**CHAPTER 10, REVIEW EXERCISES, page 695**

1. First $f(x)=\frac{1}{28}(2x+3)$ is nonnegative on [0,4]. Next,
$\int_0^4 \frac{1}{28}(2x+3)dx = \frac{1}{28}(x^2+3x)\Big|_0^4 = \frac{1}{28}(16+12)=1.$

2. First, $f(x)=\frac{3}{16}\sqrt{x}\geq 0$ on [0,4]. Next, $\int_0^4 \frac{3}{16}x^{1/2}\,dx = \frac{1}{8}x^{3/2}\Big|_0^4 = \frac{1}{8}(8)=1.$

3. First, $f(x)=\frac{1}{4}>0$ on [7,11]. Next, $\int_7^{11}\frac{1}{4}dx = \frac{1}{4}x\Big|_7^{11} = \frac{1}{4}(11-7)=1.$

4. First, $f(x)=\dfrac{4}{x^5}>0$ on $[1,\infty)$. Next,
$$\int_1^\infty 4x^{-5}\,dx = \lim_{b\to\infty}\int_1^b 4x^{-5}dx = \lim_{b\to\infty} -\frac{1}{x^4}\Big|_1^b = \lim_{b\to\infty}\left(-\frac{1}{b^4}+1\right)=1.$$

5. $\int_0^9 kx^2\,dx = \dfrac{k}{3}x^3\Big|_0^9 = \dfrac{k}{3}(729)=1.$ Therefore, $k=\dfrac{1}{243}.$

6. $\int_1^{16} kx^{-1/2}\,dx = 2kx^{1/2}\Big|_1^{16} = 2k(4-1)=6k=1$ implies $k=\dfrac{1}{6}.$

7. $\int_1^3 kx^{-2}\,dx = \dfrac{2k}{3}=1.$ Therefore, $k=\dfrac{3}{2}.$

8. $$\int_1^\infty kx^{-2.5}\,dx = \lim_{b\to\infty}\int_1^b kx^{-2.5}\,dx = \lim_{b\to\infty} -\frac{k}{1.5x^{1.5}}\Big|_1^b$$
$$= \lim_{b\to\infty}\left[-\frac{k}{1.5b^{1.5}}+\frac{k}{1.5}\right] = \frac{k}{1.5}=1 \text{ implies } k=1.5.$$

9. a. $\int_2^4 \frac{2}{21}x\,dx = \frac{1}{21}x^2\Big|_2^4 = \frac{16}{21}-\frac{4}{21}=\frac{12}{21}=\frac{4}{7}.$

b. $\int_4^4 \frac{2}{21}x\,dx = \frac{1}{21}x^2\Big|_4^4 = 0.$

c. $\int_3^4 \frac{2}{21}x\,dx = \frac{1}{21}x^2\Big|_3^4 = \frac{16}{21}-\frac{9}{21}=\frac{7}{21}=\frac{1}{3}.$

10. a. $P(2 \le x \le 4) = \int_2^4 \frac{1}{4}\,dx = \frac{1}{4}x\Big|_2^4 = \frac{1}{4}(4-2) = \frac{1}{2}.$

b. $P(x \le 3) = \int_1^3 \frac{1}{4}\,dx = \frac{1}{4}x\Big|_1^3 = \frac{1}{4}(3-1) = \frac{1}{2}.$

c. $P(x \ge 2) = \int_2^5 \frac{1}{4}\,dx = \frac{1}{4}x\Big|_2^5 = \frac{1}{4}(5-2) = \frac{3}{4}.$

11. a. $P(1 \le x \le 3) = \frac{3}{16}\int_1^3 x^{1/2}\,dx = \frac{1}{8}x^{3/2}\Big|_1^3 = \frac{1}{8}(3\sqrt{3}-1) \approx 0.52.$

b. $P(x \le 3) = \frac{3}{16}\int_0^3 x^{1/2}dx = \frac{1}{8}x^{3/2}\Big|_0^3 = \frac{1}{8}(3\sqrt{3}-0) \approx 0.65.$

c. $P(x = 2) = \frac{3}{16}\int_2^2 x^{1/2}\,dx = 0.$

12. a. $P(x \le 10) = \int_1^{10} x^{-2}\,dx = -\frac{1}{x}\Big|_1^{10} = -\frac{1}{10} + 1 = \frac{9}{10}.$

b. $P(2 \le x \le 4) = \int_2^4 x^{-2}\,dx = -\frac{1}{x}\Big|_2^4 = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}.$

c. $P(x \ge 2) = \int_2^\infty x^{-2}\,dx = \lim_{b\to\infty}\int_2^b x^{-2}\,dx = \lim_{b\to\infty} -\frac{1}{x}\Big|_2^b = \frac{1}{2}.$

