BAND-STOP FILTERS

A band-stop filter (also known as band-reject, band-elimination, or notch filter) suppresses a band of frequencies of a signal, leaving intact the low- and high-frequency bands. A band-stop filter specification can be expressed as shown in Fig. 1. In the stopband from f_3 Hz to f_4 Hz, the attenuation is

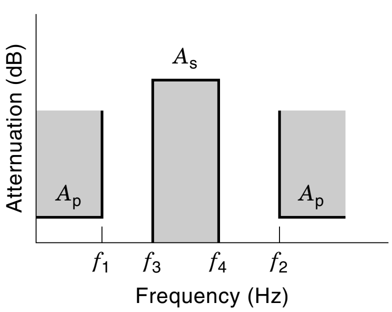

Figure 1. Band-stop filter specification.

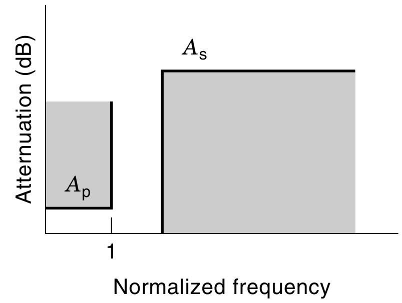

Figure 2. Normalized low-pass requirements.

at least A_s dB. In the passbands (below f_1 Hz and above f_2 Hz), the maximum attenuation is A_p dB. The bands from f_1 to f_3 and from f_4 to f_2 are called the *transition bands*. The filter requirement is said to be geometrically symmetrical if $f_1 f_2 = f_3 f_4$.

An approach to designing a circuit (a band-stop filter) with a frequency response that satisfies the band-stop requirements shown in Fig. 1 is described below. It consists of two steps: the approximation of the requirements by a transfer function, and the synthesis of the transfer function.

In the approximation part of the design process, it is desirable to find a transfer function with a frequency response that satisfies the band-stop requirements. To find that transfer function, first convert the band-stop requirements into the normalized low-pass requirements. For the case that the band-stop requirements are symmetrical, the corresponding normalized low-pass requirements are shown in Fig. 2. The normalized passband frequency $F_p = 1$ and the passband attenuation is A_p dB. The normalized stopband frequency is

$$F_s = \frac{f_2 - f_1}{f_4 - f_3}$$

and the stopband attenuation is A_s dB.

With such low-pass requirements, we can obtain the corresponding low-pass transfer function $T_{LP}(s)$. (See LOW-PASS FILTERS for more information about how to obtain the transfer function.) The band-stop filter transfer function $T_{BS}(s)$ is obtained by making the transformation

$$T_{BS}(s) = T_{LP}(s)\Big|_{s=\frac{Bs}{s^2+(2\pi f_0)^2}}$$

where B is the bandwidth of the band-stop filter defined as

$$B = 2\pi(f_2 - f_1)$$

and f_0 is the center frequency of the band-stop requirement defined as

$$f_0 = \sqrt{f_1 f_2} = \sqrt{f_3 f_4}$$

To use this method when the requirement is not symmetrical, for the case that $f_1 f_2 > f_3 f_4$, we form a more stringent requirement by either decreasing f_2 or increasing f_4, so that the symmetrical condition is met. The band-stop transfer function that satisfies the new requirements must also satisfy

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the original requirements. In case $f_1 f_2 < f_3 f_4$, we either increase f_1 or decrease f_3 and then apply the same procedure.

A simple example is provided to illustrate this concept. For the band-stop requirements $A_s = 25$ dB, $A_p = 3.01$ dB, $f_1 = 1$ kHz, $f_2 = 100$ kHz, $f_3 = 8$ kHz, and $f_4 = 12.5$ kHz; the corresponding normalized low-pass requirements are: $A_s = 25$ dB, $A_p = 3.01$ dB, $F_p = 1$, and $F_s = 22$. Choosing a single-pole Butterworth approximation, the low-pass transfer function for the normalized low-pass requirements is

$$T_{\mathrm{LP}}(s) = \frac{1}{s+1}$$

which meets the stopband requirements easily. The band-stop transfer function is obtained by the transformation

$$T_{\mathrm{BS}}(s) = \left.\frac{1}{S+1}\right|_{S=\frac{2\pi(100\times10^3 - 1\times10^3)s}{s^2+(2\pi 100\times10^3)(2\pi 1\times10^3)}}$$

which simplifies to

$$T_{\mathrm{BS}}(s) = \frac{s^2 + 3.948\times10^9}{s^2 + 6.220\times10^5 s + 3.948\times10^9}$$

