Chapter 6

6–1.

$$f_X(x) = \frac{1}{4}; \quad 0 < x < 4$$

$$P\left(\frac{1}{2} < X < \frac{7}{4}\right) = \int_{1/2}^{7/4} \frac{dx}{4} = \frac{5}{16}, \quad P\left(\frac{9}{4} < X < \frac{27}{8}\right) = \int_{9/4}^{27/8} \frac{dx}{4} = \frac{9}{32}$$

6–2.

$$f_X(x) = \frac{4}{34}; \quad \frac{143}{4} < x < \frac{177}{4}$$

$$P(X < 40) = \int_{143/4}^{40} \frac{4}{34}\,dx = \frac{1}{2}$$

$$P(40 < X < 42) = \int_{40}^{42} \frac{4}{34}\,dx = \frac{4}{17}$$

6–3.

$$f_X(x) = \frac{1}{2}, \quad 0 \le x \le 2$$

$$F_Y(y) = P(Y \le y) = P\left(X \le \frac{y-5}{2}\right) = \int_0^{\frac{y-5}{2}} \frac{dx}{2} = \frac{y-5}{4}$$

So

$$f_Y(y) = \frac{1}{4}, \quad 5 < y < 9$$

6–4. The p.d.f. of profit, X, is

$$f_X(x) = \frac{1}{2000}; \quad 0 < x < 2000$$

$$Y = \text{Brokers Fees} = 50 + 0.06X$$

$$\begin{aligned} F_Y(y) &= P(50 + 0.06X \le y) \\ &= P\left(X \le \frac{y-50}{0.06}\right) \\ &= \int_0^{\frac{y-50}{0.06}} \frac{dx}{2000} = \frac{y-50}{120} \end{aligned}$$

$$f_Y(y) = \frac{1}{120}; \quad 50 < y < 170$$

6–5.

$$\begin{aligned} M_X(t) &= E(e^{tX}) = \int_{\alpha}^{\beta} e^{tx}\,\frac{dx}{\beta-\alpha} \\ &= \left.\frac{e^{tx}}{(\beta-\alpha)t}\right|_{\alpha}^{\beta} \\ &= \frac{e^{t\beta}-e^{t\alpha}}{(\beta-\alpha)t} \end{aligned}$$

Using L'Hôspital's rule when necessary, we obtain

$$\begin{aligned} E(X) &= M_X'(0) = \frac{1}{\beta-\alpha}\left[t^{-1}\beta e^{t\beta} - t^{-2}e^{t\beta} - t^{-1}\alpha e^{t\alpha} + t^{-2}e^{t\alpha}\right]_{t=0} \\ &= \frac{1}{\beta-\alpha}\left[\beta^2 e^{t\beta} - \beta^2 e^{t\beta}/2 - \alpha^2 e^{t\alpha} + \alpha^2 e^{t\alpha}/2\right]_{t=0} \\ &= \frac{\beta+\alpha}{2} \end{aligned}$$

and

$$\begin{aligned} E(X^2) &= M_X''(0) \\ &= \frac{1}{\beta-\alpha}\Big[t^{-1}\beta^2 e^{t\beta} - \beta e^{t\beta}t^{-2} + 2e^{t\beta}t^{-3} - t^{-2}\beta e^{t\beta} \\ &\qquad -t^{-1}\alpha^2 e^{t\alpha} + \alpha e^{t\alpha}t^{-2} + t^{-2}\alpha e^{t\alpha} - 2e^{t\alpha}t^{-3}\Big]_{t=0} \\ &= \frac{1}{\beta-\alpha}\left[\frac{\beta^3-\alpha^3}{3}\right] \end{aligned}$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{(\beta-\alpha)^2}{12}$$

6–6.

$$E(X) = \frac{\beta+\alpha}{2} = 0 \Rightarrow \beta+\alpha = 0$$

$$V(X) = \frac{(\beta-\alpha)^2}{12} = 1 \Rightarrow \beta^2 - 2\alpha\beta + \alpha^2 = 12$$

$$\Rightarrow \alpha = -\sqrt{3}, \quad \beta = +\sqrt{3}$$

6–7. The CDF for Y is

y	$F_Y(y)$
$y < 1$	0
$1 \le y < 2$	0.3
$2 \le y < 3$	0.5
$3 \le y < 4$	0.9
$y > 4$	1

Generate realizations of $u_i \sim$ Uniform[0,1] random numbers as described in Section 6–6; use these in the inverse as $y_i = F_Y^{-1}(u_i)$, $i = 1, 2, \ldots$. For example, if $u_1 = 0.623$, then $y_1 = F_Y^{-1}(0.623) = 3$.