13. $\mu = \frac{1}{5}\int_2^7 x\,dx = \frac{1}{10}x^2\Big|_2^7 = \frac{1}{10}(49-4) = \frac{9}{2}.$

$\text{Var}(x) = \frac{1}{5}\int_2^7 x^2\,dx - \left(\frac{9}{2}\right)^2 = \frac{1}{15}x^3\Big|_2^7 - (4.5)^2 = \frac{1}{15}(343-8) - (4.5)^2 \approx 2.083.$

$\sigma = \sqrt{2.083} \approx 1.44.$

14. $\mu = \frac{1}{28}\int_0^4 x(2x+3)dx = \frac{1}{28}\int_0^4 2x^2 + 3x\,dx = \frac{2x^3}{3} + \frac{3x^2}{2}\Big|_0^4 = \frac{50}{21}.$

$\text{Var}(x) = \int_0^4 \frac{1}{28}x^2(2x-3)\,dx - (\mu)^2 = \frac{1}{28}\int_0^4 (2x^3+3x^2)\,dx - \left(\frac{50}{21}\right)^2 = \frac{1}{28}\left(\frac{x^4}{2} + x^3\right)\Big|_0^4 - \left(\frac{50}{21}\right)^2$

$= \frac{1}{28}\left(\frac{256}{2} + 64\right) - \left(\frac{50}{21}\right)^2 \approx 1.1882.$

$\sigma = \sqrt{1.1882} \approx 1.0900.$

15. $\mu = \frac{1}{4}\int_{-1}^1 x(3x^2+1)dx = \frac{1}{4}\int_{-1}^1 (3x^3+x)dx = \frac{1}{4}\left(\frac{3}{4}x^4 + \frac{1}{2}x^2\right)\Big|_{-1}^1 = 0.$

$\text{Var}(x) = \int_{-1}^{1} x^2(3x^2 - 1)\,dx - 0 = \frac{1}{4}\int_{-1}^{1}(3x^4 + x^2)\,dx = \frac{1}{4}\left(\frac{3}{5}x^5 + \frac{1}{3}x^3\right)\Big|_{-1}^{1} = \frac{7}{15}.$

$\sigma = \sqrt{\frac{7}{15}} \approx 0.6831.$

16. $\mu = \int_1^{\infty} 4x^{-4}\,dx = \lim_{b\to\infty}\int_1^b 4x^{-4}\,dx = \lim_{b\to\infty} -\frac{4}{3x^3}\Big|_1^b = \lim_{b\to\infty}\left(-\frac{4}{3b^3} + \frac{4}{3}\right) = \frac{4}{3}.$

$\text{Var}(x) = \int_1^{\infty} 4x^{-3}\,dx - \left(\frac{4}{3}\right)^2 = \lim_{b\to\infty}\int_1^b 4x^{-3}\,dx - \frac{16}{9} = \lim_{b\to\infty} -\frac{2}{x^2}\Big|_1^b - \frac{16}{19}$

$= \lim_{b\to\infty}\left(-\frac{2}{b^2} + 2\right) - \frac{16}{9} = 2 - \frac{16}{9} = \frac{2}{9}.$

$\sigma = \sqrt{\frac{2}{9}} \approx 0.4714.$

17. $P(Z < 2.24) = 0.9875.$

18. $P(Z > -1.24) = 1 - 0.1075 \approx 0.8925.$

19. $P(0.24 \le Z \le 1.28) = P(Z \le 1.28) - P(Z \le 0.24) = 0.8997 - 0.5948 = 0.3049.$

20. $P(-1.37 \le Z \le 1.37) = P(Z \le 1.37) - P(Z \le -1.37) = 0.9147 - 0.0853 = 0.8294.$

21. $f$ is nonnegative in $D$. Next,

$$\iint_D f(x,y)\,dA = \int_0^8\int_0^4 \frac{1}{64}x^{1/2}y^{1/3}\,dx\,dy = \frac{1}{64}\int_0^8\left[\frac{2}{3}x^{3/2}y^{1/3}\Big|_0^4\right]dy = \frac{1}{64}\cdot\frac{2}{3}\cdot 8\int_0^8 y^{1/3}\,dy$$

$$= \frac{1}{12}\left[\frac{3}{4}y^{4/3}\Big|_0^8\right] = \frac{1}{16}\cdot 16 = 1$$

and this proves that $f$ is a joint probability density function on $D$.

