BRIDGE CIRCUITS

Bridges are the most commonly used circuits in measurement techniques. They enable accurate static measurements of resistance, capacitance, or inductance. Measurement accuracy is provided by the null-balance method of output indication, and by the fact that the bridge circuit configuration allows comparison of unknown components with precise standard units. This resulted in development of bridge instruments as complete units of laboratory equipment. The balance in bridges is highly sensitive with respect to variations of the bridge components, and this brought about the widespread use of bridge configurations in transducer and sensor applications. In addition, the bridge circuits may be found as "working" circuitry in electric filters where they provide the flexibility inachievable for other filter configurations, in radio receivers and transmitters where the bridge approach is used to design stable sinusoidal oscillators, and elsewhere in electronic hardware, where they are met in a wide variety of circuits used for determination of impedance, reactance, frequency, and oscillation period. The number of circuits based on the bridge configuration is increasing, and this article describes the elements of the general theory of bridge circuits, and outlines some of their above-mentioned basic applications with more stress on measurement and transducer ones.

The circuit [Fig. 1(a)] including four arms with impedances, Z_1, Z_2, Z_3, Z_4, an element (in applications called "balance detector" or "balance indicator") with impedance Z_o, and a voltage source of value E_g and output impedance Z_g is an example of the so-called bridge circuit. Figure 1(b) shows the equivalent "lattice" form of this circuit. This is the simplest circuit, for which the currents in the impedances cannot be found using the circuit reduction based on parallel or series connection of two or more impedances. To find these currents, one has to write, for example, a system of three loop equations. As a result, this circuit, which is not very complicated, is frequently used for demonstration of general (1) (mesh, loop, and nodal analysis) and special methods (2) (wye-delta transformation) of circuit analysis. The calculation of the current I_o in the impedance Z_o is a favorite example for demonstration of Thévenin and Norton theorems (1,3).

Most technical applications of this bridge circuit are based on a simple relationship that exists among the circuit arm

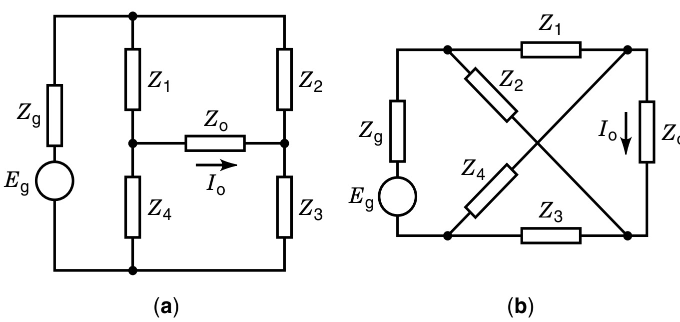

Figure 1. (a) Bridge circuit and (b) its lattice form.

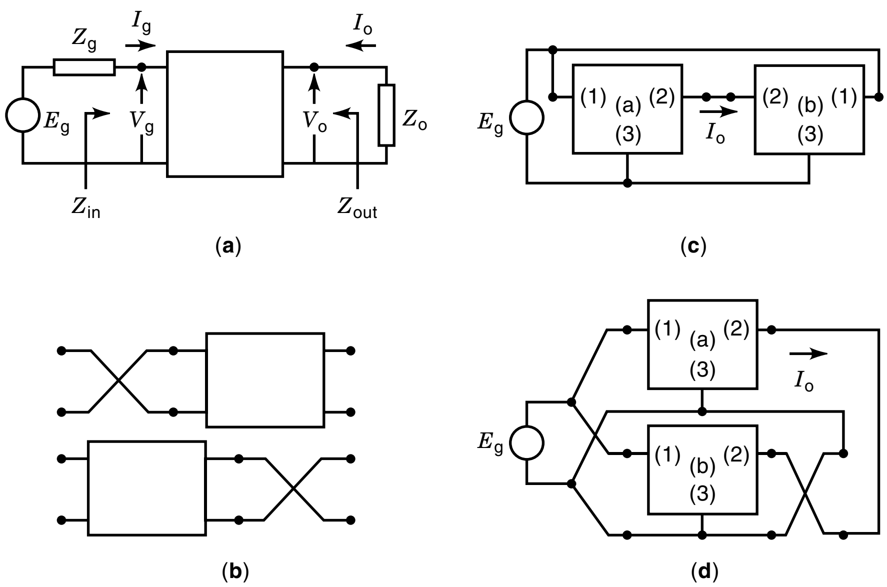

Figure 2. (a) Bridge circuit as a transmission system; (b) two-ports with crossed input or output wires; (c) two subcircuits in a bridge: (d) their parallel-series connection is clearly seen.

