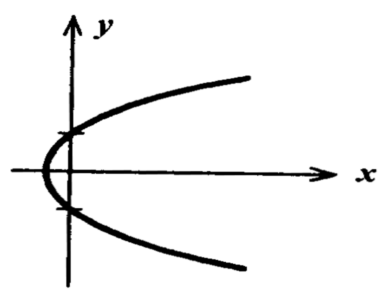
CHAPTER 2 REVIEW, page 158

1. a. $9 - x \geq 0$ gives $x \leq 9$ and the domain is $(-\infty, 9]$.
   b. $2x^2 - x - 3 = (2x - 3)(x + 1)$, and $x = 3/2$ or $x = -1$.
   Since the denominator of the given expression is zero at these points, we see that the domain of f cannot include these points and so the domain of f is
   $(-\infty, -1) \cup (-1, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$.

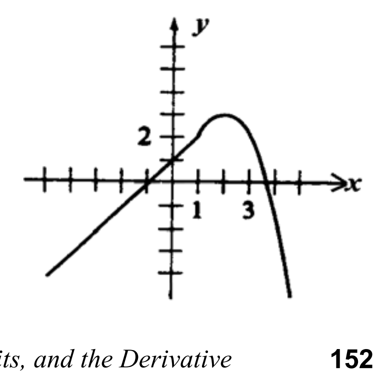
2. a. $f(-2) = 3(4) + 5(-2) - 2 = 0$.
   b. $f(a + 2) = 3(a + 2)^2 + 5(a + 2) - 2 = 3a^2 + 12a + 12 + 5a + 10 - 2$
   $= 3a^2 + 17a + 20$.
   c. $f(2a) = 3(2a)^2 + 5(2a) - 2 = 12a^2 + 10a - 2$.
   d. $f(a + h) = 3(a + h)^2 + 5(a + h) - 2 = 3a^2 + 6ah + 3h^2 + 5a + 5h - 2$.

3. a.



   b. For each value of $x > 0$, there are two values of y. We conclude that y is not a function of x. Equivalently, the function fails the vertical line test.
   c. Yes. For each value of y, there is only 1 value of x.

4.



5. a. $f(x)g(x) = \dfrac{2x+3}{x}$

b. $\dfrac{f(x)}{g(x)} = \dfrac{1}{x(2x+3)}$

c. $f(g(x)) = \dfrac{1}{2x+3}$.

d. $g(f(x)) = 2\left(\dfrac{1}{x}\right) + 3 = \dfrac{2}{x} + 3.$

6. $\lim_{x\to 0}(5x-3) = 5(0) - 3 = -3.$

7. $\lim_{x\to 1}(x^2+1) = [(1)^2+1] = 1+1 = 2.$

8. $\lim_{x\to -1}(3x^2+4)(2x-1) = (3+4)(-2-1) = -21.$

9. $\lim_{x\to 3}\dfrac{x-3}{x+4} = \dfrac{3-3}{3+4} = 0.$

10. $\lim_{x\to 2}\dfrac{x+3}{x^2-9} = \dfrac{2+3}{4-9} = -1.$

11. $\lim_{x\to -2}\dfrac{x^2-2x-3}{x^2+5x+6}$ does not exist. (The denominator is 0 at $x = -2$.)

12. $\lim_{x\to 3}\sqrt{2x^3-5} = \sqrt{2(27)-5} = 7.$

13. $\lim_{x\to 3}\dfrac{4x-3}{\sqrt{x+1}} = \dfrac{12-3}{\sqrt{4}} = \dfrac{9}{2}.$

14. $\lim_{x\to 1^+}\dfrac{x-1}{x(x-1)} = \lim_{x\to 1^+}\dfrac{1}{x} = 1.$

15. $\lim_{x\to 1^-}\dfrac{\sqrt{x}-1}{x-1} = \lim_{x\to 1^-}\dfrac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^-}\dfrac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^-}\dfrac{1}{\sqrt{x}+1} = \dfrac{1}{2}.$

16. $\lim_{x\to\infty}\dfrac{x^2}{x^2-1} = \lim_{x\to\infty}\dfrac{1}{1-\dfrac{1}{x^2}} = 1.$

17. $\lim_{x\to -\infty}\dfrac{x+1}{x} = \lim_{x\to -\infty}\left(1+\dfrac{1}{x}\right) = 1.$

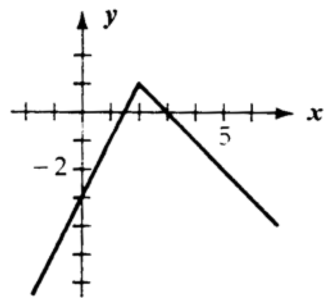
18. $\lim_{x\to\infty}\dfrac{3x^2+2x+4}{2x^2-3x+1} = \lim_{x\to\infty}\dfrac{3+\dfrac{2}{x}+\dfrac{4}{x^2}}{2-\dfrac{3}{x}+\dfrac{1}{x^2}} = \dfrac{3}{2}.$

19. $\lim_{x\to -\infty}\dfrac{x^2}{x+1} = \lim_{x\to -\infty} x\cdot\dfrac{1}{1+\dfrac{1}{x}} = -\infty$, so the limit does not exist.

