12-1 $\underset{\sim}{x}(t) = 10 + \underset{\sim}{\nu}(t) \qquad R_\nu(\tau) = 2\,\delta(\tau) \qquad E\{\underset{\sim}{\nu}(t)\} = 0$

$$E\{\underset{\sim}{n}_T\} = E\{\underset{\sim}{x}(t)\} = 10 \qquad C_x(\tau) = 2\delta(\tau)$$

From (12-5)

$$\sigma^2_{n_T} = \frac{1}{2T}\int_{-T}^{T} C_x(\tau)\left(1 - \frac{|\tau|}{2T}\right) d\tau = \frac{1}{2T}\int_{-T}^{T} 2\delta(\tau)\left(1 - \frac{|\tau|}{2T}\right)d\tau = \frac{1}{T}$$

12-2 The process $\underset{\sim}{x}(t)$ is normal (note correction) and such that

$$F(x,x;\tau) \longrightarrow F^2(x) \quad \text{as} \quad \tau \to \infty \qquad (i)$$

We shall show that it is mean-ergodic. It suffices to show that [see (12-10)]

$$C(\tau) \longrightarrow 0 \qquad \text{as} \qquad \tau \to \infty$$

Proof. We can assume (scaling and centering) that $\eta = 0$ $C(0) = 1$. With this assumption, the RVs $\underset{\sim}{x}(t+\tau)$ and $\underset{\sim}{x}(t)$ are $N(0,0;1,1;r)$ where $r = r(\tau) = C(\tau)$ is the autocovariance of $\underset{\sim}{x}(t)$. Hence,

$$f(x_1,x_2;\tau) = \frac{1}{2\pi\sqrt{1-r^2}} \exp\left\{-\frac{1}{2(1-r^2)}(x_1^2 - 2rx_1x_2 + x_2^2)\right\}$$

$$= \frac{1}{2\pi\sqrt{1-r^2}} \exp\left\{-\frac{1}{2(1-r^2)}(x_1 - rx_2)^2\right\} e^{-x_2^2/2}$$

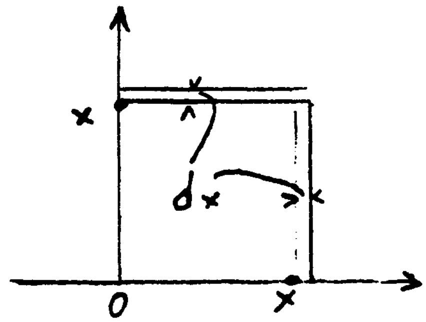
Clearly, $f(x,y) = f(y,x)$, hence, (see figure)

$$F(x+dx,\ x+dx;\tau) - F(x,x,\tau) = 2\int_{-\infty}^{x} f(\xi,x)\,d\xi\,dx$$

$$= \frac{1}{\pi\sqrt{1-r^2}} \int_{-\infty}^{x} \exp\left\{-\frac{1}{2(1-r^2)}(\xi - xr)^2\right\} d\xi\ e^{-x^2/2}\,dx$$

Furthermore,

$$F^2(x+dx) - F^2(x) = 2\,F(x)f(x)dx$$



From the above and (i) it follows that

$$G\left(\frac{x-rx}{\sqrt{1-r^2}}\right) \xrightarrow[\tau\to\infty]{} G(x)$$

Hence, $r(\tau) \to 0$ as $\tau \to \infty$

12-3 If $\underset{\sim}{x}(t)$ is normal, then [see (12-27)]

$$C_{zz}(\tau) = R_x(\lambda+\tau)R_x(\lambda-\tau) + R_x^2(\tau) \qquad \underset{\sim}{z}(t) = \underset{\sim}{x}(t+\lambda)\underset{\sim}{x}(t)$$

If, therefore, $R_x(\tau) = 0$ for $|\tau| > a$, then $C_{zz}(\tau) = 0$ for $|\tau| > \lambda + a$.

