Dave's Math Tables

Dave's Math Tables: *Questions & Comments* UP

(Math | Questions & Comments)

Please Help:
This site's Math Message Board needs people to help in answering math questions. Please see How to Help.

If you have a math question, there are a number of routes you may take:

(1) Post your question on this site's Math Message Board. This is a web-based newsgroup (discussion group) for math talk, question, and answers. There are sections for algebra, trig, geometry, calculus, elementary math, and general discussions.

(2) Send your question to Ask Dr. Math. This is a highly-popular, high-quality math question answering site run by Swarthmore College as part of the Math Forum.

(3) I am currently very busy, so I can not personally respond to math problems and questions; please use the above *Math Message Board* instead, where a number of qualified mathematicians, teachers, and students will be able to take your questions. If you do, though, have technical questions with regards to this site or need to reach me, the following methods are available:

(A) Type your question/comment here. You *must* include your e-mail if you desire a response.

SendComments

(B) E-mail me at sismspec@ix.netcom.com.

(C) My AOL Instant Messenger screen name is $C_6H_{10}CH_3$.

(D) My ICQ is 7664967. This is another very popular instant messenger service.

Dave's Math Tables: *Number Notation* UP

(**Math** | **General** | **NumberNotation**)

Hierarchy of Numbers

```
0(zero) 1(one) 2(two)   3(three) 4(four)
5(five) 6(six) 7(seven) 8(eight) 9(nine)
10^1(ten)  10^2(hundred)  10^3(thousand)

name             American-French  English-German
million          10^6             10^6
billion          10^9             10^12
trillion         10^12            10^18
quadrillion      10^15            10^24
quintillion      10^18            10^30
sextillion       10^21            10^36
septillion       10^24            10^42
octillion        10^27            10^48
nonillion        10^30            10^54
decillion        10^33            10^60
undecillion      10^36            10^66
duodecillion     10^39            10^72
tredecillion     10^42            10^78
quatuordecillion 10^45            10^84
quindecillion    10^48            10^90
sexdecillion     10^51            10^96
septendecillion  10^54            10^102
octodecillion    10^57            10^108
novemdecillion   10^60            10^114
vigintillion     10^63            10^120
----------------------------------------
googol           10^100
googolplex       10^googol = 10^(10^100)
----------------------------------------
```

SI Prefixes

Number	Prefix	Symbol
10^{1}	deka-	da
10^{2}	hecto-	h
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E
10^{21}	zeta-	Z
10^{24}	yotta-	Y

Number	Prefix	Symbol
10^{-1}	deci-	d
10^{-2}	centi-	c
10^{-3}	milli-	m
10^{-6}	micro-	u (greek mu)
10^{-9}	nano-	n
10^{-12}	pico-	p
10^{-15}	femto-	f
10^{-18}	atto-	a
10^{-21}	zepto-	z
10^{-24}	yocto-	y

Roman Numerals

I=1	V=5	X=10	L=50	C=100	D=500	M=1 000
	$\overline{V}$=5 000	$\overline{X}$=10 000	$\overline{L}$=50 000	$\overline{C}$ = 100 000	$\overline{D}$=500 000	$\overline{M}$=1 000 000

Examples:

1 = I 2 = II 3 = III 4 = IV 5 = V 6 = VI 7 = VII 8 = VIII 9 = IX 10 = X	11 = XI 12 = XII 13 = XIII 14 = XIV 15 = XV 16 = XVI 17 = XVII 18 = XVIII 19 = XIX 20 = XX 21 = XXI	25 = XXV 30 = XXX 40 = XL 49 = XLIX 50 = L 51 = LI 60 = LX 70 = LXX 80 = LXXX 90 = XC 99 = XCIX

Number Base Systems

```
decimal binary ternary oct hex
      0      0       0   0   0
      1      1       1   1   1
      2     10       2   2   2
      3     11      10   3   3
      4    100      11   4   4
      5    101      12   5   5
      6    110      20   6   6
      7    111      21   7   7
      8   1000      22  10   8
      9   1001     100  11   9
     10   1010     101  12   A
     11   1011     102  13   B
     12   1100     110  14   C
     13   1101     111  15   D
     14   1110     112  16   E
     15   1111     120  17   F
     16  10000     121  20  10
     17  10001     122  21  11
     18  10010     200  22  12
     19  10011     201  23  13
     20  10100     202  24  14
```

Java Base Conversion Calculator *(This converts non-integer values & negative bases too!)*

(For Microsoft 2+/Netscape 2+/Javascript web browsers only)

from base	to base	value to convert
10	16	256
calculate	100	

Caution: due to CPU restrictions, some rounding has been known to occur for numbers spanning greater than 12 base10 digits, 13 hexadecimal digits or 52 binary digits. Just like a regular calculator, rounding can occur.

Dave's Math Tables: *Weights and Measures* UP

(Math | General | Weights and Measures | Lengths)

Unit Conversion Tables for Lengths & Distances

A note on the metric system:
Before you use this table, convert to the base measurement first, in that convert centi-meters to meters, convert kilo-grams to grams. In this way, I don't have to list every imaginable combination of metric units.

The notation 1.23**E + 4** stands for 1.23 x 10^{+4} = 0.000123.

from \ to	= __ feet	= __ inches	= __ meters	= __ miles	= __ yards
foot		12	0.3048	(1/5280)	(1/3)
inch	(1/12)		0.0254	(1/63360)	(1/36)
meter	3.280839...	39.37007...		6.213711...**E - 4**	1.093613...
mile	5280	63360	1609.344		1760
yard	3	36	0.9144	(1/1760)	

To use: Find the unit to convert **from** in the left column, and multiply it by the expression under the unit to convert **to**.
Examples: foot = 12 inches; 2 feet = 2x12 inches.

Useful Exact Length Relationships

mile = 1760 yards = 5280 feet
yard = 3 feet = 36 inches
foot = 12 inches
inch = 2.54 centimeters

Dave's Math Tables: *Weights and Measures*

(Math | General | Weights and Measures | Areas)

Unit Conversion Tables for Areas

A note on the metric system:
Before you use this table **convert to the base measurement first**, in that convert **centi-meters to meters**, convert **kilo-grams to grams**. In this way, I don't have to list every imaginable combination of metric units.

The notation 1.23**E + 4** stands for 1.23 x 10^{+4} = 0.000123.

from \ to	= __ acres	= __ feet^2	= __ inches^2	= __ meters^2	= __ miles^2	= __ yards^2
acre		43560	6272640	4046.856...	(1/640)	4840
foot^2	(1/43560)		144	0.09290304	(1/27878400)	(1/9)
inch^2	(1/6272640)	(1/144)		6.4516**E - 4**	3.587006**E - 10**	(1/1296)
meter^2	2.471054...**E - 4**	10.76391...	1550.0031		3.861021...**E - 7**	1.195990...
mile^2	640	27878400	2.78784**E + 9**	2.589988...**E + 6**		3097600
yard^2	(1/4840)	9	1296	0.83612736	3.228305...**E - 7**	

To use: Find the unit to convert **from** in the left column, and multiply it by the expression under the unit to convert **to**.
Examples: foot^2 = 144 inches^2; 2 feet^2 = 2x144 inches^2.

