CHAPTER 7 REVIEW EXERCISES, page 534

1. Let $u = 2x$ and $dv = e^{-x}\,dx$ so that $du = 2\,dx$ and $v = -e^{-x}$. Then

$$\int 2xe^{-x}\,dx = uv - \int v\,du = -2xe^{-x} + 2\int e^{-x}\,dx$$
$$= -2xe^{-x} - 2e^{-x} + C = -2(1+x)e^{-x} + C.$$

2. Let $u = x$ and $dv = e^{4x}\,dx$, so that $du = dx$ and $v = \frac{1}{4}e^{4x}$. Then

$$\int xe^{4x}\,dx = \tfrac{1}{4}xe^{4x} - \tfrac{1}{4}\int e^{4x}\,dx = \tfrac{1}{4}xe^{4x} - \tfrac{1}{16}e^{4x} + C$$
$$= \tfrac{1}{16}(4x-1)e^{4x} + C.$$

3. Let $u = \ln 5x$ and $dv = dx$, so that $du = \frac{1}{x}\,dx$ and $v = x$. Then

$$\int \ln 5x\,dx = x\ln 5x\,dx - \int dx = x\ln 5x - x + C = x(\ln 5x - 1) + C.$$

4. Let $u = \ln 2x$ and $dv = dx$, so that $du = \frac{1}{x}\,dx$ and $v = x$. Then

$$\int_1^4 \ln 2x\ dx = x\ \ln 2x\Big|_1^4 - \int_1^4 dx = 4\ln 8 - \ln 2 - \left[x\Big|_1^4\right] = 4\ln 8 - \ln 2 - 3.$$

5. Let $u = x$ and $dv = e^{-2x}\,dx$ so that $du = dx$ and $v = -\frac{1}{2}e^{-2x}$. Then

$$\int_0^1 xe^{-2x}\,dx = -\tfrac{1}{2}xe^{-2x}\Big|_0^1 + \tfrac{1}{2}\int_0^1 e^{-2x}\,dx = -\tfrac{1}{2}e^{-2} - \tfrac{1}{4}e^{-2x}\Big|_0^1$$
$$= -\tfrac{1}{2}e^{-2} - \tfrac{1}{4}e^{-2} + \tfrac{1}{4} = \tfrac{1}{4}(1 - 3e^{-2}).$$

6. Let $u = x$ and $dv = e^{2x}\,dx$ so that $du = dx$ and $v = \frac{1}{2}e^{2x}$.

$$\int_0^2 xe^{2x}\,dx = \tfrac{1}{2}xe^{2x}\Big|_0^2 - \tfrac{1}{2}\int_0^2 e^{2x}\,dx = e^4 - \tfrac{1}{4}e^{2x}\Big|_0^2 = e^4 - \tfrac{1}{4}e^4 + \tfrac{1}{4} = \tfrac{1}{4}(1 + 3e^4).$$

7. $f(x) = \int f'(x)\,dx = \int \frac{\ln x}{\sqrt{x}}\,dx.$ To evaluate the integral, we integrate by parts with $u = \ln x$, $dv = x^{-1/2}\,dx$, $du = \frac{1}{x}\,dx$ and $v = 2x^{1/2}\,dx$. Then

$$\int \frac{\ln x}{x^{1/2}}\,dx = 2x^{1/2}\ln x - \int 2x^{-1/2}\,dx = 2x^{1/2}\ln x - 4x^{1/2} + C$$
$$= 2x^{1/2}(\ln x - 2) + C = 2\sqrt{x}(\ln x - 2) + C.$$

But $f(1)=-2$ and this gives $2\sqrt{1}(\ln 1-2)+C=-2,$ or $C=2$. Therefore, $f(x)=2\sqrt{x}(\ln x-2)+2.$

8. $f'(x)=xe^{-3x};$

Let $u=x,\ dv=e^{-3x}\,dx,\ du=dx,\ v=-\frac{1}{3}e^{-3x}.$ Then

$$f(x)=uv-\int v\,du=-\frac{1}{3}xe^{-3x}+\frac{1}{3}\int e^{-3x}\,dx=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}+C.$$

