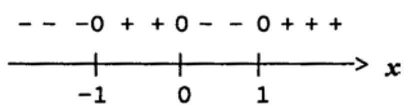
CHAPTER 4 REVIEW, page 330

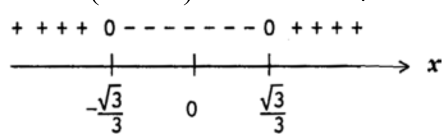
1. a. $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$. $f'(x) = x^2 - 2x + 1 = (x - 1)^2$. $f'(x) = 0$ gives $x = 1$, the critical point of f. Now, $f'(x) > 0$ for all $x \neq 1$. Thus, f is increasing on $(-\infty,1) \cup (1,\infty)$.
   b. Since $f'(x)$ does not change sign as we move across the critical point $x = 1$, the First Derivative Test implies that $x = 1$ does not give rise to a relative extremum of f.
   c. $f''(x) = 2(x - 1)$. Setting $f''(x) = 0$ gives $x = 1$ as a candidate for an inflection point of f. Since $f''(x) < 0$ for $x < 1$, and $f''(x) > 0$ for $x > 1$, we see that f is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$.
   d. The results of (c) imply that $(1, -\frac{17}{3})$ is an inflection point.

2. a. $f(x) = (x - 2)^3$; $f'(x) = 3(x - 2)^2 > 0$ for all $x \neq 2$. Therefore, f is increasing on $(-\infty,2) \cup (2,\infty)$.
   b. There are no relative extrema.
   c. $f''(x) = 6(x - 2)$. Since $f''(x) < 0$ if $x < 2$ and $f''(x) > 0$ if $x > 2$, we see that f is concave downward on $(-\infty,2)$ and concave upward on $(2,\infty)$,
   d. The results of (c) show that (2,0) is an inflection point.

3. a. $f(x) = x^4 - 2x^2$. $f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x + 1)(x - 1)$. The sign diagram of f' shows that f is decreasing on $(-\infty,-1) \cup (0,1)$ and increasing on $(-1,0) \cup (1,\infty)$.

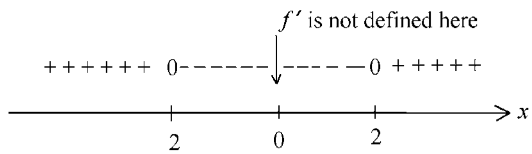


   b. The results of (a) and the First Derivative Test show that (-1,-1) and (1,-1) are relative minima and (0,0) is a relative maximum.
   c. $f''(x) = 12x^2 - 4 = 4(3x^2 - 1) = 0$ if $x = \pm\sqrt{3}/3$. The sign diagram



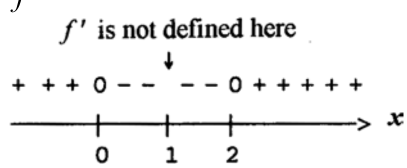
   shows that f is concave upward on $(-\infty, -\sqrt{3}/3) \cup (\sqrt{3}/3, \infty)$ and concave downward on $(-\sqrt{3}/3, \sqrt{3}/3)$.
   d. The results of (c) show that $(-\sqrt{3}/3, -5/9)$ and $(\sqrt{3}/3, -5/9)$ are inflection points.

4. $f(x) = x + \frac{4}{x}$. $f'(x) = 1 - \frac{4}{x^2} = \frac{x^2-4}{x^2} = \frac{(x-2)(x+2)}{x^2}$. Setting $f'(x) = 0$ gives $x = -2$ and $x = 2$ as critical points of f. $f'(x)$ is undefined at $x = 0$ as well. The sign diagram for f' is



a. f is increasing on $(-\infty,-2) \cup (2,\infty)$ and decreasing on $(-2,0) \cup (0,2)$.
b. $f(-2) = -4$ is a relative maximum and $f(2) = 4$ is a relative minimum.
c. Next, we compute $f''(x) = \frac{8}{x^3}$. Since $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$, we see that f is concave downward on $(-\infty,0)$ and concave upward on $(0,\infty)$.
d. There are no inflection points. Note that $x = 0$ is not in the domain of f and is therefore not a candidate for an inflection point.

5. a. $f(x) = \dfrac{x^2}{x-1}$. $f'(x) = \dfrac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \dfrac{x^2-2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$.
The sign diagram of f'



shows that f is increasing on $(-\infty,0) \cup (2,\infty)$ and decreasing on $(0,1) \cup (1,2)$.
b. The results of (a) show that (0,0) is a relative maximum and (2,4) is a relative minimum.

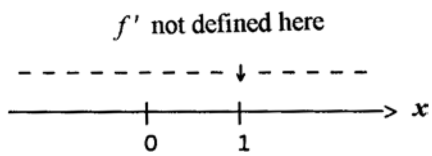
c.
$$f''(x) = \frac{(x-1)^2(2x-2) - x(x-2)2(x-1)}{(x-1)^4} = \frac{2(x-1)[(x-1)^2 - x(x-2)]}{(x-1)^4}$$
$$= \frac{2}{(x-1)^3}.$$
Since $f''(x) < 0$ if $x < 1$ and $f''(x) > 0$ if $x > 1$, we see that f is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$.
d. Since $x = 1$ is not in the domain of f, there are no inflection points.

