

transform — library for integral transforms

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Introduction

The `transform` library provides some integral transformations.

The package functions are called using the package name `transform` and the name of the function. E.g., use

```
>> transform::fourier(exp(-t^2), t, s)
```

to compute the Fourier transform of e^{-t^2} with respect to `t` at the point `s`. This mechanism avoids naming conflicts with other library functions. If this is found to be inconvenient, then the routines of the `transform` package may be exported via `export`. E.g., after calling

```
>> export(transform, fourier)
```

the function `transform::fourier` may be called directly:

```
>> fourier(exp(-t^2), t, s)
```

All routines of the `transform` package are exported simultaneously by

```
>> export(transform)
```

The functions available in the `transform` library can be listed with:

```
>> info(transform)
```

`transform::fourier`, `transform::invfourier` – Fourier and inverse Fourier transform

`transform::fourier(f, t, s)` computes the Fourier transform $\int_{-\infty}^{\infty} f e^{ist} dt$ of the expression $f = f(t)$ with respect to the variable t at the point s .

`transform::invfourier(F, S, T)` computes the inverse Fourier transform $\frac{1}{2\pi} \int_{-\infty}^{\infty} F e^{-iST} dS$ of the expression $F = F(S)$ with respect to the variable S at the point T .

Call(s):

- ⌘ `transform::fourier(f, t, s)`
- ⌘ `transform::invfourier(F, S, T)`

Parameters:

- `f, F` — arithmetical expressions
- `t, S` — identifiers (the transformation variables)
- `s, T` — arithmetical expressions (the evaluation points)

Return Value: an arithmetical expression

Overloadable by: `f, F`

Related Functions: `numeric::fft`, `numeric::invfft`

Details:

- ⌘ An unevaluated function call is returned, if no explicit representation of the transform is found.
- ⌘ `transform::invfourier(F, S, T)` is computed as

$$\text{transform} :: \text{fourier}(F, S, -T)/2/\text{PI}.$$

This result is returned, if no explicit representation of the transformation is found.

- ⌘ The discrete Fourier transform is implemented by the functions `numeric::fft` and `numeric::invfft`.
-

Example 1. The following call produces the Fourier transform as an expression in the variable `s`:

```
>> transform::fourier(exp(-t^2), t, s)
```

$$\frac{1}{2} \sqrt{\frac{s}{4}}$$

>> transform::invfourier(%, s, t)

$$\exp(-t^2)$$

Note that the Fourier transform can be evaluated directly at a specific point such as $s = 2a$ or $s = 5$:

>> transform::fourier(t*exp(-a*t^2), t, s),
 transform::fourier(t*exp(-a*t^2), t, 2*a),
 transform::fourier(t*exp(-a*t^2), t, 2)

$$\frac{1}{2} \sqrt{\frac{s}{4a}}, \frac{1}{2} \sqrt{\frac{1}{a}}$$

Example 2. An unevaluated call is returned, if no explicit representation of the transform is found:

>> transform::fourier(besselJ(0, 1/(1 + t^2)), t, s)

$$\text{transform::fourier} \left(\text{besselJ} \left(0, \frac{1}{t^2 + 1} \right), t, s \right)$$

>> transform::invfourier(%, s, t)

$$\text{besselJ} \left(0, \frac{1}{t^2 + 1} \right)$$

Note that the inverse transform is related to the direct transform:

>> transform::invfourier(unknown(s), s, t)

$$\frac{\text{transform::fourier}(\text{unknown}(s), s, -t)}{2 \text{ PI}}$$

Example 3. The distribution `dirac` is handled:

```
>> transform::fourier(t^3, t, s)
```

$$2 \int_{-\infty}^{\infty} \text{dirac}(s, 3)$$

```
>> transform::invfourier(%, s, t)
```

$$t^3$$

```
>> transform::fourier(heaviside(t - t0), t, s)
```

$$\frac{\exp(i s t_0)}{s} \int_{-\infty}^{\infty} \text{dirac}(s) + \frac{i}{s}$$

Example 4. The Fourier transform of a function is related to the Fourier transform of its derivative:

```
>> transform::fourier(diff(f(t), t), t, s)
```

$$-i s \text{ transform::fourier}(f(t), t, s)$$

Background:

☞ Reference: F. Oberhettinger, “Tables of Fourier Transforms and Fourier Transforms of Distributions”, Springer, 1990.