Since $f(0)=0$, $-\frac{1}{9}+C=0$ and $C=\frac{1}{9}.$ Therefore,

$$f(x)=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}+\frac{1}{9}.$$

9. Using Formula 4 with $a = 3$ and $b = 2$, we obtain

$$\int \frac{x^2}{(3+2x)^2}\,dx=\frac{1}{8}\left[3+2x-\frac{9}{3+2x}-6\ln|3+2x|\right]+C.$$

10. Using Formula 5 with $a = 3$ and $b = 2$, we have

$$\int \frac{2x}{\sqrt{2x+3}}\,dx=2\int \frac{x}{\sqrt{2x+3}}\,dx=2\cdot\frac{2}{3(4)}(2x-6)\sqrt{2x+3}+C$$

$$=\frac{2}{3}(x-3)\sqrt{2x+3}+C.$$

11. Use Formula 24 with $a = 4$ and $n = 2$, obtaining $\int x^2e^{4x}\,dx=\frac{1}{4}x^2e^{4x}-\frac{1}{2}\int xe^{4x}\,dx.$

Use Formula 23 to obtain

$$\int x^2e^{4x}\,dx=\tfrac{1}{4}x^2e^{4x}-\tfrac{1}{2}\left[\tfrac{1}{16}(4x-1)e^{4x}\right]+C$$

$$=\tfrac{1}{32}(8x^2-4x+1)e^{4x}+C.$$

12. Use Formula 18 with $a = 5$. We obtain $\int \frac{dx}{(x^2-25)^{3/2}}=-\frac{x}{25\sqrt{x^2-25}}+C.$

13. Use Formula 17 with $a = 2$ obtaining $\int \frac{dx}{x^2\sqrt{x^2-4}} = \frac{\sqrt{x^2-4}}{4x} + C.$

14. First, we make the subtitution $u = 2x$ so that $du = 2\ dx$, or $dx = \frac{1}{2}du$.

Then with $x = \frac{1}{2}u$, we have $\int 8x^3 \ln 2x\, dx = \int 8(\frac{u}{2})^3 \ln u(\frac{1}{2}du) = \frac{1}{2}\int u^3 \ln u\, du.$

Then use Formula 27 with $n = 3$, obtaining

$$\int u^3 \ln u\, du = \frac{u^4}{16}(4\ln u - 1) + C.$$

Therefore, $\int 8x^3 \ln 2x\, dx = \frac{1}{2}\cdot\frac{(2x)^4}{16}(4\ln 2x - 1) + C\ = \frac{1}{2}x^4(4\ln 2x - 1) + C.$

15. $$\int_0^\infty e^{-2x}\, dx = \lim_{b\to\infty}\int_0^b e^{-2x}\, dx = \lim_{b\to\infty}(-\tfrac{1}{2}e^{-2x})\Big|_0^b = \lim_{b\to\infty}(-\tfrac{1}{2}e^{-2b} + \tfrac{1}{2}) = \tfrac{1}{2}.$$

16. $$\int_{-\infty}^0 e^{3x}\, dx = \lim_{a\to-\infty} e^{3x}dx = \lim_{a\to-\infty}(\tfrac{1}{3}e^{3x})\Big|_a^0 = \lim_{a\to-\infty}(\tfrac{1}{3} - \tfrac{1}{3}e^{3a}) = \tfrac{1}{3.}$$

17. $$\int_3^\infty \frac{2}{x}dx = \lim_{b\to\infty}\int_3^b \frac{2}{x}dx = \lim_{b\to\infty} 2\ \ln x\Big|_3^b = \lim_{b\to\infty}(2\ln b - 2\ln 3) = \infty.$$