6–8.

$$\begin{aligned} f_X(x) &= \frac{1}{4}; \quad 0 < x < 4 \\ &= 0; \quad \text{otherwise} \end{aligned}$$

The roots of $y^2 + 4xy + (x+1) = 0$ are real if $16x^2 - 4(x+1) \ge 0$, or if

$$(x - \frac{1}{8})^2 - \frac{17}{64} \ge 0$$

or where $x \le \frac{1}{8}(1 - \sqrt{17})$ or $x \ge \frac{1}{8}(1 + \sqrt{17})$

$$\begin{aligned} P\left(X \le \frac{1}{8}(1 - \sqrt{17})\right) &= 0 \\ P\left(X \ge \frac{1}{8}(1 + \sqrt{17})\right) &= \int_{\frac{1}{8}(1+\sqrt{17})}^{4} \frac{dx}{4} = \frac{1}{32}(31 - \sqrt{17}) \end{aligned}$$

6–9.

$$M_X(t) = \int_0^\infty e^{tx} \lambda e^{-\lambda x}\, dx = \lambda \int_0^\infty e^{x(t-\lambda)}\, dx,$$

which converges if $t < \lambda$. Thus for $t < \lambda$,

$$M_X(t) = \frac{\lambda}{t - \lambda}\left[e^x(t-\lambda)\right]_{x=0}^{\infty} = \frac{\lambda}{\lambda - t} = \frac{1}{1 - \lambda/t},\ t < \lambda$$

$$E(X) = M_X'(0) = \left[\lambda(\lambda - t)^{-2}\right]_{t=0} = \frac{1}{\lambda}$$

$$E(X^2) = M_X''(0) = \left[2\lambda(\lambda - t)^{-3}\right]_{t=0} = \frac{2}{\lambda^2}$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

6–10. X denotes life length,

$$E(X) = \frac{1}{\lambda} = 3 \Rightarrow \lambda = \frac{1}{3}$$

P^* denotes profit

$$\begin{aligned} P^* &= 1000, \quad X > 1 \\ &= 750, \quad X \leq 1 \end{aligned}$$

$$\begin{aligned} E(P^*) &= 1000P(X > 1) + 750P(X \leq 1) \\ &= 1000e^{-1/3} + 750(1 - e^{-1/3}) = \$929.13 \end{aligned}$$

6–11.

$$P(X < \frac{1}{2}) = \int_0^{1/2} \frac{1}{3}e^{-x/3}dx = 1 - e^{(-1/3)(1/2)} = 0.154$$

or 15.4% experience failure in the first six months.

6–12. $P^* =$ Profit, $T =$ Life Length

$$\begin{aligned} P^* &= rY - dY, \quad T > Y \\ &= rT - dY, \quad T \leq Y \end{aligned}$$

$$\begin{aligned} E(P^*) &= (rY - dY)P(T \geq Y) - dYP(T \leq Y) + r\int_0^Y t\theta e^{-\theta t}\,dt \\ &= r(\theta^{-1} - \theta^{-1}e^{-\theta Y}) - dY \end{aligned}$$

$$\frac{dE(P^k)}{dY} = re^{-\theta Y} - d = 0 \Rightarrow Y = -\theta^{-1}\ell\text{n}(\frac{d}{r})$$

For Y to be positive, $0 < \frac{d}{r} < 1$.

6–13. X = Life Length,

$$E(X) = \frac{1}{\lambda} = 3 \Rightarrow \lambda = \frac{1}{3}$$

$$P(X < 1) \;=\; 1 - e^{-1/3} \;=\; 0.283$$

28.3% of policies result in a claim.