6. a. $f(x)=\sqrt{x-1}$. $f'(x) = \frac{1}{2}(x-1)^{-1/2} = \dfrac{1}{2\sqrt{x-1}}$.

Since $f'(x) > 0$ if $x > 1$, we see that f is increasing on $(1,\infty)$.
b. Since there are no critical points in $(1,\infty)$, f has no relative extrema.
c. $f''(x) = -\dfrac{1}{4}(x-1)^{-3/2} = -\dfrac{1}{4(x-1)^{3/2}} < 0$ if $x > 1$, and so f is concave downward on $(1,\infty)$.
d. There are no inflection points since $f''(x) \neq 0$ for all x in $(1,\infty)$.

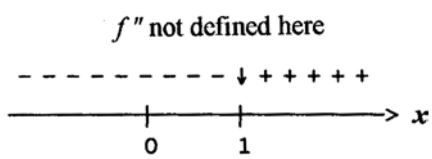
7. $f(x) = (1-x)^{1/3}$. $f'(x) = -\dfrac{1}{3}(1-x)^{-2/3} = -\dfrac{1}{3(1-x)^{2/3}}$.

The sign diagram for f' is



a. f is decreasing on $(-\infty,1) \cup (1,\infty)$.
b. There are no relative extrema.
c. Next, we compute $f''(x) = -\dfrac{2}{9}(1-x)^{-5/3} = -\dfrac{2}{9(1-x)^{5/3}}$.

The sign diagram for f'' is



We find f is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$.
d. $x = 1$ is a candidate for an inflection point of f. Referring to the sign diagram for f'', we see that $(1,0)$ is an inflection point.

8. $f(x) = x\sqrt{x-1} = x(x-1)^{1/2}$.

a. $f'(x) = x(\frac{1}{2})(x-1)^{-1/2} + (x-1)^{1/2} = \frac{1}{2}(x-1)^{-1/2}[x+2(x-1)] = \dfrac{3x-2}{2(x-1)^{1/2}}$.

Setting $f'(x) = 0$ gives $x = 2/3$. But, this point lies outside the domain of f which is $[1,\infty)$. Thus, there are no critical points of f. Now, $f'(x) > 0$ for all $x \in (1,\infty)$ so f is increasing there.

b. Since there are no critical points, f has no relative minimum.

c. $$f''(x) = \frac{1}{2}\left[\frac{(x-1)^{1/2}(3)-(3x-2)\frac{1}{2}(x-1)^{-1/2}}{(x-1)}\right]$$

$$= \frac{1}{2}\left[\frac{\frac{1}{2}(x-1)^{-1/2}[6(x-1)-(3x-2)]}{(x-1)}\right] = \frac{3x-4}{4(x-1)^{3/2}}.$$

Next, $f''(x) = 0$ implies that $x = 4/3$. $f''(x) < 0$ if $x < 4/3$, and $f''(x) > 0$ if $x > 4/3$, so f is concave downward on $(1, \frac{4}{3})$ and concave upward on $(\frac{4}{3}, \infty)$.

d. From the results of (c), we conclude that $(\frac{4}{3}, \frac{4\sqrt{3}}{9})$ is an inflection point of f.

9. a. $f(x) = \dfrac{2x}{x+1}$. $f'(x) = \dfrac{(x+1)(2)-2x(1)}{(x+1)^2} = \dfrac{2}{(x+1)^2} > 0$ if $x \neq -1$.

Therefore f is increasing on $(-\infty,-1) \cup (-1,\infty)$.

b. Since there are no critical points, f has no relative extrema.

c. $f''(x) = -4(x+1)^{-3} = -\dfrac{4}{(x+1)^3}$. Since $f''(x) > 0$ if $x < -1$ and $f''(x) < 0$ if $x > -1$, we see that f is concave upward on $(-\infty,-1)$ and concave downward on $(-1,\infty)$.

d. There are no inflection points since $f''(x) \neq 0$ for all x in the domain of f.

10. $f(x) = -\dfrac{1}{1+x^2}$; $f'(x) = \dfrac{2x}{(1+x^2)^2}$.

Setting $f'(x) = 0$ gives $x = 0$ as the only critical point of f. For $x < 0$, $f'(x) < 0$ and for $x > 0$, $f'(x) > 0$. Therefore,

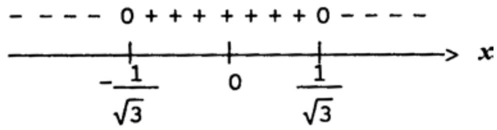
a. f is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.

b. f has a relative minimum at $f(0) = -1$.

c. Next, we compute

$$f''(x) = \frac{(1+x^2)^2(2)-2x(2)(1+x^2)(2x)}{(1+x^2)^4} = \frac{2(1+x^2)(1+x^2-4x^2)}{(1+x^2)^4} = -\frac{2(3x^2-1)}{(1+x^2)^3}$$

and we see that $x = \pm 1/\sqrt{3}$ are candidates for inflection points of f. The sign diagram for f'' is

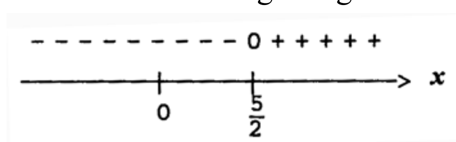


Therefore f is concave downward on $(-\infty, -1/\sqrt{3}) \cup (1/\sqrt{3}, \infty)$ and concave upward on $(-1/\sqrt{3}, 1/\sqrt{3})$.

d. $(-1/\sqrt{3}, -3/4)$ and $(1/\sqrt{3}, -3/4)$ are inflection points of f.

