CHAPTER 13

13-1
$$\hat{\underline{s}}(t - \tfrac{T}{2}) = a\,\underline{s}(t) + b\,\underline{s}(t - T)$$

$$\underline{s}(t - \tfrac{T}{2}) - [a\,\underline{s}(t) + b\,\underline{s}(t - T)] \perp \underline{s}(t),\ \underline{s}(t - T)$$

$$R(T/2) = a\,R(0) + b\,R(T)$$

$$a = b = \frac{R(T/2)}{R(0) + R(T)} = \frac{e^{-1/2}}{1 + e^{-1}}$$

$$R(T/2) = a\,R(T) + b\,R(0)$$

$$P = E\{[\underline{s}(t - \tfrac{T}{2}) - \hat{\underline{s}}(t - \tfrac{T}{2})]\,\underline{s}(t - \tfrac{T}{2})\}$$

$$= R(0) - aR(T/2) - bR(T/2) = R(0) - \frac{R^2(T/2)}{R(0) + R(T)} = \frac{I}{1 + e^{-1}}$$

13-2
$$\int_0^T \underline{s}(t)dt - [a\,\underline{s}(0) + b\,\underline{s}(T)] \perp \underline{s}(0),\ \underline{s}(T)$$

$$\int_0^T R(t)dt = aR(0) + bR(T) \qquad \int_0^T R(T - t)dt = aR(T) + bR(0)$$

The above two integrals are equal. Hence,

$$a = b = \frac{\int_0^T R(t)dt}{R(0) + R(T)}$$

13-3 $$\hat{\mathbf{s}}'(t) = a\,\mathbf{x}(t) + b\,\mathbf{x}(t-\tau)$$

$$\mathbf{s}'(t) - [a\,\mathbf{x}(t) + b\,\mathbf{x}(t-\tau)] \perp \mathbf{x}(t),\ \mathbf{x}(t-\tau)$$

$$R_{s'x}(0) = a\,R_{xx}(0) + b\,R_{xx}(\tau) \qquad R_{s'x}(0) = R_{s's}(0) = 0$$

$$R_{s'x}(\tau) = a\,R_{xx}(\tau) + b\,R_{xx}(0) \qquad R_{xx}(\tau) = R_{ss}(\tau) + R_{\nu\nu}(\tau)$$

For small τ

$$R_{s'x}(\tau) = R_{s's}(\tau) = R'_{ss}(\tau) \simeq \tau\,R''_{ss}(0) \qquad R_{xx}(\tau) \simeq R_{xx}(0) + \tau^2 R''_{xx}(0)/2$$

Hence,

$$a = -b + O(\tau^2) \qquad \tau R''_{ss}(0) = a\,\tau^2\,R''_{xx}(0)/2 + O(\tau^3)$$

13-4 It suffices to show that, for any m,

$$E\left\{\left[\mathbf{x}(t) - \sum_{n=-\infty}^{\infty} \frac{\sin(\sigma t - n\pi)}{\sigma t - n\pi}\,\mathbf{x}(nT)\right]\mathbf{x}(mT)\right\} = 0$$

The left side equals

$$R(t - mT) - \sum_{n=-\infty}^{\infty} \frac{\sin(\sigma t - n\pi)}{\sigma t - n\pi}\,R(nT - mT)$$

From the sampling theorem (10-140) it follows that this is zero because the Fourier transform

$$e^{-jmT}S(\omega)$$

of $R(t - mT)$ is zero for $|\omega| > \sigma$.

13-5 Since

$$\hat{E}\{\mathbf{x}(t+\lambda)|\mathbf{s}(t)\} = a\mathbf{s}(t) \qquad a = R(\lambda)/R(0)$$

it follows from the assumption that

$$\mathbf{s}(t+\lambda) - a\,\mathbf{s}(t) \perp \mathbf{s}(t-\tau)$$

Hence

$$R(\lambda+\tau) = \frac{R(\lambda)}{R(0)} R(\tau) \qquad (i)$$

The only continuous function satisfying the above is an exponential. This is easily shown if we assume that $R(\lambda)$ is differentiable for $\lambda > 0$. Differentiating (i) with respect to λ and setting $\lambda = 0^+$, we obtain

$$R'(\tau) + \alpha R(\tau) = 0 \qquad \alpha = -R'(0^+)/R(0) \qquad \tau > 0$$

This yields $R(\tau) = Ie^{-\alpha\tau}$ for $\tau > 0$.

