CHAPTER 8 REVIEW EXERCISES, page 618

1. $f(0,1) = 0$; $f(1,0) = 0$; $f(1,1) = \dfrac{1}{1+1} = \dfrac{1}{2}$.
   $f(0,0)$ does not exist because the point (0,0) does not lie in the domain of f.

2. $f(1,1) = \dfrac{e}{1+\ln 1} = e$; $f(1,2) = \dfrac{e^2}{1+\ln 2}$; $f(2,1) = \dfrac{2e}{1+\ln 2}$;.
   $f(1,0)$ does not exist because the point (0,0) does not lie in the domain of f.

3. $h(1,1,0) = 1 + 1 = 2$; $h(-1,1,1) = -e - 1 = -(e + 1)$;
   $h(1,-1,1) = -e - 1 = -(e + 1)$.

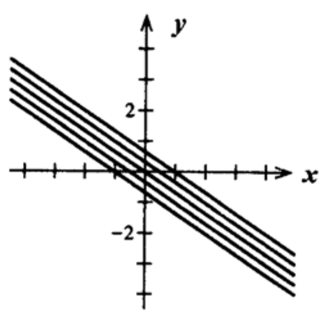
4. The domain of f is the set of all ordered pairs (u,v) of real numbers such that $u \geq 0$ and $u \neq v$.

5. $D = \{(x,y) | y \neq -x\}$

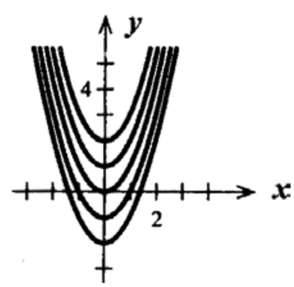
6. $D = \{(x,y) \mid x \leq 1, y \geq 0\}$

7. The domain of f is the set of all ordered triplets (x,y,z) of real numbers such that $z \geq 0$ and $x \neq 1$, $y \neq 1$, and $z \neq 1$.

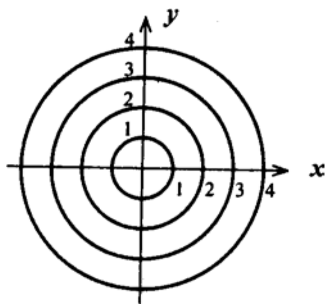
8. $2x + 3y = z$



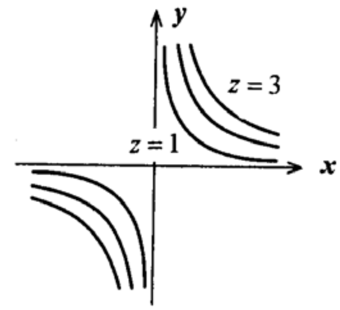
9. $z = y - x^2$



10. $z = \sqrt{x^2 + y^2}$



11. $z = e^{xy}$



12. $f(x,y) = x^2y^3 + 3xy^2 + \dfrac{x}{y}$; $f_x = 2xy^3 + 3y^2 + \dfrac{1}{y}$; $f_y = 3x^2y^2 + 6xy - \dfrac{x}{y^2}$.

13. $f(x,y) = x\sqrt{y} + y\sqrt{x}$; $f_x = \sqrt{y} + \dfrac{y}{2\sqrt{x}}$; $f_y = \dfrac{x}{2\sqrt{y}} + \sqrt{x}$

14. $f(u,v) = \sqrt{uv^2 - 2u}$; $f_u = \frac{1}{2}(uv^2 - 2u)^{1/2}(v^2 - 2) = \dfrac{v^2 - 2}{2\sqrt{uv^2 - 2u}}$.

$$f_v = \tfrac{1}{2}(uv^2 - 2u)^{-1/2}(2uv) = \frac{uv}{\sqrt{uv^2 - 2u}}.$$

15. $f(x,y) = \dfrac{x - y}{y + 2x}$. $f_x = \dfrac{(y+2x) - (x-y)(2)}{(y+2x)^2} = \dfrac{3y}{(y+2x)^2}$.