6–14. No.

$$P(X \leq 2) \;=\; 1 - e^{-2\lambda}$$

$$P(X \leq 3) \;=\; 1 - e^{-3\lambda}$$

$$1 - e^{-2\lambda} \;=\; \frac{2}{3}(1 - e^{-3\lambda}) \;\Rightarrow\; 1 \;=\; 3e^{-2\lambda} - 2e^{-3\lambda}$$

Only $\lambda = 0$ satisfies this condition; but we must have $\lambda > 0$, so there is no value of λ for which

$$P(X \leq 2) \;=\; \frac{2}{3}P(X \leq 3)$$

6–15.

$$\begin{aligned} C_I &= C, \quad X > 15; \\ &= C + Z, \quad X \leq 15 \\ C_{II} &= 3C, \quad X > 15 \\ &= 3C + Z, \quad X \leq 15 \end{aligned}$$

$$\begin{aligned} E(C_I) &= CP(X > 15) + (C + Z)P(X \leq 15) \\ &= Ce^{-15/25} + (C + Z)(1 - e^{-15/25}) \doteq C + 0.4512Z \\ E(C_{II}) &= 3CP(X > 15) + (3C + Z)P(X \leq 15) \\ &= 3Ce^{-15/25} + (3C + Z)(1 - e^{-15/35}) \doteq 3C + 0.3426Z \end{aligned}$$

$$E(C_{II}) - E(C_I) = 2C - 0.1026Z$$

which favors process I if $C > 0.0513Z$.

6–16.

$$P(X > x+s|X > x) \;=\; P(X > s) \;=\; P(X > 10000) \;=\; e^{-10000/20000} \;=\; 0.6064$$

and $P(X < 30000|X > 20000) \;=\; 0.3936$.

6–17.

$$\Gamma(p) \;=\; \int_0^\infty x^{p-1}e^{-x}dx$$

Let $x = y^2 \Rightarrow \frac{dx}{dy} = 2y$. So $\Gamma(p) \;=\; \int_0^\infty y^{2p-1}e^{-y^2}dy$ and $\Gamma(\frac{1}{2}) \;=\; 2\int_0^\infty e^{-y^2}dy$.

$$\begin{aligned}\left(\Gamma(\frac{1}{2})\right)^2 \;&=\; 4\int_0^\infty e^{-y^2}\,dy \cdot \int_0^\infty e^{-x^2}\,dx \\ &=\; 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy\end{aligned}$$

Let $x = \rho\cos(\theta)$ and $y = \rho\sin(\theta)$. So

$$\begin{aligned}\left(\Gamma(\frac{1}{2})\right)^2 \;&=\; 4\int_0^{\pi/2}\int_0^\infty \rho e^{-\rho^2}\,d\rho\,d\theta \\ &=\; 2\int_0^{\pi/2} d\theta \;=\; \pi\end{aligned}$$

So $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

6–18. Integrate by parts with $u = x^{n-1}, dv = e^{-x}dx$

$$\begin{aligned}\Gamma(n) \;&=\; \int_0^\infty x^{n-1}e^{-x}dx \;=\; \left[-e^{-x}x^{n-1}\right]_0^\infty + (n-1)\int_0^\infty x^{n-2}e^{-x}dx \\ &=\; 0 + (n-1)\int_0^\infty x^{n-2}e^{-x}dx \;=\; (n-1)\cdot\Gamma(n-1)\end{aligned}$$

Repeatedly using the approach above, we get

$$\begin{aligned}\Gamma(n) \;&=\; (n-1)\cdot\Gamma(n-1) \;=\; (n-1)(n-2)\cdot\Gamma(n-2) \\ &=\; \cdots \;=\; (n-1)(n-2)\cdots\Gamma(1).\end{aligned}$$

Since $\Gamma(1) = \int_0^\infty e^{-x}dx = 1$, we have $\Gamma(n) = (n-1)!$

6–19.

$$Y = X_1 + \cdots + X_{10}$$

$$g(x_i) = \begin{cases} 7e^{-7x_i} & \text{if } x_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} P(Y > 1) &= \int_1^\infty \frac{7}{\Gamma(10)}(7y)^9 e^{-7y} dy \\ &= \sum_{k=0}^{9} e^{-7\cdot 1}\frac{7^k}{k!} = 0.8305 \end{aligned}$$

6–20.

$$\lambda = 6;\ t = 4 \Rightarrow \lambda t = 24$$

$$P(X \geq 24) = 1 - P(X \leq 23) = 1 - \sum_{x=0}^{23} \frac{e^{-24}24^x}{x!}$$

6–21.