13-6 Given:

$$E\{\mathbf{y}_n\} = 0 \qquad \mathbf{x}_n = \mathbf{y}_1 + \cdots + \mathbf{y}_n = \mathbf{x}_{n-1} + \mathbf{y}_n$$

Furthermore the RVs $\mathbf{y}_n$ are independent. Hence, $\mathbf{y}_n$ is independent of $\mathbf{x}_{n-1}, \ldots, \mathbf{x}_1$. This yields

$$E\{\mathbf{x}_n|\mathbf{x}_{n-1},\ldots,\mathbf{x}_1\} = E\{\mathbf{x}_{n-1} + \mathbf{y}_n|\mathbf{x}_{n-1},\ldots,\mathbf{x}_1\}$$

$$= E\{\mathbf{x}_{n-1}|\mathbf{x}_{n-1},\ldots,\mathbf{x}_1\} + E\{\mathbf{y}_n\} = \mathbf{x}_{n-1}$$

13-7 (a) If $\hat{E}\{\mathbf{x}_n | \mathbf{x}_{n-1},\dots,\mathbf{x}_1\} = \mathbf{x}_{n-1}$, then

$$= \mathbf{x}_n - \mathbf{x}_{n-1}, \perp \mathbf{x}_{n-1},\dots,\mathbf{x}_1$$

From this it follows that the RVs $\mathbf{y}_n = \mathbf{x}_n - \mathbf{x}_{n-1}$ are orthogonal and

$$\mathbf{x}_n = \mathbf{y}_n + \mathbf{x}_{n-1} = \mathbf{y}_n + \mathbf{y}_{n-1} + \dots + \mathbf{y}_1 \qquad \text{(i)}$$

Conversely, if (i) is true and the RVs $\mathbf{y}_n$ are orthogonal, then

$$\mathbf{x}_n - \mathbf{x}_{n-1} = \mathbf{y}_n \perp \mathbf{x}_{n-1},\dots,\mathbf{x}_1$$

(b) $E\{\mathbf{x}_n^2\} = E\{[(\mathbf{x}_n - \mathbf{x}_{n-1}) + \mathbf{x}_{n-1}]^2\}$

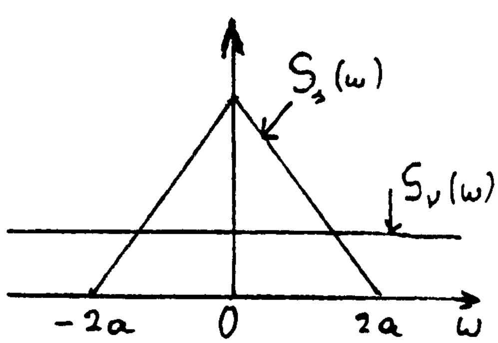
$$= E\{(\mathbf{x}_n - \mathbf{x}_{n-1})^2\} + E\{\mathbf{x}_{n-1}^2\} \geq E\{\mathbf{x}_{n-1}^2\}$$

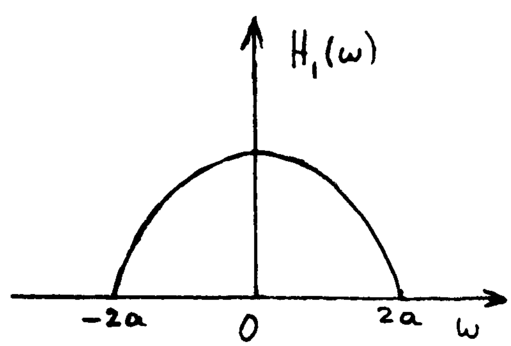
for any n.