impedances so that the current (or voltage) of the detector impedance has a zero value. One can easily see that in the circuits of Fig. 1 the condition $I_o = 0$ (or the "balance" condition) is achieved when

$$Z_1Z_3 = Z_2Z_4 \tag{1}$$

In measurements, this relationship allows one to calculate one of the impedances if three others are known. In transducers, it is used in inverse sense—that is, if $I_o \neq 0$ then Eq. (1) is violated as well. The deflection δI_o of the current I_o from zero (the value and sign) is used to evaluate the deviation of Z_1, Z_2, Z_3, Z_4 or their combinations from their nominal values satisfying Eq. (1). If these impedances are dependent on some physical variables (which are called measurands), then δI_o provides information on these physical variables.

The simplicity of Eq. (1) and its independence of Z_o and Z_g (which in many applications are not well specified) make the bridge measurements of physical variables reliable and sensitive. This feature brought about the widespread use of bridge circuits in instrumentation and, recently, in microsensors.

By analogy, all circuits (of usually simple configurations) where a certain relationship between the elements results in a zero current in a given element, or zero voltage between a given pair of nodes, are called bridge circuits here.

BRIDGE CIRCUIT BALANCE CONDITIONS

Let us consider the bridge circuit as a passive two-port connected between the impedances Z_o and Z_g [Fig. 2(a)] and assume the balance conditions. Investigation of the systems of parameters applied for two-port description (4) allows one to formulate some specific relationships pertaining to bridge circuits.

The four terminal quantities for this two-port are related by the equations

$$\begin{aligned} V_g &= a_{11}V_0 - a_{12}I_0 \\ I_g &= a_{21}V_0 - a_{22}I_0 \end{aligned} \tag{2}$$

The terms a_{11}, a_{12}, a_{21}, and a_{22}, (called "chain parameters") are the terms of $\boldsymbol{a}$-parameter matrix. Equations (2) show that the two-port does not transmit voltages and currents from left to right if one of the following conditions is satisfied:

$$a_{11} = \infty \quad a_{12} = \infty \quad a_{21} = \infty \quad a_{22} = \infty \tag{3}$$

The investigation of the terms $\boldsymbol{g}$-, $\boldsymbol{y}$-, $\boldsymbol{z}$-, and $\boldsymbol{h}$-matrices (4) allows one to formulate other specific relationships pertaining to the balanced bridge circuits. In these circuits, correspondingly, the parameters of other two-port matrices have the values

$$g_{21} = 0 \quad y_{21} = 0 \quad z_{21} = 0 \quad h_{21} = 0 \tag{4}$$

From the other side, the bridge is a reciprocal circuit, and if Eq. (2) is satisfied then the conditions

$$g_{12} = 0 \quad y_{12} = 0 \quad h_{12} = 0 \quad z_{12} = 0 \tag{5}$$

are also, correspondingly, satisfied, and the circuit will not transmit voltages and currents as well from right to left.

In addition, for reciprocal circuits the following relationship exists among the chain parameters:

$$|a| = a_{11}a_{22} - a_{12}a_{21} = 1 \tag{6}$$

Now let one consider the input and output impedances

$$Z_{in} = \frac{a_{11}Z_0 + a_{12}}{a_{21}Z_0 + a_{22}} \quad Z_{out} = \frac{a_{22}Z_g + a_{12}}{a_{21}Z_g + a_{11}} \tag{7}$$

One easily finds that if Eqs. (6) is valid and one of the conditions of Eq. (3) is satisfied, these impedances are given by one of the following expressions:

$$Z_{in} = \frac{a_{11}}{a_{21}} \quad Z_{in} = \frac{a_{12}}{a_{22}} \quad Z_{out} = \frac{a_{22}}{a_{21}} \quad Z_{out} = \frac{a_{12}}{a_{11}} \tag{8}$$

Hence, the condition of balance indeed is independent of Z_o and Z_g (they are "not seen" from the input and output terminals) if the two-port is a linear one.