11. $f(x) = x^2 - 5x + 5$

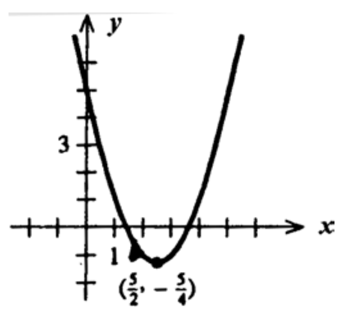
1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 5 as the y-intercept.
3. $\lim_{x\to-\infty} (x^2 - 5x + 5) = \lim_{x\to\infty} (x^2 - 5x + 5) = \infty$.
4. There are no asymptotes because f is a quadratic function.
5. $f'(x) = 2x - 5 = 0$ if $x = 5/2$. The sign diagram



shows that f is increasing on $(\frac{5}{2}, \infty)$ and decreasing on $(-\infty, \frac{5}{2})$.

6. The First Derivative Test implies that $(\frac{5}{2}, -\frac{5}{4})$ is a relative minimum.
7. $f''(x) = 2 > 0$ and so f is concave upward on $(-\infty, \infty)$.
8. There are no inflection points.

The graph of f follows.



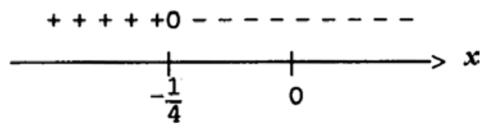
12. $f(x) = -2x^2 - x + 1$

1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.

3. $\lim_{x\to-\infty} (-2x^2 - x + 1) = \lim_{x\to\infty} (-2x^2 - x + 1) = -\infty$.

4. There are no asymptotes because f is a polynomial function.

5. $f'(x) = -4x - 1 = 0$ if $x = -1/4$. The sign diagram of f'



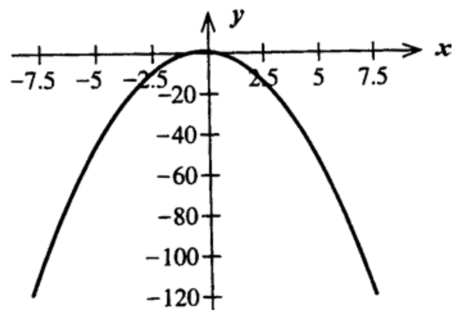
shows that f is increasing on $(-\infty, -\frac{1}{4})$ and decreasing on $(-\frac{1}{4}, \infty)$.

6. The results of (5) show that $(-\frac{1}{4}, \frac{9}{8})$ is a relative maximum.

7. $f''(x) = -4 < 0$ for all x in $(-\infty, \infty)$ and so f is concave downward on $(-\infty, \infty)$.

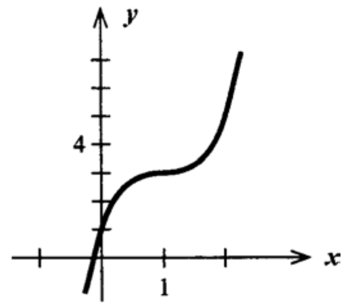
8. There are no inflection points.

The graph of f follows.



13. $g(x) = 2x^3 - 6x^2 + 6x + 1$.

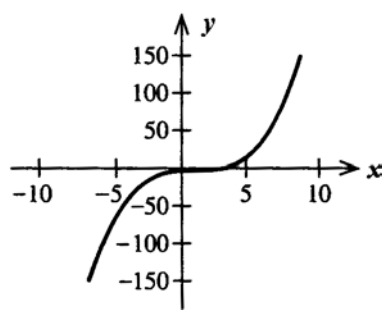
1. The domain of g is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.
3. $\lim_{x\to-\infty} g(x) = -\infty$, $\lim_{x\to\infty} g(x) = \infty$.
4. There are no vertical or horizontal asymptotes.
5. $g'(x) = 6x^2 - 12x + 6 = 6(x^2 - 2x + 1) = 6(x - 1)^2$. Since $g'(x) > 0$ for all $x \neq 1$, we see that g is increasing on $(-\infty,1) \cup (1,\infty)$.
6. $g'(x)$ does not change sign as we move across the critical point $x = 1$, so there is no extremum.
7. $g''(x) = 12x - 12 = 12(x - 1)$. Since $g''(x) < 0$ if $x < 1$ and $g''(x) > 0$ if $x > 1$, we see that g is concave upward on $(1,\infty)$ and concave downward on $(-\infty,1)$.
8. The point $x = 1$ gives rise to the inflection point (1,3).
9. The graph of g follows.



