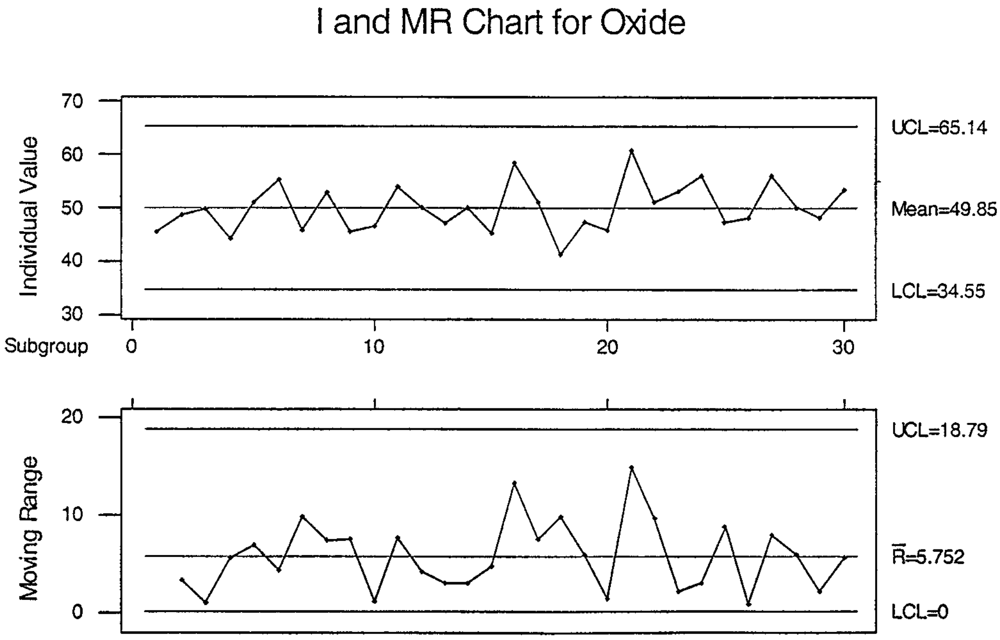
$$PCR = \frac{15 - 5}{6(3.23)} = 0.516$$

17–7. (a) Normal probability plot



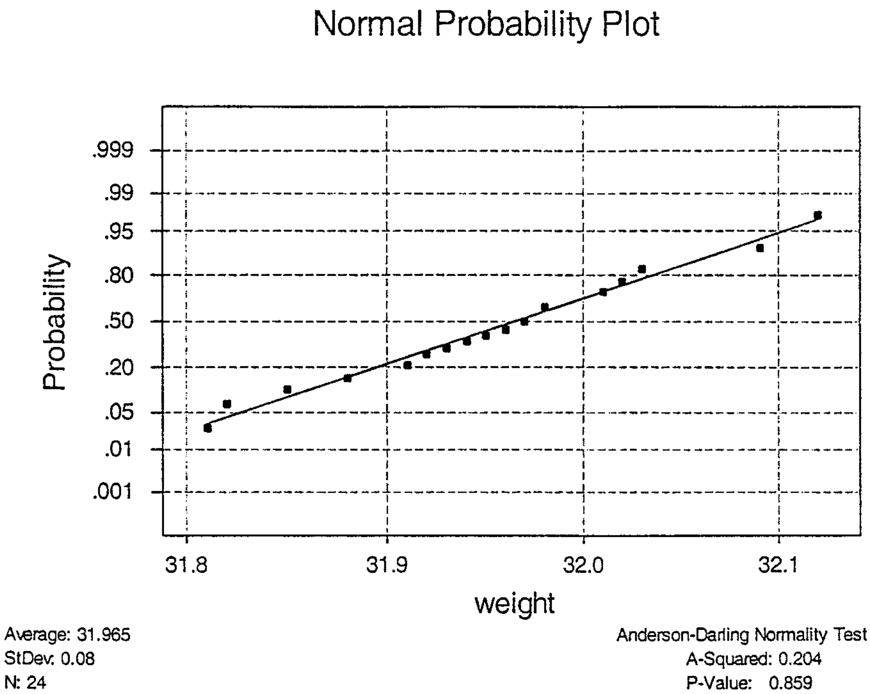
The normality assumption appears to be satisfied.

