



**Coursework Assignment 1 - Semester 2 2006/7**

**Module code: MA2005N**

**Module title: Graphs and Networks**

**Module leader: Amir Khossousi**

**INSTRUCTION:**

This individual coursework assignment has a 20% weighting. You are required to answer all questions. Up to 5 marks will be awarded for clarity of solution and presentation. Your solution need not be word-processed.

You must submit the following declaration as part of your assignment.

|                                                                                    |                    |
|------------------------------------------------------------------------------------|--------------------|
| Surname:                                                                           | Other Names:       |
| ID No:                                                                             | Course code_MA2005 |
| Student Declaration: <i>"I declare that the work submitted is solely my own"</i> . |                    |
| Your Signature                                                                     |                    |

Submit your answers (including this sheet) on A4 paper stapled together (**not in folders**).

To be submitted by Tuesday 27 March at the Undergraduate Registry, Tower Building.

You are advised to keep a copy of your completed work before submission.

1. By applying the Havel-Hakimi method, determine whether the following sequences are graphic. Draw simple graphs for any that are.

- (i) 2, 2, 3, 4, 4, 5
- (ii) 4, 5, 6, 7, 7, 7, 7
- (iii) 4, 5, 6, 6, 6, 6, 6, 7.

[9 marks]

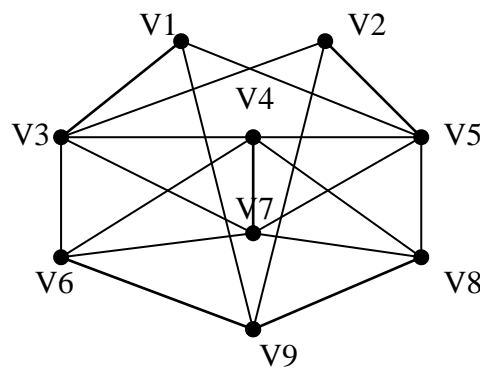
2. The graph  $G$  with vertex set  $\{v_1, v_2, v_3, v_4, v_5\}$  has adjacency matrix,  $A$ , and incidence matrix,  $M$ , where

$$M = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

- (i) Using the incidence matrix, draw the graph of  $G$ .
- (ii) Determine the adjacency matrix  $A$  and, by calculating  $A^2$ , find the number of walks of length 2 between any two vertices.
- (iii) Calculate the number of walks of length 4 from  $v_2$  to  $v_4$ .

[9 marks]

3. Let  $H$  be the following graph.



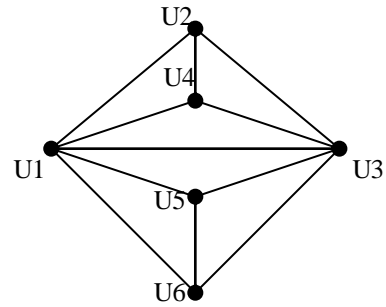
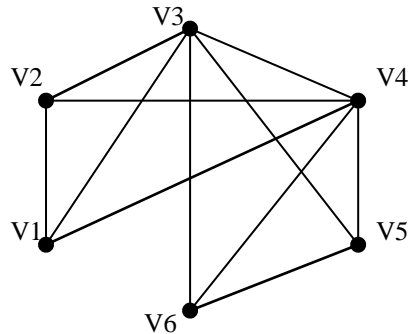
Determine, giving reasons for your answers,

- (i) the vertex connectivity and edge connectivity of  $H$ ;
- (ii) whether  $H$  is Eulerian, semi-Eulerian, or neither;
- (iii) whether  $H$  is Hamiltonian, semi-Hamiltonian, or neither;
- (iv) whether  $H$  has an open trail that is not a path.

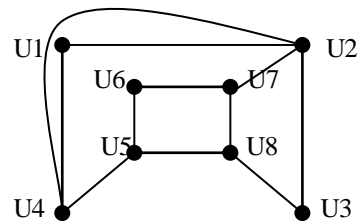
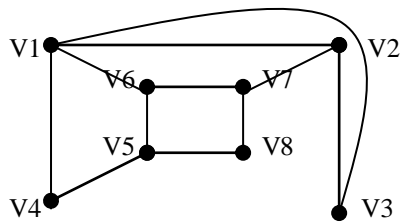
[10 marks]

4. Determine whether the two graphs in each of the following pairs are isomorphic. For each pair give either an isomorphism or a reason why no isomorphism exists.

(i)



(ii)



[6 marks]

5. The table below shows the distances between pairs of nodes of a network that have direct connections. The symbol  $\infty$  is used to indicate the nodes that are not directly connected. Apply Floyd's shortest path algorithm to find the shortest route and its distance between any two distinct nodes in the network.

|   | 1        | 2        | 3  | 4        | 5        |
|---|----------|----------|----|----------|----------|
| 1 | -        | 4        | 18 | 15       | $\infty$ |
| 2 | 4        | -        | 12 | $\infty$ | 5        |
| 3 | 18       | 12       | -  | 2        | 6        |
| 4 | 15       | $\infty$ | 2  | -        | 3        |
| 5 | $\infty$ | 5        | 6  | 3        | -        |

[11 marks]