12-4 If $\underset{\sim}{x}(t) = \underset{\sim}{a}\, e^{j(\omega t + \underset{\sim}{\phi})}$ then the time-average

$$\frac{1}{2T}\int_{-T}^{T} \underset{\sim}{x}(t+\tau)\underset{\sim}{x}^*(t)dt = e^{j\omega\tau}\,|\underset{\sim}{a}|^2$$

12-5 If $\underset{\sim}{z}(t) = \underset{\sim}{x}(t+\lambda)\underset{\sim}{y}(t)$, then

$$C_{zz}(\tau) = E\{\underset{\sim}{x}(t+\lambda+\tau)\underset{\sim}{y}(t+\tau)\underset{\sim}{x}(t+\lambda)\underset{\sim}{y}(t)\} - R_{xy}^2(\lambda)$$

and the result follows from (12-5).

12-6 The process $\bar{x}(t) = x(t - \theta)$ is stationary with mean $\bar{\eta}$ and covariance $\bar{C}(\tau)$ given by [see (10-176) and (10-177)]

$$\bar{\eta} = \frac{1}{T}\int_0^T \eta(t)dt \qquad \bar{C}(\tau) = \frac{1}{T}\int_0^T C(t+\tau,t)dt$$

If $R(t+\tau,t) \to \eta^2(t)$ as $\tau \to \infty$ (note correction), then

$$C(t+\tau,t) \xrightarrow[\tau\to\infty]{} 0 \qquad \text{hence} \qquad \bar{C}(\tau) \xrightarrow[\tau\to\infty]{} 0$$

This shows that [see (12-10)], $\bar{x}(t)$ is ergodic, therefore,

$$\frac{1}{2c}\int_{-c}^{c} \bar{x}(t)dt = \frac{1}{2c}\int_{-c+\theta}^{c+\theta} x(t)dt \longrightarrow \bar{\eta}$$

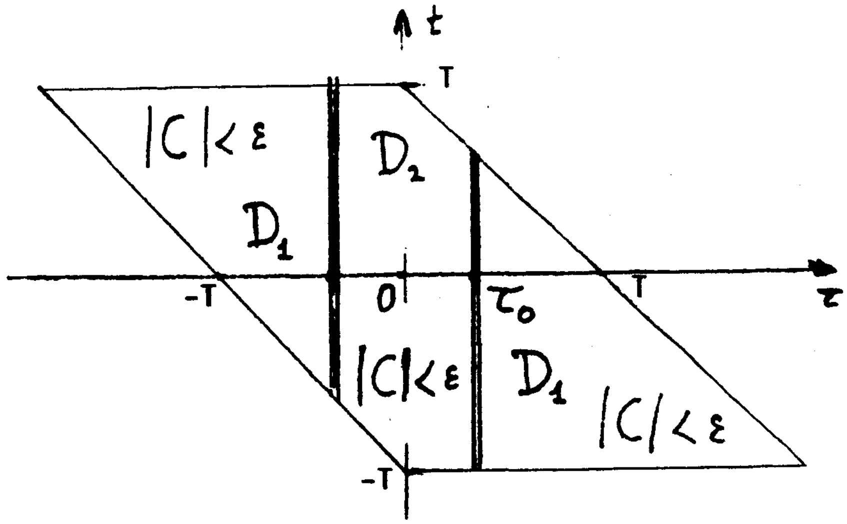
This yields the desired result because for a specific outcome, $\theta(\zeta) = \theta$ is a constant and

$$\lim_{c\to\infty} \frac{1}{2c}\int_{-c+\theta}^{c+\theta} x(t)dt = \lim_{c\to\infty} \frac{1}{2c}\int_{-c}^{c} x(t)dt$$

12-7 From (9-38) it follows that

$$4T^2\sigma_T^2 = \int_{-T}^{T}\int_{-T}^{T} C(t_1,t_2)dt_1,dt_2 = \iint_D C(t+\tau,t)d\tau dt$$

where D is the parallelogram in the figure. Given $\varepsilon > 0$, we can find a constant τ_o such that