Useful Exact Area & Length Relationships

acre = (1/640) miles^2
mile = 1760 yards = 5280 feet
yard = 3 feet = 36 inches
foot = 12 inches
inch = 2.54 centimeters

Note that when converting area units:
1 foot = 12 inches
(1 foot)^2 = (12 inches)^2 (square both sides)
1 foot^2 = 144 inches^2
The linear & area relationships are not the same!

Dave's Math Tables: *Exponential Identities* UP

(**Math** | **Algebra** | **Exponents**)

Powers

$x^a x^b = x^{(a + b)}$

$x^a y^a = (xy)^a$

$(x^a)^b = x^{(ab)}$

$x^{(a/b)}$ = b^{th} root of (x^a) = $(\sqrt[b]{x})^a$

$x^{(-a)} = 1 / x^a$

$x^{(a - b)} = x^a / x^b$

Logarithms

$y = \log_b(x)$ if and only if $x=b^y$

$\log_b(1) = 0$

$\log_b(b) = 1$

$\log_b(x*y) = \log_b(x) + \log_b(y)$

$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^n) = n \log_b(x)$

$\log_b(x) = \log_b(c) * \log_c(x) = \log_c(x) / \log_c(b)$

Dave's Math Tables: *e* UP

(**Math** | **OddsEnds** | **Constants** | **e**)

e = 2.7182818284 5904523536 0287471352 6624977572 4709369995 9574966967 6277240766 3035354759 4571382178 5251664274 ...

$e = \lim_{(n \to 0)} (1 + n)^{(1/n)}$ **or** $e = \lim_{(n \to \infty)} (1 + 1/n)^{n}$

$$e = \sum_{n=0}^{\infty} 1 / k!$$

see also Exponential Function Expansions.

Dave's Math Tables: *Exponential Expansions* UP

(Math | Calculus | Expansions | Series | Exponent)

Function	Summation Expansion	Comments
e	$e=\sum_{n=0}^{\infty} 1 / n!$ $= 1/1 + 1/1 + 1/2 + 1/6 + \ldots$	see constant e
e^{-1}	$=\sum_{n=0}^{\infty} (-1)^n / n!$ $= 1/1 - 1/1 + 1/2 - 1/6 + \ldots$	
e^x	$=\sum_{n=0}^{\infty} x^n / n!$ $= 1/1 + x/1 + x^2 / 2 + x^3 / 6 + \ldots$	

Dave's Math Tables: *Log Expansions*

(Math | Calculus | Expansions | Series | Log)

Expansions of the Logarithm Function

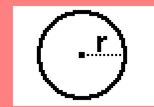
Function	Summation Expansion	Comments
ln (x)	$=\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ $= (x-1) - (1/2)(x-1)^2 + (1/3)(x-1)^3 + (1/4)(x-1)^4 + \ldots$	Taylor Series Centered at 1 $(0 < x <= 2)$
ln (x)	$=\sum_{n=1}^{\infty} \frac{((x-1) / x)^n}{n}$ $= (x-1)/x + (1/2) ((x-1) / x)^2 + (1/3) ((x-1) / x)^3 + (1/4) ((x-1) / x)^4 + \ldots$	$(x > 1/2)$
ln (x)	$=\ln(a)+\sum_{n=1}^{\infty} \frac{(x-a)^n}{n a^n}$ $= \ln(a) + (x-a) / a - (x-a)^2 / 2a^2 + (x-a)^3 / 3a^3 - (x-a)^4 / 3a^4 + \ldots$	Taylor Series $(0 < x <= 2a)$
ln (x)	$=2\sum_{n=1}^{\infty} \frac{((x-1)/(x+1))^{(2n-1)}}{(2n-1)}$ $= 2 [(x-1)/(x+1) + (1/3)((x-1)/(x+1))^3 + (1/5) ((x-1)/(x+1))^5 + (1/7) ((x-1)/(x+1))^7 + \ldots]$	$(x > 0)$

Expansions Which Have Logarithm-Based Equivalents

Summantion Expansion	Equivalent Value	Comments
$\sum_{n=1}^{\infty} \frac{x^n}{n}$ $= x + (1/2)x^2 + (1/3)x^3 + (1/4)x^4 + \ldots$	= - ln (x + 1)	(-1 < x <= 1)
$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ $= - x + (1/2)x^2 - (1/3)x^3 + (1/4)x^4 + \ldots$	= - ln(x)	(-1 < x <= 1)
$\sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$ $= x + (1/3)x^3 + (1/ 5)x^5 + (1/7)x^7 + \ldots$	$= \frac{\ln ((1+x)/(1-x))}{2}$	(-1 < x < 1)

Dave's Math Tables: *Circles* UP

(**Math** | **Geometry** | **Circles**)



a circle

Definition: A circle is the locus of all points equidistant from a central point.

Definitions Related to Circles

arc: a curved line that is part of the circumference of a circle
chord: a line segment within a circle that touches 2 points on the circle.
circumference: the distance around the circle.
diameter: the longest distance from one end of a circle to the other.
origin: the center of the circle
pi (π): A number, 3.141592..., equal to (the circumference) / (the diameter) of any circle.
radius: distance from center of circle to any point on it.
sector: is like a slice of pie (a circle wedge).
tangent of circle: a line perpendicular to the radius that touches ONLY one point on the circle.

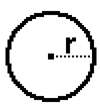
diameter = 2 x radius of circle

Circumference of Circle = **PI x diameter** = 2 PI x radius

where **PI = π = 3.141592...**

Area of Circle:

area = PI r^2



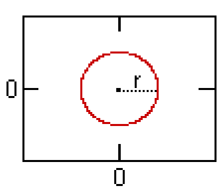
Length of a Circular Arc: (with central angle θ)

if the angle θ is in degrees, then length = θ x (PI/180) x r
if the angle θ is in radians, then length = r x θ

Area of Circle Sector: (with central angle θ)

if the angle θ is in degrees, then area = (θ/360) x PI r^2
if the angle θ is in radians, then area = (θ/2) x PI r^2

Equation of Circle: (cartesian coordinates)



for a circle with center **(j, k)** and radius **(r):**

$(x-j)^2 + (y-k)^2 = r^2$

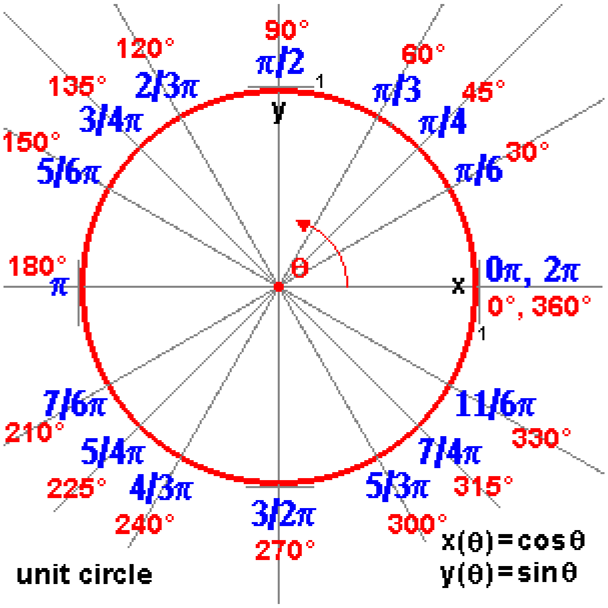
Equation of Circle: (polar coordinates)

for a circle with center (0, 0): **$r(\theta)$ = radius**

for a circle with center with polar coordinates: (c, α) and radius **a**:
$r^2 - 2cr\cos(\theta - \alpha) + c^2 = a^2$

Equation of a Circle: (parametric coordinates)

for a circle with origin (j, k) and radius r:
$x(t) = r\cos(t) + j \qquad y(t) = r\sin(t) + k$

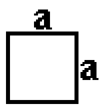


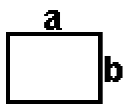
Dave's Math Tables: *Areas, Volumes, Surface Areas* UP

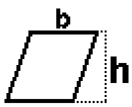
(Math | Geometry | AreasVolumes)

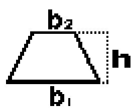
(*pi* = π = 3.141592...)

