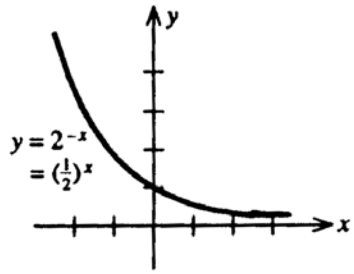
CHAPTER 5 REVIEW EXERCISES, page 395

1. a-b



Since $y = \left(\frac{1}{2}\right)^x = \frac{1}{2^x} = 2^{-x}$, it has the same graph as that of $y = 2^{-x}$.

2. If $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$, then $\log_{2/3}\left(\frac{27}{8}\right) = -3.$

3. $16^{-3/4} = 0.125$ is equivalent to $-\frac{3}{4} = \log_{16} 0.125$.

4. $\log_4(2x + 1) = 2$, $(2x + 1) = 4^2 = 16$, $2x = 15$, or $x = \frac{15}{2}$.

5.
$$\ln(x-1) + \ln 4 = \ln(2x+4) - \ln 2$$
$$\ln(x-1) - \ln(2x+4) = -\ln 2 - \ln 4 = -(\ln 2 + \ln 4)$$
$$\ln\left(\frac{x-1}{2x+4}\right) = -\ln 8 = \ln \tfrac{1}{8}.$$
$$\left(\frac{x-1}{2x+4}\right) = \frac{1}{8}$$
$$8x - 8 = 2x + 4$$
$$6x = 12, \text{ or } x = 2.$$

CHECK: l.h.s. $\ln(2-1) + \ln 4 = \ln 4$

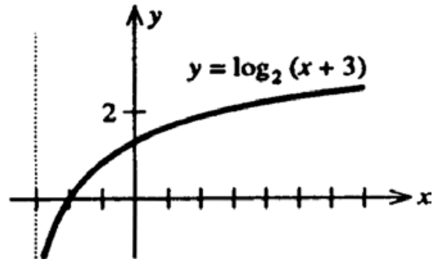
r.h.s $\ln(4+4) - \ln 2 = \ln 8 - \ln 2 = \ln \frac{8}{2} = \ln 4.$

6. $\ln 30 = \ln 2\times3\times5 = \ln 2 + \ln 3 + \ln 5 = x + y + z.$

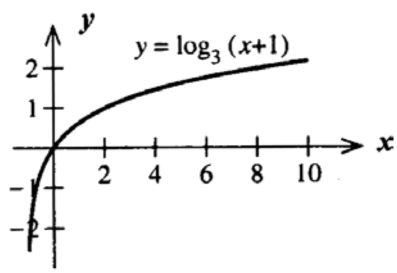
7. $\ln 3.6 = \ln \frac{36}{10} = \ln 36 - \ln 10 = \ln 6^2 - \ln 2\cdot 5 = 2\ln 6 - \ln 2 - \ln 5$
$= 2(\ln 2 + \ln 3) - \ln 2 - \ln 5 = 2(x + y) - x - z = x + 2y - z.$

8. $\ln 75 = \ln(3 \cdot 5^2) = \ln 3 + 2\ln 5 = y + 2z$.

9. We first sketch the graph of $y = 2^{x-3}$. Then we take the reflection of this graph with respect to the line $y = x$.



10. We first sketch the graph of $y = 3^{x-1}$. Then we take the reflection of this graph with respect to the line $y = x$.



11. $f(x) = xe^{2x}; f'(x) = e^{2x} + xe^{2x}(2) = (1 + 2x)e^{2x}$.

12. $f(t) = \sqrt{t}e^t + t;\ f'(t) = \frac{1}{2}t^{-1/2}e^t + t^{1/2}e^t + 1 = \dfrac{e^t}{2\sqrt{t}} + \sqrt{t}e^t + 1.$

13. $g(t) = \sqrt{t}e^{-2t};\ g'(t) = \frac{1}{2}t^{-1/2}e^{-2t} + \sqrt{t}e^{-2t}(-2) = \dfrac{1-4t}{2\sqrt{t}e^{2t}}.$

14. $g(x) = e^x(1 + x^2)^{1/2}$.

$$g'(x) = e^x\frac{d}{dx}(1+x^2)^{1/2} + (1+x^2)^{1/2}\frac{d}{dx}e^x = e^x\tfrac{1}{2}(1+x^2)^{-1/2}(2x) + (1+x^2)^{1/2}e^x$$

$$= e^x(1+x^2)^{-1/2}(x+1+x^2) = \frac{e^x(x^2+x+1)}{\sqrt{1+x^2}}.$$

15. $y = \dfrac{e^{2x}}{1+e^{-2x}};\ y' = \dfrac{(1+e^{-2x})e^{2x}(2) - e^{2x}\cdot e^{-2x}(-2)}{(1+e^{-2x})^2} = \dfrac{2(e^{2x}+2)}{(1+e^{-2x})^2}.$

16. $f(x) = e^{2x^2-1}$; $f'(x) = e^{2x^2-1}(4x) = 4x\,e^{2x^2-1}$.

