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1. $f'(x) = \frac{d}{dx}(3x^5 - 2x^4 + 3x^2 - 2x + 1) = 15x^4 - 8x^3 + 6x - 2.$

2. $f'(x) = \frac{d}{dx}(4x^6 + 2x^4 + 3x^2 - 2) = 24x^5 + 8x^3 + 6x.$

3. $g'(x) = \frac{d}{dx}(-2x^{-3} + 3x^{-1} + 2) = 6x^{-4} - 3x^{-2}.$

4. $f'(t) = \frac{d}{dt}(2t^2 - 3t^3 - t^{-1/2}) = 4t - 9t^2 + \frac{1}{2}t^{-3/2}.$

5. $g'(t) = \frac{d}{dt}\ (2t^{-1/2} + 4t^{-3/2} + 2) = -t^{-3/2} - 6t^{-5/2}.$

6. $h'(x) = \frac{d}{dx}\left(x^2 + \frac{2}{x}\right) = 2x - \frac{2}{x^2}.$

7. $f'(t) = \frac{d}{dt}(t + 2t^{-1} + 3t^{-2}) = 1 - 2t^{-2} - 6t^{-3} = 1 - \frac{2}{t^2} - \frac{6}{t^3}.$

8. $g'(s) = \frac{d}{ds}(2s^2 - 4s^{-1} + 2s^{-1/2}) = 4s + 4s^{-2} - s^{-3/2} = 4s + \frac{4}{s^2} - \frac{1}{s^{3/2}}.$

9. $h'(x) = \frac{d}{dx}(x^2 - 2x^{-3/2}) = 2x + 3x^{-5/2} = 2x + \frac{3}{x^{5/2}}.$

10. $f(x) = \frac{x+1}{2x-1}.\quad f'(x) = \frac{(2x-1)(1) - (x+1)(2)}{(2x-1)^2} = -\frac{3}{(2x-1)^2}.$

11. $g(t) = \frac{t^2}{2t^2+1}.$

$$g'(t) = \frac{(2t^2+1)\frac{d}{dt}(t^2) - t^2\frac{d}{dt}(2t^2+1)}{(2t^2+1)^2}$$

$$= \frac{(2t^2+1)(2t) - t^2(4t)}{(2t^2+1)^2} = \frac{2t}{(2t^2+1)^2}.$$

12. $h(t) = \frac{t^{1/2}}{t^{1/2}+1}.\quad h'(t) = \frac{(t^{1/2}+1)\frac{1}{2}t^{-1/2} - t^{1/2}(\frac{1}{2}t^{-1/2})}{(t^{1/2}+1)^2} = \frac{1}{2\sqrt{t}(\sqrt{t}+1)^2}.$

13. $f(x) = \dfrac{\sqrt{x}-1}{\sqrt{x}+1} = \dfrac{x^{1/2}-1}{x^{1/2}+1}$.

$$f'(x) = \frac{(x^{1/2}+1)(\frac{1}{2}x^{-1/2}) - (x^{1/2}-1)(\frac{1}{2}x^{-1/2})}{(x^{1/2}+1)^2}$$
$$= \frac{\frac{1}{2}+\frac{1}{2}x^{-1/2} - \frac{1}{2}+\frac{1}{2}x^{-1/2}}{(x^{1/2}+1)^2} = \frac{x^{-1/2}}{(x^{1/2}+1)^2} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}.$$

14. $f(t) = \dfrac{t}{2t^2+1}$. $f'(t) = \dfrac{(2t^2+1)(1) - t(4t)}{(2t^2+1)^2} = \dfrac{1-2t^2}{(2t^2+1)^2}$.

15. $f(x) = \dfrac{x^2(x^2+1)}{x^2-1}$.

$$f'(x) = \frac{(x^2-1)\frac{d}{dx}(x^4+x^2) - (x^4+x^2)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$
$$= \frac{(x^2-1)(4x^3+2x) - (x^4+x^2)(2x)}{(x^2-1)^2}$$
$$= \frac{4x^5+2x^3-4x^3-2x-2x^5-2x^3}{(x^2-1)^2}$$
$$= \frac{2x^5-4x^3-2x}{(x^2-1)^2} = \frac{2x(x^4-2x^2-1)}{(x^2-1)^2}.$$

16. $f'(x) = 3(2x^2+x)^2 \dfrac{d}{dx}(2x^2+x) = 3(4x+1)(2x^2+x)^2$.