14. $g(x) = \frac{1}{3}x^3 - x^2 + x - 3$

1. The domain of g is $(-\infty, \infty)$.
2. Setting $x = 0$ gives -3 as the y-intercept.
3. $\lim_{x\to-\infty} (\frac{1}{3}x^3 - x^2 + x - 3) = -\infty$ and $\lim_{x\to\infty} (\frac{1}{3}x^3 - x^2 + x - 3) = \infty$
4. There are no asymptotes because $g(x)$ is a polynomial.
5. $g'(x) = x^2 - 2x + 1 = (x - 1)^2 = 0$ if $x = 1$, a critical point of g. Observe that $g'(x) > 0$ if $x \neq 1$, and so g is increasing on $(-\infty,1) \cup (1,\infty)$.
6. The results of (5) show that there are no relative extrema.
7. $g'(x) = 2x - 2 = 2(x - 1) = 0$ if $x = 1$. Observe that $g'(x) < 0$ if $x < 1$ and $g''(x) > 0$ if $x > 1$ and so g is concave downward on $(-\infty,1)$ and is concave upward on $(1,\infty)$.
8. The results of (7) show that $(1,-\frac{8}{3})$ is an inflection point.

The graph of g follows.



15. $h(x) = x\sqrt{x-2}$.

1. The domain of h is $[2,\infty)$.
2. There are no y-intercepts. Next, setting $y = 0$ gives 2 as the x-intercept.
3. $\lim_{x\to\infty} x\sqrt{x-2} = \infty$.

4. There are no asymptotes.

5. $h'(x) = (x-2)^{1/2} + x(\frac{1}{2})(x-2)^{-1/2} = \frac{1}{2}(x-2)^{-1/2}[2(x-2)+x]$

$$= \frac{3x-4}{2\sqrt{x-2}} > 0 \quad \text{on } [2,\infty)$$

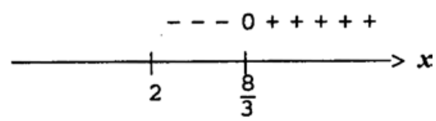
and so h is increasing on $[2,\infty)$.

6. Since h has no critical points in $(2,\infty)$, there are no relative extrema.

7. $h''(x) = \frac{1}{2}\left[\frac{(x-2)^{1/2}(3) - (3x-4)\frac{1}{2}(x-2)^{-1/2}}{x-2}\right]$

$$= \frac{(x-2)^{-1/2}[6(x-2)-(3x-4)]}{4(x-2)} = \frac{3x-8}{4(x-2)^{3/2}}.$$

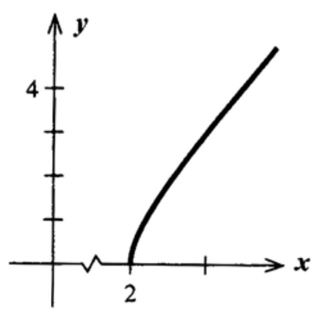
The sign diagram for h''



shows that h is concave downward on $(2,\frac{8}{3})$ and concave upward on $(\frac{8}{3},\infty)$.

8. The results of (7) tell us that $(\frac{8}{3},\frac{8\sqrt{6}}{9})$ is an inflection point.

The graph of h follows.



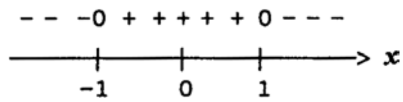
16. $h(x) = \dfrac{2x}{1+x^2}$.

1. The domain of h is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 0 as the y-intercept.
3. $\lim_{x\to-\infty} \frac{2x}{1+x^2} = \lim_{x\to\infty} \frac{2x}{1+x^2} = 0.$

4. The results of (3) tell us that $y = 0$ is a horizontal asymptote.

5. $h'(x) = \dfrac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2} = \dfrac{2(1-x^2)}{(1+x^2)^2} = \dfrac{2(1-x)(1+x)}{(1+x^2)^2}$.

The sign diagram of h'



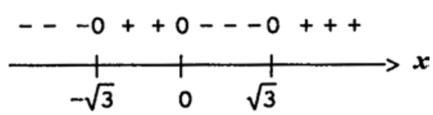
tells us that h is decreasing on $(-\infty,-1) \cup (1,\infty)$ and increasing on $(-1,1)$.

6. The results of (6) show that $(-1,-1)$ is a relative minimum and $(1,1)$ is a relative maximum.

7. $h''(x) = 2\left[\dfrac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)(2x)}{(1+x^2)^4}\right]$

$$= \frac{4x(1+x^2)[-(1+x^2) - 2(1-x^2)]}{(1+x^2)^4} = \frac{4x(x^2-3)}{(1+x^2)^3}.$$

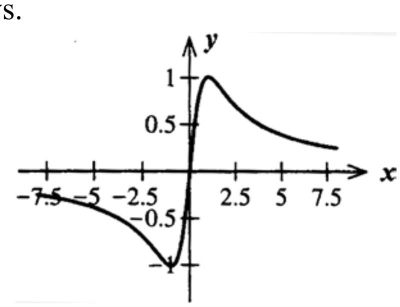
The sign diagram of h''



shows that h is concave downward on $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$ and concave upward on $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$.