17. $f(x) = xe^{-x^2}$; $f'(x) = e^{-x^2} + xe^{-x^2}(-2x) = (1 - 2x^2)e^{-x^2}$.

18. $g(x) = (1 + e^{2x})^{3/2}$; $g'(x) = \frac{3}{2}(1+e^{2x})^{1/2} \cdot e^{2x}(2) = 3e^{2x}(1+e^{2x})^{1/2}$.

19. $f(x) = x^2e^x + e^x$;
$f'(x) = 2xe^x + x^2e^x + e^x = (x^2 + 2x + 1)e^x = (x + 1)^2e^x$.

20. $g(t) = t \ln t$; $g'(t) = \ln t + t(\frac{1}{t}) = \ln t + 1$

21. $f(x) = \ln(e^{x^2} + 1)$; $f'(x) = \dfrac{e^{x^2}(2x)}{e^{x^2}+1} = \dfrac{2xe^{x^2}}{e^{x^2}+1}$.

22. $f(x) = \dfrac{x}{\ln x}$. $f'(x) = \dfrac{\ln x \frac{d}{dx}x - x\frac{d}{dx}\ln x}{(\ln x)^2} = \dfrac{\ln x - x\cdot\frac{1}{x}}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$.

23. $f(x) = \dfrac{\ln x}{x+1}$. $f'(x) = \dfrac{(x+1)\left(\frac{1}{x}\right) - \ln x}{(x+1)^2} = \dfrac{1 + \frac{1}{x} - \ln x}{(x+1)^2} = \dfrac{x - x\ \ln x + 1}{x(x+1)^2}$.

24. $y = (x + 1)e^x$; $y' = e^x + (x+1)e^x = (x+2)e^x$.

25. $y = \ln(e^{4x} + 3)$; $y' = \dfrac{e^{4x}(4)}{e^{4x}+3} = \dfrac{4e^{4x}}{e^{4x}+3}$.

26. $f(r) = \dfrac{re^r}{1+r^2}$; $f'(r) = \dfrac{(1+r^2)(e^r + re^r) - re^r(2r)}{(1+r^2)^2} = \dfrac{(r^3 - r^2 + r + 1)e^r}{(1+r^2)^2}$.

27. $f(x) = \dfrac{\ln x}{1+e^x}$;

$$f'(x) = \frac{(1+e^x)\frac{d}{dx}\ln x - \ln x\frac{d}{dx}(1+e^x)}{(1+e^x)^2} = \frac{(1+e^x)\left(\frac{1}{x}\right) - (\ln x)e^x}{(1+e^x)^2}$$

$$= \frac{1+e^x - xe^x \ln x}{x(1+e^x)^2} = \frac{1+e^x(1-x\ln x)}{x(1+e^x)^2}.$$

28. $g(x) = \dfrac{e^{x^2}}{1+\ln x}$; $\quad g'(x) = \dfrac{(1+\ln x)e^{x^2}(2x) - e^{x^2}\left(\dfrac{1}{x}\right)}{(1+\ln x)^2} = \dfrac{(2x^2+2x^2\cdot \ln x - 1)e^{x^2}}{x(1+\ln x)^2}$

29. $y = \ln(3x+1)$; $y' = \dfrac{3}{3x+1}$;

$$y'' = 3\frac{d}{dx}(3x+1)^{-1} = -3(3x+1)^{-2}(3) = -\frac{9}{(3x+1)^2}.$$

30. $y = x\ln x$; $y' = \ln x + x\left(\dfrac{1}{x}\right) = \ln x + 1$ and $y'' = \dfrac{1}{x}$.

31. $h'(x) = g'(f(x))f'(x)$. But $g'(x) = 1 - \dfrac{1}{x^2}$ and $f'(x) = e^x$.

So $f(0) = e^0 = 1$ and $f'(0) = e^0 = 1$. Therefore,

$h'(0) = g'(f(0))f'(0) = g'(1)f'(0) = 0 \cdot 1 = 0.$

32. $h'(1) = g'[f(1)]f'(1)$ by the Chain Rule.

Now, $g'(x) = \dfrac{(x-1)-(x+1)}{(x-1)^2} = -\dfrac{2}{(x-1)^2}$;

$f'(x) = \dfrac{1}{x}$. Now, $f(1) = 0$. So $h'(1) = -\dfrac{2}{(-1)^2} \cdot 1 = -2$.

