Intervals of convergence of power series

Worked examples

1.

$$\sum_{n=1}^{\infty} \frac{(x-1)^{n+1}}{3^n} = \frac{(x-1)^2}{3^1} + \frac{(x-1)^3}{3^2} + \frac{(x-1)^4}{3^3} + \cdots$$

The initial term is $a = (x-1)^2/3$.

The common ratio is $r = (x-1)/3$.

Therefore the sum of the series is:

$$\frac{a}{1-r} = \frac{(x-1)^2/3}{1-(x-1)/3}$$

which simplifies to

$$\frac{(x-1)^2}{4-x}$$

The interval of convergence $-1 < r < 1$ is

$$-1 < (x-1)/3 < 1$$

which it is better to write as

$$-2 < x < 4.$$

2.

$$\sum_{n=0}^{\infty} \frac{2^n\,(x+4)^{n+2}}{6} = \frac{(x+4)^2}{6} + \frac{2\,(x+4)^3}{6} + \frac{4\,(x+4)^4}{6} + \frac{8\,(x+4)^5}{6} + \cdots$$

has initial term $a = \frac{1}{6}(x+4)^2$ and common ratio $r = 2(x+4)$ so

$$\frac{a}{1-r} = \frac{(x+4)^2/6}{1-2(x+4)} = \frac{1}{6}\,\frac{(x+4)^2}{-2x-7}$$

which has interval of convergence $-1 < 2(x+4) < 1$, that is,

$$-4.5 < x < -3.5$$