The set of conditions of Eqs. (3) and (4) can be used, to some extent, for synthesis of bridge circuits (5). A bridge circuit can frequently be represented as a regular connection (4) of k two-ports. Then the conditions of Eqs. (3) and (4) are modified into

$$\sum_{i=1}^{i=k} g_{21}^{(i)} = 0 \quad \text{or} \quad \sum_{i=1}^{i=k} \frac{1}{a_{11}^{(k)}} = 0 \tag{9}$$

for parallel-series connection of these two-ports. They are modified into

$$\sum_{i=1}^{i=k} y_{21}^{(i)} = 0 \quad \text{or} \quad \sum_{i=1}^{i=k} \frac{1}{a_{12}^{(k)}} = 0 \tag{10}$$

for parallel connection of two-ports. Then they will give

$$\sum_{i=1}^{i=k} z_{21}^{(i)} = 0 \quad \text{or} \quad \sum_{i=1}^{i=k} \frac{1}{a_{21}^{(k)}} = 0 \tag{11}$$

for series connection of two-ports. Finally, the conditions of Eqs. (3) and (4) will be modified into

$$\sum_{i=1}^{i=k} h_{21}^{(i)} = 0 \quad \text{or} \quad \sum_{i=1}^{i=k} \frac{1}{a_{22}^{(k)}} = 0 \tag{12}$$

for series-parallel connection of two-ports.

Some bridge circuits include a two-port with input or output crossed wires [Fig. 2(b)]. Such a two-port is described by an $\boldsymbol{a}$ matrix with terms that are negatives of the initial two-port matrix terms. If the initial two-port is described by a $\boldsymbol{z}$, $\boldsymbol{y}$, $\boldsymbol{h}$, or $\boldsymbol{g}$ matrix, then only the diagonal terms of these matrices should change their sign.

Many bridge circuits can be represented as a connection of two two-ports shown in Fig. 2(c), where the bridge output branch is a wire carrying the current I_o. This connection can be redrawn as shown in Fig. 2(d). Then the condition of balance ($I_o = 0$) for this circuit can be written as $g_{21}^{(a)} = g_{21}^{(b)}$ or as $a_{11}^{(a)} = a_{11}^{(b)}$.

The following three bridges serve as examples. The circuit of the twin-T bridge [Fig. 3(a)] is a parallel connection of two T circuits. The parameter $y_{21}^{(i)}$ ($i = 1, 2$) for each of these circuits can be easily calculated, and their sum $y_{21}^{(1)} + y_{21}^{(2)}$, in accordance with Eq. (10) gives the balance condition

$$Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2} + Z_4 + Z_6 + \frac{Z_4 Z_6}{Z_5} = 0 \tag{13}$$

The ordinary bridge can be represented as a series-parallel connection of two simple two-ports [Fig. 3(b)]. Calculating the $a_{22}^{(i)}$ ($i = 1, 2$) parameters and using Eq. (11), one obtains

$$\frac{Z_1}{Z_1 + Z_2} = \frac{Z_4}{Z_4 + Z_3} \tag{14}$$

from which Eq. (1) follows immediately.

The double bridge [Fig. 3(c)] is easily recognized as the connection of two two-ports shown in Fig. 2(c). Equating the parameters $a_{11}^{(1)}$ (left part) and $a_{11}^{(2)}$ (right part), one can find the balance condition

$$\frac{Z_6(Z_5 + Z_7) + (Z_1 + Z_4)(Z_5 + Z_6 + Z_7)}{Z_6 Z_7 + Z_4(Z_5 + Z_6 + Z_7)} = \frac{Z_2 + Z_3}{Z_3} \tag{15}$$

for this bridge.