13-8 The Fourier transform $S_s(\omega)$ of the function

$$R_s(\tau) = A\,\frac{\sin^2 a\tau}{\tau^2}$$

is a triangle as shown





And since $S_\nu(\omega) = N$, (13-16) yields

$$H_1(\omega) = \frac{S_s(\omega)}{S_s(\omega) + S_\nu(\omega)} = \frac{Aa\pi(1 - |\omega|/2a)}{Aa\pi(1 - |\omega|/2a) + N}$$

We show next that

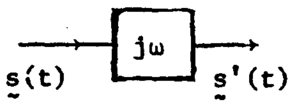
$$H_2(\omega) = j\omega H_1(\omega)$$

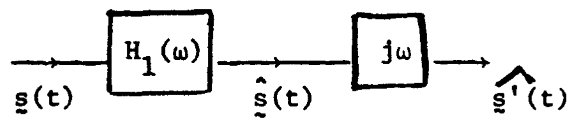
Proof

$$\varepsilon(t) = s(t) \quad \hat{s}'(t) \perp x(\xi) \quad \text{all } t,\xi. \quad \text{Hence } R_{\varepsilon x}(\tau) = 0.$$

This yields [see (9-131)]

$$R_{\varepsilon' x}(\tau) = R'_{\varepsilon x}(\tau) = 0, \text{ hence } s'(t) - \hat{s}'(t) \perp x(\xi)$$





In other words, the estimate of $s'(t)$ equals the derivative of the estimate $\hat{s}(t)$ of $s(t)$. This follows from Prob. 13-9 with $T(\omega) = j\omega$.

13-9 We wish to show that the estimator of

$$y(t) = \int_{-\infty}^{\infty} p(t-\alpha)s(\alpha)\,d\alpha \qquad p(t) \leftrightarrow T(\omega)$$

equals

$$\hat{y}(t) = \int_{-\infty}^{\infty} p(t-\alpha)\hat{s}(\alpha)\,d\alpha$$

where $\hat{s}(t)$ is the estimator of $s(t)$.

Proof. Clearly

$$E\{[s(t) - \hat{s}(t)]x(\xi)\} = 0 \qquad \text{all } t,\xi$$

Hence

$$E\{[y(t) - \hat{y}(t)]x(\xi)\}$$

$$= \int_{-\infty}^{\infty} p(t-\alpha)E\{[s(\alpha) - \hat{s}(\alpha)]x(\xi)\}\,d\alpha = 0$$

13-10 [See (13-46) and beyond]

(a) $S(s) = \frac{1}{s^4+1} = \frac{1}{(s^2+\sqrt{2}\,s+1)(s^2-\sqrt{2}\,s+1)}$

(b) $L(s) = \frac{1}{(s+\alpha)^2+\beta^2} \qquad \ell(t) = \frac{1}{\beta}\, e^{-\alpha t} \sin\beta t U(t) \qquad \alpha = \beta = \frac{1}{\sqrt{2}}$

(c) $h_i(t) = \frac{1}{\beta}\, e^{-\alpha\lambda}\, e^{-\alpha t} \sin\beta(t+\lambda) U(t)$

$$H_i(s) = \frac{1}{\beta}\, e^{-\alpha\lambda}\, \frac{(s+\alpha)\sin\beta\lambda + \beta\cos\beta\lambda}{(s+\alpha)^2+\beta^2}$$

$$H(s) = \frac{H_i(s)}{L(s)} = b_0 + b_1 s \qquad b_0 = e^{-\alpha\lambda}(\cos\beta\lambda + \frac{\alpha}{\beta}\sin\beta\lambda) \qquad b_1 = \frac{\sin\beta\lambda}{\beta}\, e^{-\alpha\lambda}$$

13-11 (a) The given equation is the Wiener-Hopf equation (13-40) for the prediction problem with $\lambda = \ln 2$. We can, therefore, use the method described after (13-46):

$$S(s) = \frac{3}{1-s^2} + \frac{22}{9-s^2} = \frac{49-25s^2}{(1-s^2)(9-s^2)}$$

$$L(s) = \frac{7+5s}{(1+s)(3+s)} \qquad \ell(t) = (e^{-t} + 4e^{-3t})U(t)$$

$$h_i(t) = (e^{-\ln 2}e^{-t} + 4e^{-3\ln 2}e^{-3t})U(t)$$

$$H_i(s) = \frac{1/2}{1+s} + \frac{4/8}{3+s} = \frac{2+s}{(1+s)(3+s)} \qquad H(s) = \frac{2+s}{7+5s}$$

(b)

$$H(s) = \frac{N(s)}{D(s)} \qquad \frac{N(s) - 2^s D(s)}{D(s)} L(s)L(-s) = Y(s)$$

