CHAPTER 12, REVIEW EXERCISES, page 808

1. $120° = \frac{2\pi}{3}$ rad.

2. $450° = \frac{450}{180}\pi = \frac{5\pi}{2}$ rad.

3. $-225° = -\frac{225}{180}\pi = -\frac{5\pi}{4}$ rad.

4. $\frac{11\pi}{6}$ rad $= 11\left(\frac{\pi}{6}\right) = 11(30) = 330°$.

5. $-\frac{5\pi}{2}$ rad $= -\frac{5}{2}(180) = -450°$.

6. $-\frac{7\pi}{4}$ rad $= -\frac{7}{4}(180) = -315°$.

7. $\cos\theta = \frac{1}{2}$ implies that $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$.

8. $\cot\theta = -\sqrt{3}$ implies that $\theta = 150°$ or $330°$.

9. $f(x) = \sin 3x$; $f'(x) = 3\cos 3x$.

10. $f(x) = 2\cos\frac{x}{2}$; $f'(x) = -\sin\frac{x}{2}$.

11. $f(x) = 2\sin x - 3\cos 2x$.
$f'(x) = 2\cos x + 3(\sin 2x)(2) = 2(\cos x + 3\sin 2x)$.

12. $f(x) = \sec^2\sqrt{x}$.
$$f'(x) = 2\sec\sqrt{x}\,\frac{d}{dx}\sec\sqrt{x} = 2\sec\sqrt{x}(\sec\sqrt{x}\,\tan\sqrt{x})\frac{d}{dx}\sqrt{x}$$
$$= (\sec^2\sqrt{x}\,\tan\sqrt{x})x^{-1/2} = \frac{\sec^2\sqrt{x}\,\tan\sqrt{x}}{\sqrt{x}}.$$

13. $f(x) = e^{-x}\tan 3x$.
$f'(x) = -e^{-x}\tan 3x + e^{-x}(\sec^2 3x)(3) = e^{-x}(3\sec^2 3x - \tan 3x)$.

14. $f(x) = (1 - \csc 2x)^2$.
$$f'(x) = 2(1-\csc 2x)\frac{d}{dx}(1-\csc 2x) = 2(1-\csc 2x)(\csc 2x\cot 2x)(2)$$
$$= 4(1-\csc 2x)(\csc 2x\cot 2x).$$

15. $f(x) = 4\sin x\cos x$.
$f'(x) = 4[\sin x(-\sin x) + (\cos x)(\cos x)] = 4(\cos^2 x - \sin^2 x) = 4\cos 2x$.

16. $f(x) = \dfrac{\cos x}{1-\cos x}$.

$$f'(x) = \frac{(1-\cos x)(-\sin x) - \cos x(\sin x)}{(1-\cos x)^2} = -\frac{\sin x}{(1-\cos x)^2}.$$

17. $f(x) = \dfrac{1-\tan x}{1-\cot x}$.

$$f'(x) = \frac{(1-\cot x)(-\sec^2 x) - (1-\tan x)(\csc^2 x)}{(1-\cot x)^2}$$

$$= \frac{-\sec^2 x + \sec^2 x \cot x - \csc^2 x + \csc^2 x \tan x}{(1-\cot x)^2}$$

$$= \frac{(\cot x - 1)\sec^2 x - (1-\tan x)\csc^2 x}{(1-\cot x)^2}.$$

18. $f(x) = \ln(\cos^2 x)$; $f'(x) = \dfrac{(2\cos x)(-\sin x)}{\cos^2 x} = -2\tan x.$

19. $f(x) = \sin(\sin x)$; $f'(x) = \cos(\sin x)\cdot\cos x.$

20. $f(x) = e^{\sin x}\cdot\cos x$.

$$f'(x) = e^{\sin x}(-\sin x) + (\cos x)e^{\sin x}\frac{d}{dx}(\sin x) = e^{\sin x}(-\sin x + \cos^2 x).$$

21. $f(x) = \tan^2 x$. $f'(x) = 2\tan x \sec^2 x$.