$|C(t+\tau,t)| < \varepsilon$ for $|\tau| > \tau_o$ (uniform continuity). Furthermore, if $C(t,t) < P$ then

$$C^2(t_1,t_2) \le C(t_1,t_1)C(t_2,t_2) < P^2$$

Thus,

$|C| < \varepsilon$ in D_1 and $|C| < P$ in D_2

The area of D_1 is less than $4T^2$; the area of D_2 is less than $4\tau_o T$. Hence

$$\sigma_T^2 < \varepsilon + \frac{\tau_o}{T} \xrightarrow[T\to\infty]{} \varepsilon$$

And since ε is arbitrary, we conclude that $\sigma_T \to 0$.

12-8 It follows from (6-234) with $\underset{\sim}{x}(t) = \underset{\sim}{x}$, $\underset{\sim}{x}(t+\lambda) = \underset{\sim}{y}$

$$\eta_1 = \eta_2 = 0 \qquad \sigma_1^2 = \sigma_2^2 = R(0) \qquad r\sigma_1,\sigma_2 = R(\lambda)$$

(b) The proof is based on the identity

$$E\{\underset{\sim}{y}|M\} = E\,\{E\{\underset{\sim}{y}|\underset{\sim}{x}\}|M\} \qquad M = \{\underset{\sim}{x}(t)\in D\} \qquad \text{(i)}$$

Proof Suppose first that D consists of the union of open intervals. In this case, if $x\in D$, then for small δ the interval $(x,x+\delta)$ is a subset of D, hence

$$\{x \le \underset{\sim}{x} < x + dx,\ M\} = \{x \le \underset{\sim}{x} < x + dx\}$$

for $x\in D$ and $\{\emptyset\}$ otherwise. This yields

$$f(x|M)\,dx = \frac{P\{x\le x<x+dx\}}{P(M)} = \frac{1}{p} f(x)dx \qquad p = P(M)$$

for $x\in D$ and 0 otherwise. Similarly, $f(x,y|M) = f(x,y)/p$ for $x\in D$ and 0 otherwise. From the above it follows that

$$E\{E\{\underset{\sim}{y}|\underset{\sim}{x}|M\}\} = \int_D \left(\int_{-\infty}^{\infty} yf(y|x)dy\right) f_x(x|M)dx$$

$$= \int_D \int_{-\infty}^{\infty} \frac{yf(x,y)f(x)}{f(x)\,p}\, dydx = \int_D \int_{-\infty}^{\infty} yf(x,y|M)dydx = E\{\underset{\sim}{y}|M\}$$

If D has isolated points, we replace each $x \in D$ by an open interval $(x-\varepsilon,\ x+\varepsilon)$ forming an open set D_ε. Clearly, $D_\varepsilon \to D$ as $\varepsilon \to 0$ and (i) follows if at the isolated points x_i of D, $E\{\underset{\sim}{y}|x_i\}$ is interpreted as a limit.

Since $E\{\underset{\sim}{x}(t+\lambda)|\underset{\sim}{x}(t)\} = R(\lambda)\underset{\sim}{x}(t)/R(0)$, (i) yields

$$E\{\underset{\sim}{x}(t+\lambda)|M\} = E\{E\{\underset{\sim}{x}(t+\lambda)|\underset{\sim}{x}(t)\}|M\} = E\left\{\frac{R(\lambda)}{R(0)}\underset{\sim}{x}(t)|M\right\} = \frac{R(\lambda)}{R(0)}\ \bar{x}$$

(c) We select for D the interval (a,b) and we form the samples $\underset{\sim}{x}(nT)$, $\underset{\sim}{x}(nT+\lambda)$ of a single realization of $\underset{\sim}{x}(t)$ retaining only the pairs $\underset{\sim}{x}(t_i)$, $\underset{\sim}{x}(t_i+\lambda)$ such that $a < \underset{\sim}{x}(t_i) < b$. Using (5-51), we obtain

$$E\{\underset{\sim}{x}(t+\lambda)|\ a < \underset{\sim}{x}(t) < b\} = \frac{R(\lambda)}{R(0)}\bar{x} \simeq \frac{1}{N}\sum_{i=1}^{N} \underset{\sim}{x}(t_i+\lambda)$$

where $\bar{x} = E\{\underset{\sim}{x}(t)|a<\underset{\sim}{x}(t)<b\}$. This approximation is satisfactory if N is large and $R(\tau)\simeq 0$ for $\tau > T$.