$$f_y = \frac{(y+2x)(-1) - (x-y)}{(y+2x)^2} = \frac{-3x}{(y+2x)^2}.$$

16. $g(x,y) = \dfrac{xy}{x^2 + y^2}$;

$$g_x = \frac{(x^2+y^2)y - xy(2x)}{(x^2+y^2)^2} = \frac{y^3 - x^2y}{(x^2+y^2)^2} = \frac{y(y^2 - x^2)}{(x^2+y^2)^2} = \frac{y(y-x)(y+x)}{(x^2+y^2)^2};$$

$$g_y = \frac{(x^2+y^2)x - xy(2y)}{(x^2+y^2)^2} = \frac{x(x^2 - y^2)}{(x^2+y^2)^2} = \frac{x(x-y)(x+y)}{(x^2+y^2)^2};$$

17. $h(x,y) = (2xy + 3y^2)^5; h_x = 10y(2xy + 3y^2)^4$; $h_y = 10(x + 3y)(2xy + 3y^2)^4$.

18. $f(x,y)=(xe^y+1)^{1/2}$; $f_x=\frac{1}{2}(xe^y+1)^{-1/2}e^y=\dfrac{e^y}{2(xe^y+1)^{1/2}}$;

$f_y=\frac{1}{2}(xe^y+1)^{-1/2}xe^y=\dfrac{xe^y}{2(xe^y+1)^{1/2}}$

19. $f(x,y)=(x^2+y^2)e^{x^2+y^2}$;

$f_x=2xe^{x^2+y^2}+(x^2+y^2)(2x)e^{x^2+y^2}=2x(x^2+y^2+1)e^{x^2+y^2}$.

$f_y=2ye^{x^2+y^2}+(x^2+y^2)(2y)e^{x^2+y^2}=2y(x^2+y^2+1)e^{x^2+y^2}$.

20. $f(x,y)=\ln(1+2x^2+4y^4)$; $f_x=\dfrac{4x}{1+2x^2+4y^4}$; $f_y=\dfrac{16y^3}{1+2x^2+4y^4}$.

21. $f(x,y)=\ln\left(1+\dfrac{x^2}{y^2}\right)$. $f_x=\dfrac{\frac{2x}{y^2}}{1+\frac{x^2}{y^2}}=\dfrac{2x}{x^2+y^2}$; $f_y=\dfrac{-\frac{2x^2}{y^3}}{1+\frac{x^2}{y^2}}=-\dfrac{2x^2}{y(x^2+y^2)}$.

22. $f(x,y)=x^3-2x^2y+y^2+x-2y$; $f_x=3x^2-4xy+1$; $f_y=-2x^2+2y-2$;
Therefore, $f_{xx}=6x-4y$, $f_{xy}=f_{yx}=-4x$, $f_{yy}=2$.

23. $f(x,y)=x^4+2x^2y^2-y^4$; $f_x=4x^3+4xy^2$; $f_y=4x^2y-4y^3$;

$f_{xx}=12x^2+4y^2$, $f_{xy}=8xy=f_{yx}$, $f_{yy}=4x^2-12y^2$.

24. $f_x=3(2x^2+3y^2)^2(4x)=12x(2x^2+3y^2)^2$;

$f_y=3(2x^2+3y^2)^2(6y)=18y(2x^2+3y^2)^2$;

$$f_{xx}=12(2x^2+3y^2)^2+12x(2)(2x^2+3y^2)(4x)$$
$$=12(2x^2+3y^2)^2[(2x^2+3y^2)+8x^2]=12(2x^2+3y^2)$$
$$f_{xy}=12x(2)(2x^2+3y^2)(6y)=144xy(2x^2+3y^2)(10x^2+3y^2)$$
$$f_{yy}=18(2x^2+3y^2)^2+18y(2)(2x^2+3y^2)(6y)$$
$$=18(2x^2+3y^2)[(2x^2+3y^2)+12y^2]$$
$$=18(2x^2+3y^2)(2x^2+15y^2)$$

25. $g(x,y)=\dfrac{x}{x+y^2}$; $g_x=\dfrac{(x+y^2)-x}{(x+y^2)^2}=\dfrac{y^2}{(x+y^2)^2}$, $g_y=\dfrac{-2xy}{(x+y^2)^2}$.

