MA2006 Ex wk 2

1. Use mathematical induction to prove the following:

   a) $2 + 6 + 10 + \ldots + (4n - 2) = 2n^2$ $\quad n > 0$

   b) $1 + 3 + 6 + \ldots + n(n+1)/2 = n(n+1)(n+2)/6$ $\quad n \geq 1$

   c) $1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ $\quad n \geq 1$

   d) $1*3 + 2*4 + 3*5 + \ldots + n(n+2) = n(n+1)(2n+7)/6$ $\quad n \geq 1$

   e) $(-2)^0 + (-2)^1 + (-2)^2 + \ldots + (-2)^n = (1 - (-2)^{n+1})/3$ $\quad n \geq 0$

   f) $2^n < n!$ for all $n \geq 4$

   g) $2^{3n} - 1$ is divisible by 7, for all $n \geq 0$

   h) $n^3 - n$ is divisible by 3, for all $n \geq 0$.

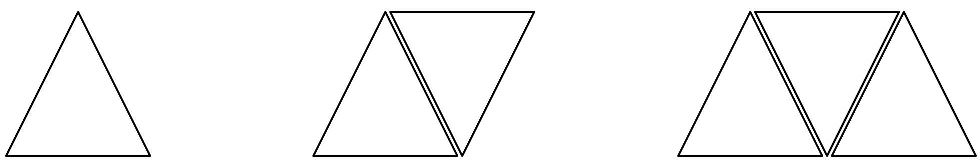
2.
```
TOTAL := 0
    While TRUE do
        Begin
         TOTAL := TOTAL + 1
         TOTAL :=  TOTAL + 4
        End
```

   (i) Let $T(i)$ ($i \geq 0$) be the value of *TOTAL* at the end of iteration i.
   Complete the following recursive definition of T: $\mathbf{N} \rightarrow \mathbf{N}$

   $T(0) = 0$, $T(i + 1) =$.

   (ii) Using your definition in (i) prove, by induction, that $T(n) = 5n$ ($n \geq 0$).

3. Consider the following, triangles joined along a single edge:



$k = 1$ $\quad$ $k = 2$ $\quad$ $k = 3$ …

Prove the following statement:

For $k \geq 1$, k triangles joined along a single edge have $E(k) = 2k + 1$ edges.

4. *SUM* := 0
COUNT := 0

WHILE true do

BEGIN
COUNT := *COUNT* + 1
SUM := *SUM* + *COUNT*
END

(i) Let $C(i)$ be the value of *COUNT* at the end of iteration i.

Let $S(i)$ be the value of *SUM* at the end of iteration i.

Complete the following recursive definitions of C: $\mathbf{N} \rightarrow \mathbf{N}$ and S: $\mathbf{N} \rightarrow \mathbf{N}$

$C(0) =$, $C(i) =$

and $S(0) =$, $S(i) =$.

(ii) Using your definitions in (i) prove, by induction, that $C(n) = n$ and $S(n) = n(n + 1)/2$.

5. Use the Peano Axioms and the definitions of addition and multiplication to evaluate or prove the following. In each example you should give a justification for every step in the evaluation or proof

5.1. 0 * 1.

5.2. 0 + 2 given that 0 + 1 = l.

5.3. (2*1) + 1 given that 0 + 2 = 2 + 0.

5.4. 4 + 4 given that 4 + 2 = 6.

5.5. (1 * 0) + (0 * 1) given the answer to question 1.

5.6. 2 + 1 = 1 + 2 given that + is associative.