17. $f(x) = (3x^3-2)^8; f'(x) = 8(3x^3-2)^7(9x^2) = 72x^2(3x^3-2)^7$.

18. $h'(x) = 5(x^{1/2}+2)^4 \dfrac{d}{dx}x^{1/2} = 5(x^{1/2}+2)^4 \cdot \dfrac{1}{2}x^{-1/2} = \dfrac{5(\sqrt{x}+2)^4}{2\sqrt{x}}$.

19. $f'(t) = \dfrac{d}{dt}(2t^2+1)^{1/2} = \dfrac{1}{2}(2t^2+1)^{-1/2}\dfrac{d}{dt}(2t^2+1)$

$$= \frac{1}{2}(2t^2+1)^{-1/2}(4t) = \frac{2t}{\sqrt{2t^2+1}}.$$

20. $g(t)=\sqrt[3]{1-2t^3}=(1-2t^3)^{1/3}$.

$g'(t)=\frac{1}{3}(1-2t^3)^{-2/3}(-6t^2)=-2t^2(1-2t^3)^{-2/3}$.

21. $s(t)=(3t^2-2t+5)^{-2}$

$$s'(t)=-2(3t^2-2t+5)^{-3}(6t-2)=-4(3t^2-2t+5)^{-3}(3t-1)$$
$$=-\frac{4(3t-1)}{(3t^2-2t+5)^3}.$$

22. $f(x)=(2x^3-3x^2+1)^{-3/2}$.

$f'(x)=-\frac{3}{2}(2x^3-3x^2+1)^{-5/2}(6x^2-6x)=-9x(x-1)(2x^3-3x^2+1)^{-5/2}$

23. $h(x)=\left(x+\dfrac{1}{x}\right)^2=(x+x^{-1})^2$.

$$h'(x)=2(x+x^{-1})(1-x^{-2})=2\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$$
$$=2\left(\frac{x^2+1}{x}\right)\left(\frac{x^2-1}{x^2}\right)=\frac{2(x^2+1)(x^2-1)}{x^3}.$$

24. $h(x)=\dfrac{1+x}{(2x^2+1)^2}$.

$$h'(x)=\frac{(2x^2+1)^2(1)-(1+x)2(2x^2+1)(4x)}{(2x^2+1)^4}$$
$$=\frac{(2x^2+1)[(2x^2+1)-8x-8x^2]}{(2x^2+1)^4}=-\frac{6x^2+8x-1}{(2x^2+1)^3}.$$

25. $h'(t)=(t^2+t)^4\dfrac{d}{dt}(2t^2)+2t^2\dfrac{d}{dt}(t^2+t)^4$

$$=(t^2+t)^4(4t)+2t^2\cdot 4(t^2+t)^3(2t+1)$$
$$=4t(t^2+t)^3[(t^2+t)+4t^2+2t]=4t^2(5t+3)(t^2+t)^3.$$

26. $f(x)=(2x+1)^3(x^2+x)^2$.

$$f'(x) = (2x+1)^3 \cdot 2(x^2+x)(2x+1) + (x^2+x)^2 3(2x+1)^2(2)$$
$$= 2(2x+1)^2(x^2+x)[(2x+1)^2 + 3(x^2+x)]$$
$$= 2(2x+1)^2(x^2+x)(7x^2+7x+1).$$

27. $g(x) = x^{1/2}(x^2-1)^3.$

$$g'(x) = \frac{d}{dx}[x^{1/2}(x^2-1)^3] = x^{1/2} \cdot 3(x^2-1)^2(2x) + (x^2-1)^3 \cdot \tfrac{1}{2}x^{-1/2}$$
$$= \tfrac{1}{2}x^{-1/2}(x^2-1)^2[12x^2 + (x^2-1)]$$
$$= \frac{(13x^2-1)(x^2-1)^2}{2\sqrt{x}}.$$

28. $f(x) = \dfrac{x}{(x^3+2)^{1/2}}.$

$$f'(x) = \frac{(x^3+2)^{1/2}(1) - x \cdot \tfrac{1}{2}(x^3+2)^{-1/2} \cdot 3x^2}{x^3+2}$$
$$= \frac{\tfrac{1}{2}(x^3+2)^{-1/2}[2(x^3+2) - 3x^3]}{x^3+2} = \frac{4-x^3}{2(x^3+2)^{3/2}}.$$