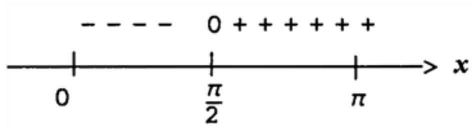
So the slope is $f'(\frac{\pi}{4}) = 2(1)(\sqrt{2})^2 = 4,$ and equation of the tangent line is

$$y - 1 = 4(x - \tfrac{\pi}{4}), \text{ or } y = 4x + 1 - \pi.$$

22. $y = \cos^2 x$; $y' = -2\cos x \sin x$.

Setting $y'(0) = 0,$ we obtain $x = 0, \frac{\pi}{2}, \pi$ as the critical points of f.

The sign diagram for f' is show below.



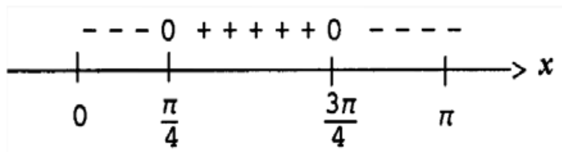
We conclude that

a. f is increasing on $(\frac{\pi}{2}, \pi)$ and f is decreasing on $(0, \frac{\pi}{2})$.

b. The relative minimum value of f is $f(\frac{\pi}{2}) = 0$.

Next, $y'' = 2\sin^2 x - 2\cos^2 x = -2(\cos^2 x - \sin^2 x) = -2\cos 2x$.

Setting $y'' = 0$, we obtain $x = \frac{\pi}{4}$ and $\frac{3\pi}{4}$. The sign diagram for f'' follows.

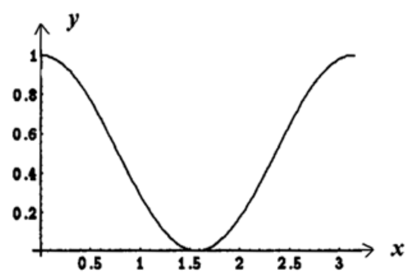


We conclude that

c. f is concave upward on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and f is concave downward on $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi)$.

d. The inflection points of f are $(\frac{\pi}{4}, \frac{1}{2})$ and $(\frac{3\pi}{4}, \frac{1}{2})$.

The graph of f follows.



23. Let $u = \frac{2}{3}x$ so that $du = \frac{2}{3}dx$, or $dx = \frac{3}{2}du$. So

$$\int \cos\tfrac{2}{3}x\,dx = \tfrac{3}{2}\int \cos u\,du = \tfrac{3}{2}\sin u + C = \tfrac{3}{2}\sin\tfrac{2}{3}x + C.$$

24. $\int \sin^2 2x\,dx = \frac{1}{2}\int (1 - \cos 4x)\,dx = \frac{1}{2}x - \frac{1}{8}\sin 4x + C.$

25. $\int x\csc x^2 \cot x^2\,dx$. Put $u = x^2$, then $du = 2x\,dx$. Then

$$\int x\csc x^2 \cot x^2\,dx = \tfrac{1}{2}\int \csc u \cot u\,du$$
$$= -\tfrac{1}{2}\csc u + C = -\tfrac{1}{2}\csc x^2 + C.$$

26. Let $u = x$ and $dv = \cos x\,dx$ so $du = dx$ and $v = \sin x$. Then integrating by parts, we have
$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C.$$

27. Let $u = \sin x$ so that $du = \cos x\,dx$. Then
$$\int \sin^2 x\cos x\,dx = \int u^2 du = \tfrac{1}{3}\sin^3 x + C.$$

28. $\int e^x \sin e^x\,dx = -\cos e^x + C.$

29. Let $u = \sin x$ so that $du = \cos x\,dx$. Then
$$\int \frac{\cos x}{\sin^2 x}dx = \int \frac{du}{u^2} = \int u^{-2}\,du = -\frac{1}{u} + C = -\frac{1}{\sin x} + C = -\csc x + C.$$

30. $\int_0^{\pi/4} \tan x\,dx = -\ln|\cos x|\Big|_0^{\pi/4} = -\ln\dfrac{\sqrt{2}}{2}.$