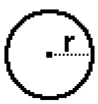
Areas

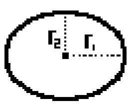
square = a^2 

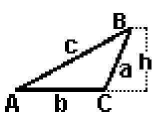
rectangle = ab 

parallelogram = bh 

trapezoid = $h/2 (b_1 + b_2)$ 

circle = *pi* r^2 

ellipse = *pi* $r_1 r_2$ 

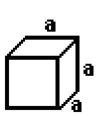
triangle = (1/2) b h 

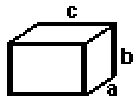
equilateral triangle = $[\sqrt{3}/2]\, a^2 = \sqrt{3/4}\, a^2$

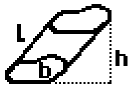
triangle given SAS = (1/2) a b sin C

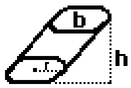
triangle given a,b,c = $\sqrt{s(s-a)(s-b)(s-c)}$ when s = (a+b+c)/2 **(Heron's formula)**

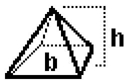
Volumes

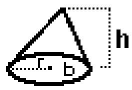
cube = a^3 

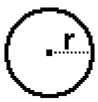
rectangular prism = a b c 

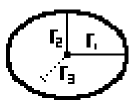
irregular prism = **b** h 

cylinder = **b** h = *pi* r^2 h 

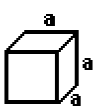
pyramid = (1/3) **b** h 

cone = (1/3) **b** h = 1/3 *pi* r^2 h 

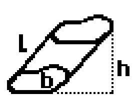
sphere = (4/3) *pi* r^3 

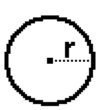
ellipsoid = (4/3) *pi* r_1 r_2 r_3 

Surface Area

cube = 6 a^2 

prism:
(lateral area) = perimeter(**b**) L

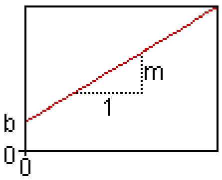
(total area) = perimeter(**b**) L + 2**b** 

sphere = 4 *pi* r^2 

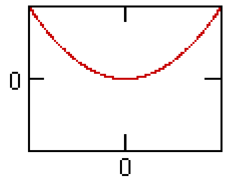
Dave's Math Tables: *Algebraic Graphs*

(**Math** | **OddsEnds** | **Graphs** | **Algebra**)

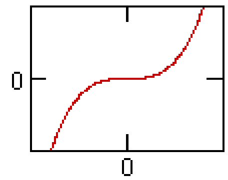
$y=mx+b$



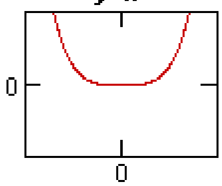
$y=x^2$



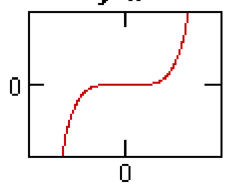
$y=x^3$



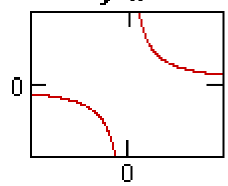
$y=x^4$



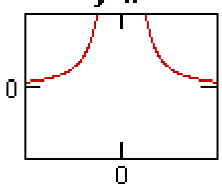
$y=x^5$



$y=x^{-1}$



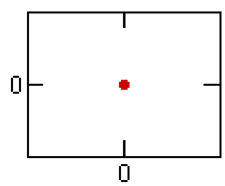
$y=x^{-2}$



Conic Sections

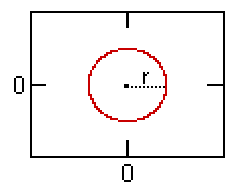
(see also Conic Sections)

Point



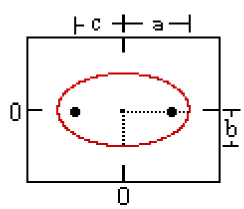
x^2 + y^2 = 0

Circle



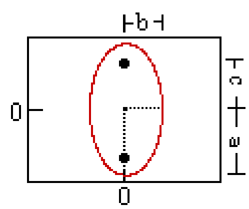
x^2 + y^2 = r^2

Ellipse



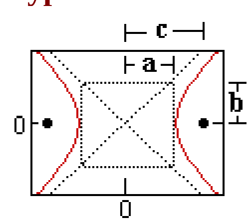
x^2 / a^2 + y^2 / b^2 = 1

Ellipse



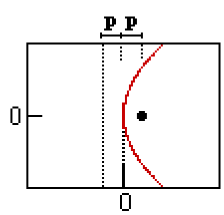
x^2 / b^2 + y^2 / a^2 = 1

Hyperbola

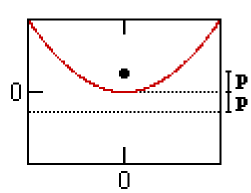


x^2 / a^2 - y^2 / b^2 = 1

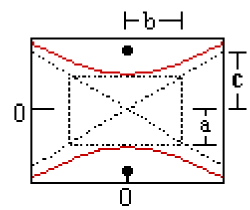
Parabola



Parabola

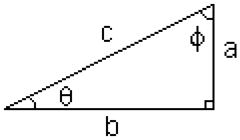


Hyperbola



$4px = y^{2}$	$4py = x^{2}$	$y^{2} / a^{2} - x^{2} / b^{2} = 1$
For any of the above with a center at (j, k) instead of (0,0), replace each x term with (x-j) and each y term with (y-k) to get the desired equation.		