Since $Y(s)$ is analytic for $\underline{Re}\, s \leq 0$, all roots of $D(s)$ must be cancelled by the zeros of $L(s)$, hence, $D(s) = 7+5s$. Similarly the poles $s = -1$ and $s = -3$ of $L(s)$ must be cancelled by the zeros of the term $N(s) - 2^s D(s)$. With $N(s) = As + B$, this yields

$$N(-1) - 2^{-1}D(-1) = -A + B - 2^{-1}(7-5) = 0 \qquad A = 1$$

$$N(-3) - 2^{-3}D(-3) = -3A + B - 2^{-3}(7-15) = 0 \qquad B = 2$$

$$H(s) = \frac{2+s}{7+5s}$$

(c) The Laplace transform of the function $R(\tau)$ in (a) equals

$$\frac{49-25s^2}{9-10s^2+s^4}$$

Hence (convolution theroem), the inverse transform $y(t)$ of $Y(s)$ equals

$$y(t) = \int_0^\infty h(\alpha)R(t-\alpha)d\alpha - R(t + \ln 2)$$

From the analyticity of $Y(s)$ for $\underline{Re}\, s < 0$ it follows that $y(t) = 0$ for $t > 0$. Therefore, (b) gives a direct method for solving the Wiener-Hopf equation (13-40).

13-12 (a) The given equation is identical with equation (13-22) for the prediction problem with $r=1$. We can, therefore, use the method in (13-31)-(13-33):

$$S(z) = \frac{3}{5-2w} + \frac{8}{10-3w} = \frac{70-25w}{(5-2w)(10-3w)} \qquad w = z+\frac{1}{z}$$

$$L(z) = \frac{a - bz^{-1}}{(2-z^{-1})(3-z^{-1})} \qquad a = \sqrt{30}+\sqrt{5} \simeq 7.75 \qquad b = \sqrt{30}-\sqrt{5} \simeq 3.25$$

$$\ell[0] = \frac{a}{6} \simeq 1.3 \qquad H(z) = 1 - \frac{\ell[0]}{L(z)} \simeq \frac{0.41\,z^{-1} - 0.167\,z^{-2}}{1-0.42z^{-1}}$$

(b)

$$H(z) = \frac{N(z)}{D(z)} \qquad \frac{N(z) - zD(z)}{D(z)} L(z)L(z^{-1}) = Y(z)$$

Since $Y(z)$ is analytic for $|z|<1$, all roots of $D(z)$ must be cancelled by the zeros of $L(z)$, hence, $D(z) = 1-0.42\,z^{-1}$. Similarly, the poles $z=1/2$ and $z=1/3$ of $L(z)$ must be cancelled by the zeros of the term $N(z)-zD(z)$. With $N(z) = A+Bz^{-1}$, this yields

$$N(\tfrac{1}{2}) - \tfrac{1}{2}D(\tfrac{1}{2}) \simeq A+2B-0.08 = 0 \qquad A \simeq 0.42$$

$$N(\tfrac{1}{3}) - \tfrac{1}{3}D(\tfrac{1}{3}) \simeq A+3B+0.09 = 0 \qquad B = -0.17$$

$$H(z) \simeq \frac{0.42-0.17z^{-1}}{1-0.42z^{-1}}$$

The z transform of the sequence R_m in (a) equals

$$\frac{70-25w}{6w^2-35w+50} \qquad w = z+z^{-1}$$

Hence, the inverse transform y_n of $Y(z)$ equals

$$y_n = \sum_{k=0}^{n} h_k R_{n-k} - R_{n+1}$$

From the analyticity of $Y(z)$ for $|z|<1$ it follows that $y_n = 0$ for $n \geq 0$. Therefore, (b) gives a direct method for solving (13-22).

13-13 A predictor is a stable function H(z) vanishing at ∞. Since $H(z) \to 0$ 1 as $z \to \infty$, we conclude that $E_N(z) = 1 - H(z) \to 1$ and $E_N(z)H_a(z) \to 1$ as $z \to \infty$. From this and (13-25) it follows that the difference $1 - E_N(z)H_a(z)$ is a predictor and the MS error equals

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} | E_N(e^{j\omega}) |^2 S(\omega)d\omega = P$$

because $|A_a(e^{j\omega})| = 1$.