31. $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\cos x}{1-\cos^2 x}dx = \int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^2 x}dx = \int_{\pi/6}^{\pi/2} \sin^{-2} x\cos x\,dx$
$$= -\frac{1}{\sin x}\bigg|_{\pi/6}^{\pi/2} = -1 + 2 = 1.$$

32. $\displaystyle\int_0^{\pi} \sin x(1+\cos^3 x)\,dx = \int_0^{\pi} (\sin x + \sin x\cos^3 x)\,dx$
$$= -\cos x - \tfrac{1}{4}\cos^4 x\Big|_0^{\pi} = (1-\tfrac{1}{4}) - (-1-\tfrac{1}{4}) = 2.$$

33. $\displaystyle A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx = -\cos x - \sin x\Big|_{\pi/4}^{5\pi/4}$
$$= \left(\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}\right) - \left(-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\right) = 2\sqrt{2} \text{ sq units.}$$

34. $P(t) = 400 + 50\sin\left(\frac{\pi}{12}\right)$; $P'(t) = 50\left(\frac{\pi}{12}\right)\cos\frac{\pi}{12}$. Therefore,

$$P'(4) = \frac{50\pi}{12}\cos\frac{\pi}{3} = \frac{50\pi}{12}\cdot\frac{1}{2} = \frac{25\pi}{12};$$

that is, the fox population is increasing at the rate of $\frac{25\pi}{12} \approx 6.5$ foxes per month.

Next, $p(t) = 3000 + 500\cos\left(\frac{\pi t}{12}\right)$; $p'(t) = -500\left(\frac{\pi}{12}\right)\sin\frac{\pi t}{12}$.

Therefore $p'(4) = -\frac{500\pi}{12}\sin\frac{\pi}{3} = -\left(\frac{500\pi}{12}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{125\sqrt{3}\pi}{6}$; that is, the rabbit population is decreasing at the rate of $\frac{125\sqrt{3}\pi}{6}$, or 113.4 rabbits per month.

35. $R(t) = 60 + 37\sin^2\left(\frac{\pi t}{12}\right)$.

$R'(t) = 37(2)\sin\left(\frac{\pi t}{12}\right)\cdot\cos\left(\frac{\pi t}{12}\right)\left(\frac{\pi}{12}\right) = \frac{37\pi}{6}\sin\left(\frac{\pi t}{6}\right)$ $\qquad [\sin 2\theta = 2\sin\theta\cos\theta]$

Setting $R'(t) = 0$ gives $\frac{\pi t}{6} = 0, \pi, 2\pi, \ldots$. So $t = 0, 6, 12, \ldots$.
Therefore, $t = 6$ is a critical point in (0,12). Now

$$R'(t) = \frac{37\pi}{6}\cos\left(\frac{\pi t}{6}\right)\cdot\frac{\pi}{6} = \frac{37\pi^2}{36}\cos\left(\frac{\pi t}{6}\right).$$

Since $R''(6) = -\frac{37\pi^2}{36} < 0$, the Second Derivative Test implies that the occupancy rate is highest when $t = 6$ (the beginning of December).Next, we set $R''(t) = 0$ giving $\frac{\pi t}{6} = \frac{\pi}{2}$, or $t = 3$. You can show that $R''(3) < 0$ and so $R'(t)$ is maximized at $t = 3$. So the occupancy rate is increasing most rapidly at the beginning of September.

36. The average occupancy rate is

$$\frac{1}{12}\int_0^{12}\left[60 + 37\sin^2\left(\frac{\pi t}{12}\right)\right]dt = \frac{1}{12}\int_0^{12} 60\,dt + \frac{1}{12}\int_0^{12} 37\sin^2\left(\frac{\pi t}{12}\right)dt$$

$$= 60 + \frac{37}{24}\int_0^{12}\left[1 - \cos\left(\frac{\pi t}{6}\right)\right]dt = 60 + \frac{37}{24}\left[t - \left(\frac{6}{\pi}\right)\sin\frac{37}{24}\left(\frac{\pi t}{6}\right)\right]\Bigg|_0^{12}$$

$$= 60 + \frac{37}{24}[12 - \left(\frac{6}{\pi}\right)(0)] \approx 78.5, \text{ or } 78.5 \text{ percent.}$$