(b)



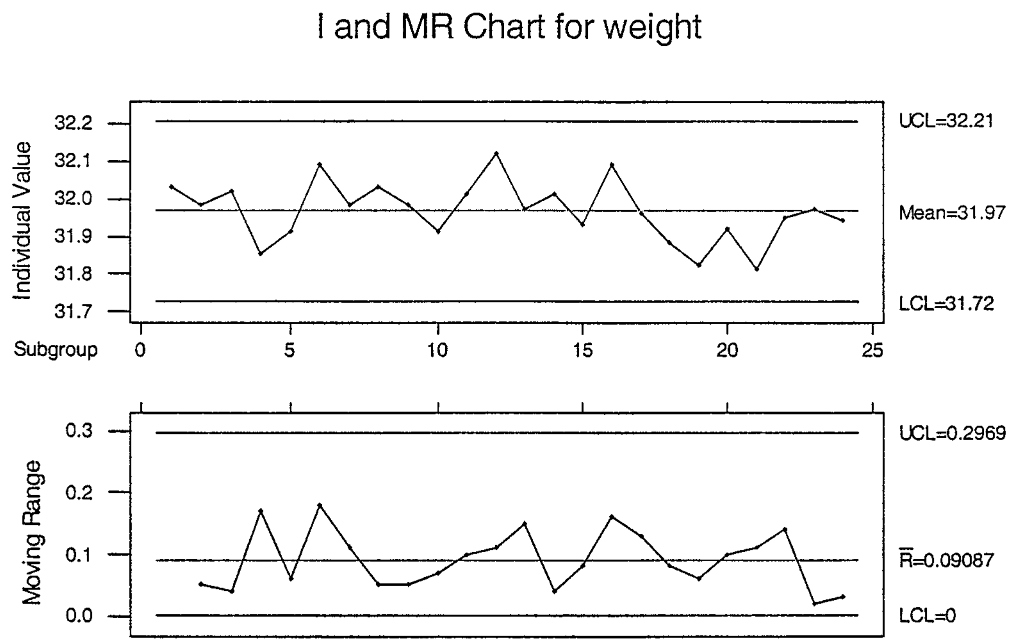
The process appears to be in control.

17–8. (a) Normal probability plot



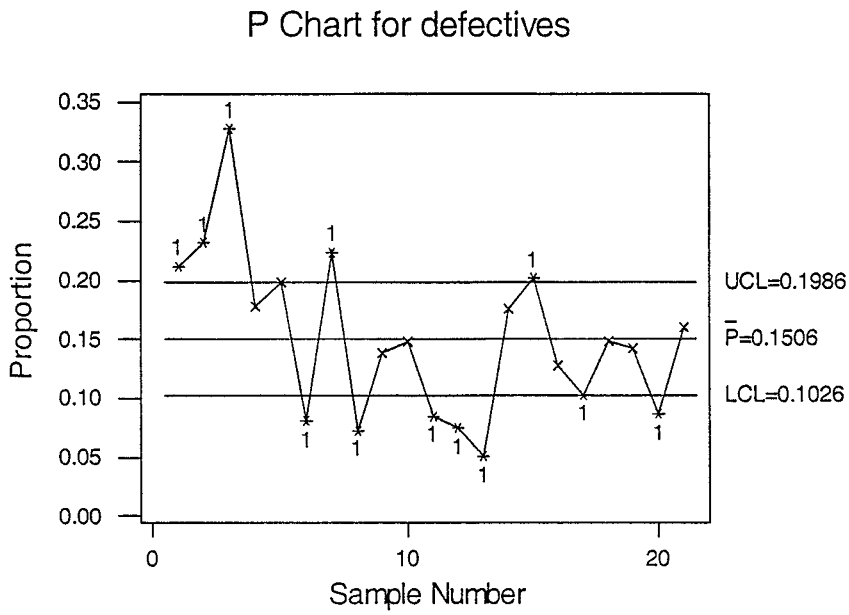
The normality assumption appears to be satisfied.