13-14 As we know, if

$$\underset{\sim}{s}[n] = a_1\underset{\sim}{s}[n-1] + \cdots + a_m\underset{\sim}{s}[n-m] + \underset{\sim}{\varepsilon}[n]$$

where $\underset{\sim}{\varepsilon}[n]$ is white noise, then the one-step predictor of $\underset{\sim}{s}[n]$ equals

$$\hat{\underset{\sim}{s}}_1[n] = a_1\underset{\sim}{s}[n-1] + \cdots + a_m\underset{\sim}{s}[n-m]$$

We wish to show that the sum

$$\hat{\underset{\sim}{s}}_2[n] = a_1\hat{\underset{\sim}{s}}_1[n-1] + a_2\underset{\sim}{s}[n-2] + \cdots + a_m\underset{\sim}{s}[n-m]$$

is its two-step predictor. It suffices to show that

$$\underset{\sim}{s}[n] - \hat{\underset{\sim}{s}}_2[n] \perp \underset{\sim}{s}[n-k] \qquad k \geq 2$$

<u>Proof</u>

$$\underset{\sim}{s}[n] - \hat{\underset{\sim}{s}}_2[n] = a_1\left(\underset{\sim}{s}_1[n-1] - \hat{\underset{\sim}{s}}_1[n-1]\right) + \underset{\sim}{\varepsilon}[n]$$

This completes the proof because

$$\underset{\sim}{s}_1[n-1] - \hat{\underset{\sim}{s}}_1[n-1] \perp \underset{\sim}{s}(n-k], \quad k \geq 2 \text{ and } \underset{\sim}{\varepsilon}[n] \perp \underset{\sim}{s}[n-k] \quad k \geq 1.$$

13-15 The Nth order MS estimation error P_N equals [see (13-66)]

$$P_N = \frac{\Delta_{N+1}}{\Delta_N}$$

This tends to the MS estimation error in (13-34). Hence,

$$\lim_{N\to\infty} \ln P_N = \frac{1}{2\sigma}\int_{-\sigma}^{\sigma} \ln S(\omega)d\omega = \lim_{N\to\infty} \ln \frac{\Delta_{N+1}}{\Delta_N}$$

To complete the proofs, we use (14-129)

$$\lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \ln \frac{\Delta_{n+1}}{\Delta_n} = \lim_{N\to\infty} \ln \frac{\Delta_{N+1}}{\Delta_N}$$

and the result follows because

$$\frac{1}{N}\sum_{n=1}^{N} (\ln \Delta_{n+1} - \ln \Delta_n) = \frac{\ln\Delta_{N+1}}{N} - \frac{\ln\Delta_1}{N}$$

and the last term tends to zero as $N\to\infty$.

13-16 $P_0 = R[0] = 15$ $\qquad R[1] = 10$ $\qquad R[2] = 5$ $\qquad R[3] = 0$

We use Levinson's algorithm [see (13-67)]

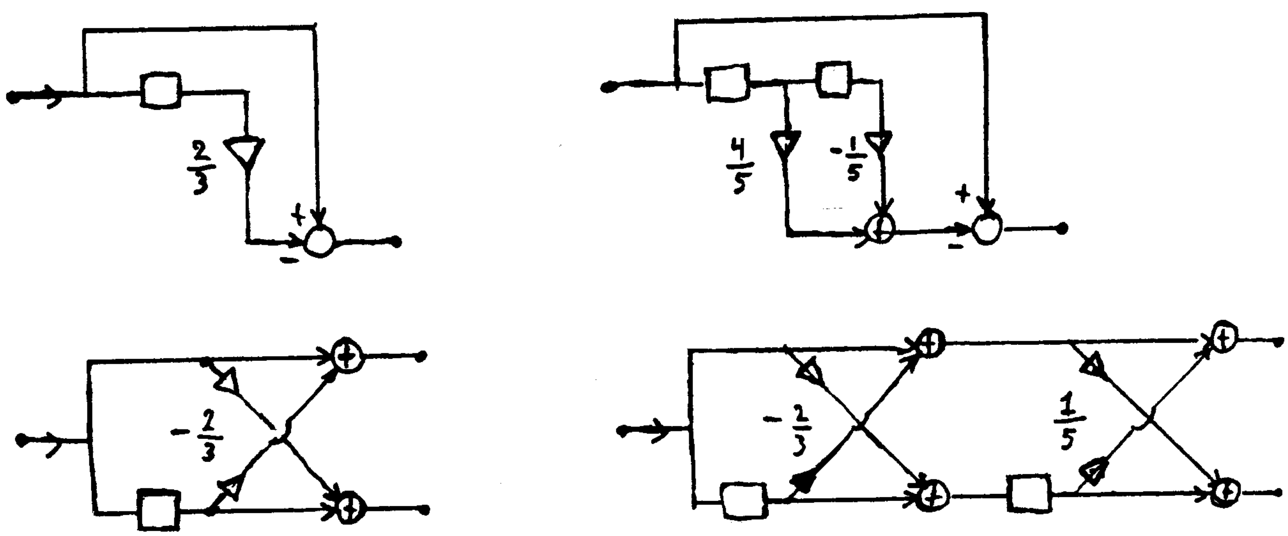
$$P_0K_1 = R[1] \qquad K_1 = a_1^1 = \frac{2}{3} \qquad P_1 = (1 - k_1^2)P_0 = \frac{25}{3}$$