8. The results of (6) also tell us that $(-\sqrt{3},-\frac{\sqrt{3}}{2})$ and $(\sqrt{3},\frac{\sqrt{3}}{2})$ are inflection points.

The graph of h follows.



17. $f(x) = \dfrac{x-2}{x+2}$.

1. The domain of f is $(-\infty,-2) \cup (-2,\infty)$.

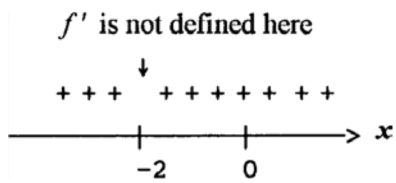
2. Setting $x = 0$ gives -1 as the y-intercept. Setting $y = 0$ gives 2 as the x-intercept.

3. $\lim_{x\to-\infty} \frac{x-2}{x+2} = \lim_{x\to\infty} \frac{x-2}{x+2} = 1.$

4. The results of (3) tell us that $y = 1$ is a horizontal asymptote. Next, observe that the denominator of $f(x)$ is equal to zero at $x = -2$, but its numerator is not equal to zero there. Therefore, $x = -2$ is a vertical asymptote.

5. $$f'(x) = \frac{(x+2)(1)-(x-2)(1)}{(x+2)^2} = \frac{4}{(x+2)^2}.$$

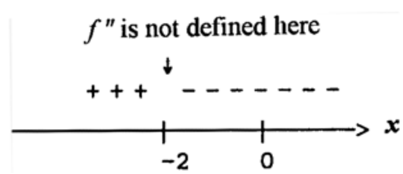
The sign diagram of f'



tells us that f is increasing on $(-\infty,-2) \cup (-2,\infty)$.

6. The results of (5) tells us that there are no relative extrema.

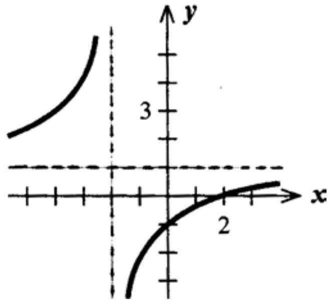
7. $f''(x) = -\frac{8}{(x+2)^3}$. The sign diagram of f'' follows



and it shows that f is concave upward on $(-\infty,-2)$ and concave downward on $(-2,\infty)$.

8. There are no inflection points.

The graph of f follows.



18. $f(x) = x - \frac{1}{x}$.

1. The domain of f is $(-\infty,0) \cup (0,\infty)$.

2. There are no y-intercepts. Next, setting $y = 0$ gives $\frac{x^2 - 1}{x} = \frac{(x+1)(x-1)}{x} = 0$ and so the x-intercepts are -1 and 1.

3. $\lim_{x \to -\infty}\left(x - \frac{1}{x}\right) = -\infty$ and $\lim_{x \to \infty}\left(x - \frac{1}{x}\right) = \infty$.

4. There are no horizontal asymptotes. But from $f(x) = \frac{x^2 - 1}{x}$ we see that the denominator of $f(x)$ is equal to zero at $x = 0$. Since the numerator is not equal to zero there, we conclude that $x = 0$ is a vertical asymptote.

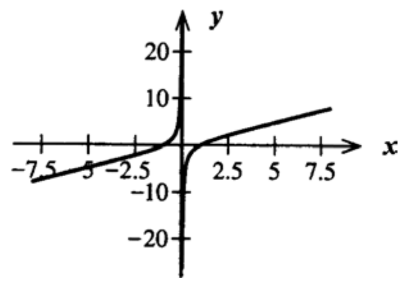
5. $f'(x) = 1 + \frac{1}{x^2} = \frac{x^2 + 1}{x^2} > 0$ for all $x \neq 0$. Therefore, f is increasing on $(-\infty,0) \cup (0,\infty)$.

6. The results of (5) show that f has no relative extrema.

7. $f''(x) = -\frac{2}{x^3}$. Observe that $f''(x) > 0$ if $x < 0$ and $f''(x) < 0$ if $x > 0$. Therefore, f is concave upward on $(-\infty,0)$ and concave downward on $(0,\infty)$.

8. There are no inflection points.

The graph of f follows.



19. $\lim_{x\to-\infty} \frac{1}{2x+3} = \lim_{x\to\infty} \frac{1}{2x+3} = 0$ and so $y = 0$ is a horizontal asymptote. Since the denominator is equal to zero at $x = -3/2$, but the numerator is not equal to zero there, we see that $x = -3/2$ is a vertical asymptote.

20. $\lim_{x\to-\infty} \frac{2x}{x+1} = \lim_{x\to\infty} \frac{2x}{x+1} = 2$ and so $y = 2$ is a horizontal asymptote. Since the denominator is equal to zero at $x = -1$, but the numerator is not equal to zero there, we see that $x = -1$ is a vertical asymptote.