(b)



The process appears to be in control.

17–9.



The process is out of control.

17–10. $\overline{p} = 0.05 \qquad UCL = 0.05 + 3(0.0218) = 0.115$

$$P(\overline{X} < 0.115 | \mu = 0.08) = 1 - P\left(Z < \frac{0.115 - 0.08}{0.0654}\right) = 1 - P(Z < 0.535)$$

$$= 1 - 0.7036 = 0.2964 \quad \text{Probability of detecting shift on first sample following shift}$$

$$P(\text{detecting before 3rd sample}) = 1 - (0.7036)^2 = 0.5049$$

17–11. For the detection probability to equal 0.5, the magnitude of the shift must bring the fraction nonconforming exactly to the upper control limit. That is, $\delta = k\sqrt{p(1-p)/n}$, where δ is the magnitude of the shift. Solving for n gives $n = (k/\delta)^2 p(1-p)$. For example, if $k = 3$, $p = 0.01$ (the in-control fraction nonconforming), and $\delta = 0.04$, then $n = (3/0.04)^2(0.01)(0.99) = 56$.

17–12. (a) $PCR = 1.5$

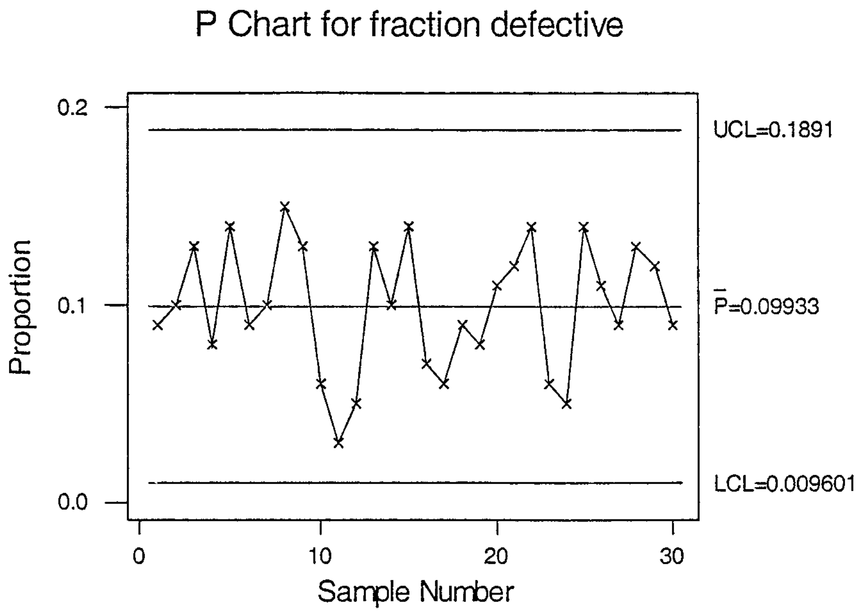
(b) About 7 defective parts per million.

(c) $PCR_k = 1$ PCR unchanged.

(d) About 0.135 percent defective.

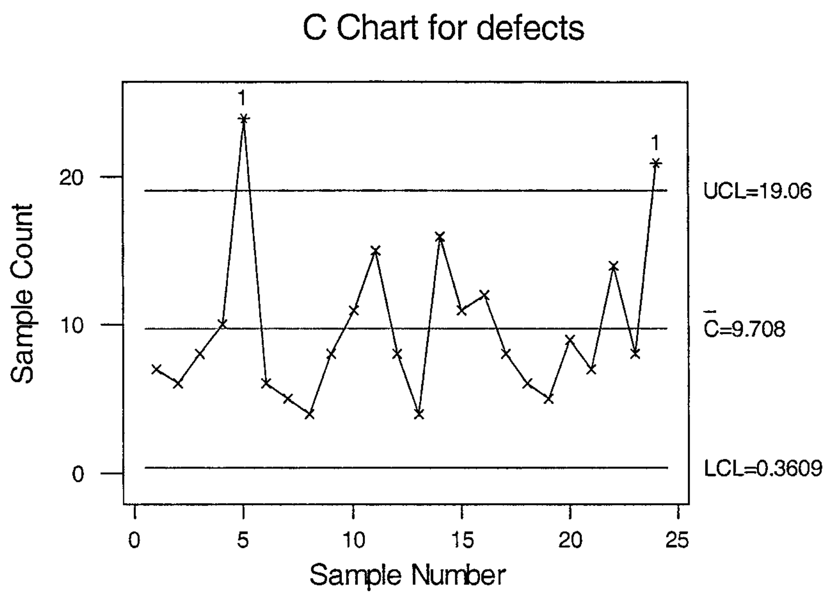
17–13. Center the process at $\mu = 100$. The probability that a shift to $\mu = 105$ will be detected on the first sample following the shift is about 0.15. A p-chart with $n = 7$ would perform about as well.