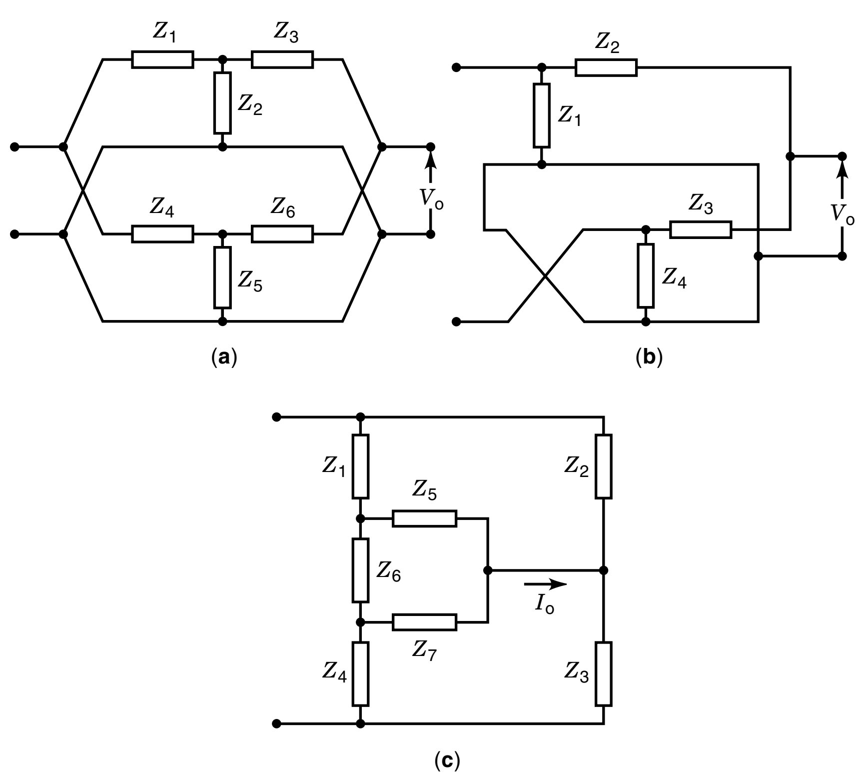

Figure 3. Examples of bridge circuits: (a) twin-T bridge; (b) simple bridge redrawn as a series-parallel connection of two two-ports; (c) double bridge.

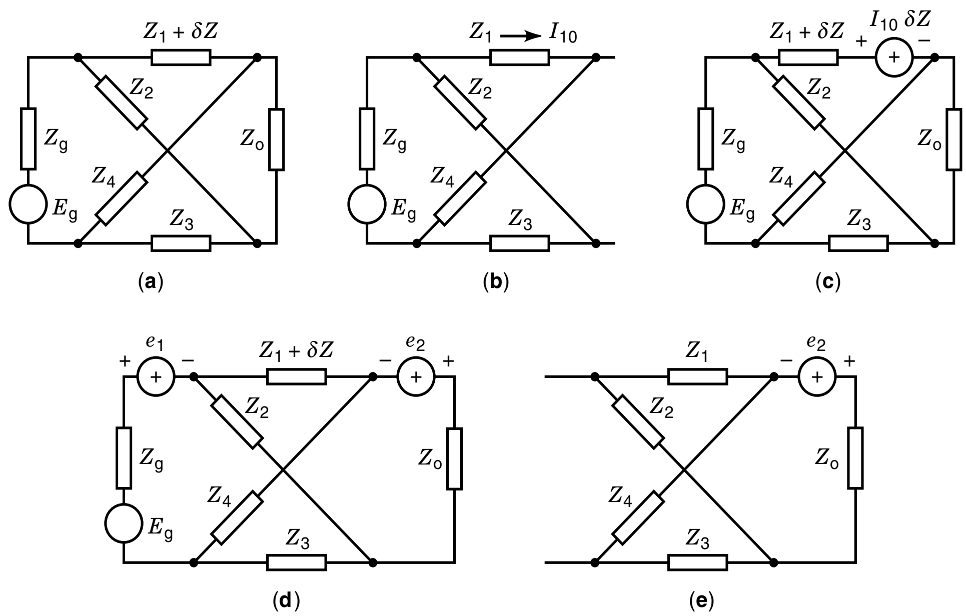

Figure 4. Calculation of sensitivity in a simple bridge circuit: (a) initial circuit; (b) circuit in balance; (c) introduction of compensating source: (d) extraction of external sources in autonomous two-port; (e) circuit for calculation of current variation.

SENSITIVITY

An important parameter of the bridge circuit is sensitivity. It is usually calculated for the balanced bridge condition. One defines the sensitivity of the bridge output (or balanced) current as

$$S_{\rm i} = \frac{dI_0}{dZ_{\rm k}} \approx \frac{\delta I_0}{\delta Z_{\rm k}} \tag{16}$$

where the derivative is determined at the point $I_{\rm o} = 0$, and the sensitivity of the bridge balanced voltage as

$$S_{\rm v} = \frac{dV_0}{dZ_{\rm k}} \approx \frac{\delta V_0}{\delta Z_{\rm k}} \tag{17}$$

with the derivative determined at $V_{\rm o} = 0$. Here $Z_{\rm k}$ is the element of the bridge circuit that varies (it is frequently called a "tuning element"). The right sides of Eqs. (16) and (17) show that variations are used for practical calculations of the sensitivities. In addition, $\delta V_{\rm o} = Z_{\rm o}\delta i_{\rm o}$ so that

$$S_{\rm v} = Z_0 S_{\rm i} \tag{18}$$

and calculation of only one sensitivity suffices.