Dave's Math Tables: *Trigonometric Identities*



sin(theta) = a / c	csc(theta) = 1 / sin(theta) = c / a
cos(theta) = b / c	sec(theta) = 1 / cos(theta) = c / b
tan(theta) = sin(theta) / cos(theta) = a / b	cot(theta) = 1/ tan(theta) = b / a

---

sin(-x) = -sin(x)
csc(-x) = -csc(x)
cos(-x) = cos(x)
sec(-x) = sec(x)
tan(-x) = -tan(x)
cot(-x) = -cot(x)

sin$^{\wedge 2}$(x) + cos$^{\wedge 2}$(x) = 1	tan$^{\wedge 2}$(x) + 1 = sec$^{\wedge 2}$(x)	cot$^{\wedge 2}$(x) + 1 = csc$^{\wedge 2}$(x)
sin(x ± y) = sin x cos y ± cos x sin y		
cos(x ± y) = cos x cosy ∓ sin x sin y		

tan(x ± y) = (tan x ± tan y) / (1 ∓ tan x tan y)

sin(2x) = 2 sin x cos x

cos(2x) = cos$^{\wedge 2}$(x) - sin$^{\wedge 2}$(x) = 2 cos$^{\wedge 2}$(x) - 1 = 1 - 2 sin$^{\wedge 2}$(x)

tan(2x) = 2 tan(x) / (1 - tan$^{\wedge 2}$(x))

sin$^{\wedge 2}$(x) = 1/2 - 1/2 cos(2x)

cos$^{\wedge 2}$(x) = 1/2 + 1/2 cos(2x)

sin x - sin y = 2 sin((x - y)/2) cos((x + y)/2)

cos x - cos y = -2 sin((x-y)/2) sin((x + y)/2)

Trig Table of Common Angles

angle	**0**	**30**	**45**	**60**	**90**
sin^2(a)	0/4	1/4	2/4	3/4	4/4
cos^2(a)	4/4	3/4	2/4	1/4	0/4
tan^2(a)	0/4	1/3	2/2	3/1	**4/0**

Given Triangle abc, with angles A,B,C; a is opposite to A, b oppositite B, c opposite C:

a/sin(A) = b/sin(B) = c/sin(C) **(Law of Sines)**

c^2 = a^2 + b^2 - 2ab cos(C)

b^2 = a^2 + c^2 - 2ac cos(B) **(Law of Cosines)**

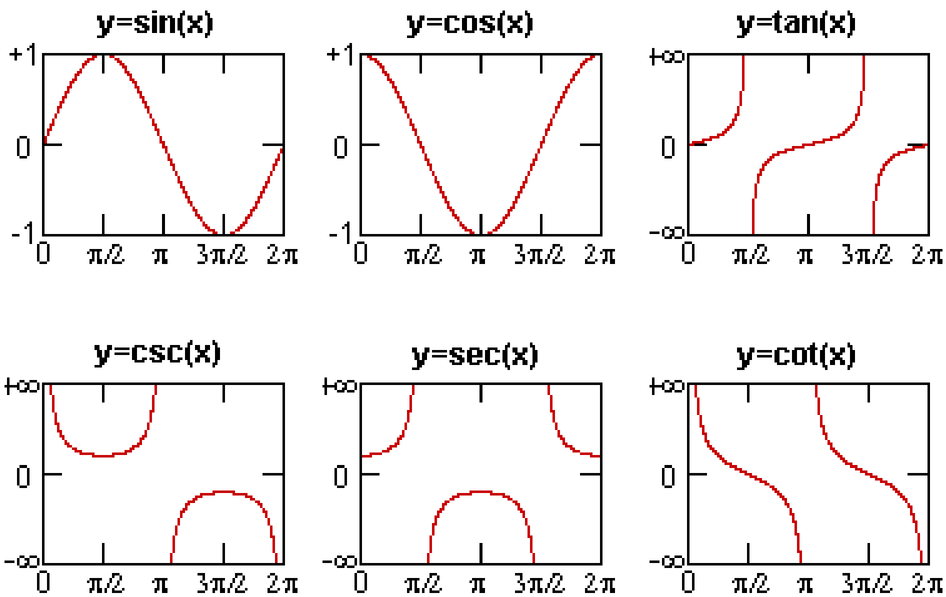
a^2 = b^2 + c^2 - 2bc cos(A)

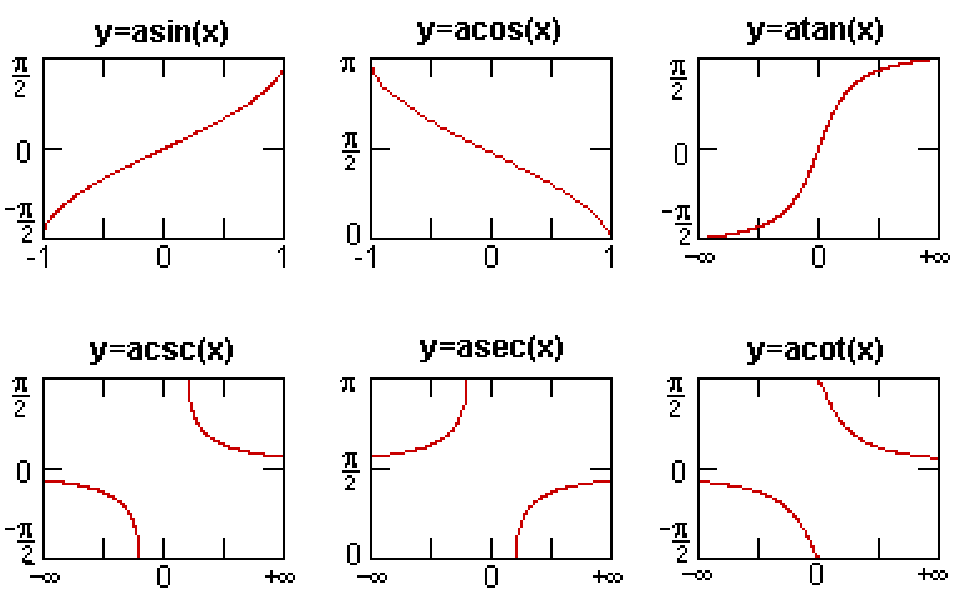
(a - b)/(a + b) = tan 1/2(A-B) / tan 1/2(A+B) **(Law of Tangents)** --*not neccessary with the above*

Dave's Math Tables: *Trigonometric Graphs*

UP

(Math | OddsEnds | Graphs | Trig)





Dave's Math Tables: *Series Properties* UP

(Math | Calculus | Expansions | Series | Properties)

Semi-Formal Definition of a "Series":

A series $\sum_{n=a}^{b} a_n$ is the *indicated* sum of all values of a_n when n is set to each integer from a to b inclusive; namely, the indicated sum of the values $a_a + a_{a+1} + a_{a+2} + \ldots + a_{b-1} + a_b$.

Definition of the "Sum of the Series":

The "sum of the series" is the *actual result* when all the terms of the series are summed.

Note the difference: "1 + 2 + 3" is an example of a "series," but "6" is the actual "sum of the series."