17–14.

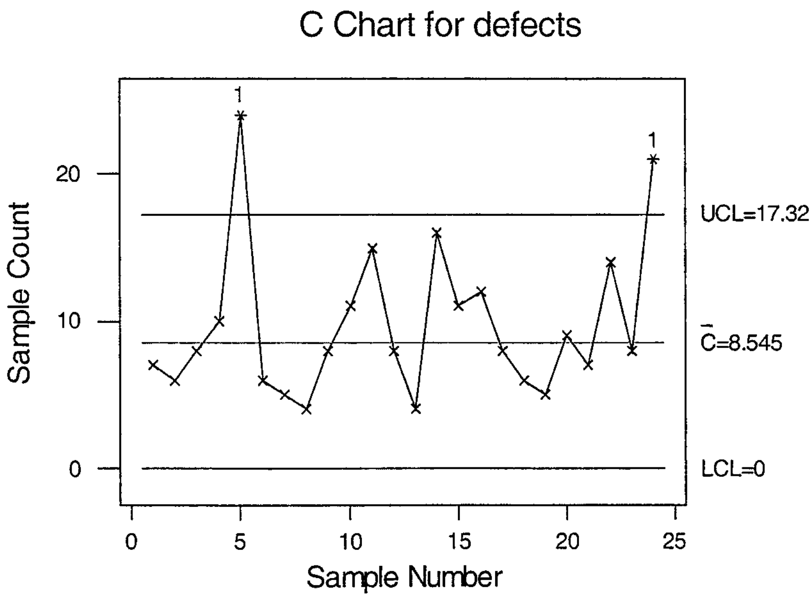


The process appears to be in control.

17–15.

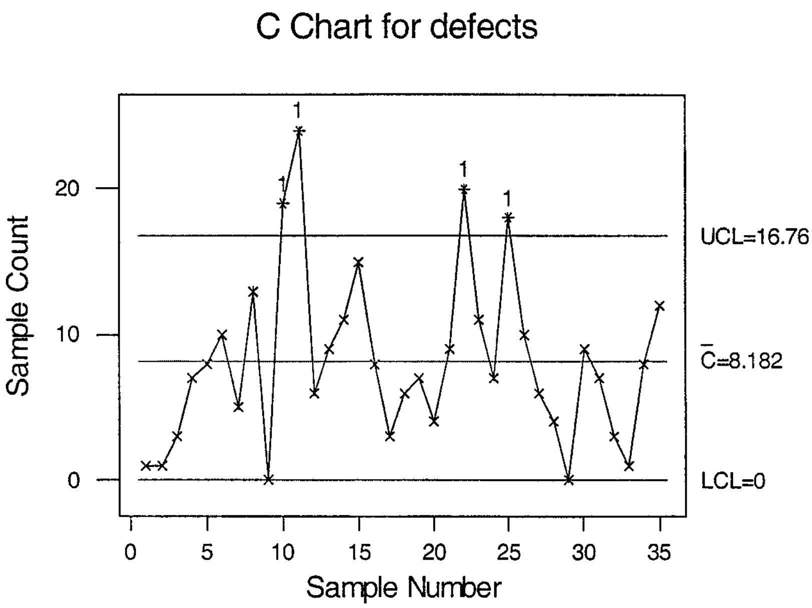


The process is out of control. Removing two out-of-control points and revising the limits results in:



The process is now in control.

17–16.