18. $$\int_2^\infty \frac{1}{(x+2)^{3/2}}dx = \lim_{b\to\infty}\int_2^b (x+2)^{-3/2}\, dx = \lim_{b\to\infty} -2(x+2)^{-1/2}\Big|_2^b$$

$$= \lim_{b\to\infty}\left[-\frac{2}{(b+2)^{1/2}} + 1\right] = 1.$$

19. $$\int_2^\infty \frac{dx}{(1+2x)^2} = \lim_{b\to\infty}\int_2^b (1+2x)^{-2}\, dx = \lim_{b\to\infty}(\tfrac{1}{2})(-1)(1+2x)^{-1}\Big|_2^b$$

$$= \lim_{b\to\infty}\left(-\frac{1}{2(1+2b)} + \frac{1}{2(5)}\right) = \frac{1}{10}.$$

20. $$\int_1^\infty 3e^{1-x}\, dx = \lim_{b\to\infty}\int_1^b 3e^{1-x}\, dx = \lim_{b\to\infty} -3e^{1-x}\Big|_1^b = \lim_{b\to\infty}(-3e^{1-b} + 3) = 3.$$

21. $\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2}; x_0 = 1,\ x_1 = \frac{3}{2},\ x_2 = 2,\ x_3 = \frac{5}{2},\ x_4 = 3.$

Trapezoidal Rule:

$$\int_1^3 \frac{dx}{1+\sqrt{x}} \approx \frac{\frac{1}{2}}{2}\left[\frac{1}{2}+\frac{2}{1+\sqrt{1.5}}+\frac{2}{1+\sqrt{2}}+\frac{2}{1+\sqrt{2.5}}+\frac{1}{1+\sqrt{3}}\right] \approx 0.8421.$$

Simpson's Rule

$$\int_1^3 \frac{dx}{1+\sqrt{x}} \approx \frac{\frac{1}{2}}{3}\left[\frac{1}{2}+\frac{4}{1+\sqrt{1.5}}+\frac{2}{1+\sqrt{2}}+\frac{4}{1+\sqrt{2.5}}+\frac{1}{1+\sqrt{3}}\right] \approx 0.8404.$$

22. $\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}; x_0 = 0,\ x_1 = \frac{1}{4},\ x_2 = \frac{2}{4},\ x_3 = \frac{3}{4},\ x_4 = \frac{4}{4}.$

Trapezoidal Rule:

$$\int_0^1 e^{x^2}\,dx \approx \frac{\frac{1}{4}}{2}\left[1+2e^{(0.25)^2}+2e^{(0.5)^2}+2e^{(0.75)^2}+e\right] \approx 1.491$$

Simpson's Rule:

$$\int_0^1 e^{x^2}\,dx \approx \frac{\frac{1}{4}}{3}\left[1+4e^{(0.25)^2}+2e^{(0.5)^2}+4e^{(0.75)^2}+e\right] \approx 1.464$$

23. $\Delta x = \frac{1-(-1)}{4} = \frac{1}{2};\ x_0 = -1,\ x_1 = -\frac{1}{2},\ x_2 = 0,\ x_3 = \frac{1}{2},\ x_4 = 1.$

Trapezoidal Rule:

$$\int_{-1}^1 \sqrt{1+x^4}\,dx \approx \frac{0.5}{2}\left[\sqrt{2}+2\sqrt{1+(-0.5)^4}+2+2\sqrt{1+(0.5)^4}+\sqrt{2}\right] \approx 2.2379.$$

Simpson's Rule:

$$\int_{-1}^1 \sqrt{1+x^4}\,dx \approx \frac{0.5}{3}\left[\sqrt{2}+4\sqrt{1+(-0.5)^4}+2+4\sqrt{1+(0.5)^4}+\sqrt{2}\right]$$
$$\approx 2.1791.$$

24. Here $a = 1$, $b = 3$, $n = 4$ and $\Delta x = 0.5$, $x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$, and $x_4 = 3$.