29. $h(x) = \dfrac{(3x+2)^{1/2}}{4x-3}.$

$$h'(x) = \frac{(4x-3)\tfrac{1}{2}(3x+2)^{-1/2}(3) - (3x+2)^{1/2}(4)}{(4x-3)^2}$$
$$= \frac{\tfrac{1}{2}(3x+2)^{-1/2}[3(4x-3) - 8(3x+2)]}{(4x-3)^2} = -\frac{12x+25}{2\sqrt{3x+2}(4x-3)^2}.$$

30. $f(t) = \dfrac{(2t+1)^{1/2}}{(t+1)^3}.$

$$f'(t) = \frac{(t+1)^3 \tfrac{1}{2}(2t+1)^{-1/2}(2) - (2t+1)^{1/2} \cdot 3(t+1)^2(1)}{(t+1)^6}$$
$$= \frac{(2t+1)^{-1/2}(t+1)^2[(t+1) - 3(2t+1)]}{(t+1)^6} = -\frac{5t+2}{\sqrt{2t+1}(t+1)^4}.$$

31. $f(x) = 2x^4 - 3x^3 + 2x^2 + x + 4.$

$$f'(x) = \frac{d}{dx}(2x^4 - 3x^3 + 2x^2 + x + 4) = 8x^3 - 9x^2 + 4x + 1.$$

$$f''(x) = \frac{d}{dx}(8x^3 - 9x^2 + 4x + 1) = 24x^2 - 18x + 4 = 2(12x^2 - 9x + 2).$$

32. $g(x) = x^{1/2} + x^{-1/2}$. $g'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

$$g''(x) = -\tfrac{1}{4}x^{-3/2} + \tfrac{3}{4}x^{-5/2} = -\frac{1}{4x^{3/2}} + \frac{3}{4x^{5/2}}.$$

33. $h(t) = \dfrac{t}{t^2 + 4}$. $h'(t) = \dfrac{(t^2 + 4)(1) - t(2t)}{(t^2 + 4)^2} = \dfrac{4 - t^2}{(t^2 + 4)^2}.$

$$h''(t) = \frac{(t^2 + 4)^2(-2t) - (4 - t^2)2(t^2 + 4)(2t)}{(t^2 + 4)^4}$$

$$= \frac{-2t(t^2 + 4)[(t^2 + 4) + 2(4 - t^2)]}{(t^2 + 4)^4} = \frac{2t(t^2 - 12)}{(t^2 + 4)^3}.$$

34. $f(x) = (x^3 + x + 1)^2.$

$$f'(x) = 2(x^3 + x + 1)(3x^2 + 1) = 2(3x^5 + 3x^3 + 3x^2 + x^3 + x + 1)$$

$$= 2(3x^5 + 4x^3 + 3x^2 + x + 1).$$

$$f''(x) = 2(15x^4 + 12x^2 + 6x + 1).$$

35. $f'(x) = \dfrac{d}{dx}(2x^2 + 1)^{1/2} = \dfrac{1}{2}(2x^2 + 1)^{-1/2}(4x) = 2x(2x^2 + 1)^{-1/2}.$

$$f''(x) = 2(2x^2 + 1)^{-1/2} + 2x \cdot (-\tfrac{1}{2})(2x^2 + 1)^{-3/2}(4x)$$

$$= 2(2x^2 + 1)^{-3/2}[(2x^2 + 1) - 2x^2] = \frac{2}{(2x^2 + 1)^{3/2}}.$$

36. $f(t) = t(t^2 + 1)^3.$

$$f'(t) = (t^2+1)^3 + t\cdot 3(t^2+1)^2(2t) = (t^2+1)^2[(t^2+1)+6t^2]$$
$$= (t^2+1)^2(7t^2+1).$$
$$f''(t) = (t^2+1)(14t) + (7t^2+1)(2)(t^2+1)(2t)$$
$$= 2t(t^2+1)[7(t^2+1)+2(7t^2+1)] = 6t(t^2+1)(7t^2+3).$$

37. $6x^2 - 3y^2 = 9$ so $12x - 6y\dfrac{dy}{dx} = 0$ and $-6y\dfrac{dy}{dx} = -12x$.
Therefore, $\dfrac{dy}{dx} = \dfrac{-12x}{-6y} = \dfrac{2x}{y}$.

38. $2x^3 - 3xy = 4$. $6x^2 - 3y - 3x\dfrac{dy}{dx} = 0$, so $\dfrac{dy}{dx} = \dfrac{2x^2 - y}{x}$.

39. $y^3 + 3x^2 = 3y$, so $3y^2y' + 6x = 3y'$, $3y^2y' - 3y' = -6x$,
and $y'(3y^2 - 3) = -6x$. Therefore, $y' = -\dfrac{6x}{3(y^2-1)} = -\dfrac{2x}{y^2-1}$.