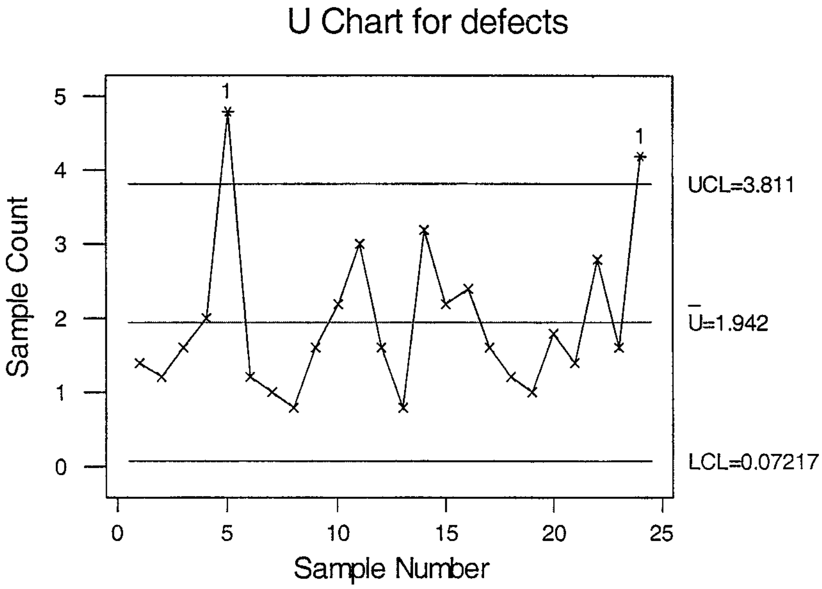


The process appears to be out of control.

17–17. $UCL = 16.485$; detection probability $= 0.434$

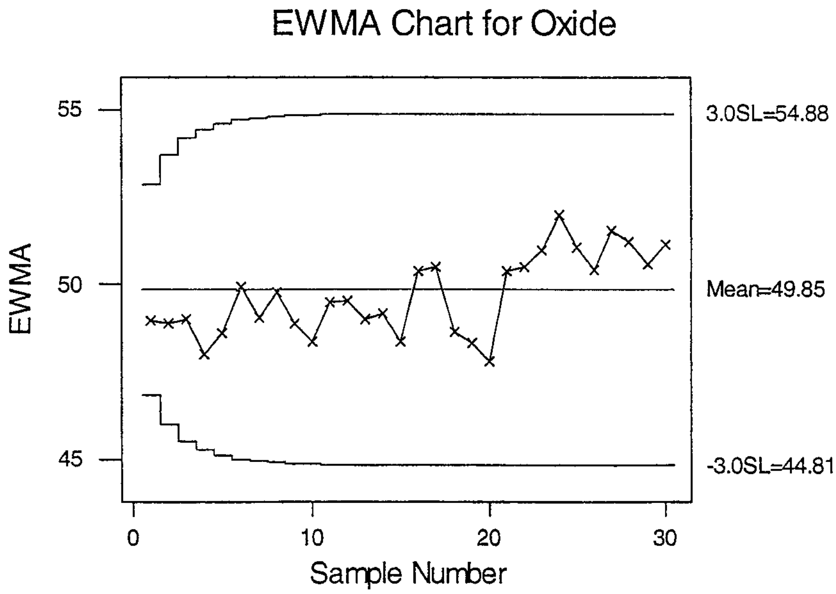
17–18. $UCL = 19.487$, $CL = 10$, $LCL = 0.513$

17–19.