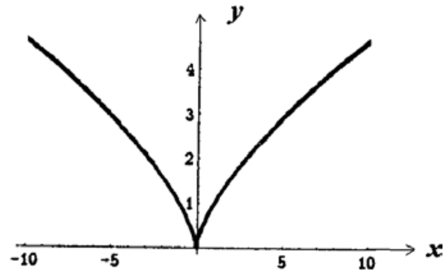
21. $\lim_{x\to-\infty} \frac{5x}{x^2-2x-8} = \lim_{x\to\infty} \frac{5x}{x^2-2x-8} = 0$ and so $y = 0$ is a horizontal asymptote. Next, note that the denominator is zero if $x^2 - 2x - 8 = (x - 4)(x + 2) = 0$, or $x = -2$ or $x = 4$. Since the numerator is not equal to zero at these points, we see that $x = -2$ and $x = 4$ are vertical asymptotes.

22. $\lim_{x\to-\infty} \frac{x^2+x}{x^2-x} = \lim_{x\to\infty} \frac{x^2+x}{x^2-x} = 1$, we see that $y = 1$ is a horizontal asymptote. Next observe that the denominator is equal to zero at $x = 0$ or $x = 1$. Since the numerator is not equal to zero at $x = 1$, we see that $x = 1$ is a vertical asymptote.

23. $f(x) = 2x^2 + 3x - 2$; $f'(x) = 4x + 3$. Setting $f'(x) = 0$ gives $x = -3/4$ as a critical point of f. Next, $f''(x) = 4 > 0$ for all x, so f is concave upward on $(-\infty, \infty)$. Therefore, $f(-\frac{3}{4}) = -\frac{25}{8}$ is an absolute minimum of f. There is no absolute maximum.

24. $g(x) = x^{2/3}$. $g'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$ and so $x = 0$ is a critical point. Since $g'(x) < 0$ if $x < 0$ and $g'(x) > 0$ if $x > 0$, we see that (0,0) is a relative minimum. The graph of g

shows that (0,0) is an absolute minimum.



25. $g(t) = \sqrt{25 - t^2} = (25 - t^2)^{1/2}$. Differentiating $g(t)$, we have

$$g'(t) = \tfrac{1}{2}(25 - t^2)^{-1/2}(-2t) = -\frac{t}{\sqrt{25 - t^2}}.$$

Setting $g'(t) = 0$ gives $t = 0$ as a critical point of g. The domain of g is given by solving the inequality $25 - t^2 \geq 0$ or $(5 - t)(5 + t) \geq 0$ which implies that $t \in [-5,5]$. From the table

t	-5	0	5
$g(t)$	0	5	0

we conclude that $g(0) = 5$ is the absolute maximum of g and $g(-5) = 0$ and $g(5) = 0$ is the absolute minimum value of g.

26. $f(x) = \frac{1}{3}x^3 - x^2 + x + 1$; $f'(x) = x^2 - 2x + 1 = (x - 1)^2$. Therefore, $x = 1$ is a critical point of f.

x	0	1	2
$f(x)$	1	4/3	5/3

From the table, we see that $f(0) = 1$ is the absolute minimum value and $f(2) = 5/3$ is the absolute maximum value of f.

27. $h(t) = t^3 - 6t^2$. $h'(t) = 3t^2 - 12t = 3t(t - 4) = 0$ if $t = 0$ or $t = 4$, critical points of h. But only $t = 4$ lies in (2,5).

t	2	4	5
$h(t)$	-16	-32	-25

From the table, we see that there is an absolute minimum at (4,-32) and an absolute maximum at (2,-16).

28. $g(x) = \dfrac{x}{x^2+1}$. $g'(x) = \dfrac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} = 0$, if $x = \pm 1$. But only the critical point $x = 1$ lies in (0,5).

x	0	1	5
$g(x)$	0	1/2	5/26

From the table, we see that (0,0) is an absolute minimum and $(1, \frac{1}{2})$ is an absolute maximum.

29. $f(x) = x - \dfrac{1}{x}$ on [1,3]. $f'(x) = 1 + \dfrac{1}{x^2}$. Since $f'(x)$ is never zero, f has no critical point.

x	1	3
$f(x)$	0	$\frac{8}{3}$

We see that $f(1) = 0$ is the absolute minimum value and $f(3) = 8/3$ is the absolute maximum value.

30. $h(t) = 8t - \dfrac{1}{t^2}$ on [1,3]. $h'(t) = 8 + \dfrac{2}{t^3} = \dfrac{8t^3+2}{t^3} = 0$ if $t = -\frac{1}{4^{1/3}}$. But this critical point does not lie in (1,3). Furthermore, $t = 0$ is not a critical point of h nor does it lie in (1,3).

t	1	3
$h(t)$	7	$\frac{215}{9}$

From the table, we se that (1,7) gives an absolute minimum of h and $(3,\frac{215}{9})$ gives an absolute maximum of h.

31. $f(s) = s\sqrt{1-s^2}$ on [-1,1]. The function f is continuous on [-1,1] and differentiable on (-1,1). Next,

$$f'(s) = (1-s^2)^{1/2} + s(\tfrac{1}{2})(1-s^2)^{-1/2}(-2s) = \frac{1-2s^2}{\sqrt{1-s^2}}.$$

Setting $f'(s) = 0$, we have $s = \pm\sqrt{2}/2,$ giving the critical points of f. From the table

x	-1	$-\sqrt{2}/2$	$\sqrt{2}/2$	1
$f(x)$	0	-1/2	1/2	0

we see that $f(-\sqrt{2}/2) = -1/2$ is the absolute minimum value and $f(\sqrt{2}/2) = 1/2$ is the absolute maximum value of f.