$$\begin{aligned} M_X(t) &= E(e^{tX}) = \int_0^\infty e^{tx} \frac{\lambda}{\Gamma(r)}(\lambda x)^{r-1} e^{-\lambda x} dx \\ &= \frac{\lambda^r}{\Gamma(r)} \int_0^\infty x^{r-1} e^{-x(\lambda - t)} dx \end{aligned}$$

The integral converges if $(\lambda - t) > 0$ or $\lambda > t$. Let $u = x(\lambda - t)$, $\frac{dx}{du} = \frac{1}{(\lambda - t)}$. So

$$\begin{aligned} M_X(t) &= \frac{\lambda^r}{\Gamma(r)} \int_0^\infty \left(\frac{u}{\lambda - t}\right)^{r-1} e^{-u} (\lambda - t)^{-1}\, du \\ &= \left(\frac{\lambda}{\lambda - t}\right)^r \cdot \frac{1}{\Gamma(r)} \int_0^\infty u^{r-1} e^{-u}\, du \\ &= \left(\frac{\lambda}{\lambda - t}\right)^r \cdot \frac{1}{\Gamma(r)} \cdot \Gamma(r) = \left(\frac{\lambda}{\lambda - t}\right)^r \\ &= (1 - (t/\lambda))^{-r}, \text{ where } \lambda > t \end{aligned}$$

6–22.

$$\begin{aligned} P(Y > 24) &= \int_{24}^\infty \frac{0.25}{\Gamma(4)}\, (0.25y)^3 e^{-0.25y}\, dy \\ &= \sum_{k=0}^{3} e^{-0.25\cdot 24} \frac{(0.25 \cdot 24)^k}{k!} = 0.1512 \end{aligned}$$

6–23. $E(X) = r/\lambda = 40$, $V(X) = r/\lambda^2 = 400$

$\lambda = 0.1$, $r = 4$

$$\begin{aligned} P(X < 20) &= \int_0^{20} \frac{0.1}{\Gamma(4)}(0.1x)^3 e^{-0.1x}\,dx \\ &= 1 - \sum_{k=0}^{3} e^{-0.1\cdot 20}\frac{(0.1\cdot 20)^k}{k!} \;=\; 0.1429 \end{aligned}$$

$$P(X < 60) = 1 - \sum_{k=0}^{3} e^{-0.1\cdot 60}\frac{(0.1\cdot 60)^k}{k!} \;=\; 0.8488$$

6–24.

$$E(X) = \frac{\lambda^r}{\Gamma(r)}\int_0^\infty x(x-u)^{r-1}e^{-\lambda(x-u)}dx$$

Let $y = \lambda(x-u) \Rightarrow \frac{dx}{dy} = \frac{1}{\lambda}$

$$\begin{aligned} E(X) &= \frac{\lambda^{r-2}}{\Gamma(r)}\int_0^\infty (y+\lambda u)(y/\lambda)^{r-1}e^{-y}\,dy \\ &= \frac{1}{\lambda\Gamma(r)}\int_0^\infty y^r e^{-y}dy + \frac{\lambda u}{\lambda\Gamma(r)}\int_0^\infty y^{r-1}e^{-y}\,dy \\ &= \frac{\Gamma(r+1)}{\lambda\Gamma(r)} + \frac{u\Gamma(r)}{\Gamma(r)} \\ &= \frac{r}{\lambda} + u \end{aligned}$$

6–26.

$$f_X(x) = \frac{\Gamma(\lambda+r)}{\Gamma(\lambda)\Gamma(r)}x^{\lambda-1}(1-x)^{r-1}$$

$\lambda = r = 1$ gives

$$\begin{aligned} f_X(x) &= \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)}x^0(1-x)^0 \\ &= \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

6–27. $\lambda = 2$, $r = 1$

$$\begin{aligned} f_X(x) &= \frac{\Gamma(3)}{\Gamma(2)\Gamma(1)} x(1-x)^0 \\ &= \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$\lambda = 1$, $r = 2$

$$\begin{aligned} f_X(x) &= \frac{\Gamma(3)}{\Gamma(1)\Gamma(2)} x^0(1-x) \\ &= \begin{cases} 2(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

6–29. See solution to 3-22.