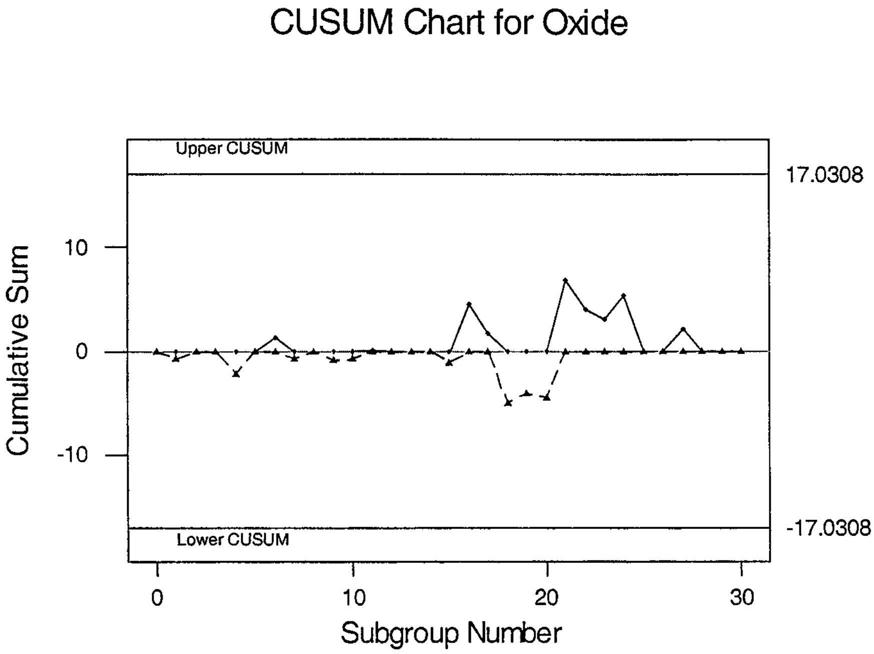


Since the sample sizes are equal, the c and u charts are equivalent.

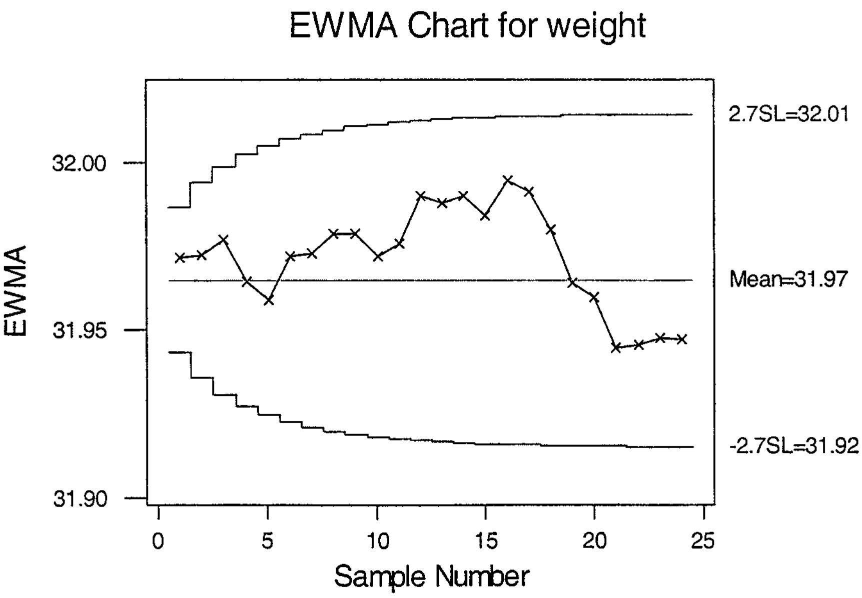
17–20.



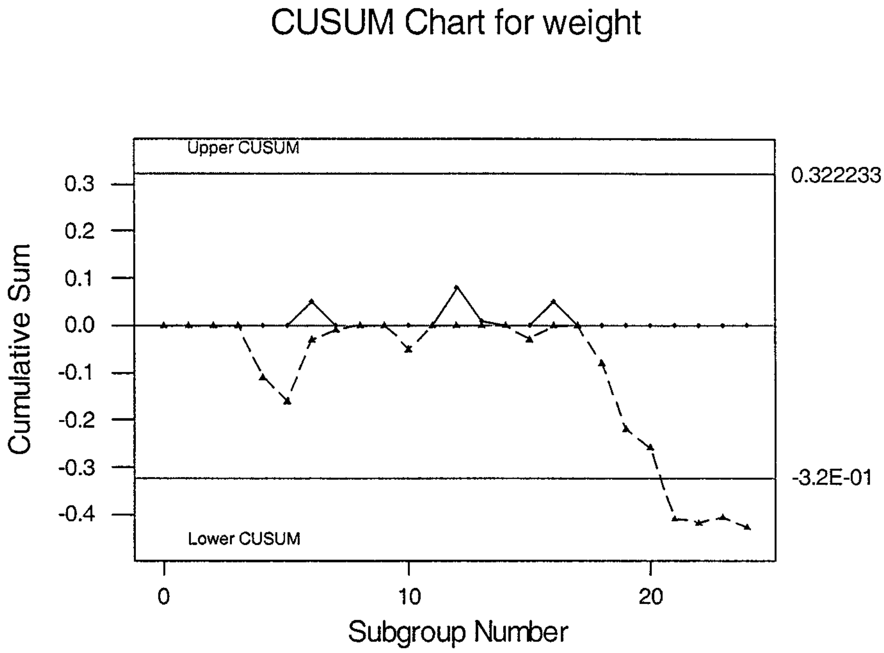
17–21.