Therefore, $g_{xx} = -2y^2(x+y^2)^{-3} = -\dfrac{2y^2}{(x+y^2)^3}$,

$$g_{yy} = \frac{(x+y^2)^2(-2x)+2xy(2)(x+y^2)2y}{(x+y^2)^4} = \frac{2x(x^2+y^2)[-x-y^2+4y^2]}{(x+y^2)^4}$$
$$= \frac{2x(3y^2-x)}{(x+y^2)^3}.$$

and
$$g_{xy} = \frac{(x+y^2)2y-y^2(2)(x+y^2)2y}{(x+y^2)^4} = \frac{2(x+y^2)[xy+y^3-2y^3]}{(x+y^2)^4}$$
$$= \frac{2y(x-y^2)}{(x+y^2)^3} = g_{yx}.$$

26. $g(x,y) = e^{x^2+y^2}$; $g_x = 2xe^{x^2+y^2}$, $g_y = 2ye^{x^2+y^2}$;

$g_{xx} = 2e^{x^2+y^2} + (2x)^2 e^{x^2+y^2} = 2(1+2x^2)e^{x^2+y^2}$

$g_{xy} = 4xye^{x^2+y^2} = g_{yx}$; $g_{yy} = 2e^{x^2+y^2} + (2y)^2 e^{x^2+y^2} = 2(1+2y^2)e^{x^2+y^2}$

27. $h(s,t) = \ln\left(\dfrac{s}{t}\right)$. Write $h(s,t) = \ln s - \ln t$. Then $h_s = \dfrac{1}{s}$, $h_t = -\dfrac{1}{t}$.

Therefore, $h_{ss} = -\dfrac{1}{s^2}$, $h_{st} = h_{ts} = 0$, $h_{tt} = \dfrac{1}{t^2}$.

28. $f(x,y,z) = x^3y^2z + xy^2z + 3xy - 4z$; $f_x(1,1,0) = 3x^2yz + y^2z + 3y\big|_{(1,1,0)} = 3$;

$f_y(1,1,0) = 2x^3yz + 2xyz + 3x\big|_{(1,1,0)} = 3$; $f_z(1,1,0) = x^3y^2 + xy^2 - 4\big|_{(1,1,0)} = -2.$

29. $f(x,y) = 2x^2 + y^2 - 8x - 6y + 4$; To find the critical points of f, we solve the system $\begin{cases} f_x = 4x - 8 = 0 \\ f_y = 2y - 6 = 0 \end{cases}$ obtaining $x = 2$ and $y = 3$. Therefore, the sole critical point of f is (2,3). Next, $f_{xx} = 4, f_{xy} = 0, f_{yy} = 2.$ Therefore,

$$D = f_{xx}(2,3)f_{yy}(2,3) - f_{xy}(2,3)^2 = 8 > 0.$$

Since $f_{xx}(2,3) > 0$, we see that $f(2,3) = -13$ is a relative minimum.

30. $x(x,y) = x^2 + 3xy + y^2 - 10x - 20y + 12$. We solve the system

$$\begin{cases} f_x = 2x + 3y - 10 = 0 \\ f_y = 3x + 2y - 20 = 0 \end{cases} \text{ or } \begin{cases} 2x + 3y = 10 \\ 3x + 2y = 20 \end{cases}$$

obtaining $x = 8$ and $y = -2$ so that (8,–2) is the only critical point of f. Next, we compute $f_{xx} = 2, f_{xy} = 3$, and $f_{yy} = 2$.

Therefore, $D = f_{xx}(8,-2)f_{yy}(8,-2) - f_{xy}^2(8,-2) = (2)(2) - 3^2 = -5 < 0$.

Since $D < 0$, we see that (8,–2) gives rise to a saddle point of f.