6–30.

$$E(X) = \int_0^\infty x \, \frac{\beta}{\delta} \left(\frac{x-\gamma}{\delta} \right)^{\beta-1} e^{-(\frac{x-\gamma}{\delta})^\beta} \, dx$$

First let $y = \frac{x-\gamma}{\delta} \Rightarrow dx = \delta dy$

$$\begin{aligned} E(X) &= \int_0^\infty (\delta y + \gamma) \, \frac{\beta}{\delta} \, y^{\beta-1} e^{-y^\beta} \delta \, dy \\ &= \beta \int_0^\infty (\delta y + \gamma) y^{\beta-1} e^{-y^\beta} \, dy \end{aligned}$$

Let $u = y^\beta \Rightarrow dy = \beta^{-1} y^{-\beta+1} du$

$$\begin{aligned} E(X) &= \beta \int_0^\infty (\delta u^{1/\beta} + \gamma) u^{(1-1/\beta)} e^{-u} \beta^{-1} u^{(1/\beta-1)} \, du \\ &= \gamma \int_0^\infty e^{-u} \, du + \int_0^\infty \delta u^{1/\beta} e^{-u} \, du \\ &= \gamma + \delta \Gamma \left(1 + \frac{1}{\beta} \right) \end{aligned}$$

Using the same approach

$$E(X^2) = \int_0^\infty x^2 \, \frac{\beta}{\delta} \left(\frac{x-\gamma}{\delta} \right)^{\beta-1} e^{-(\frac{x-\gamma}{\delta})} \, dx$$

Let $y = \frac{x-\gamma}{\delta} \Rightarrow dx = \delta dy$

$$E(X^2) = \int_0^\infty (\delta y + \gamma)^2 \frac{\beta}{\delta} y^{\beta-1} e^{-y^\beta} \delta \, dy$$

$u = y^\beta \Rightarrow dy = \beta^{-1} y^{1-\beta}$

$$\begin{aligned} E(X^2) &= \int_0^\infty (\delta u^{1/\beta} + \gamma)^2 \frac{\beta}{\delta} u^{1-1/\beta} e^{-u} \delta \, \beta^{-1} u^{1/\beta - 1} \, du \\ &= \delta^2 \, \Gamma\left(1 + \frac{2}{\beta}\right) + 2\gamma\delta \, \Gamma\left(1 + \frac{1}{\beta}\right) + \gamma^2 \end{aligned}$$

So

$$V(X) = E(X^2) - [E(X)]^2 = \delta^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$$

6–31.

$$F(x) = 1 - e^{-(\frac{x-\gamma}{\delta})^\beta}$$

$$F(1.5) = 1 - e^{-(\frac{1.5-1}{0.5})^2} \doteq 0.63$$

6–32.

$$F(x) = 1 - e^{-(\frac{x-0}{400})^{1/3}}$$

$$1 - F(600) = e^{-(\frac{600}{400})^{1/3}} \doteq 0.32$$

6–33.

$$F(x) = 1 - e^{-(\frac{x-0}{400})^{1/2}}$$

$$1 - F(800) = 1 - e^{-2^{1/2}} \doteq 0.24$$

6–34. The graphs are identical to those shown in Figure 6–8.

6–35.

$$F(x) = 1 - e^{-(\frac{x-0}{200})^{1/4}}$$

(a) $1 - F(1000) = 1 - e^{-5^{1/4}} \doteq 0.22$
(b) $0 + 200\Gamma(5) = 200 \cdot 24 = 4800 = E(X)$

6–36. $P^* =$ Profit

$$\begin{aligned} P^* &= \$100; \quad x \geq 8760 \\ &= -\$50; \quad x < 8760 \end{aligned}$$

$$\begin{aligned} E(P^*) &= -50 \int_0^{8760} 20000^{-1} e^{-20000^{-1} x}\, dx + 100 \int_{8760}^{\infty} 20000^{-1} e^{-20000^{-1} x}\, dx \\ &= -50(1 - e^{-876/2000}) + 100 e^{-876/2000} \\ &= \$46.80/\text{set} \end{aligned}$$

6–37. $r/\lambda = 20$

$(r/\lambda^2)^{1/2} = 10$

$r = 4,\ \lambda = 0.2$

$$P(X \leq 15) \;=\; F(15) \;=\; 1 - \sum_{k=0}^{3} e^{-3} 3^k / k! \;=\; 0.3528$$

6–38. (a) Use Table XV, Col. 1 with scaling and Equation 6–35.