12-9 (a) From (7-61) with $E\{w(t)\} = C_{xy}(\lambda)$:

$$R_{ww}(\tau) = C_{xy}(\lambda+\tau)C_{xy}(\lambda-\tau) + C_{xx}(\tau)C_{yy}(\tau) + C_{xy}^2(\lambda) = C_{ww}(\tau) + C_{xy}^2(\lambda)$$

(b) It follows from (a) that if

$$C_{xx}(\tau) \to 0 \qquad C_{yy}(\tau) \to 0 \qquad C_{xy}(\sigma) \to 0$$

then $C_{ww}(\tau) \to 0$ as $|\tau| \to \infty$; hence [see (12-10)] the process $\underset{\sim}{x}(t)$ and $\underset{\sim}{y}(t)$ are covariance ergodic.

12-10 From (10B-1) with $g(x) = 1$:

$$\left| \int_a^b f(x)dx \right|^2 \le \int_a^b |f(x)|^2 dt \int_a^b 1^2 \times dx = (b-a) \int_a^b |f(x)|^2 dx$$

12-11 We use as estimate of η the time average $\hat{\eta}_T$ in (12-1): As we know (see Example 12-4)

$$E\{\eta_T\} = \eta \qquad \sigma_T^2 = \frac{5}{2T}$$

We wish to find ε such that

$$P\{\eta - \varepsilon < \eta_T < \eta + \varepsilon\} = 0.95$$

(a) From (5-88):

$$0.95 = P\{|\eta_T - \eta| \le \varepsilon\} \le 1 - \frac{\sigma_T^2}{\varepsilon^2} \qquad \varepsilon = \varepsilon_a \le \frac{\sigma_T}{\sqrt{0.05}} = \frac{50}{T}$$

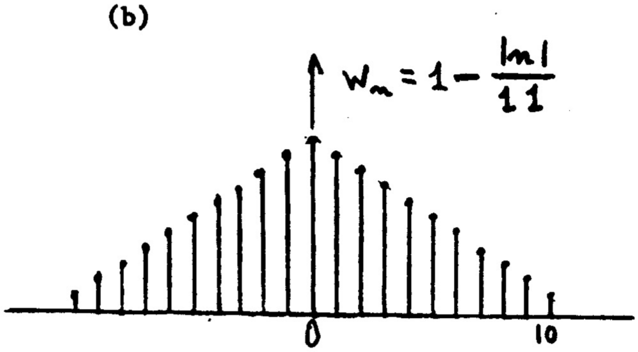
(b) If $\nu(t)$ is normal, then η_T is normal; hence,

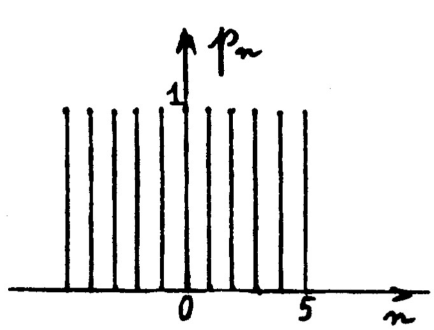
$$0.95 = 2G\left(\frac{\varepsilon}{\sigma_T}\right) - 1 \qquad G\left(\frac{\varepsilon}{\sigma_T}\right) = 0.975 \qquad \frac{\varepsilon}{\sigma_T} = z_{0.975}$$

This yields $\varepsilon = \varepsilon_b \simeq \sqrt{10/5} = \varepsilon_a\sqrt{5}$

12-12 (a) It follows from the convolution theorem for Fourier series

(b)