31. $f(x,y) = x^3 - 3xy + y^2$. We solve the system of equations $\begin{cases} f_x = 3x^2 - 3y = 0 \\ f_y = -3x + 2y = 0 \end{cases}$

obtaining $x^2 - y = 0$, or $y = x^2$. Then $-3x + 2x^2 = 0$, and $x(2x - 3) = 0$, and $x = 0$, or $x = 3/2$ and $y = 0$, or $y = 9/4$. Therefore, the critical points are (0,0) and $(\frac{3}{2}, \frac{9}{4})$.

Next, $f_{xx} = 6x, f_{xy} = -3$, and $f_{yy} = 2$ and $D(x,y) = 12x - 9 = 3(4x - 3)$. Therefore, $D(0,0) = -9$ so (0,0) is a saddle point. $D(\frac{3}{2}, \frac{9}{4}) = 3(6 - 3) = 9 > 0$, and $f_{xx}(\frac{3}{2}, \frac{9}{4}) > 0$ and therefore, $f(\frac{3}{2}, \frac{9}{4}) = \frac{27}{8} - \frac{81}{8} + \frac{81}{16} = -\frac{27}{16}$ is the relative minimum value.

32. $f(x,y) = x^3 + y^2 - 4xy + 17x - 10y + 8$. To find the critical points of f, we solve the system $\begin{cases} f_x = 3x^2 - 4y + 17 = 0 \\ f_y = 2y - 4x - 10 = 0 \end{cases}$. From the second equation, we have $y = 2x + 5$ which, when substituted into the first equation gives

$$3x^2 - 8x - 20 + 17 = 0$$

or $$3x^2 - 8x - 3 = (3x + 1)(x - 3) = 0.$$

The solutions are $x = -1/3$ or 3. Therefore, the critical points of f are $(-\frac{1}{3}, \frac{13}{3})$ and (3,11). Next, we compute $f_{xx} = 6x, f_{xy} = -4, f_{yy} = 2$ and so

$$D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = 12x - 16.$$

Since $D(-\frac{1}{3}, \frac{13}{3}) = -20 < 0$, we see that $(-\frac{1}{3}, \frac{13}{3})$ gives a saddle point. Since $D(3,11) = 20 > 0$ and $f_{xx}(3,11) = 18 > 0$, we see that (3,11) affords a relative minimum of f.

33. $f(x,y) = f(x,y) = e^{2x^2+y^2}$. To find the critical points of f, we solve the system

$$\begin{cases} f_x = 4xe^{2x^2+y^2} = 0 \\ f_y = 2ye^{2x^2+y^2} = 0 \end{cases}$$

giving (0,0) as the only critical point of *f*. Next,

$$f_{xx} = 4(e^{2x^2+y^2} + 4x^2e^{2x^2+y^2}) = 4(1+4x^2)e^{2x^2+y^2}$$
$$f_{xy} = 8xye^{2x^2+y^2}$$
$$f_{yy} = 2(1+2y^2)e^{2x^2+y^2}.$$

Therefore, $D = f_{xx}(0,0)f_{yy}(0,0) - f_{xy}^2(0,0) = (4)(2) - 0 = 8 > 0$, and so (0,0) gives a relative minimum of *f* since $f_{xx}(0,0) > 0$. The minimum value of *f* is $f(0,0) = e^0 = 1$.

34. We solve the system

$$\begin{cases} f_x = \dfrac{2x-2}{x^2+y^2-2x-2y+4} = 0 \\ f_y = \dfrac{2y-2}{x^2+y^2-2x-2y+4} = 0 \end{cases}$$

obtaining $x = 1$ and $y = 1$ and giving (1,1) as the only critical point of *f*. Next, we compute

$$f_{xx} = \frac{(x^2+y^2-2x-2y+4)(2)-(2x-2)^2}{(x^2+y^2-2x-2y+4)^2}$$
$$f_{xy} = \frac{(x^2+y^2-2x-2y+4)(0)-(2x-2)(2y-2)}{(x^2+y^2-2x-2y+4)^2} = \frac{-4(x-1)(y-1)}{(x^2+y^2-2x-2y+4)^2}$$
$$f_{yy} = \frac{(x^2+y^2-2x-2y+4)(2)-(2y-2)^2}{(x^2+y^2-2x-2y+4)^2}.$$

In particular, $f_{xx}(1,1) = \frac{2}{2^2} = \frac{1}{2}$, $f_{xy}(1,1) = 0$, and $f_{yy}(1,1) = 1$. Therefore, $D = f_{xx}(1,1)f_{yy}(1,1) - f_{xy}(1,1) = \frac{1}{2} > 0$, and since $f_{xx}(1,1) = 1 > 0$, we conclude that (1,1) gives rise to a relative minimum of *f*. The relative minimum of *f* is $f(1,1) = \ln 2$.