32. $f(x) = \dfrac{x^2}{x-1}$. Observe that $\lim\limits_{x\to 1^-}\dfrac{x^2}{x-1} = -\infty$ and $\lim\limits_{x\to 1^+}\dfrac{x^2}{x-1} = \infty$. Therefore, there are no absolute extrema.

33. We want to maximize $P(x) = -x^2 + 8x + 20$. Now, $P'(x) = -2x + 8 = 0$ if $x = 4$, a critical point of P. Since $P''(x) = -2 < 0$, the graph of P is concave downward. Therefore, the critical point $x = 4$ yields an absolute maximum. So, to maximize profit, the company should spend \$4000 on advertising per month.

34. a. $f'(t) = \dfrac{d}{dt}(0.157t^2 + 1.175t + 2.03) = 0.314t + 1.75 > 0$ on (0, 6) and so f is increasing on (0, 6).

b. $f'(t) = \dfrac{d}{dt}(0.314t + 1.175) = 0.314 > 0$ and so f is concave upward on (0, 6).

c. Online travel spending is increasing at an increasing rate over the years in question.

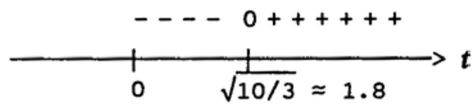
35. a. $I(t) = \dfrac{50t^2 + 600}{t^2 + 10}$.

$$I'(t) = \frac{(t^2+10)(100t) - (50t^2+600)(2t)}{(t^2+10)^2} = -\frac{200t}{(t^2+10)^2} < 0$$ on (0,10) and so I is decreasing on (0,10).

b. $$I''(t) = -200\left[\frac{(t^2+10)^2(1) - t(2)(t^2+10)(2t)}{(t^2+10)^4}\right]$$

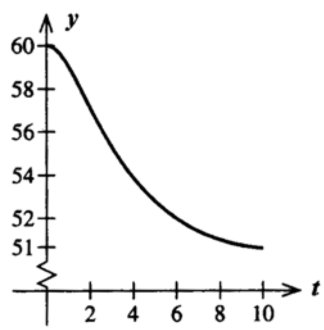
$$= \frac{-200(t^2+10)[(t^2+10)-4t^2]}{(t^2+10)^4} = -\frac{200(10-3t^2)}{(t^2+10)^3}.$$

The sign diagram of I'' (for $t > 0$)



shows that I is concave downward on $(0, \sqrt{10/3})$ and concave upward on $(\sqrt{10/3}, \infty)$.

c.



d. The rate of decline in the environmental quality of the wildlife was increasing the first 1.8 years. After that time the rate of decline decreased.

36. The revenue is $R(x) = px = x(-0.0005x^2 + 60) = -0.0005x^3 + 60x$. Therefore, the total profit is $P(x) = R(x) - C(x) = -0.0005x^3 + 0.001x^2 + 42x - 4000$. $P'(x) = -0.0015x^2 + 0.002x + 42$. Setting $P'(x) = 0$, we have $3x^2 - 4x - 84{,}000 = 0$. Solving for x, we find

$$x = \frac{4 \pm \sqrt{16 - 4(3)(84{,}000)}}{2(3)} = \frac{4 \pm 1004}{6} = 168, \text{ or } -167.$$

We reject the negative root. Next, $P''(x) = -0.003x + 0.002$ and

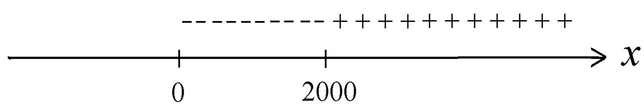
$P''(168) = -0.003(168) + 0.002 = -0.502 < 0$. By the Second Derivative Test, $x = 168$ gives a relative maximum. Therefore, the required level of production is 168 DVDs.

37. a. $C(x) = 0.001x^2 + 100x + 4000$.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{0.001x^2 + 100x + 4000}{x} = 0.001x + 100 + \frac{4000}{x}.$$

b. $$\overline{C}'(x) = 0.001 - \frac{4000}{x^2} = \frac{0.001x^2 - 4000}{x^2} = \frac{0.001(x^2 - 4{,}000{,}000)}{x^2}.$$

Setting $\overline{C}'(x) = 0$ gives $x = \pm 2000$. We reject the negative root.



The sign diagram of $\overline{C}'$ shows that $x = 2000$ gives rise to a relative minimum of $\overline{C}$. Since $\overline{C}''(x) = \dfrac{8000}{x^3} > 0$ if $x > 0$, we see that $\overline{C}$ is concave upward on $(0,\infty)$. So

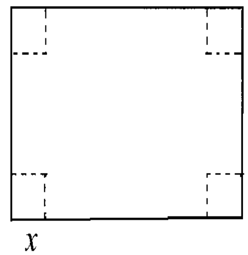
$x = 2000$ yields an absolute minimum. So the required production level is 2000 units.