33. $y = (2x^3+1)(x^2+2)^3$. $\quad \ln y = \ln(2x^3+1) + 3\ln(x^2+2)$.

$$\frac{y'}{y} = \frac{6x^2}{2x^3+1} + \frac{3(2x)}{x^2+2} = \frac{6x^2(x^2+2)+6x(2x^3+1)}{(2x^3+1)(x^2+2)}$$

$$= \frac{6x^4+12x^2+12x^4+6x}{(2x^3+1)(x^2+2)} = \frac{18x^4+12x^2+6x}{(2x^3+1)(x^2+2)}.$$

Therefore, $y' = 6x(3x^3+2x+1)(x^2+2)^2$.

34. $f(x) = \dfrac{x(x^2-2)^2}{x-1}$. $\ln f(x) = \ln x + 2\ln(x^2-2) - \ln(x-1)$. So

$$\frac{f'(x)}{f(x)} = \frac{1}{x} + \frac{2(2x)}{x^2-2} - \frac{1}{x-1} = \frac{(x^2-2)(x-1)+4x^2(x-1)-x(x^2-2)}{x(x-1)(x^2-2)}$$

$$= \frac{4x^3 - 5x^2 + 2}{x(x-1)(x^2-2)}.$$

$$f'(x) = \frac{4x^3 - 5x^2 + 2}{x(x-1)(x^2-2)} \cdot \frac{x(x^2-2)^2}{x-1} = \frac{(4x^3 - 5x^2 + 2)(x^2-2)}{(x-1)^2}.$$

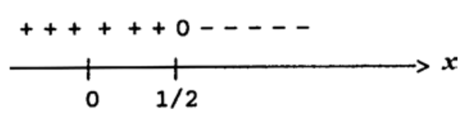
35. $y = e^{-2x}$. $y' = -2e^{-2x}$ and this gives the slope of the tangent line to the graph of $y = e^{-2x}$ at any point (x, y). In particular, the slope of the tangent line at $(1, e^{-2})$ is $y'(1) = -2e^{-2}$. The required equation is $y - e^{-2} = -2e^{-2}(x - 1)$ or $y = \frac{1}{e^2}(-2x + 3)$.

36. $y = xe^{-x}$; $y' = e^{-x} + xe^{-x}(-1) = (1 - x)e^{-x}$. The slope of the tangent line at $(1, e^{-1})$, where $x = 1$, is 0. Therefore, a required equation is $y = 1/e$.

37. $f(x) = xe^{-2x}$.

We first gather the following information on f.

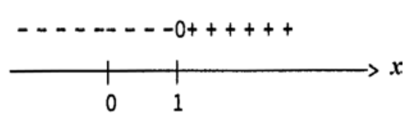
1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 0 as the y-intercept.
3. $\lim_{x \to -\infty} xe^{-2x} = -\infty$ and $\lim_{x \to \infty} xe^{-2x} = 0$.
4. The results of (3) show that $y = 0$ is a horizontal asymptote.
5. $f'(x) = e^{-2x} + xe^{-2x}(-2) = (1-2x)e^{-2x}$. Observe that $f'(x) = 0$ if $x = 1/2$, a critical point of f. The sign diagram of f'



shows that f is increasing on $(-\infty, \frac{1}{2})$ and decreasing on $(\frac{1}{2}, \infty)$.

6. The results of (5) show that $(\frac{1}{2}, \frac{1}{2}e^{-1})$ is a relative maximum.

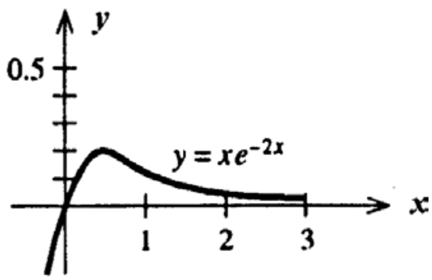
7. $f''(x) = -2e^{-2x} + (1-2x)e^{-2x}(-2) = 4(x-1)e^{-2x}$ and is equal to zero if $x = 1$. The sign diagram of f''



shows that the graph of f is concave downward on $(-\infty, 1)$ and concave upward on

$(1,\infty)$.

The graph of f follows.

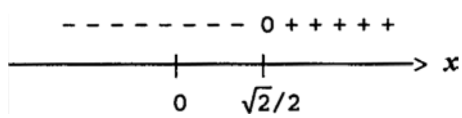


38. $f(x) = x^2 - \ln x$.

We first gather the following information on f.