Algebraic Definition:

$$\sum_{n=a}^{b} a_n = a_a + a_{a+1} + a_{a+2} + \ldots + a_{b-1} + a_b$$

Summation Arithmetic:

$$\sum_{n=a}^{b} c\, a_n = c \sum_{n=a}^{b} a_n \quad \textit{(constant c)}$$

$$\sum_{n=a}^{b} a_n + \sum_{n=a}^{b} b_n = \sum_{n=a}^{b} a_n + b_n$$

$$\sum_{n=a}^{b} a_n - \sum_{n=a}^{b} b_n = \sum_{n=a}^{b} a_n - b_n$$

Summation Identities on the Bounds:

$$\sum_{n=a}^{b} a_n + \sum_{n=b+1}^{c} a_n = \sum_{n=a}^{c} a_n$$

$$\sum_{n=a}^{b} a_n = \sum_{n=a-c}^{b-c} a_{n+c}$$

$$\sum_{n=a}^{b} a_n = \sum_{n=a/c}^{b/c} a_{nc}$$

|
(similar relations exist for subtraction and division as generalized below for any operation g)
|

$$\sum_{n=a}^{b} a_n = \sum_{n=g(a)}^{g(b)} a_{g^{-1}(c)}$$

Dave's Math Tables: *Power Summations* UP

(**Math** | **Calculus** | **Expansions** | **Series** | **Power**)

Summation	Expansion	Equivalent Value	Comments
$\sum_{k=1}^{n} k$	$= 1 + 2 + 3 + 4 + .. + n$	$= (n^2 + n) / 2$ $= (1/2)n^2 + (1/2)n$	sum of 1st n integers
$\sum_{k=1}^{n} k^2$	$= 1 + 4 + 9 + 16 + .. + n^2$	$= (1/6)n(n+1)(2n+1)$ $= (1/3)n^3 + (1/2)n^2 + (1/6)n$	sum of 1st n squares
$\sum_{k=1}^{n} k^3$	$= 1 + 8 + 27 + 64 + .. + n^3$	$= (1/4)n^4 + (1/2)n^3 + (1/4)n^2$	sum of 1st n cubes
$\sum_{k=1}^{n} k^4$	$= 1 + 16 + 81 + 256 + .. + n^4$	$= (1/5)n^5 + (1/2)n^4 + (1/3)n^3 - (1/30)n$	
$\sum_{k=1}^{n} k^5$	$= 1 + 32 + 243 + 1024 + .. + n^5$	$= (1/6)n^6 + (1/2)n^5 + (5/12)n^4 - (1/12)n^2$	
$\sum_{k=1}^{n} k^6$	$= 1 + 64 + 729 + 4096 + .. + n^6$	$= (1/7)n^7 + (1/2)n^6 + (1/2)n^5 - (1/6)n^3 + (1/42)n$	
$\sum_{k=1}^{n} k^7$	$= 1 + 128 + 2187 + 16384 + .. + n^7$	$= (1/8)n^8 + (1/2)n^7 + (7/12)n^6 - (7/24)n^4 + (1/12)n^2$	
$\sum_{k=1}^{n} k^8$	$= 1 + 256 + 6561 + 65536 + .. + n^8$	$= (1/9)n^9 + (1/2)n^8 + (2/3)n^7 - (7/15)n^5 + (2/9)n^3 - (1/30)n$	

$\sum_{k=1}^{n} k^9$	= 1 + 512 + 19683 + 262144 + .. + n^9	= $(1/10)n^{10} + (1/2)n^9 + (3/4)n^8 - (7/10)n^6 + (1/2)n^4 - (3/20)n^2$	
$\sum_{k=1}^{n} k^{10}$	= 1 + 1024 + 59049 + 1048576 + .. + n^{10}	= $(1/11)n^{11} + (1/2)n^{10} + (5/6)n^9 - n^7 + n^5 - (1/2)n^3 + (5/66)n$	

Dave's Math Tables: *Power Summations #2* UP

(**Math** | **Calculus** | **Expansions** | **Series** | **Power2)**

Summation	Expansion	Equivalent Value	Comments
$\sum_{n=1}^{\infty} 1/n$	= 1 + 1/2 + 1/3 + 1/4 + ...	diverges to ∞	see the gamma constant
$\sum_{n=1}^{\infty} 1/n^2$	= 1 + 1/4 + 1/9 + 1/16 + ...	= (1/6) PI ^{2} = 1.64493406684822...	see Expansions of PI
$\sum_{n=1}^{\infty} 1/n^3$	= 1 + 1/8 + 1/27 + 1/81 + ...	= 1.20205690315031...	see the Unproved Theorems
$\sum_{n=1}^{\infty} 1/n^4$	= 1 + 1/16 + 1/81 + 1/256 + ...	= (1/90) PI ^{4} = 1.08232323371113...	see Expansions of PI
$\sum_{n=1}^{\infty} 1/n^5$	= 1 + 1/32 + 1/243 + 1/1024 + ...	= 1.03692775514333...	see the Unproved Theorems
$\sum_{n=1}^{\infty} 1/n^6$	= 1 + 1/64 + 1/729 + 1/4096 + ...	= (1/945) PI ^{6} = 1.017343061984449...	see Expansions of PI
$\sum_{n=1}^{\infty} 1/n^7$	= 1 + 1/128 + 1/2187 + 1/16384 + ...	= 1.00834927738192...	see the Unproved Theorems
$\sum_{n=1}^{\infty} 1/n^8$	= 1 + 1/256 + 1/6561 + 1/65536 + ...	= (1/9450) PI ^{8} = 1.00407735619794...	see Expansions of PI

$\sum_{n=1}^{\infty} 1/n^{9}$	= 1 + 1/512 + 1/19683 + 1/262144 + ...	= 1.00200839282608...	see the Unproved Theorems
$\sum_{n=1}^{\infty} 1/n^{10}$	= 1 + 1/1024 + 1/59049 + 1/1048576 + ...	$= (1/93555)\ PI^{10} =$ 1.00099457512781...	see Expanisions of PI
$\sum_{n=1}^{\infty} 1/(2n)^{n}$	$= 1 + 1/(2n)^{2} + 1/(2n)^{3} + 1/(2n)^{4} + ...$	$= (-1)^{n-1} (2^{2n} B_{(2n)} PI^{2n}) / (2(2n)!)$	see Expanisions of PI

Dave's Math Tables: *Geometric Summations*

UP

(Math | Calculus | Expansions | Series | Geometric)

Summation	Expansion	Convergence	Comments
$\sum_{n=0}^{n-1} r^n$	$= 1 + r + r^2 + r^3 + .. + r^{n-1}$ (first n terms)	for $r \neq 1$, $= (1 - r^n) / (1 - r)$ for $r = 1$, $= nr$	Finite Geometric Series
$\sum_{n=0}^{\infty} r^n$	$= 1 + r + r^2 + r^3 + ...$	for $\|r\| < 1$, converges to $1 / (1 - r)$ for $\|r\| >= 1$ diverges	Infinite Geometric Series

See also the Geometric Series Convergence in the Convergence Tests.

Dave's Math Tables: *Series Convergence Tests* UP

(Math | Calculus | Expansions | Series | Convergence Tests)

Definition of Convergence and Divergence in Series

The n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = a_1 + a_2 + a_3 + ... + a_n$. If the sequence of these partial sums $\{S_n\}$ converges to L, then the sum of the series converges to L. If $\{S_n\}$ diverges, then the sum of the series diverges.

Operations on Convergent Series

If $\sum a_n = A$, and $\sum b_n = B$, then the following also converge as indicated:

$$\sum ca_n = cA$$
$$\sum (a_n + b_n) = A + B$$
$$\sum (a_n - b_n) = A - B$$

Alphabetical Listing of Convergence Tests

Absolute Convergence

If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then the series $\sum_{n=1}^{\infty} a_n$ also converges.