Trapezoidal Rule:

$$\int_1^3 \frac{e^x}{x}dx \approx \frac{0.5}{2}\left(e^1+\frac{2e^{1.5}}{1.5}+\frac{2e^2}{2}+\frac{2e^{2.5}}{2.5}+\frac{e^3}{3}\right)$$
$$\approx 0.25(2.7182818 + 5.9755854 + 7.389056 + 9.7459952 + 6.695179)$$
$$\approx 8.1310.$$

Simpson's Rule:

$$\int_1^3 \frac{e^x}{x}\,dx \approx \frac{0.5}{3}\left(e^1 + \frac{4e^{1.5}}{1.5} + \frac{2e^2}{2} + \frac{4e^{2.5}}{2.5} + \frac{e^3}{3}\right)$$

$\approx$ 0.1666667(2.7182818 + 11.951171 + 7.389056 + 19.49199 + 6.695179)

$\approx$ 8.041.

25. The producer's surplus is given by $PS = \bar{p}\bar{x} - \int_0^{\bar{x}} s(x)\,dx$, where $\bar{x}$ is found by solving the equation $2\sqrt{25+x^2} = 13$. Then $\sqrt{25+x^2} = 13$, $25 + x^2 = 169$, and $x = \pm 12$. So $\bar{x} = 12$. Therefore, $PS = (26)(12) - 2\int_0^{12} (25+x^2)^{1/2}\,dx$.

Using Formula 7 with a = 5, we obtain

$$PS = (26)(12) - 2\int_0^{12} (25+x^2)^{1/2}\,dx$$

$$= 312 - 2\left(\tfrac{x}{2}(25+x^2)^{1/2} + \tfrac{25}{2}\ln\left|x + (25+x^2)^{1/2}\right|\right|_0^{12}$$

$$= 312 - 2[6(13) + \tfrac{25}{2}\ln(12+13) - \tfrac{25}{2}\ln 5] \approx 115.76405,$$

or \$1,157,641.

26. Let $u = t$ and $dv = e^{-0.05t}$ so that $du = 1$ and $v = -20e^{-0.05t}$, and integrate by parts obtaining

$$S(t) = -20te^{-0.05t} + \int 20e^{-0.05t}\,dt = -20te^{-0.05t} - 400e^{-0.05t} + C$$

$$= -20te^{-0.05t} - 400e^{-0.05t} + C = -20e^{-0.05t}(t+20) + C.$$

The initial condition implies $S(0) = 0$ giving $-20(20) + C = 0$, or $C = 400$.
Therefore, $S(t) = -20e^{-0.05t}(t + 20) + 400$.
By the end of the first year, the number of units sold is given by

$S(12) = -20e^{-0.6}(32) + 400 = 48.761$, or 48,761 cartridges.

27. If $p = 30$, we have $2\sqrt{325 - x^2} = 30$, $\sqrt{325 - x^2} = 15$, or $325 - x^2 = 225$, $x^2 = 100$, or $x = \pm 10$. So the equilibrium point is (10, 30).

$$CS = \int_0^{10} 2\sqrt{325 - x^2}\,dx - (30)(10).$$

To evaluate the integral using Simpson's Rule with n = 10, we have

$\Delta x = \frac{10-0}{10} = 1;\ x_0 = 0, x_1 = 1, x_2 = 2, ..., x_{10} = 10.$

$$2\int_0^{10} \sqrt{325-x^2}\,dx$$
$$\approx \tfrac{2}{3}\left[\sqrt{325} + 4\sqrt{325-1} + 2\sqrt{325-4} + \cdots + 4\sqrt{325-81} + \sqrt{325-100}\right]$$

Therefore, $CS \approx$ 341.0 - 300 $\approx$ 41.1 , or \$41,100.

28. Trapezoidal Rule:

$A = \frac{100}{2}$ [0 + 480 + 520 + 600 + 680 + 680 + 800 + 680 + 600 + 440 + 0]
= 274,000, or 274,000 sq ft.

Simpson's Rule:

$A = \frac{100}{3}$ [0 + 960 + 520 + 1200 + 680 + 1360 + 800 + 1360 + 600 + 880 + 0]
= 278,667, or 278,667 sq ft.

29. We want the present value of a perpetuity with m = 1, P = 10,000, and r = 0.09. We find $PV = \frac{(1)(10{,}000)}{0.09} \approx 111{,}111$ or approximately \$111,111.