MA2008 Groups and Vector Spaces

Teaching Location	City and North
Teaching Semester	Spring
Module Level	I
Home Academic Department	CCTM
Module Leader	Dr A. Tonks
Module Web Site	MA2008
Teaching Mode	day
Module Title	Groups and Vector Spaces
Timeslot	City: Monday am North: Monday pm
Credit Rating For Module	15

Module Summary

This module provides an introduction to abstract algebra and algebraic structures, including an introduction to group theory with emphasis on certain concrete families of finite groups, and building on algebra which students have seen at certificate level to introduce vector spaces, linear transformations and their properties.

Prerequsites:

MA1010C Linear Algebra or MA1010N Linear Algebra or MA1007C Maths Techniques 2 or MA1007N Maths Techniques 2

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Assessment type	Description of item	Weighting	Week due
CST	Class Test	25	7
EXU	Unseen Examination	75	13

Module Aims

The aim of this module is to introduce students to the abstract algebraic structures of groups and of vector spaces, which arise from the ideas of symmetries and of vector and matrix calculus respectively. These two primary examples of algebraic structures have applications across science and engineering, and also provide a firm foundation of necessary basic algebraic notions for the student to further their study mathematical study.

Syllabus

Vector Spaces

Revision of arithmetic of matrices and vectors in $\mathbf{R}^n$ and $\mathbf{C}^n$ and application to systems of equations
Formal definition of a vector space. Examples.
Linear combinations and spanning. Dependence and independence of vectors.
Subspaces of vector spaces. Bases and dimensions. Standard basis for $\mathbf{R}^n$ and $\mathbf{C}^n$.
Quotient spaces.
Linear transformations, matrix representation relative to a given basis. Change of basis.
Kernel and image; first isomorphism theorem (rank/nullity theorem).
Eigenvalues, eigenspaces and bases of eigenvectors; application to diagonalisation.
Cayley-Hamilton.

Group Theory

Revision of notions of binary operation, associativity.
Formal definition of a group. Worked examples: symmetries of a regular polygon.
Generators, orders. Abelian groups, cyclic groups, free groups, relations in groups.
Subgroups. Cosets and Theorem of Lagrange. Normal subgroups; quotient groups.
Homomorphisms and isomorphisms. Kernel and image; first isomorphism theorem.
Permuation groups: symmetric and alternating groups. Cayley's theorem.
Groups of matrices.

Learning And Teaching

Lectures will be used to formally introduce the various concepts, ideas and applications of group theory (11 hours) and vector spaces and abstract linear algebra (11 hours) and will provide a focal point for the module. Tutorial sessions (22 hours) will give students guidance in completing weekly exercises and maintain a careful balance of emphasis between theory and practice. Attendance at both lectures and tutorials will be regarded as compulsory. Students will work on the tutorial examples sheets between sessions (44 hours) either individually or in small informal groups; these examples sheets will be purely formative in nature, allowing for quick feedback on students' progress and understanding. Students who do not complete the exercises will be at a great disadvantage in the progress test and final examination since the questions will be of a similar nature. Tutorial handouts may in addition suggest relevant computer algebra (such as MAPLE for linear algebra and permutations, or possibly GAP for advanced group theory) resources for further matrix work more open-ended problems intended to stimulate the student to individual research, as well as indicating preparatory and directed reading for the next lecture (33 hours). Students will be expected to spend time on unsupervised work, for example, directed reading (30 Hours).

Learning Outcomes

On successful completion of this module, students should be able to
1. perform basic calculations in familiar groups including C_n (or Z_n), D_n, S_n, A_n.
2. perform calculations with vectors and transformations using suitable matrix representations
3. construct homomorphisms between groups and vector spaces and determine their properties
4. appreciate notions and theory of subgroups/subspaces and quotient groups/quotient spaces and apply them correctly to well defined problems
5. understand both the concrete examples and the concept of linear independence and be able to apply them to find bases from given spanning sets
6. determine whether a transformation of a vector spaces may be represented by a diagonal matrix with respect to an appropriate basis

Assessment Strategy

There will be an unseen test during the course and this will provide an opportunity for students to monitor their progress and understanding of the material covered in the first half of the course. There will be an unseen examination at the end of the course (LO1-7, A3). Students are expected to attend lectures and tutorials and complete weekly tutorial exercise sheets, but these will be of a purely formative nature.

Bibliography

O'Gorman, S.P. et al.; **Lecture Notes in Groups.** *University of North London.*
Jordan, C and Jordan, D; **Groups.** *Arnold (1994)*
Fraleigh, J.B.; **A first course in abstract algebra.** *Addison-Wesley (1976)*
Green, J.A.; **Sets and Groups.** *Routledge and Kegan Paul. (1965)*
Fraleigh, J. et al.; **Linear Algebra.** *Addison-Wesley (1994)*
Lipschutz, S. and Lipson, M., Schaum's **Outline of Linear Algebra.** *McGraw-Hill (2000)*

Other links to useful online material will be made available on WebCT