$$P_1K_2 = R[2] - R[1]a_1^1 = -\frac{5}{3} \qquad K_2 = -\frac{1}{5}$$

$$a_1^2 = a_1^1 - K_2a_1^1 = \frac{4}{5} \qquad a_2^2 = -\frac{1}{5} \qquad P_2 = 8$$

$$P_2K_3 = R[3] - R[2]a_1^2 - R[1]a_2^2 \qquad K_3 = -\frac{1}{4}$$

$$a_1^3 = \frac{3}{4} \qquad a_2^3 = 0 \qquad a_3^3 = -\frac{1}{4} \qquad P_3 = 7.5$$



13-17 $P_0 = R[0] = 5 \qquad K_1 = 0.4 \qquad K_2 = 0.6 \qquad K_3 = 0.8$

$R[1] = P_0K_1 = 2 \qquad a_1^1 = 0.4 \qquad P_1 = 4.2$

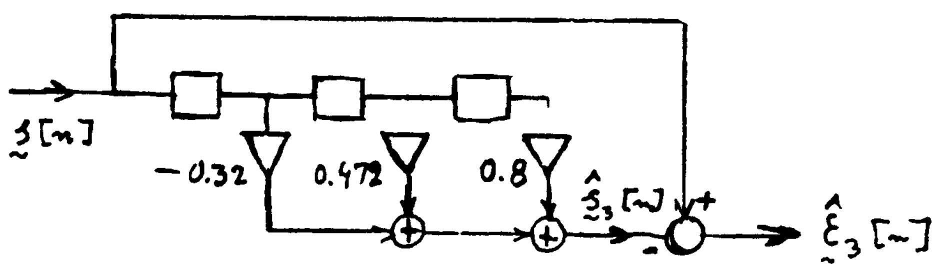
$R[2] = R[1]a_1^1 + P_1K_2 = 3.32$

$a_1^2 = 0.16 \qquad a_2^2 = 0.6 \qquad P_2 = 2.688$

$R[3] = R[2]a_1^2 + R[1]a_2^2 + P_2K_3 = 3.8816$

$a_1^3 = a_1^2 - K_3a_2^2 = -0.32 \qquad a_2^3 = a_2^2 - K_3a_1^2 = 0.472 \qquad a_3^3 = 0.8$

$a_3^3 = 0.8 \qquad P_3 = 0.968$



13-18
$$S_x(s) = \frac{4\lambda}{-s^2+4\lambda^2} + N = \frac{N(-s^2+c^2)}{-s^2+4\lambda^2}$$

$$\Gamma_x(s) = \frac{s+2\lambda}{\sqrt{N}\,(s+c)} \qquad c = 2\lambda\sqrt{1+\frac{1}{\lambda N}}$$

and (13-104) yields

$$H_x(s) = \frac{c-2\lambda}{s+c} \qquad h_x(t) = (c-2\lambda)e^{-ct}U(t)$$

13-19 (a) $\hat{\varepsilon}_{N+m}[n+m] \perp \underset{\sim}{s}[n-k] \qquad k = -m+1, \ldots, 0, \ldots, N$

$$\hat{\varepsilon}_N[n] = \underset{\sim}{s}[n] - a_1 \underset{\sim}{s}[n-1] - \cdots - a_N \underset{\sim}{s}[n-N]$$