With p_n as above, $w_n = \frac{1}{11} p_n * p_n$

$$P(\omega) = \sum_{n=-5}^{5} e^{-jnT\omega} = \frac{\sin 5.5\omega T}{\sin 0.5\omega T} \qquad W(\omega) = \frac{1}{11} P^2(\omega)$$

12-13

$$\underset{\sim}{X}_T(\omega) = \frac{1}{\sqrt{2T}} \int_{-T}^{T} x(t)e^{-j\omega t}dt \qquad \underset{\sim}{S}_T(\omega) = |X_T(\omega)|^2$$

and

$$\Gamma(u,v) = \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R(t_1 - t_2)e^{-j(u\,t_1 + v\,t_2)}\,dt_1 dt_2 \qquad \text{(i)}$$

as in (9-173) and (9-174) yield

$$E\{\underset{\sim}{S}_T(\omega)\} = \Gamma(\omega, -\omega)$$

$$\text{Var } \underset{\sim}{S}_T(\omega) = |\Gamma(\omega, -\omega)|^2 + |\Gamma(\omega,\omega)|^2 \geq E^2\{\underset{\sim}{S}_T(\omega)\}$$

$$\text{Var } \underset{\sim}{S}_T(0) = 2|\Gamma(0,0)|^2 = 2\{E^2\{\underset{\sim}{S}_T(0)\}$$

The remaining part of the problem is more difficult. We outline the proof (For details see Papoulis, Signal Analysis). From (i) and the convolution theorem it follows that

$$\Gamma(u,v) = \int_{-\infty}^{\infty} \frac{\sin T(u + v - \alpha)}{\pi\, T(u + v - \alpha)} \frac{\sin T\alpha}{\alpha} S(v - \alpha)d\alpha$$

If $S(\omega)$ is nearly constant in an interval of length $1/T$, then it can be taken outside the integral sign. Hence,

$$\Gamma(u,v) \simeq S(v) \int_{-\infty}^{\infty} \frac{\sin T(u + v - \alpha)}{\pi\, T(u + v - \alpha)} \frac{\sin T\alpha}{\alpha} d\alpha = S(v) \frac{\sin T(u + v)}{T(u + v)}$$

This yields

$$\Gamma(\omega,-\omega) \simeq S(\omega) < \Gamma(\omega,\omega) \simeq S(\omega) \frac{\sin 2T\omega}{T\omega} \underset{\omega T\to\infty}{\longrightarrow} 0$$

$$\text{Var } \underset{\sim}{S}_T(\omega) \leq 2\, E^2\{\underset{\sim}{S}_T(\omega)\} \underset{\omega T\to\infty}{\longrightarrow} E^2\{\underset{\sim}{S}_T(\omega)\}$$

12-14 The function

$$\underset{\sim}{X}_c(\omega) = \int_{-T}^{T} c(t)\underset{\sim}{x}(t)e^{-j\omega t}dt$$

is the Fourier transform of the product

$$c(t)\underset{\sim}{x}_T(t) \qquad \underset{\sim}{x}_T(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases}$$

Hence, the function

$$2\,T\,\underset{\sim}{S}_T(\omega) = |\underset{\sim}{X}_c(\omega)|^2$$

is the Fourier transform of

$$\underset{\sim}{c}(t)\underset{\sim}{x}_T(t) * c(-t)\underset{\sim}{x}_T(-t)$$

$$= \int_{-T+|\tau|/2}^{T-|\tau|/2} c(t+\tfrac{\tau}{2})\underset{\sim}{x}_T(t+\tfrac{\tau}{2})c(t-\tfrac{\tau}{2})\underset{\sim}{x}(t-\tfrac{\tau}{2})dt$$

12-15 Since $C(-\tau) = C(\tau)$, it follows from (12-28) that for large T,

$$\text{Var } \underset{\sim}{R}_T(\lambda) \simeq \frac{1}{2T}\int_{-\infty}^{\infty} [C(\lambda+\tau)C(\lambda-\tau) + C^2(\tau)]\, d\tau$$