40. $x^2 + 2x^2y^2 + y^2 = 10$.
$2x + 4xy^2 + 2x^2(2yy') + 2yy' = 0,$
$2yy'(2x^2 + 1) = -2x(1 + 2y^2)$
$$y' = -\frac{x(1+2y^2)}{y(2x^2+1)}.$$

41. $x^2 - 4xy - y^2 = 12$ so $2x - 4xy' - 4y - 2yy' = 0$ and $y'(-4x - 2y) = -2x + 4y$.
So $y' = \dfrac{-2(x-2y)}{-2(2x+y)} = \dfrac{x-2y}{2x+y}$.

42. $3x^2y - 4xy + x - 2y = 6$.
$$6xy + 3x^2y' - 4y - 4xy' + 1 - 2y' = 0$$
$$y'(3x^2 - 4x - 2) = 4y - 6xy - 1$$
$$y' = \frac{4y - 6xy - 1}{3x^2 - 4x - 2}.$$

43.

$$df = f'(x)dx = (2x - 2x^{-3})dx = \left(2x - \frac{2}{x^3}\right)dx = \frac{2(x^4 - 1)}{x^3}dx$$

44. $df = f'(x)dx = \frac{d}{dx}(x^3+1)^{-1/2}dx = -\frac{1}{2}(x^3+1)^{-3/2}(3x^2)dx = -\frac{3x^2}{2(x^3+1)^{3/2}}dx$

45. a. $df = f'(x)dx = \frac{d}{dx}(2x^2+4)^{1/2}dx = \frac{1}{2}(2x^2+4)^{-1/2}(4x) = \frac{2x}{\sqrt{2x^2+4}}dx$

b. $\Delta f \approx df\Big|_{\substack{x=4 \\ dx=0.1}} = \frac{2(4)(0.1)}{\sqrt{2(16)+4}} = \frac{0.8}{6} = \frac{8}{60} = \frac{2}{15}.$

c. $\Delta f = f(4.1) - f(4) = \sqrt{2(4.1)^2+4} - \sqrt{2(16)+4} = 0.1335$

From (b), $\Delta f \approx \frac{2}{15} \approx 0.1333.$

46. Take $y = f(x) = x^{1/3}$, $x = 27$. Then $\Delta x = dx = 26.8 - 27 = -0.2$

$\Delta y \approx dy = f'(x)\Delta x = \frac{1}{3}x^{-2/3}\Big|_{x=27} \cdot (-0.2) = \frac{1}{3(9)}(-0.2) = -\frac{2}{270} = -\frac{1}{135}.$

Therefore, $\sqrt[3]{26.8} - \sqrt[3]{27} = \Delta y = -\frac{1}{135}$; $\sqrt[3]{26.8} = \sqrt[3]{27} - \frac{1}{135} = 3 - \frac{1}{135} \approx 2.9926.$

47. $f(x) = 2x^3 - 3x^2 - 16x + 3$ and $f'(x) = 6x^2 - 6x - 16.$

a. To find the point(s) on the graph of *f* where the slope of the tangent line is equal to –4, we solve

$6x^2 - 6x - 16 = -4$, $6x^2 - 6x - 12 = 0$, $6(x^2 - x - 2) = 0$

$6(x - 2)(x + 1) = 0$

and $x = 2$ or $x = -1$. Then $f(2) = 2(2)^3 - 3(2)^2 - 16(2) + 3 = -25$ and $f(-1) = 2(-1)^3 - 3(-1)^2 - 16(-1) + 3 = 14$ and the points are (2,–25) and (–1,14).

b. Using the point-slope form of the equation of a line, we find

that $y - (-25) = -4(x - 2)$, $y + 25 = -4x + 8$, or $y = -4x - 17$

and $y - 14 = -4(x + 1)$, or $y = -4x + 10$

are the equations of the tangent lines at (2,–25) and (–1,14).

48. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 4x + 1.$ $f'(x) = x^2 + x - 4.$

a. Set $x^2 + x - 4 = -2$, $x^2 + x - 2 = (x + 2)(x - 1) = 0$, so $x = -2$ or 1. Therefore, the points are $(-2, \frac{25}{3})$ and $(1, -\frac{13}{6})$.

b. $y - \frac{25}{3} = -2(x + 2)$, or $y = -2x + \frac{13}{3}$, and $y + \frac{13}{6} = -2(x - 1)$, or $y = -2x - \frac{1}{6}$.