Alternating Series Test

If for all n, a_n is positive, non-increasing (i.e. $0 < a_{n+1} <= a_n$), and approaching zero, then the alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n-1} a_n$$

both converge.
If the alternating series converges, then the remainder $R_N = S - S_N$ (where S is the exact sum of the infinite series and S_N is the sum of the first N terms of the series) is bounded by $|R_N| <= a_{N+1}$

Deleting the first N Terms

If N is a positive integer, then the series

$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=N+1}^{\infty} a_n$

both converge or both diverge.

Direct Comparison Test

If 0 <= a_n <= b_n for all n greater than some positive integer N, then the following rules apply:

If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Geometric Series Convergence

The geometric series is given by

$\sum_{n=0}^{\infty} a\, r^n = a + a\, r + a\, r^2 + a\, r^3 + \ldots$

If |r| < 1 then the following geometric series converges to a / (1 - r).

If |r| >= 1 then the above geometric series diverges.

Integral Test

If for all n >= 1, f(n) = a_n, and f is positive, continuous, and decreasing then

$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} a_n$

either both converge or both diverge.

If the above series converges, then the remainder $R_N = S - S_N$ (where S is the exact sum of the infinite series and S_N is the sum of the first N terms of the series) is bounded by 0< = R_N <= $\int$(N..∞) f(x) dx.

Limit Comparison Test

If lim (n-->

(a_n / b_n) = L,

where a_n, b_n > 0 and L is finite and positive, then the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

n^{th}-Term Test for Divergence

If the sequence $\{a_n\}$ does not converge to zero, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

p-Series Convergence

The p-series is given by

$$\sum_{n=1}^{\infty} 1/n^p = 1/1^p + 1/2^p + 1/3^p + \ldots$$

where $p > 0$ by definition.
If $p > 1$, then the series converges.
If $0 < p <= 1$ then the series diverges.

Ratio Test

If for all n, $n \neq 0$, then the following rules apply:
Let $L = \lim (n \to \infty) \mid a_{n+1} / a_n \mid$.
If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
If $L = 1$, then the test in *inconclusive*.

Root Test

Let $L = \lim (n \to \infty) \mid a_n \mid^{1/n}$.
If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
If $L = 1$, then the test in *inconclusive*.

Taylor Series Convergence

If f has derivatives of all orders in an interval I centered at c, then the Taylor series converges as indicated:

$$\sum_{n=0}^{\infty} (1/n!)\ f^{(n)}(c)\ (x - c)^n = f(x)$$

if and only if lim (n-->∞) R_n = 0 for all x in I.

The remainder $R_N = S - S_N$ of the Taylor series (where S is the exact sum of the infinite series and S_N is the sum of the first N terms of the series) is equal to $(1/(n+1)!)\ f^{(n+1)}(z)\ (x - c)^{n+1}$, where z is some constant between x and c.

Dave's Math Tables: *Table of Integrals* UP

(Math | Calculus | Integrals | Table Of)

Power of x.

$\int x^n \, dx = x^{(n+1)} / (n+1) + C$ ($n \neq -1$) Proof	$\int 1/x \, dx = \ln\|x\| + C$

Exponential / Logarithmic

$\int e^x \, dx = e^x + C$ Proof	$\int b^x \, dx = b^x / \ln(x) + C$ Proof, Tip!
$\int \ln(x) \, dx = x \ln(x) - x + C$ Proof	

Trigonometric

$\int \sin x \, dx = -\cos x + C$ Proof	$\int \csc x \, dx = - \ln\|\csc x + \cot x\| + C$ Proof
$\int \cos x \, dx = \sin x + C$ Proof	$\int \sec x \, dx = \ln\|\sec x + \tan x\| + C$ Proof
$\int \tan x \, dx = -\ln\|\cos x\| + C$ Proof	$\int \cot x \, dx = \ln\|\sin x\| + C$ Proof

Trigonometric Result

$\int \cos x \, dx = \sin x + C$ Proof	$\int \csc x \cot x \, dx = - \csc x + C$ Proof
$\int \sin x \, dx = -\cos x + C$ Proof	$\int \sec x \tan x \, dx = \sec x + C$ Proof
$\int \sec^2 x \, dx = \tan x + C$ Proof	$\int \csc^2 x \, dx = - \cot x + C$ Proof

Inverse Trigonometric

$\int \arcsin x \, dx = x \arcsin x + \sqrt{(1-x^2)} + C$
$\int \text{arccsc}\, x \, dx = x \arccos x - \sqrt{(1-x^2)} + C$
$\int \arctan x \, dx = x \arctan x - (1/2) \ln(1+x^2) + C$

Inverse Trigonometric Result

$\int \frac{dx}{\sqrt{(1 - x^2)}} = \arcsin x + C$
$\int \frac{dx}{x\sqrt{(x^2 - 1)}} = \text{arcsec}\lvert x\rvert + C$
$\int \frac{dx}{1 + x^2} = \arctan x + C$

Useful Identities

arccos x = π/2 - arcsin x
(-1 <= x <= 1)

arccsc x = π/2 - arcsec x
(|x| >= 1)

arccot x = π/2 - arctan x
(for all x)

Hyperbolic

$\int \sinh x\, dx = \cosh x + C$ Proof	$\int \text{csch}\, x\, dx = \ln \lvert\tanh(x/2)\rvert + C$ Proof
$\int \cosh x\, dx = \sinh x + C$ Proof	$\int \text{sech}\, x\, dx = \arctan (\sinh x) + C$
$\int \tanh x\, dx = \ln (\cosh x) + C$ Proof	$\int \coth x\, dx = \ln \lvert\sinh x\rvert + C$ Proof

---

Click on Proof for a proof/discussion of a theorem.

To solve a more complicated integral, see The Integrator at http://integrals.com.

Dave's Math Tables: *Integral Identities* UP

(**Math** | **Calculus** | **Integrals** | **Identities**)

Formal Integral Definition:

$\int_a^b f(x)\,dx = \lim_{(d \to 0)} \sum (k=1..n)\, f(X_{(k)})\,(x_{(k)} - x_{(k-1)})$ *when...*

$a = x_0 < x_1 < x_2 < \ldots < x_n = b$

$d = \max(x_1 - x_0, x_2 - x_1, \ldots, x_n - x_{(n-1)})$

$x_{(k-1)} \le X_{(k)} \le x_{(k)} \quad k = 1, 2, \ldots, n$

$\int_a^b F'(x)\,dx = F(b) - F(a)$ **(Fundamental Theorem for integrals of derivatives)**

$\int a\, f(x)\,dx = a \int f(x)\,dx$ *(if a is constant)*

$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int_a^b f(x)\,dx = \int f(x)\,dx \mid (a\ b)$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

$\int f(u)\,du/dx\,dx = \int f(u)\,du$ **(integration by substitution)**

Dave's Math Tables: *Table Derivatives* UP

(Math | Calculus | Derivatives | Table Of)

Power of x.