(b) $\check{\underset{\sim}{\varepsilon}}_{N+m}[n-m] \perp \underset{\sim}{s}[n+k] \qquad k = -m+1, \ldots, 0, \ldots, N$

$$\check{\varepsilon}_N[n] = \underset{\sim}{s}[n] - a_1 \underset{\sim}{s}[n+1] - \cdots - a_N \underset{\sim}{s}[n+N]$$

(c) $\check{\underset{\sim}{\varepsilon}}_{N+m}[n-N-m] \perp \underset{\sim}{s}[n+k] \qquad k = -N-m+1, \ldots, -N, \ldots, 0$

$$\hat{\underset{\sim}{\varepsilon}}_N[n] = \underset{\sim}{s}[n] - a_1 \underset{\sim}{s}[n-1] - \cdots - a_N \underset{\sim}{s}[n-N]$$

13-20

$$S_s(\omega) = \frac{2}{\omega^2 + 0.04} \qquad S_x(\omega) = \frac{5\omega^2 + 2.2}{\omega^2 + 0.04} \qquad L_x(s) = \sqrt{5}\,\frac{s+0.66}{s+0.2}$$

(a) From (13-16)

$$H(\omega) = \frac{2}{5\omega^2 + 2.2}$$

(b) From (13-104)

$$H_x(s) = 1 - \sqrt{5}\,\Gamma_x(s) = \frac{0.46}{s+0.66}$$

(c) Using (13-48)

$$L_s(s) = \frac{\sqrt{2}}{s+0.2} \qquad \ell(t) = \sqrt{2}\, e^{-0.2t}\, U(t)$$

$$h_i(t) = \sqrt{2}\, e^{-0.2(t+2)} U(t) \qquad H_i(s) = \frac{\sqrt{2}\, e^{-0.4}}{s+0.2}$$

$$H(s) = e^{-0.4} \qquad \hat{\underset{\sim}{s}}(t+2) = e^{-0.4} \underset{\sim}{s}(t)$$

(d) [see (13-99) and beyond]

$$S_{sx}(s) = \frac{1}{0.04 - s^2} \qquad \Gamma_x(s) = \frac{s+0.2}{\sqrt{5}(s+0.66)} \qquad S_{si_x}(s) = \sqrt{20}\,\frac{0.66-s}{s+0.2}$$

$$R_{si_x}(\tau) = \sqrt{20}\left[\delta(\tau) + \frac{0.86}{s+0.2}\right] \qquad h_{i_x}(\tau) = 0.86\sqrt{20}\; e^{-0.2(t+2)}U(t)$$

$$H_{i_x}(s) = \frac{0.86\ 20\ e^{-0.4}}{s+0.2} \qquad H_x(s) = \frac{1.72\ e^{-0.4}}{s+0.66}$$

13-21 As in Example 13-2 with $N_0 = 1.8$, $N = 5$, $a = 0.8$

$$S_s(z) = \frac{1.8}{(1-0.8\ z^{-1})(1-0.8z)} \qquad L_s(z) = \frac{\sqrt{1.8}}{1-0.8\ z^{-1}}$$

$$S_x(z) = \frac{8(1-0.5\ z^{-1})(1-0.5z)}{(1-0.8\ z^{-1})(1-0.8z)} \qquad L_x(z) = \frac{\sqrt{8}(1-0.5\ z^{-1}}{1-0.8\ z^{-1}}$$

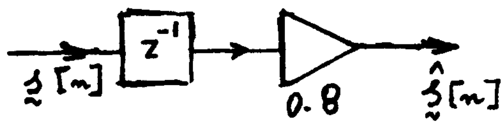
(a)
$$H(z) = \frac{9}{40(1-0.52\ z^{-1})(1-0.5z)} \qquad h[n] = 2 \times 2^{-|n|}$$

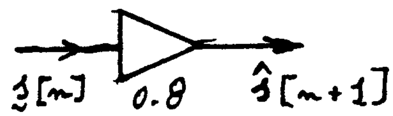
(b) From (13-114) with $\ell_x[0] = \sqrt{8}$

$$H_x(z) = \frac{3/8}{1-0.5\ z^{-1}} \qquad h_x[n] = \frac{3}{8} \times 0.5\ U[n]$$