35. We form the Lagrangian function $F(x,y,\lambda) = -3x^2 - y^2 + 2xy + \lambda(2x + y - 4)$. Next, we solve the system

$$\begin{cases} F_x = 6x + 2y + 2\lambda = 0 \\ F_y = -2y + 2x + \lambda = 0\,. \\ F_\lambda = 2x + y - 4 = 0 \end{cases}$$

Multiplying the second equation by 2 and subtracting the resultant equation from the first equation yields $6y - 10x = 0$ so $y = 5x/3$. Substituting this value of y into the third equation of the system gives $2x + \frac{5}{3}x - 4 = 0$. So $x = \frac{12}{11}$ and consequently $y = \frac{20}{11}$. So $(\frac{12}{11}, \frac{20}{11})$ gives the maximum value for f subject to the given constraint.

36. We form the Lagrangian function

$$F(x,y,\lambda) = 2x^2 + 3y^2 - 6xy + 4x - 9y + 10 + \lambda(x + y - 1).$$

Next, we solve the system

$$\begin{cases} F_x = 4x - 6y + 4 + \lambda = 0 \\ F_y = 6y - 6x - 9 + \lambda = 0 \\ F_\lambda = x + y - 1 = 0. \end{cases}$$

Subtracting the second equation from the first, we obtain $10x - 12y + 13 = 0$. Adding this equation to the equation obtained by multiplying the third equation in the system by 12, we obtain $22x - 1 = 0$ or $x = \frac{1}{22}$. Therefore $y = \frac{21}{22}$ and so the point $(\frac{1}{22}, \frac{21}{22})$ gives the minimum value of f subject to the given constraint.

37. The Lagrangian function is $F(x,y,\lambda) = 2x - 3y + 1 + \lambda(2x^2 + 3y^2 - 125)$. Next, we solve the system of equations

$$\begin{cases} F_x = 2 + 4\lambda x = 0 \\ F_y = -3 + 6\lambda y = 0 \\ F_\lambda = 2x^2 + 3y^2 - 125 = 0. \end{cases}$$

Solving the first equation for x gives $x = -1/2\lambda$. The second equation gives $y = 1/2\lambda$. Substituting these values of x and y into the third equation gives

$$2\left(-\frac{1}{2\lambda}\right)^2 + 3\left(\frac{1}{2\lambda}\right)^2 - 125 = 0$$

$$\frac{1}{2\lambda^2} + \frac{3}{4\lambda^2} - 125 = 0$$

$$2 + 3 - 500\lambda^2 = 0, \text{ or } \lambda = \pm\frac{1}{10}.$$

Therefore, $x = \pm 5$ and $y = \pm 5$ and so the critical points of f are $(-5,5)$ and $(5,-5)$. Next, we compute

$$f(-5,5) = 2(-5) - 3(5) + 1 = -24.$$
$$f(5,-5) = 2(5) - 3(-5) + 1 = 26.$$

So f has a maximum value of 26 at $(5,-5)$ and a minimum value of -24 at $(-5,5)$.

38. Form the Lagrangian function $F(x,y,\lambda) = e^{x-y} + \lambda(x^2 + y^2 - 1)$. Next, we solve the system

$$\begin{cases} F_x = e^{x-y} + 2\lambda x = 0 \\ F_y = -e^{x-y} + 2\lambda y = 0 \\ F_\lambda = x^2 + y^2 - 1 = 0 \end{cases}.$$

Adding the first two equations, we obtain $2\lambda(x + y) = 0$. Since $\lambda \neq 0$, (otherwise we have $e^{x-y} = 0$ which is impossible), we find $y = -x$. Substituting this value of y into the third equation of the system gives $2x^2 - 1 = 0$, or $x = \pm\sqrt{2}/2$. The corresponding values of y are $\pm\sqrt{2}/2$. We see that $(-\sqrt{2}/2, \sqrt{2}/2)$ gives rise to a minimum of f with value $e^{-\sqrt{2}}$, whereas $(\sqrt{2}/2, -\sqrt{2}/2)$ gives rise to a maximum of f with value $e^{\sqrt{2}}$.