CUSUM Chart for Oxide
Upper CUSUM
17.0308
10
0
-10
Cumulative Sum
-17.0308
Lower CUSUM
0
10
20
30
Subgroup Number

17–22.


EWMA Chart for weight
2.7SL=32.01
32.00
EWMA
Mean=31.97
31.95
-2.7SL=31.92
31.90
0
5
10
15
20
25
Sample Number

17–23.



17–24. (a)

$$R(t) = \int_t^\infty f(x)\,dx = \begin{cases} 1 & \text{if } t < \alpha \\ \frac{\beta - t}{\beta - \alpha} & \text{if } \alpha \le t \le \beta \\ 0 & \text{if } t > \beta \end{cases}$$

(b) $\displaystyle\int_0^\infty R(t)\,dt = \frac{\alpha + \beta}{2}$

(c) $\displaystyle h(t) = \frac{f(t)}{R(t)} = \frac{1}{\beta - t}, \quad \alpha \le t \le \beta.$

(d) $\displaystyle H(t) = \int_0^t h(t)\,dt = -\ell\text{n}\left(\frac{\beta - t}{\beta - \alpha}\right)$

$$e^{-H(t)} = \frac{\beta - t}{\beta - \alpha} = R(t).$$

17–25. $R_S(t) = e^{-\lambda_s t}$, $\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 = 7.6 \times 10^{-2}$

(a) $R_S(60) = e^{-7.6\times 10^{-2}\times 60} = 0.0105$

(b) $MTTF = 1/\lambda_s = \frac{1}{7.6\times 10^{-2}} = 13.16$ hours

17–26. $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0.002$

(a) $R(1000) = \sum_{k=2}^{5} \binom{5}{k} (0.367)^k (0.633)^{5-k} = 0.6056$

(b) $R(1000) = 1 - (0.633)^5 = 0.8984$

17–27. $R(1000) = \sum_{k=0}^{3} \frac{e^{-1}(1)^k}{k!} = 0.98104$

17–28. $\lambda = 1/160 = 6.25 \times 10^{-3}$

17–29. 0.84, 0.85

17–30. If $\hat{\theta}$ is the maximum likelihood estimator of θ and $\phi = g(\theta)$ is a single-valued function of θ, then $\hat{\phi} = g(\hat{\theta})$ is the MLE of ϕ. To prove this, note that $L(\theta)$, the likelihood function, has a maximum at $\theta = \hat{\theta}$. Furthermore, $\theta = g^{-1}(\phi)$, so the likelihood function is $L[g^{-1}(\phi)]$, which has a maximum at $\hat{\theta} = g^{-1}(\phi)$ or at $\phi = g(\hat{\theta})$. In the problem stated, R is of the form $g(\theta) = e^{-t/\theta}$, so the problem is solved.

17–31. (a) 3842

(b) $[913.63, \infty)$

17–32. (a) $\hat{R}(300) = e^{-300/\hat{\theta}} = e^{-300/3842} = 0.9249$

$\hat{R}_L(300) = e^{-300/\hat{\theta}_L} = e^{-300/913.63} = 0.72$

(b) $\hat{L}_{0.9} = 3842\,\ell\text{n}(1/0.9) = 404.795$