(c) From (13-33) with $\ell[0] = \sqrt{1.8}$

$$H\ (z) = 1 - \frac{\ell[0]}{L_s(z)} = 0.8\ z^{-1}$$





(d) The power spectrum of the estimate $\hat{\mathbf{s}}_0[n]$ of $\mathbf{s}[n]$ obtained in (b) equals

$$S_{\hat{s}_0}(z) = S_x(z)H_x(z)H_x(z^{-1}) = \frac{9/8}{(1-0.8\ z^{-1})(1-0.8\ z)}$$

Hence, $$L_{\hat{s}_0}(z) = \frac{\sqrt{9/8}}{1-0.8\ z^{-1}}$$

Therefore, the pure predictor of $\hat{s}_0[n]$ equals [see (13-33)]

$$\hat{H}^1(z) = 1 - \frac{\ell[0]}{L(z)} = 0.8\ z^{-1}$$

And (13-117) yields

$$H_x^1(z) = H_x^0(z)\hat{H}_1(z) = \frac{0.3\,z^{-1}}{1-0.5\ z^{-1}}$$

13-22

$$R_s[m] = 5 \times 0.8^{|m|} \longleftrightarrow \frac{1.8}{(1-0.8\, z^{-1})(1-0.8\, z)}$$

Hence, as in (13-135) with $V_n = 1.8$, $N_n = 5$. And (13-143) yields

$$F_n = 0.64\, F_{n-1} + V_n\, G_{n-1} \qquad F_0 = V_0 N_0 = 9$$

$$5\, G_n = 0.64\, F_{n-1} + 6.5\, G_{n-1} \qquad G_0 = V_0 + N_0 = 6.8$$

Solving, we obtain

$$F_n = 12(1.6)^n - 3(0.4)^n \qquad G_n = 6.4(1.6)^n + 0.4(0.4)^n$$

$$P_n = \frac{F_n}{G_n} \xrightarrow[n \to \infty]{} \frac{12}{6.4} = 1.875$$

This agrees with Prob. 13-21c because the MS error of the Wiener filter equals

$$P = R_s(0) - \sum_{k=0}^{\infty} R_s[k]h_x[k] = 5 - \sum_{k=0}^{\infty} 5 \times 0.8^m \times \frac{3}{8} \times 0.5^m = 1.875$$

13-23 $R_s(\tau) = 5\,e^{-0.2|\tau|}$ $\qquad R(\tau) = \frac{10}{3}\,\delta(\tau)$

$$S_s(\omega) = \frac{2}{\omega^2 + 0.2^2} \qquad A(t) = 0.2 \qquad V(t) = 2 \qquad N(t) = \frac{10}{3}$$

From (13-159)

$$F'(t) = 0.2\,F(t) + 2G(t) \qquad G'(t) = 0.3\,F(t) + 0.2G(t)$$

Case 1. If $\underset{\sim}{s}(0) = 0$, then $P(0) = F(0) = 0$, $G(0) = 1$
Solving, we obtain

$$P(t) = \frac{F(t)}{G(t)} = \frac{1.25\,e^{0.8t} - 1.25\,e^{-0.8t}}{0.625\,e^{0.8t} + 0.375\,e^{-0.8t}}$$

Case 2. If $\underset{\sim}{s}(t)$ is stationary, then $F(0) = P(0) = R_s(0) = 5$

$$P(t) = \frac{F(t)}{G(t)} = \frac{5\,e^{0.8t} + 3\,e^{-0.8t}}{2.5\,e^{0.8t} - 0.9\,e^{-0.8t}}$$

13-24 The sequences $\hat{q}_N[n]$ and $\check{q}_N[n]$ are the responses of the filters

$$\hat{E}_N(z) = 1 - \sum_{k=1}^{N} a_k^N z^{-k} \qquad \check{E}_N(z) = z^{-N}\hat{E}_N(1/z)$$

respectively, with input R[m] (see Fig. 13-11a). Hence,

$$\hat{q}_N[m] = R[m] - \sum_{k=1}^{N} R[m-k]a_k^N$$

$$\check{q}_N[m] = \hat{q}_N[N-m] = R[m-N] - \sum_{k=1}^{N} R[m-N+k]a_k^N$$

From this and the Yule-Walker equation (13-65) it follows that

$$\hat{q}_N[m] = \check{q}_N[N-m] = 0 \text{ for } 1 \le m \le N-1$$

$$\hat{q}_N[0] = \check{q}_N[N] = P_N$$

This completes the proof.