39. $df = \frac{3}{2}(x^2 + y^4)^{1/2}(2x)dx + \frac{3}{2}(x^2 + y^4)^{1/2}(4y^3)dy$.
At (3,2), $df = 9(9+16)^{1/2}dx + 48(9+16)^{1/2}dy = 45dx + 240dy$.

40. $f(x,y) = xe^{x-y} + x\ln y$; (1,1)

$$df = (e^{x-y} + xe^{x-y} + \ln y)dx + \left(-xe^{x-y} + \frac{x}{y}\right)dy.$$

At (1,1), $df = (1+1+0)dx + (-1+1)dy = 2\,dx$.

41. $\Delta f \approx df = (4xy^3 + 6y^2x - 2y)dx + (6x^2y^2 + 6yx^2 - 2x)dy$.
With $x = 1$, $y = -1$, $dx = 0.02$, and $dy = 0.02$, we find
$\Delta f \approx (-4+6+2)(0.02) + (6-6-2)(0.02) = 0.04$.

42. $\Delta f \approx df = 3x^{-1/4}y^{1/4}dx + x^{3/4}y^{-3/4}dy = 3\left(\frac{y}{x}\right)^{1/4} dx + \left(\frac{x}{y}\right)^{3/4} dy$

with $x = 16$, $y = 81$, $dx = 1$ and $dy = -1$, we find

$$\Delta f \approx 3\left(\frac{81}{16}\right)^{1/4}(1) + \left(\frac{16}{81}\right)^{3/4}(-1) = 3\left(\frac{3}{2}\right) - \left(\frac{8}{27}\right) = \frac{227}{54}.$$

43. $$\int_{-1}^{2}\int_{2}^{4}(3x-2y)\,dx\,dy = \int_{-1}^{2} \tfrac{3}{2}x^2 - 2xy\Big|_{x=2}^{x=4} dy = \int_{-1}^{2}[(24-8y)-(6-4y)]\,dy$$
$$= \int_{-1}^{2}(18-4y)\,dy = (18y-2y^2)\Big|_{-1}^{2} = (36-8)-(-18-2) = 48.$$

44. $$\int_0^1\int_0^2 e^{-x-2y}\,dx\,dy = \int_0^1 -e^{-x-2y}\Big|_{x=0}^{x=2} dy = \int_0^1(-e^{-2-2y}+e^{-2y})\,dy$$
$$= \tfrac{1}{2}e^{-2-2y} - \tfrac{1}{2}e^{-2y}\Big|_0^1 = (\tfrac{1}{2}e^{-4} - \tfrac{1}{2}e^{-2}) - (\tfrac{1}{2}e^{-2} - \tfrac{1}{2})$$
$$= \tfrac{1}{2}(e^{-4} - 2e^{-2} + 1) = \tfrac{1}{2}(e^{-2}-1)^2.$$

45. $$\int_0^1\int_{x^3}^{x^2} 2x^2y\,dy\,dx = \int_0^1 x^2y^2\Big|_{y=x^3}^{y=x^2} dx = \int_0^1 x^2(x^4-x^6)\,dx$$
$$= \int_0^1(x^6-x^8)\,dx = \frac{x^7}{7} - \frac{x^9}{9}\Big|_0^1 = \tfrac{1}{7} - \tfrac{1}{9} = \tfrac{2}{63}.$$