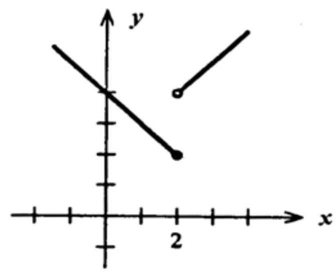
20. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(2x-3)$
$= 2(2)-3 = 4-3 = 1.$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(-x+3)$
$= -2+3 = 1.$

Therefore, $\lim_{x\to 2} f(x) = 1.$



21. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x+2) = 4;$
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(4-x) = 2.$
Therefore, $\lim_{x\to 2} f(x)$ does not exist.



22. The function is discontinuous at $x = 2$.

23. Since the denominator
$4x^2 - 2x - 2 = 2(2x^2 - x - 1) = 2(2x + 1)(x - 1) = 0$
if $x = -1/2$ or 1, we see that f is discontinuous at these points.

24. Because $\lim_{x\to -1} f(x) = \lim_{x\to -1} \dfrac{1}{(x+1)^2} = \infty$ (does not exist), we see that f is discontinuous at $x = -1$.

25. The function is discontinuous at $x = 0$.

26. a. Let $f(x) = x^2 + 2$. Then the average rate of change of y over [1,2] is
$$\frac{f(2)-f(1)}{2-1} = \frac{(4+2)-(1+2)}{1} = 3.$$
Over [1,1.5]:
$$\frac{f(1.5)-f(1)}{1.5-1} = \frac{(2.25+2)-(1+2)}{0.5} = 2.5.$$

Over [1,1.1]:

$$\frac{f(1.1)-f(1)}{1.1-1}=\frac{(1.21+2)-(1+2)}{0.1}=2.1.$$

b. Computing $f'(x)$ using the four-step process., we obtain

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{h(2x\ +\ h)}{h}=\lim_{h\to 0}(2x+h)=2.$$

Therefore, the instantaneous rate of change of f at $x = 1$ is $f'(1) = 2$, or 2 units/unit change in x.

27. $f(x)=3x+5$. Using the four-step process, we find

Step 1 $f(x+h) = 3(x+h)+5 = 3x+3h+5$

Step 2 $f(x+h)-f(x) = 3x+3h+5-3x-5 = 3h$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=\dfrac{3h}{h}=3.$

Step 4 $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}(3)=3.$

28. $f(x)=-\dfrac{1}{x}$. Now, using the four-step process, we obtain

Step 1 $f(x+h) = -\dfrac{1}{x+h}.$

Step 2 $f(x+h)-f(x) = -\dfrac{1}{x+h}-\left(-\dfrac{1}{x}\right)=-\dfrac{1}{x+h}+\dfrac{1}{x}=\dfrac{h}{x(x+h)}.$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=-\dfrac{1}{x(x+h)}.$

Step 4 $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{1}{x(x+h)}=\dfrac{1}{x^2}.$

29. $f(x)=3x+5$. We use the four-step process to obtain

Step 1 $f(x+h) = \frac{3}{2}(x+h)+5=\frac{3}{2}x+\frac{3}{2}h+5.$

Step 2 $f(x+h)-f(x) = \frac{3}{2}x+\frac{3}{2}h+5-\frac{3}{2}x-5=\frac{3}{2}h.$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=\dfrac{3}{2}.$

Step 4 $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\dfrac{3}{2}=\dfrac{3}{2}.$

Therefore, the slope of the tangent line to the graph of the function *f* at the point (-2,2) is 3/2. To find the equation of the tangent line to the curve at the point (-2,2), we use the point–slope form of the equation of a line obtaining

$$y-2=\tfrac{3}{2}[x-(-2)] \quad \text{or} \quad y=\tfrac{3}{2}x+5.$$

30. $f(x)=-x^2$. We use the four-step process to find $f'(x)$.

Step 1 $f(x+h)=-(x+h)^2=-x^2-2xh-h^2$

Step 2 $f(x+h)-f(x)=(-x^2-2xh-h^2)-(-x^2)=-2xh-h^2=h(-2x-h).$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=-2x-h$

Step 4 $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}(-2x-h)=-2x.$

The slope of the tangent line is $f'(2)=-2(2)=-4.$ An equation of the tangent line is $y-(-4)=-4(x-2)$, or $y=-4x+4$.