Since $S(\omega)$ is real, it follows from Parseval's formula and the pairs

$$C(\lambda+\tau) \leftrightarrow e^{j\lambda\omega}S(\omega) \qquad C(\lambda-\tau) \leftrightarrow e^{-j\lambda\omega}S(\omega)$$

that the above integral equals

$$\int_{-\infty}^{\infty} \left[e^{j\lambda\omega}\, S(\omega)e^{j\lambda\omega}S(\omega) + S^2(\omega)\right]\, d\omega$$

12-16 With $c = T - |\tau|/2$

$$z(t) = x(t + \frac{\tau}{2})x(t - \frac{\tau}{2}) \qquad E\{R_T(\tau)\} = R(\tau)(1 - \frac{|\tau|}{T})$$

(7-37) yields

$$E\{z(t_1)z(t_2)\} - E\{z(t_1)\}E\{z(t_2)\}$$

$$= R^2(t_1 - t_2) + R(t_1 - t_2 + \tau)R(t_1 - t_2 - \tau)$$

$$4\,T^2 \operatorname{Var} R_T(\tau) = \int_{-c}^{c}\int_{-c}^{c}[R^2(t_1 - t_2) + R(t_1 - t_2 + \tau)R(t_1 - t_2 - \tau)]dt_1 dt_2$$

$$= \int_{-2c}^{2c}[R^2(\alpha) + R(\alpha + \tau)R(\alpha - \tau)](2T - |\tau| - |\alpha|)d\alpha$$

12-17 Equating coefficients of z^k in (12-98), we obtain

$$(1-K_N^2)\,\alpha_k^{N-1} = \alpha_k^N + K_N\,\alpha_{N-k}^N$$

12-18 $R[0] = 8 \qquad R[1] = 4$

From (13-67)

$$P_0 = 8 \qquad a_1^1 = K_1 = 0.5 \qquad P_1 = (1 - K_1^2)P_0 = 6$$

$$E_1(z) = 1 - 0.5z^{-1} \qquad S(\omega) = \frac{6}{|E_1(e^{j\omega})|^2}$$

12-19 **From page** (13-67)

$$P_0 = 13 \qquad a_1^1 = K_1 = \frac{5}{13} \qquad P_1 = \frac{144}{13}$$

$$P_1 K_2 = R[2] - a_1^1 R[1] \qquad K_2 = \frac{1}{144}$$

$$a_1^2 = \frac{55}{144} \qquad a_2^2 = \frac{1}{144} \qquad P_2 = \frac{1595}{144}$$

$$S_{MEM}(\omega) = \frac{1595 \times 144}{\left|144 - 55e^{-j\omega T} - e^{-j2\omega T}\right|^2}$$

From (12-119)

$$\begin{vmatrix} 13-q & 5 & 2 \\ 5 & 13-q & 5 \\ 2 & 5 & 13-q \end{vmatrix} = 0 \qquad q_o = 14-\sqrt{51} \approx 6.86$$

Inserting the modified data 6.14, 5, 2 into the Yule-Walker equations (12-82), **we obtain**

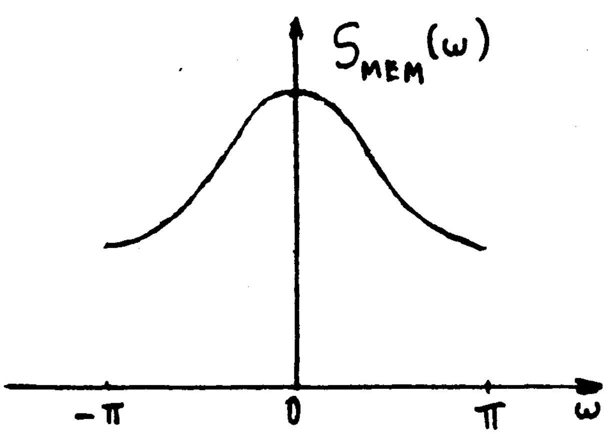
$$a_1^2 \approx 4.07 \qquad a_2^2 = -1 \qquad E_2(z) = 1 - 4.07z^{-1} + z^{-2}$$