Note that the single-pole low-pass function has been transformed to a two-pole band-stop function. The above band-stop transfer function is in the so called biquadratic form, which is an expression of the form

$$\frac{s^2 + a_1 + a_2}{s^2 + b_1 s + b_2} \tag{1}$$

There are a number of ways to synthesize and biquadratic function as an active network, such as the Friend biquad circuit (1,5), the Boctor circuit (2), and the summing four-amplifier biquad circuit (3,5). The Friend or Boctor circuit uses one operational amplifier. The summing four-amplifier biquad circuit is much easier to tune. When $a_1 = 0$, the Bainter circuit can be used. The Bainter circuit (4) is shown in Fig. 3. For higher performance circuits, see Ref. 5 for the description of different band-stop circuit topologies.

The transfer function is

$$\frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)} = \frac{s^2 + \dfrac{R_{12}}{R_{11}R_{21}R_{31}C_1C_2}}{s^2 + \left(\dfrac{1}{R_{31}C_1} + \dfrac{1}{R_{32}C_1}\right)s + \dfrac{1}{R_{22}R_{31}C_1C_2}}$$

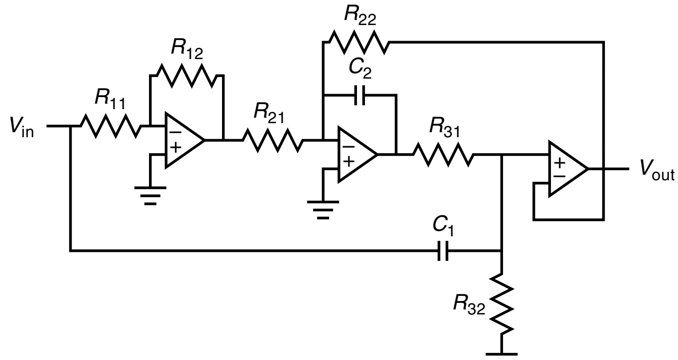

Figure 3. Bainter circuit.

Compare with Eq. (1),

$$a_1 = 0, \qquad a_2 = \frac{R_{12}}{R_{11}R_{21}R_{31}C_1C_2},$$

$$b_1 = \frac{1}{R_{31}C_1} + \frac{1}{R_{32}C_1}, \qquad b_2 = \frac{1}{R_{22}R_{31}C_1C_2}$$

Choose $C_1 = C_2 = 1$, $R_{12}/R_{11} = K$, $R_{31} = R_{32}$. Solving for the other values,

$$R_{21} = \frac{Kb_1}{2a_2}, \qquad R_{22} = \frac{b_1}{2b_2}, \qquad R_{31} = \frac{2}{b_1}$$

The impedance scaling method can be used to scale the values of R and C into the practical ranges. In general, a higher-order band-stop transfer function can be factorized into a product of biquadratic functions. Each of the biquadratic function can be synthesized by using the Bainter or other circuits. By cascading all the circuits together, the band-stop filter is realized.

BIBLIOGRAPHY

1. J. J. Friend et al., STAR, An active filter biquad section, *IEEE Trans. Circuits Syst.,* **CAS-22**: 115–121, 1975.
2. S. A. Boctor, Single amplifier functionally tunable low-pass notch filter, *IEEE Trans. Circuits Syst.,* **CAS-22**: 875–881, 1975.
3. G. Daryanani, *Principles of Active Network Synthesis and Design,* New York: Wiley, 1976.
4. J. R. Bainter, Active filter has stable notch and response can be regulated, *Electronics,* 115–117, Oct. 1975.
5. G. Moschytz and P. Horn, *Active Filter Design Handbook,* New York: Wiley, 1981.

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BANDWIDTH EFFICIENCY. See MODULATION ANALYSIS FORMULA.

BANG-BANG CONTROL. See NONLINEAR CONTROL SYSTEMS, ANALYTICAL METHODS.

BANK BRANCH AUTOMATION. See BRANCH AUTOMATION.

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