MA2008 Groups and Graphs — Sample Progress Test 2

Answer **ALL** questions

1. For all $a, b \in G(.)$ prove that $(bab^{-1})^3 = 1$ if and only if $a^3 = 1$

3 Marks

2. a) Find all the generating sets for the group Z_8 (⊕) which contain a single element

   b) List all the **proper** subgroups of the group Z_8 (⊕).

7 Marks

3 Show the group $G = \{ 1, a, a^2, b, ab, a^2b \}$ where $a^3 = b^2 = 1$ and $ab = ba$ is cyclic

6 Marks

4. Find four different cyclic subgroups of order 3 of the group
$C_3 \times C_3 = \{ 1, a, a^2, b, b^2, ab, a^2b, ab^2, a^2b^2 \}$ where $a^3 = b^3 = 1$ and $ab = ba$.

8 Marks

5. If G is the group C – {0} (.) and H is the subset of G defined by

$H = \{z : z \in C \text{ and } |z| = 1\}$

then use the Subgroup Theorem to prove that H is a subgroup of G.

6 Marks