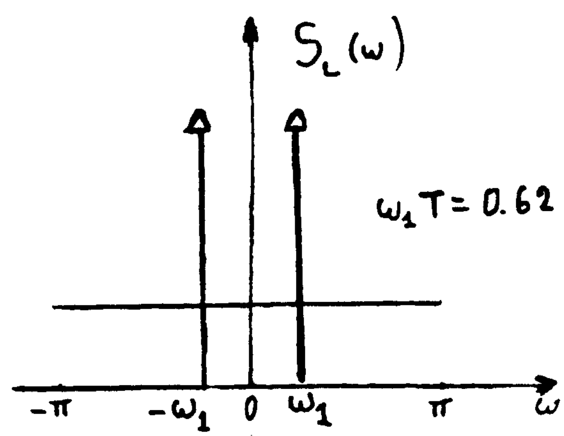
$$E_2(z) = 1 - 4.07z^{-1} + z^{-2} \qquad z_{1,2} \approx e^{\pm j0.62}$$

Solving (12-91) **we obtain**

$$R_L[m] = 6.86\ \delta[m] + 3.07 \cos 0.62m$$

$$S_L(\omega) = 6.86 + \frac{2\pi}{T} \times 3.07\ [\delta(\omega - 0.62) + \delta(\omega + 0.62)]$$





12.20 (a) Let $z = e^{j\theta_1}$ represent one of the roots of the Levinsion Polynomial $P_n(z)$ that lie on the unit circle. In that case

$$P_n\left(e^{j\theta_1}\right) = 0$$

and substituting this into the recursion equation (12-177) we get

$$|s_n| = \left|\frac{P_{n-1}\left(e^{j\theta_1}\right)}{\tilde{P}_{n-1}\left(e^{j\theta_1}\right)}\right| = 1$$

so that

$$s_n = e^{j\alpha}.$$

Let

$$P_{n-1}\left(e^{j\theta}\right) = R(\theta)\, e^{j\psi(\theta)}$$

and since $P_{n-1}(z)$ is free of zeros in $|z| \leq 1$, we have $R(\theta) > 0, 0 < \theta < 2\pi$, and once again substituting these into (12-177) we obtain

$$\begin{aligned}
\sqrt{1-s_n^2}\, P_n\left(e^{j\theta}\right) &= R(\theta)\, e^{j\psi(\theta)} - e^{j(\theta+\alpha)}\, e^{j(n-1)\theta}\, R(\theta)\, e^{-j\psi(\theta)} \\
&= R(\theta)\left[e^{j\psi(\theta)} - e^{j(n\theta+\alpha)}\, e^{-j\psi(\theta)}\right] \\
&= 2j\, R(\theta)\, e^{j(n\theta+\alpha)/2}\, sin\left(\psi(\theta) - \frac{n\theta}{2} - \frac{\alpha}{2}\right).
\end{aligned}$$

Due to the strict Hurwitz nature of $P_{n-1}(z)$, as θ varies from 0 to 2π, there is no net increment in the phase term $\psi(\theta)$, and the entire argument of the sine term above increases by $n\pi$. Consequently $P_n(e^{j\theta})$ equals zero atleast at n distinct points $\theta_1, \theta_2, \cdots \theta_n, \quad 0 < \theta_i < 2\pi$. However $P_n(z)$ is a polynomial od degree n in z and can have atmost n zeros. Thus all the above zeros are simple and they all lie on the unit circle.

(b) Suppose $P_n(z)$ and $P_{n-1}(z)$ has a common zero at $z = z_0$. Then $|z_0| > 1$ and from (12-137), we get

$$z_0\, s_n\, \tilde{P}_{n-1}(z_0) = 0$$

which gives $s_n = 0$, since $\tilde{P}_{n-1}(z_0) \neq 0$, ($\tilde{P}_{n-1}(z)$ has all its zeros in $|z| < 1$). Hence $s_n \neq 0$ implies $P_n(z)$ and $P_{n-1}(z)$ do not have a common zero.