$\frac{d}{dx} c = 0$	$\frac{d}{dx} x = 1$	$\frac{d}{dx} x^n = n\, x^{(n-1)}$ Proof

Exponential / Logarithmic

$\frac{d}{dx} e^x = e^x$ Proof	$\frac{d}{dx} b^x = b^x \ln(b)$ Proof	$\frac{d}{dx} \ln(x) = 1/x$ Proof

Trigonometric

$\frac{d}{dx} \sin x = \cos x$ Proof	$\frac{d}{dx} \csc x = -\csc x \cot x$ Proof
$\frac{d}{dx} \cos x = - \sin x$ Proof	$\frac{d}{dx} \sec x = \sec x \tan x$ Proof
$\frac{d}{dx} \tan x = \sec^2 x$ Proof	$\frac{d}{dx} \cot x = - \csc^2 x$ Proof

Inverse Trigonometric

$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{(1 - x^2)}}$	$\frac{d}{dx} \text{arccsc}\, x = \frac{-1}{\lvert x\rvert \sqrt{(x^2 - 1)}}$
$\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{(1 - x^2)}}$	$\frac{d}{dx} \text{arcsec}\, x = \frac{1}{\lvert x\rvert \sqrt{(x^2 - 1)}}$
$\frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$	$\frac{d}{dx} \text{arccot}\, x = \frac{-1}{1 + x^2}$

Hyperbolic

$\frac{d}{dx}$ sinh x = cosh x Proof	$\frac{d}{dx}$ csch x = - coth x csch x Proof
$\frac{d}{dx}$ cosh x = sinh x Proof	$\frac{d}{dx}$ sech x = - tanh x sech x Proof
$\frac{d}{dx}$ tanh x = 1 - $\tanh^2$ x Proof	$\frac{d}{dx}$ coth x = 1 - $\coth^2$ x Proof

Those with hyperlinks have proofs.

Dave's Math Tables: *Differentiation Identities*

(Math | Calculus | Derivatives | Identities)

Definitions of the Derivative:

df / dx = lim (dx -> 0) (f(x+dx) - f(x)) / dx *(right sided)*
df / dx = lim (dx -> 0) (f(x) - f(x-dx)) / dx *(left sided)*
df / dx = lim (dx -> 0) (f(x+dx) - f(x-dx)) / (2dx) *(both sided)*

$\frac{d}{dx}\int_a^x$ f(t) dt = f(x) **(Fundamental Theorem for Derivatives)**

$\frac{d}{dx}$c f(x) = c $\frac{d}{dx}$f(x) *(c is a constant)*

$\frac{d}{dx}$ (f(x) + g(x)) = $\frac{d}{dx}$ f(x) + $\frac{d}{dx}$ g(x)

$\frac{d}{dx}$ f(g(x)) = $\frac{d}{dg}$ f(g) * $\frac{d}{dx}$ g(x) **(chain rule)**

$\frac{d}{dx}$ f(x)g(x) = f'(x)g(x) + f(x)g '(x) **(product rule)**

$\frac{d}{dx}$ f(x)/g(x) = (f '(x)g(x) - f(x)g '(x)) / g$^{\wedge 2}$(x) **(quotient rule)**

Partial Differentiation Identities

if f(x(r,s), y(r,s))

df / dr = df / dx * dx / dr + df / dy * dy / dr

df / ds = df / dx * dx / ds + df / dy * dy / ds

if f(x(r,s))

df / dr = df / dx * dx / dr

df / ds = df / dx * dx / ds

directional derivative

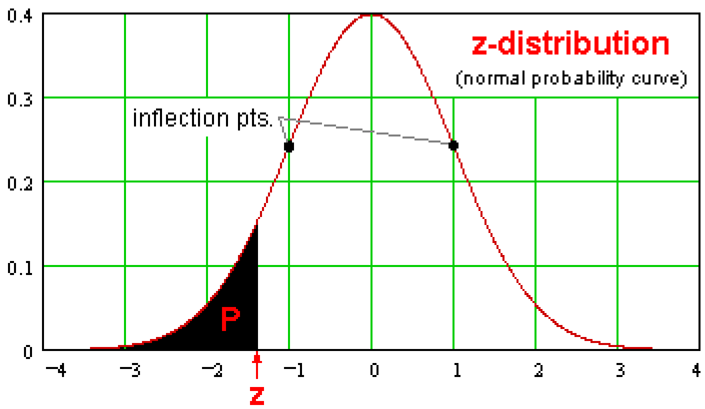
df(x,y) / d(Xi sub a) = f1(x,y) cos(a) + f2(x,y) sin(a)

(Xi sub a) = angle counter-clockwise from pos. x axis.

Dave's Math Tables: *z-distribution*

UP

(Math | Stat | Distributions | z-Distribution)



The z- is a N(0, 1) distribution, given by the equation:

$$f(z) = 1/\sqrt{(2PI)}\ e^{(-z^{\wedge 2}/2)}$$

The area within an interval (a,b) = normalcdf(a,b) = $\int_a^b e^{-z^{\wedge 2}/2}\, dz$ *(It is not integratable algebraically.)*

The Taylor expansion of the above assists in speeding up the calculation:
normalcdf($-\infty$, z) = $1/2 + 1/\sqrt{(2PI)} \sum (k=0..\infty) [((-1)^{\wedge k}\ x^{\wedge(2k+1)}) / ((2k+1)\ 2^{\wedge k}\ k!)]$

Standard Normal Probabilities:
(The table is based on the area **P** under the standard normal probability curve, below the respective **z**-statistic.)

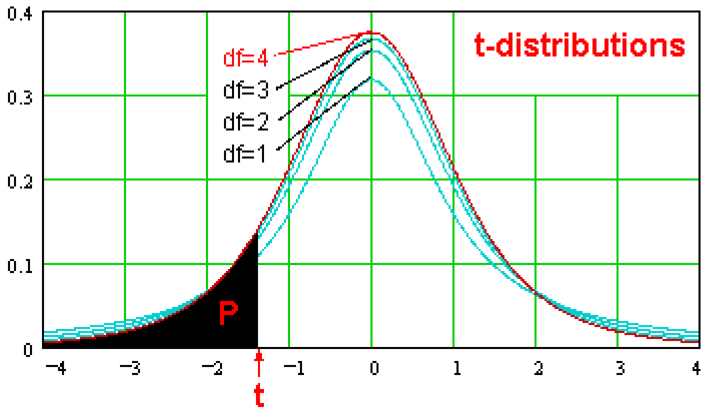
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002
-3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
-3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
-3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
-3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
-3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
-3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
-3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
-3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
-3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
-3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00103	0.00100
-2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
-2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193

-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2.0	0.02275	0.02222	0.02169	0.02118	0.02067	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04456	0.04363	0.04272	0.04181	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06425	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07214	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09852
-1.1	0.13566	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15865	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17878	0.17618	0.17361	0.17105	0.16853	0.16602	0.16354	0.16109
-0.8	0.21185	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21769	0.21476
-0.6	0.27425	0.27093	0.26763	0.26434	0.26108	0.25784	0.25462	0.25143	0.24825	0.24509
-0.5	0.30853	0.30502	0.30153	0.29805	0.29460	0.29116	0.28774	0.28434	0.28095	0.27759
-0.4	0.34457	0.34090	0.33724	0.33359	0.32997	0.32635	0.32276	0.31917	0.31561	0.31206
-0.3	0.38209	0.37828	0.37448	0.37070	0.36692	0.36317	0.35942	0.35569	0.35197	0.34826
-0.2	0.42074	0.41683	0.41293	0.40904	0.40516	0.40129	0.39743	0.39358	0.38974	0.38590
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42857	0.42465
-0.0	0.50000	0.49601	0.49202	0.48803	0.48404	0.48006	0.47607	0.47209	0.46811	0.46414

Java Normal Probability Calculator *(for Microsoft 2.0+/Netscape 2.0+/Java Script browsers only)*
To find the area **P** under the normal probability curve N(mean, standard_deviation) within the interval (left, right), type in the 4 parameters and press "Calculate". *The standard normal curve N(0,1) has a mean=0 and s.d.=1.* Use -inf and +inf for infinite limits.