49. $y = (4 - x^2)^{1/2}$. $y' = \frac{1}{2}(4 - x^2)^{-1/2}(-2x) = -\dfrac{x}{\sqrt{4 - x^2}}$.

The slope of the tangent line is obtained by letting $x = 1$, giving

$$m = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

Therefore, an equation of the tangent line is

$$y - \sqrt{3} = -\frac{\sqrt{3}}{3}(x - 1), \text{ or } y = -\frac{\sqrt{3}}{3}x + \frac{4\sqrt{3}}{3}.$$

50. $y = x(x + 1)^5$

$y' = (x+1)^5 + x \cdot 5(x+1)^4(1) = (x+1)^4[(x+1) + 5x] = (6x+1)(x+1)^4$

The slope of the tangent line is obtained by letting $x = 1$. Then

$$m = (6 + 1)(2)^4 = 112.$$

An equation of the tangent line is $y - 32 = 112(x - 1)$, or $y = 112x - 80$.

51. $f(x) = (2x - 1)^{-1}$; $f'(x) = -2(2x - 1)^{-2}$, $f''(x) = 8(2x - 1)^{-3} = \dfrac{8}{(2x - 1)^3}$.

$f'''(x) = -48(2x - 1)^4 = -\dfrac{48}{(2x - 1)^4}$.

Since $(2x - 1)^4 = 0$ when $x = 1/2$, we see that the domain of f''' is $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$.

52. $x = f(p) = -\dfrac{5}{2}p + 30$; $f'(p) = -\dfrac{5}{2}$; $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\frac{5}{2}\right)}{-\frac{5}{2}p + 30} = \dfrac{p}{12 - p}$.

a. $E(3) = \frac{3}{9} = \frac{1}{3}$ and demand is inelastic. b. $E(6) = \frac{6}{12-6} = 1$ and demand is unitary .

c. $E(9) = \frac{9}{12-9} = 3$ and demand is elastic.

53. $x = \dfrac{25}{\sqrt{p}} - 1$; $f'(p) = -\dfrac{25}{2p^{3/2}}$; $E(p) = -\dfrac{p\left(-\frac{25}{2p^{3/2}}\right)}{\frac{25}{p^{1/2}} - 1} = \dfrac{\frac{25}{2p^{1/2}}}{\frac{25-p^{1/2}}{p^{1/2}}} = \dfrac{25}{2(25 - p^{1/2})}$

Since $E(p) = 1$,

$$2(25 - p^{1/2}) = 25,$$

$$25 - p^{1/2} = \tfrac{25}{2}, \quad p^{1/2} = \tfrac{25}{2}, \text{ and } p = \tfrac{625}{4}.$$

$E(p) > 1$ and demand is elastic if $p > 156.25$; $E(p) = 1$ and demand is unitary if $p = 156.25$; and $E(p) < 1$ and demand is inelastic, if $p < 156.25$.

54. $x = 100 - 0.01p^2$; $f'(p) = -0.02p$; $E(p) = -\dfrac{p(-0.02p)}{100 - 0.01p^2} = \dfrac{p^2}{5000 - \frac{1}{2}p^2}$

a. $E(40) = \dfrac{1600}{5000 - \frac{1}{2}(1600)} = \dfrac{1600}{4200} = \dfrac{8}{21} < 1$ and so demand is inelastic.

b. Since demand is inelastic, raising the unit price slightly will cause the revenue to increase.

55. $p = 9\sqrt[3]{1000 - x}$; $\sqrt[3]{1000 - x} = \dfrac{p}{9}$; $1000 - x = \dfrac{p^3}{729}$; $x = 1000 - \dfrac{p^3}{729}$

Therefore, $x = f(p) = \dfrac{729{,}000 - p^3}{729}$ and $f'(x) = -\dfrac{3p^2}{729} = -\dfrac{p^2}{243}$.

Then $E(p) = -\dfrac{p(-\frac{p^2}{243})}{\frac{729{,}000 - p^3}{729}} = \dfrac{3p^3}{729{,}000 - p^3}$.

So $E(60) = \dfrac{3(60)^3}{729{,}000 - 60^3} = \dfrac{648{,}000}{513{,}000} = \dfrac{648}{513} > 1$, and so demand is elastic.

Therefore, raising the price slightly will cause the revenue to decrease.

