LONDON metropolitan university

Coursework Assignment - Semester 2 2007/8

Module code:	**MA2005N, MA2X05, MA2F05**
Module title:	**Graphs and Networks**
Module leader:	**Amir Khossousi**

INSTRUCTION:

This coursework assignment has a 25% weighting and contains three questions. You are required to answer all questions. Your solution may be handwritten. Up to 5 marks will be awarded for clarity of presentation.

To be submitted by Tuesday 22 April 2008 at the Undergraduate Registry, Tower Building.

You are advised to keep a copy of your completed work before submission.

You are also reminded that this is an individual coursework and a mark of zero could be awarded for plagiarized work.

1. (i) For each positive integer f, draw a **simple connected plane** graph G_f with 5 vertices and f faces if possible.

For your graph G_3 (with 5 vertices and 3 faces), draw its **dual graph** G_3^*.

Draw a **connected plane** graph F with 5 vertices and 10 faces.

[17 marks]

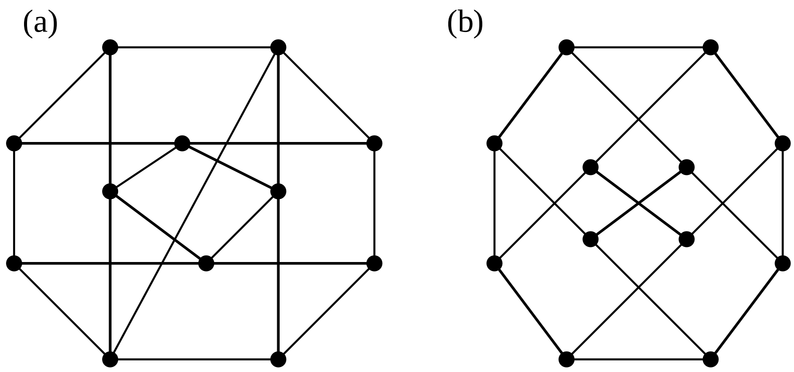
(ii) Show that, if H is a connected planar simple graph with at least three vertices and with no triangles, then

$$m \leq 2n - 4,$$

where n and m are the number of vertices and edges of H, respectively.

[8 marks]

(iii) By using the results in part (ii) above, show that only one of the following graphs is non-planar. Give a plane drawing of the one that is planar.



[10 marks]

2. The distances (km) between five cities, numbered 1 to 5, are given in the table below.

	1	**2**	**3**	**4**	**5**
1	-	4	7	1	8
2	4	-	3	7	12
3	7	3	-	2	6
4	1	7	2	-	17
5	8	12	6	17	-

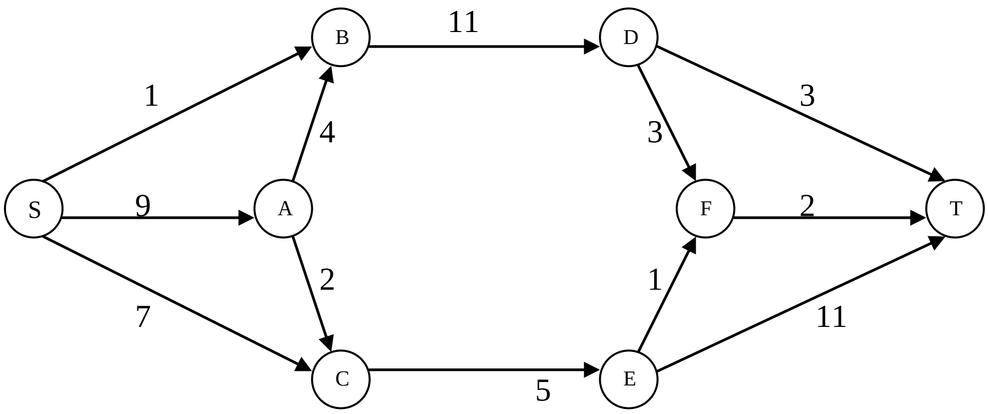
(i) Draw a network diagram to represent the information and briefly describe *Floyd's* algorithm for finding the shortest route between each pair of cities.

[8 marks]

(ii) Apply *Floyd's algorithm* to determine the shortest route and its distance between each pair of cities, clearly indicating the indirect routes and any alternative route that may exist.

[22 marks]

3. In the directed network below, the number on each arc represents the capacity of that arc.



(i) Starting with zero flow, use the maximum flow algorithm to find the maximum flow from S to T. Your solution should clearly demonstrate the labelling and flow augmenting procedures in each iteration.

[16 marks]

(ii) State the max-flow min-cut theorem, and hence identify the arcs in the minimum cut and determine its capacity.

[6 marks]

(iii) Determine, giving reasons, whether it is possible to increase the value of the maximum flow found above by increasing the capacity of one of the arcs. If such arc exists, state the maximum possible increase in the value of flow.

[8 marks]