Coursework Assignment 2 - Semester 2 2006/7

Module code:　　**MA2005N**
Module title:　　**Graphs and Networks**
Module leader:　　**Amir Khossousi**

INSTRUCTION:

This underlined individual coursework assignment has a 20% weighting. You are required to answer all questions. Up to 3 marks will be awarded for clarity of solution and presentation. Your solution need not be word-processed.

You must submit the following declaration as part of your assignment.

Surname:　　　　　　　　　　Other Names:
ID No:　　　　　　　　　Course code_MA2005
Student Declaration: *"I declare that the work submitted is solely my own"*.
Your Signature

Submit your answers (including this sheet) on A4 paper stapled together (**not in folders**).

To be submitted by Tuesday 1st May 2007 at the Undergraduate Registry, Tower Building.

You are advised to keep a copy of your completed work before submission.

1. Let G be a simple connected plane graph with m edges, n vertices and f faces.

   (i) Assuming that each vertex of G has degree of at least 3, show that

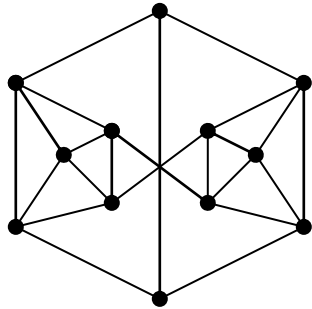 $$m \geq \frac{3}{2}n \qquad \text{and} \qquad f \geq \frac{1}{2}n+2.$$

   (6 marks)

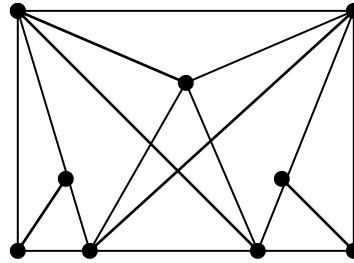   (ii) Given that each face f_i of G has at least 5 bordering edges, show that

   $n \geq \frac{3}{5}m+2$, and that G must have at least 30 edges, 20 vertices and 12 faces.

   (9 marks)

2. By deleting and/or contracting appropriate edges, prove that the graphs G_1 and G_2, given below, are non-planar.

   G_1 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ G_2

   (12 marks)

3. The table below shows the distances (in km) between the towns A, B, C, D and E.

	A	B	C	D	E
A	-	10	6	12	4
B	10	-	5	14	10
C	6	5	-	11	7
D	12	14	11	-	8
E	4	10	7	8	-

   Use the *branch-and-bound* method to find a 5-cycle through the five towns with minimum total distance travelled. Summarize your results in a tree diagram.

   (20 marks)