`transform::laplace`, `transform::invlaplace` – Laplace and inverse Laplace transform

`transform::laplace(f, t, s)` computes the Laplace transform $\int_0^{\infty} f e^{-st} dt$ of the expression $f = f(t)$ with respect to the variable t at the point s .

`transform::invlaplace(F, S, T)` computes the inverse Laplace transform of the expression $F = F(S)$ with respect to the variable S at the point T .

Call(s):

☞ `transform::laplace(f, t, s)`

☞ `transform::invlaplace(F, S, T)`

Parameters:

- f, F — arithmetical expressions
- t, S — identifiers (the transformation variables)
- s, T — arithmetical expressions (the evaluation points)

Return Value: an arithmetical expression or an unevaluated function call of domain type `transform::laplace` or `transform::invlaplace`, respectively.

Overloadable by: f, F

Details:

⚡ An unevaluated function call is returned, if no explicit representation of the transform is found.

Example 1. The following call produces the Laplace transform as an expression in the variable `s`:

```
>> transform::laplace(exp(-a*t), t, s)
```

$$\frac{1}{a + s}$$

```
>> transform::invlaplace(%, s, t)
```

$$\exp(-a t)$$

Note that the Laplace transform can be evaluated directly at a specific point such as $s = 2a$ or $s = 5$:

```
>> transform::laplace(t^10*exp(-a*t), t, s),
transform::laplace(t^10*exp(-a*t), t, 2*a),
transform::laplace(t^10*exp(-a*t), t, 1 + PI)
```

$$\frac{3628800}{(a + s)^{11}}, \frac{44800}{2187 a^{11}}, \frac{3628800}{(a + \text{PI} + 1)^{11}}$$

Some further examples:

```
>> transform::laplace(1 + exp(-a*t)*sin(b*t), t, s)
```

$$\frac{1}{s} + \frac{b}{b^2 + (a + s)^2}$$

```
>> transform::invlaplace(1/(s^3 + s^5), s, t)
```

$$\cos(t) + \frac{t^2}{2} - 1$$

```
>> transform::invlaplace(exp(-2*s)/(s^2 + 1) + s/(s^3 + 1), s, t)
```

$$\sin(t - 2) \operatorname{heaviside}(t - 2) - \frac{\exp(-t)}{3} +$$

$$\frac{\exp\left(-\frac{t}{\sqrt{2}}\right) \cos\left(\frac{t}{\sqrt{2}}\right) + 3 \sin\left(\frac{t}{\sqrt{2}}\right)}{3}$$

Example 2. An unevaluated call is returned, if no explicit representation of the transform is found:

```
>> transform::laplace(exp(-t^3), t, s)
```

$$\operatorname{transform}::\operatorname{laplace}(\exp(-t^3), t, s)$$

Note that this is not an ordinary expression, but a domain element of domain type `transform::laplace`:

```
>> domtype(%)
```

```
transform::laplace
```

The inverse of the formal transform yields the original expression:

```
>> transform::invlaplace(%2, s, t)
```

$$\exp(-t^3)$$

Example 3. The distribution `dirac` and the Heaviside function `heaviside` are handled:

```
>> transform::laplace(dirac(t - 3), t, s)
```

$$\exp(-3 s)$$

```
>> transform::invlaplace(1, s, t)
```

$$\text{dirac}(t)$$

```
>> transform::laplace(heaviside(t - PI), t, s)
```

$$\frac{\exp(-s \text{PI})}{s}$$

Example 4. The Laplace transform of a function is related to the Laplace transform of its derivative:

```
>> transform::laplace(diff(f(t), t), t, s)
```

$$s \text{ transform::laplace}(f(t), t, s) - f(0)$$