46. $$\int_1^2\int_1^x \frac{y}{x}\,dy\,dx = \int_1^2 \frac{1}{x}\left(\frac{y^2}{2}\right)\Bigg|_{y=1}^{y=x} dx = \int_1^2\left(\frac{1}{2}x - \frac{1}{2x}\right)dx = \left(\frac{1}{4}x^2 - \frac{1}{2}\ln x\right)\Bigg|_1^2$$
$$= (1-\tfrac{1}{2}\ln 2) - (\tfrac{1}{4}) = \tfrac{3}{4} - \tfrac{1}{2}\ln 2 = \tfrac{1}{4}(3-2\ln 2).$$

47. $$\int_0^2\int_0^1(4x^2+y^2)\,dy\,dx = \int_0^2 4x^2y + \tfrac{1}{3}y^3\Big|_{y=0}^{y=1} dx = \int_0^2(4x^2+\tfrac{1}{3})\,dx$$
$$= (\tfrac{4}{3}x^3 + \tfrac{1}{3}x)\Big|_0^2 = \tfrac{32}{3} + \tfrac{2}{3} = \tfrac{34}{3}.$$

48. $$V = \int_0^4\int_{y/4}^{\sqrt{y}}(x+y)\,dx\,dy = \int_0^4(\tfrac{1}{2}x^2+xy)\Big|_{x=y/4}^{x=\sqrt{y}} = \int_0^4(\tfrac{1}{2}y - y^{3/2} - \tfrac{1}{32}y^2 - \tfrac{1}{4}y^2)\,dy$$
$$= (\tfrac{1}{4}y^2 + \tfrac{2}{5}y^{5/2} - \tfrac{3}{32}y^3)\Big|_0^4 = 4 + \tfrac{64}{5} - 6 = \tfrac{54}{5} = 10\tfrac{4}{5} \text{ cu units.}$$

49. The area of R is

$$\int_0^2 \int_{x^2}^{2x} dy\,dx = \int_0^2 y\Big|_{y=x^2}^{y=2x} dx = \int_0^2 (2x - x^2)\,dx = (x^2 - \tfrac{1}{3}x^3)\Big|_0^2 = \tfrac{4}{3}.$$

Then

$$AV = \frac{1}{4/3}\int_0^2 \int_{x^2}^{2x} (xy+1)dy\,dx = \frac{3}{4}\int_0^2 \frac{xy^2}{2} + y\Big|_{x^2}^{2x} dx$$

$$= \tfrac{3}{4}\int_0^2 (-\tfrac{1}{2}x^5 + 2x^3 - x^2 + 2x)\,dx = \tfrac{3}{4}(-\tfrac{1}{12}x^6 + \tfrac{1}{2}x^4 - \tfrac{1}{3}x^3 + x^2)\Big|_0^2$$

$$= \tfrac{3}{4}(-\tfrac{16}{3} + 8 - \tfrac{8}{3} + 4) = 3.$$

50. a. $R(x,y) = px + qy = -0.02x^2 - 0.2xy - 0.05y^2 + 80x + 60y$.

b. The domain of R is the set of all points satisfying

$$0.02x + 0.1y \le 80$$
$$0.1x + 0.05y \le 60$$
$$x \ge 0,\ y \ge 0$$

c. $R(100, 300) = -0.02(100)^2 - 0.2(100)(300) - 0.05(300)^2 + 80(100)^2 + 60(300)$
$= 15{,}300$

giving the revenue of \$15,300 realized from the sale of 100 and 300 units, respectively, of 16−speed and 10−speed electric blenders.

51. $f(p,q) = 900 - 9p - e^{0.4q};\ g(p,q) = 20{,}000 - 3000q - 4p.$

We compute $\dfrac{\partial f}{\partial q} = -0.4e^{0.4q}$ and $\dfrac{\partial g}{\partial p} = -4$. Since $\dfrac{\partial f}{\partial q} < 0$ and $\dfrac{\partial g}{\partial p} < 0$ for all $p > 0$ and $q > 0$, we conclude that compact disc players and audio discs are complementary commodities.

52. $P(x,y) = -0.0005x^2 - 0.003y^2 - 0.002xy + 14x + 12y - 200.$

$\Delta P \approx dP = (-0.001x - 0.002y + 14)dx + (-0.006y - 0.002x + 12)dy.$

With $x = 1000$, $y = 1700$, $dx = 50$, and $dy = -50$, we have

$\Delta P \approx (-1 - 3.4 + 14)(50) + (-10.2 - 2 + 12)(-50) = 490$, or \$490.