1. The domain of f is $(0,\infty)$.
2. There are no y-intercepts.
3. $\lim_{x\to\infty} (x^2 - \ln x) = \infty$.
4. There are no asymptotes.
5. $f'(x) = 2x - \frac{1}{x} = \frac{2x^2 - 1}{x}$. Setting $f'(x) = 0$ gives $x = \pm \frac{\sqrt{2}}{2}$. We reject the

negative root. So $x = \frac{\sqrt{2}}{2}$ is a critical point of f. The sign diagram of f'



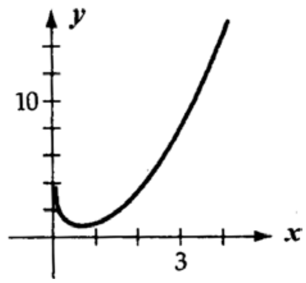
shows that f is decreasing on $(0, \frac{\sqrt{2}}{2})$ and increasing on $(\frac{\sqrt{2}}{2}, \infty)$.

6. The results of (5) show that $(\frac{\sqrt{2}}{2}, \frac{1}{2}(1+\ln 2))$ is a relative minimum of f.

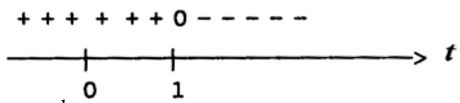
7. $f''(x) = 2 + \frac{1}{x^2}$. Observe that $f''(x) > 0$ for all x in $(0,\infty)$. So the graph of f is concave upward on $(0,\infty)$.

8. The results of (7) show that f has no inflection points.

The graph of f follows.



39. $f(t) = te^{-t}$. $f'(t) = e^{-t} + t(-e^{-t}) = e^{-t}(1 - t)$. Setting $f'(t) = 0$ gives $t = 1$ as the only critical point of f. From the sign diagram of f'



we see that $f(1) = e^{-1} = 1/e$ is the absolute maximum value of f.

40. $g(t) = \dfrac{\ln t}{t}$. $g'(t) = \dfrac{t\left(\frac{1}{t}\right) - \ln t}{t^2} = \dfrac{1 - \ln t}{t^2}$. Observe that $g'(t) = 0$ if $t = e$. But this point lies outside the interval [1, 2].

t	1	2
$\frac{\ln t}{t}$	0	$\frac{\ln 2}{2}$

From the table, we see that g has an absolute minimum at (1,0) and an absolute maximum at $(2, \frac{\ln 2}{2})$.

41. We want to find r where r satisfies the equation $8.2 = 4.5\,e^{r(5)}$. We have

$$e^{5r} = \frac{8.2}{4.5} \quad \text{or} \quad r = \frac{1}{5}\ln\left(\frac{8.2}{4.5}\right) \approx 0.12$$

and so the annual rate of return is 12 percent per year.

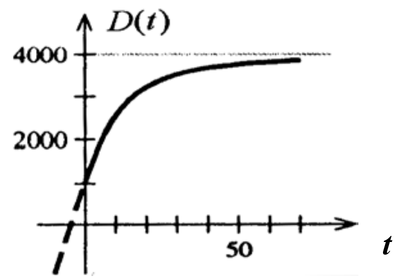
42. $P = 119346e^{-(0.1)\,4} \approx 80{,}000$, or \$80,000.

43. a. $Q(t) = 2000e^{kt}$. Now $Q(120) = 18{,}000$ gives $2000e^{120k} = 18{,}000$, $e^{120k} = 9$,

or $120k = \ln 9$. So $k = \frac{1}{120}\ln 9 \approx 0.01831$ and $Q(t) = 2000e^{0.01831t}$.
b. $Q(4) = 2000e^{0.01831(240)} \approx 161{,}992$, or approximately 162,000.

44. We have $Q(t) = Q_0e^{-kt}$, where Q_0 is the amount of radium present initially. Since the half-life of radium is 1600 years, we have $\frac{1}{2}Q_0 = Q_0e^{-1600k}$, $e^{-1600k} = \frac{1}{2}$, $-1600k = \ln\frac{1}{2} = -\ln 2$, and $k = \dfrac{\ln 2}{1600} \approx 0.0004332$.

45.



a. $D(1) = 4000 - 3000e^{-0.06} = 1175$, $D(12) = 4000 - 3000e^{-0.72} = 2540$, and $D(24) = 4000 - 3000e^{-1.44} = 3289$.
b. $\lim_{t\to\infty} D(t) = \lim_{t\to\infty}(4000 - 3000e^{-0.06t}) = 4000$.

46. We have $Q(10) = 90$ and this gives $\dfrac{3000}{1+499e^{-10k}} = 90$, $1 + 499e^{-10k} = \dfrac{3000}{90}$, $499e^{-10k} = \dfrac{2910}{90}$, $e^{-10k} = \dfrac{2910}{90(499)}$, and $k = -\dfrac{1}{10}\ln\dfrac{2910}{90(499)} \approx 0.2737$.

So $N(t) = \dfrac{3000}{1+499e^{-0.2737t}}$. The number of students who have contracted the flu by the 20th day is $N(20) = \dfrac{3000}{1+499e^{-0.2737(20)}} \approx 969.92$, or approximately 970 students.