31. a. *f* is continuous at $x=a$ because the three conditions for continuity are satisfied at $x=a$; that is,

i. $f(x)$ is defined *ii.* $\lim\limits_{x\to a}f(x)$ exists *iii.* $\lim\limits_{x\to a}f(x)=f(a)$

b. *f* is not differentiable at $x=a$ because the graph of *f* has a kink at $x=a$.

32. $S(4)=6000(4)+30{,}000=54{,}000$.

33. a. The line passes through (0, 2.4) and (5, 7.4) and has slope $m=\dfrac{7.4-2.4}{5-0}=1.$

Letting *y* denote the sales, we see that an equation of the line is

$$y-2.4=1(t-0), \text{ or } y=t+2.4.$$

We can also write this in the form $S(t)=t+2.4$.

b. The sales in 2002 were $S(3)=3+2.4=5.4$, or \$5.4 million.

34. a. $C(x)=6x+30{,}000$ b. $R(x)=10x$

c. $P(x) = R(x) - C(x) = 10x - (6x + 30{,}000) = 4x - 30{,}000$.
d. $P(6000) = 4(6000) - 30{,}000 = -6000$, or a loss of \$6000.
$P(8000) = 4(8000) - 30{,}000 = 2000$, or a profit of \$2000.
$P(12{,}000) = 4(12{,}000) - 30{,}000 = 18{,}000$, or a profit of \$18,000.

35. Substituting the first equation into the second yields
$$3x - 2(\tfrac{3}{4}x + 6) + 3 = 0 \quad \text{or} \quad \tfrac{3}{2}x - 12 + 3 = 0$$
or $x = 6$. Substituting this value of x into the first equation then gives $y = 21/2$, so the point of intersection is $(6, \tfrac{21}{2})$.

36. We solve $12x + 20{,}000 = 20x$ giving $8x = 20{,}000$, or $x = 2500$. Therefore, $y = R(2500) = 20(2500) = 50{,}000$. Therefore, the break-even point is $(2500, 50{,}000)$.

37. We solve the system
$$3x + p - 40 = 0$$
$$2x - p + 10 = 0.$$
Adding these two equations, we obtain $5x - 30 = 0$, or $x = 6$. So,
$$p = 2x + 10 = 12 + 10 = 22.$$
Therefore, the equilibrium quantity is 6000 and the equilibrium price is \$22.

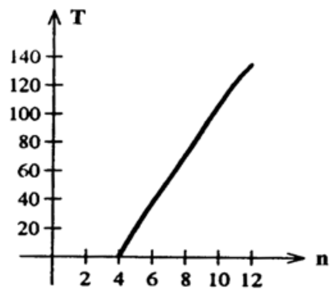
38. The child should receive $D(35) = \dfrac{500(35)}{150} = 117$, or 117 mg.

39. $R(30) = -\tfrac{1}{2}(30)^2 + 30(30) = 450$, or \$45,000.

40. $N(0) = 200(4 + 0)^{1/2} = 400$, and so there are 400 members initially.
$N(12) = 200(4 + 12)^{1/2} = 800$, and so there are 800 members a year later.

41. $T = f(n) = 4n\sqrt{n-4}$.
$$f(4) = 0,\ f(5) = 20\sqrt{1} = 20,\ f(6) = 24\sqrt{2} \approx 33.9,\ f(7) = 28\sqrt{3} \approx 48.5,$$
$$f(8) = 32\sqrt{4} = 64,\ f(9) = 36\sqrt{5} \approx 80.5,\ f(10) = 40\sqrt{6} \approx 98,$$
$$f(11) = 44\sqrt{7} \approx 116 \quad \text{and} \quad f(12) = 48\sqrt{8} \approx 135.8.$$

The graph of *f* follows:

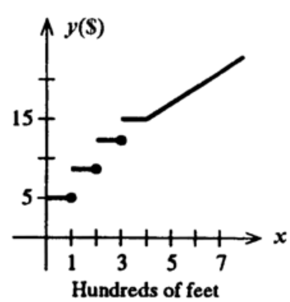


42. We solve

$$-1.1x^2+1.5x+40=0.1x^2+0.5x+15$$
$$1.2x^2-x-25=0$$
$$12x^2-10x-250=0$$
$$6x^2-5x-125=0;\quad (x-5)(6x+25)=0.$$

Therefore, $x = 5$. Substituting this value of x into the second supply equation, we have $p = 0.1(5)^2 + 0.5(5) + 15 = 20$. So the equilibrium quantity is 5000 and the equilibrium price is \$20.

43.



44. $\lim_{x\to\infty}\overline{C}(x)=\lim_{x\to\infty}\left(20+\frac{400}{x}\right)=20.$ As the level of production increases without bound, the average cost of producing the commodity steadily decreases and approaches \$20 per unit.