53. We first summarize the data:

x	y	x^2	xy
1	369	1	369
3	390	9	1170
5	396	25	1980
7	420	49	2940
9	436	81	3924
25	2011	165	10383

The normal equations are

$$5b + 25m = 2011$$

$$25b + 165m = 10383$$

The solutions are $b = 361.2$ and $m = 8.2$. Therefore, the least-square line has equation $y = 8.2x + 361.2$.

b. The average daily viewing time in 2002 (x = 11) wiill be
y = 8.2(1) + 361.2 = 451.4 = 7.52 , or 7 hr 31 min.

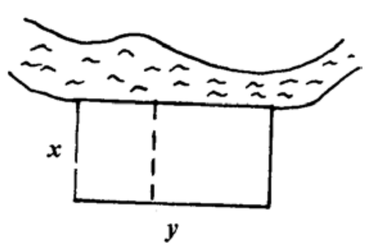
54. We want to maximize the function $R(x, y) = -x^2 - 0.5y^2 + xy + 8x + 3y + 20.$
To find the critical point of R, we solve the system

$$R_x = -2x + y + 8 = 0$$

$$R_y = -y + x + 3 = 0.$$

Adding the two equations, we obtain $-x + 11 = 0$, or $x = 11$. So $y = 14$.
Therefore, (11,14) is a critical point of R. Next, we compute
$R_{xx} = -2$, $R_{xy} = 1$, $R_{yy} = -1$. So $D(x, y) = R_{xx}R_{yy} - R_{xy}^2 = 2 - 1 = 1.$
In particular $D(11,14) = 1 > 0$. Since $R_{xx}(11,14) = -2 < 0$, we see that (11,14) gives a relative maximum of R. The nature of the problem suggests that this in fact an absolute maximum. So the company should spend \$11,000 on advertising and employ 14 agents in order to maximize its revenue.

55. Refer to the following diagram.



We want to minimize $C(x,y) = 3(2x) + 2(x) + 3y = 8x + 3y$ subject to $xy = 303{,}750$. The Lagrangian function is

$$F(x,y,\lambda) = 8x + 3y + \lambda(xy - 303{,}750).$$

Next, we solve the system

$$\begin{cases} F_x = 8 + \lambda y = 0 \\ F_y = 3 + \lambda x = 0 \\ F_\lambda = xy - 303{,}750 = 0 \end{cases}.$$

Solving the first equation for y gives $y = -8/\lambda$. The second equation gives $x = -3/\lambda$. Substituting this value into the third equation gives

$$\left(-\frac{3}{\lambda}\right)\left(-\frac{8}{\lambda}\right) = 303{,}750 \text{ or } \lambda^2 = \frac{24}{303{,}750} = \frac{4}{50{,}625},$$

or $\lambda = \pm\frac{2}{225}$. Therefore, $x = 337.5$ and $y = 900$ and so the required dimensions of the pasture are 337.5 yd by 900 yd.

56. We want to maximize the function Q subject to the constraint $x + y = 100$. We form the Lagrangian function $f(x,y,\lambda) = x^{3/4}y^{1/4} + \lambda(x + y - 100)$. To find the critical points of F, we solve

$$\begin{cases} F_x = \frac{3}{4}\left(\frac{y}{x}\right)^{1/4} + \lambda = 0 \\ F_y = \frac{1}{4}\left(\frac{x}{y}\right)^{3/4} + \lambda = 0 \\ F_\lambda = x + y - 100 = 0. \end{cases}$$

Solving the first equation for λ and substituting this value into the second equation yields $\frac{1}{4}(\frac{x}{y})^{3/4} - \frac{3}{4}(\frac{y}{x})^{1/4} = 0$, $(\frac{x}{y})^{3/4} = 3(\frac{y}{x})^{1/4}$, or $x = 3y$. Substituting this value of x into the third equation, we have $4y = 100$ or $y = 25$, and $x = 75$. Therefore, 75 units should be spent on labor and 25 units on capital.