38. $N(t) = -2t^3 + 12t^2 + 2t$. We wish to find the inflection point of the function N. Now, $N'(t) = -6t^2 + 24t + 2$ and $N''(t) = -12t + 24 = -12(t-2)$. Setting $N''(t) = 0$ gives $t = 2$. Furthermore, $N''(t) > 0$ when $t < 2$ and $N''(t) < 0$ when $t > 2$. Therefore, $t = 2$ is an inflection point of N. Thus, the average worker is performing at peak efficiency at 10 A.M.

39. $$R'(x) = k\frac{d}{dx}x(M-x) = k\left[(M-x) + x(-1)\right] = k(M-2x)$$

Setting $R'(x) = 0$ gives $M - 2x = 0$, or $x = \frac{M}{2}$, a critical point of R. Since $R''(x) = -2k < 0$, we see that $x = M/2$ affords a maximum; that is R is greatest when half the population is infected.

40. The volume is $V = f(x) = x(10-2x)^2$ cubic units for $0 \le x \le 5$.



To maximize V, we compute

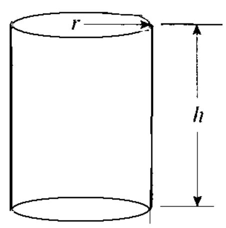
$$f'(x) = 12x^2 - 80x + 100 = 4(3x^2 - 20x + 25) = 4(3x-5)(x-5).$$

Setting $f'(x) = 0$ gives $x = 5/3$, or 5 as critical points of f. From the table

x	0	5/3	5
$f(x)$	0	2000/27≈ 74.07	0

We see that the box has a maximum volume of 74.07 cu in.

41. Suppose the radius is r and the height is h. Then the capacity is $\pi r^2 h$ and we want

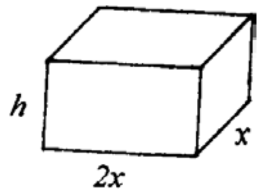


it to be 32π cu ft; that is, $\pi r^2 h = 32\pi$. Let the cost for the side by \$c/sq ft. Then the cost of construction is $C = 2\pi rhc + 2(\pi r^2)(2c) = 2\pi crh + 4\pi cr^2$. But $h = \dfrac{32\pi}{\pi r^2} = \dfrac{32}{r^2}$. Therefore,

$$C = f(r) = -\frac{64\pi c}{r^2} + 8\pi cr = \frac{-64\pi c + 8\pi cr^3}{r^2} = \frac{8\pi c(-8 + r^3)}{r^2}$$

Setting $f'(r) = 0$ gives $r^3 = 8$ or $r = 2$. Next, $f''(r) = \dfrac{128\pi c}{r^3} + 8\pi c$ and so $f''(2) > 0$. Therefore, $r = 2$ minimizes f. The required dimensions are $r = 2$ and $h = \dfrac{32}{4} = 8$. That is, its radius is 2 ft and its height is 8 ft.

42. Refer to the following picture.



$C(x) = 30(2)(2x)(x) + 20(2)(2xh + xh) = 120x^2 + 120xh$. But $x(2x)h = 4$, or $h = \dfrac{2}{x^2}$. Therefore, $C(x) = 120x^2 + 120x\left(\dfrac{2}{x^2}\right) = 120x^2 + \dfrac{240}{x}$

$$C'(x) = 240x - \frac{240}{x^2}.$$

Setting $C'(x) = 0$ gives $240x - \frac{240}{x^2} = 0$, or $x^3 = 1$. Therefore, $x = 1$.

$C''(x) = 240 + \dfrac{480}{x^3}$. In particular, $C''(1) > 0$. Therefore, the cost is minimized by taking $x = 1$. The required dimensions are 1 ft $\times$ 2 ft $\times$ 2 ft.

43. Let x denote the number of cases in each order. Then the average number of cases of beer in storage during the year is $x/2$. The storage cost is $2(x/2)$, or x dollars. Next, we see that the number of orders required is $800{,}000/x$, and so the ordering cost is

$$\frac{500(800{,}000)}{x} = \frac{400{,}000{,}000}{x}$$

dollars. Thus, the total cost incurred by the company per year is given by

$$C(x) = x + \frac{400{,}000{,}000}{x}.$$

We want to minimize C in the interval $(0, \infty)$. Now

$$C'(x) = 1 - \frac{400{,}000{,}000}{x^2}.$$

Setting $C'(x) = 0$ gives $x^2 = 400{,}000{,}000$, or $x = 20{,}000$ (we reject $x = -20{,}000$).

Next, $C''(x) = \dfrac{800{,}000{,}000}{x^3} > 0$ for all x, so C is concave upward. Thus, $x = 20{,}000$ gives rise to the absolute minimum of C. Thus, the company should order 20,000 cases of beer per order.

44. a. $f'(x) = 3x^2$ if $x \neq 0$. We see that $f'(x) > 0$ for $x < 0$ as well as for $x > 0$. In other

words $f'(x)$ does not change sign.

b. $f(0) = 2$ and is larger than $f(x)$ for x near $x = 0$. Therefore, f has a relative maximum at $x = 0$. This does not contradict the First Derivative Test because f is not continuous at $x = 0$.