12.21 Substituting $s_n = \rho^n, |\rho| < 1$ in (12-177) we get

$$\sqrt{1-\rho^{2n}}\, P_n(z) = P_{n-1}(z) - (z\rho)^n\, P_{n-1}^*(1/z^*)$$

Let $x = z\rho$ and

$$P_n(z) = P_n(x/\rho) \triangleq A_n(x)$$

so that the above iteration reduces to

$$\begin{aligned}\sqrt{1-\rho^{2n}}\, A_n(x) &= A_{n-1}(x) - x^n\, A_{n-1}^*(1/x^*) \\ &= A_{n-1}(x) - x\, \tilde{A}_{n-1}(x)\end{aligned}$$

From problem (12-20), the polynomial $A_n(x)$ has all its zeros on the unit circle (since $s_n = 1$). i.e.,

$$x_k = e^{j\theta_k} = z_k\, \rho.$$

Hence the zeros $z_k = (1/\rho)e^{j\theta_k}$ or $|z_k| = 1/\rho$. (The zeros of $P_n(z)$ lie on a circle of radius $1/\rho$).

12.22 The Levinson Polynomials $P_n(z)$ satisfy the recusion in (12-177). Define $s_n' = \lambda^n s_n$, $|\lambda| = 1$, and replacing s_n by s_n' and $P_n(z)$ by $P_n'(z)$ in (12-177) we get

$$\begin{aligned}P_n'(z) &= P_{n-1}'(z) - z s_n'\, \tilde{P}'_{n-1}(z) \\ &= P_{n-1}'(z) - (z\lambda)^n\, s_n\, P_{n-1}'^*(z)\,(1/z^*)\end{aligned}$$

Let $y = z\lambda$ and define $P_n'(y/\lambda) = A_n(y)$ so that the above recursion simplifies to

$$\begin{aligned}A_n(y) &= A_{n-1}(y) - y^n\, s_n\, A_{n-1}^*(1/y^*) \\ &= A_{n-1}(y) - y s_n\, A_{n-1}^*(y)\end{aligned}$$

and on comparing with (12-177), we notice that $A_n(y) = P_n(y) = P_n(\lambda z)$. Thus $P_n(\lambda z)$ represents the new set of Levinson Polynomials.

12.23 (a) In this case

$$S(\theta) = \left|H(e^{j\theta})\right|^2 = |1 - e^{j\theta}|^2 = 2 - e^{j\theta} - e^{-j\theta} = 2(1 - cos\theta)$$

so that $r_0 = 2, r_1 = -1, r_k = 0, |k| \geq 2$. Substituing these values into (9-196) and taking the determinant of the tridiagonal matrix $\mathbf{T}_n$ we obtain the recursion

$$|\mathbf{T}_n| = \Delta_n = 2\Delta_{n-1} - \Delta_{n-2}$$

where $\Delta_0 = 2, \Delta_1 = 3$. Let $D(z) = \sum_{n=0}^{\infty} \Delta_n \, z^n$ so that the above recursion gives

$$D(z) = \frac{2 - z}{(1 - z)^2} = \frac{1}{1 - z} + \frac{1}{(1 - z)^2} = \sum_{n=0}^{\infty} (n + 2) \, z^n$$

and hence we get

$$\Delta_n = n + 2, \qquad n \geq 0.$$

Using (12-192) and (9-196) we get

$$s_n = (-1)^{n-1} \, \frac{\Delta_n^{(1)}}{\Delta_{n-1}} = \frac{(-1)^{n-1}(-1)^n}{n + 1} = -\frac{1}{n + 1}, \quad k \geq 1.$$

(b) The new set of reflection coefficient $s'_k = -s_k$ switches around the Levinson Polynomials $P_n(z)$ and $Q(z)$, and hence it follows that they correspond to the positive-real function

$$Z'(z) = \frac{2}{1 - z}$$

which gives $r'_0 = 2, r'_k = 1, \, k \geq 1$.