Dave's Math Tables: *t-distributions* UP

(Math | Stat | Distributions | t-Distributions)

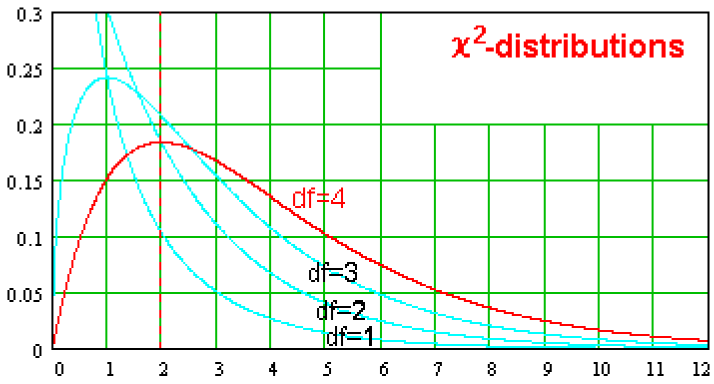


The t-distribution, with n degrees of freedom, is given by the equation:

f(t) = [Γ((n + 1)/2) (1 + x^2 / n)^(-n/2 - 1/2)] / [Γ(n/2) √(PI n)] *(See also Gamma Function.)*

Dave's Math Tables: *chi²-distribution* UP

(**Math** | **Stat** | **Distributions** | **chi²-Distributions**)



The χ^2-distribution, with n degrees of freedom, is given by the equation:

$$f(\chi^2) = (\chi^2)^{\wedge(n/2 - 1)}\ e^{\wedge(-\chi^2 / 2)}\ 2^{\wedge(-n/2)} / \Gamma(n/2)$$

The area within an interval (a, ∞) = $\int_a^\infty f(\chi^2)\, d\chi^2 = \Gamma(n/2, a/2) / \Gamma(n/2)$ *(See also Gamma function)*

Dave's Math Tables: *Fourier Series*

UP

(Math | Advanced | Fourier Series)

- **The fourier series of the function f(x)**

$$a(0) / 2 + \sum(k=1..\infty)\ (a(k) \cos kx + b(k) \sin kx)$$

$$a(k) = 1/PI \int_{-\pi}^{\pi} f(x) \cos kx\ dx$$

$$b(k) = 1/PI \int_{-\pi}^{\pi} f(x) \sin kx\ dx$$

- **Remainder of fourier series**. Sn(x) = sum of first n+1 terms at x.

$$remainder(n) = f(x) - Sn(x) = 1/PI \int_{-\pi}^{\pi} f(x+t)\ Dn(t)\ dt$$

$$Sn(x) = 1/PI \int_{-\pi}^{\pi} f(x+t)\ Dn(t)\ dt$$

$$Dn(x) = \text{Dirichlet kernel} = 1/2 + \cos x + \cos 2x + .. + \cos nx = [\ \sin(n + 1/2)x\ ] / [\ 2\sin(x/2)\ ]$$

- **Riemann's Theorem**. If f(x) is continuous except for a finite # of finite jumps in every finite interval then:

$$\lim_{(k\to\infty)} \int_a^b f(t) \cos kt\ dt = \lim_{(k\to\infty)} \int_a^b f(t) \sin kt\ dt = 0$$

- **The fourier series of the function f(x) in an arbitrary interval.**

$$A(0) / 2 + \sum(k=1..\infty)\ [\ A(k) \cos (k(PI)x / m) + B(k) (\sin k(PI)x / m)\ ]$$

$$a(k) = 1/m \int_{-m}^{m} f(x) \cos (k(PI)x / m)\ dx$$

$$b(k) = 1/m \int_{-m}^{m} f(x) \sin (k(PI)x / m)\ dx$$

- **Parseval's Theorem**. If f(x) is continuous; f(-PI) = f(PI) then

$$1/PI \int_{-\pi}^{\pi} f^{\wedge 2}(x)\ dx = a(0)^{\wedge 2} / 2 + \sum(k=1..\infty)\ (a(k)^{\wedge 2} + b(k)^{\wedge 2})$$

- **Fourier Integral of the function f(x)**

$$f(x) = \int_0^{\infty} (\ a(y) \cos yx + b(y) \sin yx\ )\ dy$$

$$a(y) = 1/PI \int_{-\infty}^{\infty} f(t) \cos ty\ dt$$

Dave's Math Tables: *Fourier Transforms* UP

(**Math** | **Advanced** | **Transforms** | **Fourier**)

Fourier Transform

- Definition of Fourier Transform

$$f(x) = 1/\sqrt{(2\pi)} \int_{-\infty}^{\infty} g(t)\, e^{(i\,tx)}\, dt$$

- Inverse Identity of Fourier Transform

$$g(x) = 1/\sqrt{(2\pi)} \int_{-\infty}^{\infty} f(t)\, e^{(-i\,tx)}\, dt$$

Fourier Sine and Cosine Transforms

- Definitions of the Transforms

$$f(x) = \sqrt{(2/\pi)} \int_{0}^{\infty} g(x) \cos(xt)\, dt \quad \textit{(Cosine Transform)}$$

$$f(x) = \sqrt{(2/\pi)} \int_{0}^{\infty} g(x) \sin(xt)\, dt \quad \textit{(Sine Transform)}$$

- Identities of the Transforms

IF f(x) is even, THEN FourierSineTransform(FourierSineTransform(f(x))) = f(x)
IF f(x) is odd, THEN FourierCosineTransform(FourierCosineTransform(f(x))) = f(x)

Under certain restrictions of continuity.

$$b(y) = 1/PI \int_{-\infty}^{\infty} f(t) \sin ty \, dt$$

$$f(x) = 1/PI \int_{0}^{\infty} dy \int_{-\infty}^{\infty} f(t) \cos (y(x-t)) \, dt$$

Special Cases of Fourier Integral

if f(x) = f(-x) then

$$f(x) = 2/PI \int_{0}^{\infty} \cos xy \, dy \int_{0}^{\infty} f(t) \cos yt \, dt$$

if f(-x) = -f(x) then

$$f(x) = 2/PI \int_{0}^{\infty} \sin xy \, dy \int_{0}^{\infty} \sin yt \, dt$$

Fourier Transforms

Fourier Cosine Transform

$$g(x) = \sqrt{(2/PI)} \int_{0}^{\infty} f(t) \cos xt \, dt$$

Fourier Sine Transform

$$g(x) = \sqrt{(2/PI)} \int_{0}^{\infty} f(t) \sin xt \, dt$$

Identities of the Transforms

If f(-x) = f(x) then

Fourier Cosine Transform (Fourier Cosine Transform (f(x))) = f(x)

If f(-x) = -f(x) then

Fourier Sine Transform (Fourier Sine Transform (f(x))) = f(x)