$$\begin{array}{ll} u_1 = 0.10480 & x_1 = 10 + u_1(10) = 11.0480 \\ u_2 = 0.22368 & x_2 = 10 + u_2(10) = 12.2368 \\ u_3 = 0.24130 & x_3 = 10 + u_3(10) = 12.4130 \\ \vdots & \vdots \\ u_{10} = 0.85475 & x_{10} = 10 + u_{10}(10) = 18.5475 \end{array}$$

(b) Use Table XV, Col. 2; $x_i = -50000\,\ell\text{n}(1 - u_i)$

$$\begin{array}{ll} u_1 = 0.15011 & x_1 = -50000\,\ell\text{n}(0.84989) = 8132.42 \\ u_2 = 0.46573 & x_2 = -50000\,\ell\text{n}(0.53427) = 31345.51 \\ u_3 = 0.48360 & x_3 = -50000\,\ell\text{n}(0.51640) = 33043.68 \\ u_4 = 0.93093 & x_4 = -50000\,\ell\text{n}(0.06907) = 133631.74 \\ u_5 = 0.39975 & x_5 = -50000\,\ell\text{n}(0.60025) = 25520.45 \end{array}$$

(c) $a = \sqrt{3} = 1.732$, $b = 4 - \ell\text{n}(4) + 3^{-1/2} = 3.191$. Now use Table XV, Col. 3.

Realization 1: $u_1 = 0.01563$, $u_2 = 0.25595$

$$y = 2\left(\frac{0.01563}{0.98437}\right)^{1.732} = 0.00153$$

$$3.191 - \ell\text{n}[(0.01563)^2 0.25595] = 12.871$$

$$y \le 12.87$$

$$x_1 = y/4 = 0.0003383$$

Realization 2: $u_1 = 0.22527$, $u_2 = 0.06243$

$$y = 2\left(\frac{0.22527}{0.77473}\right)^{1.732} = 0.2354$$

$$3.191 - \ell\text{n}[(0.22527)^2 0.06243] = 8.9456$$

$$y \le 8.9456$$

$$x_2 = y/4 = 0.05885$$

Continue for additional realizations.

Note: Since r is an integer, an alternate scheme which may be more efficient here is to let $x_i = x_{i1} + x_{i2}$, where x_{ij} is exponential with parameter $\lambda = 4$.

$$x_{ij} = -0.25\,\ell\text{n}(1 - u_{ij}),\ i = 1, 2, \ldots, 5,\ j = 1, 2$$

Realization 1: $u_1 = 0.01563$, $u_2 = 0.25595$

$$x_{11} = -0.25\,\ell\text{n}(0.98437) = 0.003938$$

$$x_{12} = -0.25\,\ell\text{n}(0.74405) = 0.073712$$

This yields $x_1 = 0.07785$. Continue for more realizations.

(d) Use Table XV, Col. 4 with scaling.

$u_1 = 0.02011$	$x_1 = 0 + 100[-\ell\text{n}(0.02011)]^2 = 390.654$
$u_2 = 0.85393$	$x_2 = 0 + 100[-\ell\text{n}(0.85393)]^2 = 15.791$
$\vdots$	$\vdots$
$u_{10} = 0.53988$	$x_{10} = 0 + 100[-\ell\text{n}(0.53988)]^2 = 61.641$

6–39. (a) Using Table XV, Col. 5, and $y = x^{0.3}$, we get

$$\begin{array}{lll} u_1 = 0.81647 & x_1 = -10\,\ell\text{n}(0.18353) = 16.954 & y_1 = 2.3376 \\ u_2 = 0.30995 & x_2 = -10\,\ell\text{n}(0.69005) = 3.7099 & y_2 = 1.4818 \\ \vdots & \vdots & \vdots \\ u_{10} = 0.53060 & x_{10} = -10\,\ell\text{n}(0.46940) = 7.5630 & y_{10} = 1.8348 \end{array}$$

(b) Using the gamma variates in Problem 6–38(c) and Table XV, Col. 3 entry #25,

$$y_1 = \frac{(0.000383)^{1/2}}{(0.28834)^{1/2}} = 0.03645$$

$$y_2 = \frac{(0.05885)^{1/2}}{(0.04839)^{1/2}} = 1.102797$$

$\vdots$

etc.