22. a. $P(0 \le x \le \frac{1}{2}, y \le 2x) = \int_0^1\int_0^{1/2} x(2-y)dx\,dy = \int_0^1\left[\frac{1}{2}x^2(2-y)\Big|_0^{1/2}\right]dy$

$= \int_0^1 \frac{1}{2}\left(\frac{1}{2}\right)^2(2-y)\,dy = -\frac{1}{16}(2-y)^2\Big|_0^1 = \frac{3}{16}.$

b. $P(\{(x,y) \mid y \le 2x\}) = \int_0^2\int_{y/2}^1 x(2-y)\,dx\,dy = \int_0^2\left[\frac{1}{2}x^2(2-y)\Big|_{y/2}^1\right]dy$

$= \frac{1}{2}\int_0^2 (2-y)(1 - \frac{y^2}{4})\,dy = \frac{1}{2}\int_0^2 (2 - \frac{y^2}{2} - y + \frac{y^3}{4})\,dy$

$$= \frac{1}{2}\left[2y - \frac{1}{6}y^3 - \frac{1}{2}y^2 + \frac{1}{k}y^4\right]_0^2 = \frac{1}{2}\left(4 - \frac{4}{3} - 2 + 1\right) = \frac{5}{6}.$$

23. a. $P(X \le 84) = P\left(Z \le \frac{84-80}{8}\right) = P(Z \le 0.5) = 0.6915.$

b. $P(X \ge 70) = P\left(\frac{70-80}{8}\right) = P(Z \ge -1.25) = P(Z \le 1.25) = 0.8944.$

c. $P(75 \le X \le 85) = P\left(\frac{75-80}{8} \le Z \le \frac{85-80}{8}\right) = P(-0.625 \le Z \le 0.625)$

$= P(Z \le 0.625) - P(Z \le -0.625) = 0.7341 - 0.2660 \approx 0.4681.$

24. a. $P(X \le 50) = P\left(Z \le \frac{50-45}{3}\right) = P(Z \le 1.667) = 0.9525.$

b. $P(X \ge 40) = P\left(Z \ge \frac{50-40}{3}\right) = P\left(Z \ge \frac{10}{3}\right) = P(Z \ge 1.33).$

$= 1 - P(Z \le 1.33) = 1 - 0.9082 = 0.0918.$

c. $P(40 \le X \le 50) = P\left(\frac{50-40}{3} \le Z \le \frac{50-45}{3}\right) = 0.9525 - 0.0916 = 0.8607.$

25. a. $P(t > 6) = \int_6^\infty \tfrac{1}{4}e^{-t/4}dt = \lim_{b\to\infty}\int_6^b -e^{-t/4}\,dt = \lim_{b\to\infty} - e^{-t/4}\Big|_6^b$

$= \lim_{b\to\infty} - e^{-b/4} + e^{-6/4} = 0.22313.$

b. $P(t < 2) = \int_0^2 \tfrac{1}{4}e^{-t/4}\,dt = -e^{-t/4}\Big|_0^2 = -e^{-1/2} + e^0 = -0.6065 + 1 = 0.39347.$

c. $\mu = \int_0^\infty \tfrac{1}{4}te^{-t/4}dt = \lim_{b\to\infty}\int_0^b \tfrac{1}{4}te^{-t/4}dt = \lim_{b\to\infty} 4(-\tfrac{1}{4}t - 1)e^{-t/4}\Big|_0^b \quad = 0 + 4 = 4.$

26. a. $\int_0^4 \tfrac{1}{5}e^{-x/5}\,dx = 1 - e^{-4/5} = 0.55067.$ b. $\int_6^\infty \tfrac{1}{5}e^{-x/5}\,dx = \lim_{b\to\infty} -e^{-x/5}\Big|_6^\infty = 0.30119.$

c. $\int_2^4 \tfrac{1}{5}e^{-x/5}\,dx = 0.22099.$