56. The number of worldwide networked *PC*s at the beginning of 1997 is given by

$$N(6) = 3.136(6)^2 + 3.954(6) + 116.468 = 253.088, \text{ or } 253.1 \text{ million.}$$

b. The rate of change of the number of worldwide networked *PCs* at any time t is given by

$$N'(t) = 6.272t + 3.954$$

In particular, the number of worldwide networked *PCs* at any time t is

$$N'(6) = 6.272(6) + 3.954 = 41.586,$$

or 41.586 million units per year.

57. $N(x) = 1000(1 + 2x)^{1/2}$. $N'(x) = 1000(\frac{1}{2})(1+2x)^{-1/2}(2) = \dfrac{1000}{\sqrt{1+2x}}$.

The rate of increase at the end of the twelfth week is $N'(12) = \dfrac{1000}{\sqrt{25}} = 200$,

or 200 subscribers/week.

58. $f(t) = 31.88(1+t)^{-0.45}$ $f'(t) = 31.88(-0.45)(1+t)^{-1.45} = -14.346(1+t)^{-1.45}$
It is changing at the rate of $f'(2) = -2.917$; that is, decreasing at the rate of 2.9 cents/minute/yr. The average price/minute at the beginning of 2000 was

$f(2) = 31.88(1+2)^{-0.45}$, or 19.45 cents/minute.

59. He can expect to live $f(100) = 46.9[1+1.09(100)]^{0.1} \approx 75.0433$, or approximately 75.04 years. $f'(t) = 46.9(0.1)(1+1.09t)^{-0.9}(1.09) = 5.1121(1+1.09t)^{-0.9}$
So the required rate of change is $f'(100) = 5.1121(1+1.09)^{-0.9} = 0.074$, or approximately 0.07 yr/yr.

60. $C(x) = 2500 + 2.2x$.

a. The marginal cost is $C'(x) = 2.2$. The marginal cost when $x = 1000$ is $C'(1000) = 2.2$. The marginal cost when $x = 2000$ is $C'(2000) = 2.2$.

b. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{2500+2.2x}{x} = 2.2 + \dfrac{2500}{x}$.

$\overline{C}'(x) = -\dfrac{2500}{x^2}$.

c. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty}\left(2.2 + \dfrac{2500}{x}\right) = 2.2$.

61. a. $R(x) = px = (-0.02x + 600)x = -0.02x^2 + 600x$

b. $R'(x) = -0.04x + 600$

c. $R'(10{,}000) = -0.04(10{,}000) + 600 = 200$ and this says that the sale of the 10,001st phone will bring a revenue of \$200.

62. a. $R(x) = px = (2000 - 0.04x)x = 2000x - 0.04x^2$.

$P(x) = R(x) - C(x)$
$= (2000x - 0.04x^2) - (0.000002x^3 - 0.02x^2 + 1000x + 120{,}000)$
$= -0.000002x^3 - 0.02x^2 + 1000x - 120{,}000$.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{0.000002x^3 - 0.02x^2 + 1000x + 120{,}000}{x}$$

$$= 0.000002x^2 - 0.02x + 1000 + \frac{120{,}000}{x}$$

b. $C'(x) = \frac{d}{dx}(0.000002x^3 - 0.02x^2 + 1000x + 120{,}000)$

$= 0.000006x^2 - 0.04x + 1000.$

$R'(x) = \frac{d}{dx}(2000x - 0.04x^2) = 2000 - 0.08x.$

$P'(x) = \frac{d}{dx}(-0.000002x^3 - 0.02x^2 + 1000x - 120{,}000)$

$= -0.000006x^2 - 0.04x + 1000$

$\overline{C}'(x) = \frac{d}{dx}(0.000002x^2 - 0.02x + 1000 + 120{,}000x^{-1})$

$= 0.000004x - 0.02 - 120{,}000x^{-2}.$

c. $C'(3000) = 0.000006(3000)^2 - 0.04(3000) + 1000 = 934$

$R'(3000) = 2000 - 0.08(3000) = 1760.$

$P'(3000) = -0.000006(3000)^2 - 0.04(3000) + 1000 = 826.$

d. $\overline{C}'(5000) = 0.000004(5000) - 0.02 - 120{,}000(5000)^{-2} = -0.0048$

$\overline{C}'(8000) = 0.000004(8000) - 0.02 - 120{,}000(8000)^{-2} = 0.0101.$

At a level of production of 5000 machines, the average cost of each additional unit is decreasing at a rate of 0.48 cents. At a level of production of 8000 machines, the average cost of each additional unit is increasing at a rate of 1 cent per unit.