The calculation of sensitivity requires a sequence of steps that can be demonstrated (Fig. 4) using the bridge of Fig. 1(b). Assume that it is required to find the sensitivity

$$S_{\rm i} = \frac{dI_0}{dZ_1} \tag{19}$$

of this bridge with respect to variation of the element Z_1 [Fig. 4(a)]. First, let us calculate the current I_{10} through this element in the condition of balance, when $I_{\rm o} = 0$. In this calculation the element $Z_{\rm o}$ can be disconnected [Fig. 4(b)] and one can find that

$$I_{10} = \frac{E_{\rm g}Z_2}{Z_{\rm g}(Z_1+Z_2)+Z_1(Z_2+Z_3)} \tag{20}$$

Let the element Z_1 vary, and let its variation be δZ (Fig. 4c). In accordance with the compensation theorem (2,5), the current $\delta I_{\rm o}$ occurring in the element $Z_{\rm o}$ can be calculated if one introduces in the branch with $Z_1 + \delta Z$ a compensating voltage source $I_{10}\delta Z$, as shown in Fig. 4(c), and consider a circuit (Fig. 4c) that is an active autonomous two-port (5) connected between the impedances $Z_{\rm g}$ and $Z_{\rm o}$. Then, this active autonomous two-port can be represented by a passive two-port (of the same structure, in this case) and two equivalent sources, which appear at the two-port terminals. This step is simply a generalization of the Thévenin-Norton theorem for two-ports. If, for example, one decides to use e_1 and e_2 connected in series with the two-port terminals [Fig. 4(d)], one can find that

$$e_2 \approx \frac{I_{10}Z_3\delta Z}{Z_2+Z_3} \tag{21}$$

(in this calculation it is assumed $Z_1 + \delta Z \approx Z_1$, and $Z_1Z_3 = Z_2Z_4$). As for e_1, there is no need of calculating it, because the bridge two-port in the circuit of Fig. 4(d) is nearly at the balance condition, and the contribution of e_1 to the current in $Z_{\rm o}$ can be neglected. Simultaneously, for the same reason, the source e_2 does not produce any current in the impedance $Z_{\rm g}$. Hence, one can calculate the current in $Z_{\rm o}$ using the "approximate" circuit of Fig. 4(e). One obtains

$$\delta I_0 \approx \frac{E_{\rm g}Z_2Z_3\delta Z}{[Z_{\rm g}(Z_1+Z_2)+Z_1(Z_2+Z_3)][Z_0(Z_2+Z_3)+Z_2(Z_3+Z_4)]} \tag{22}$$

From Eq. (22) it immediately follows that

$$S_{\rm i} \approx \frac{E_{\rm g}Z_2Z_3}{[Z_{\rm g}(Z_1+Z_2)+Z_1(Z_2+Z_3)][Z_0(Z_2+Z_3)+Z_2(Z_3+Z_4)]} \tag{23}$$

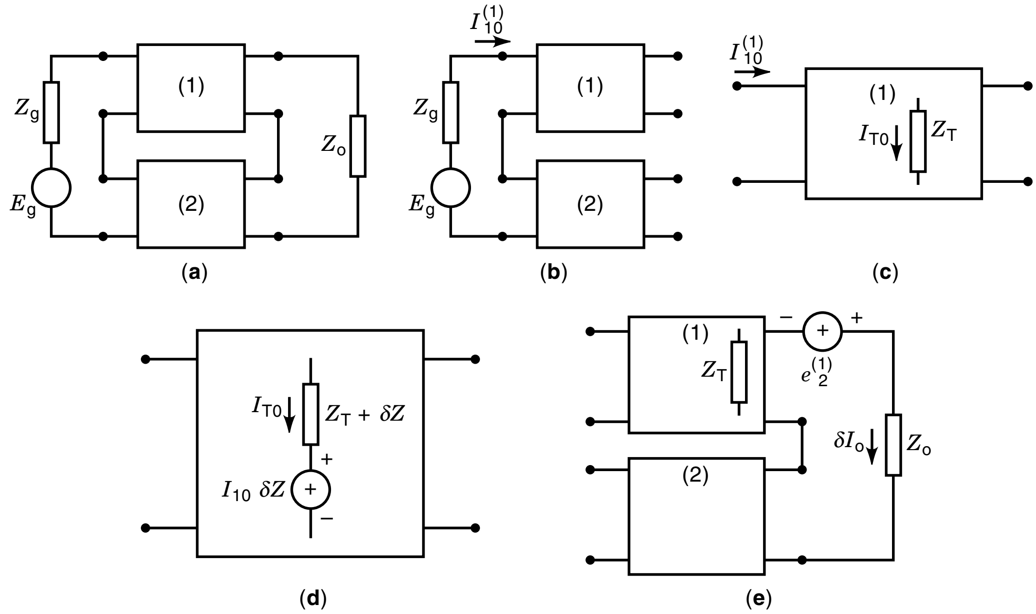

Figure 5. Calculation of sensitivity in series connection of two two-ports: (a) initial circuit; (b) circuit in balance; (c) current in tuning element; (d) introduction of compensating source; (e) extraction of external sources in autonomous two-port and the circuit for calculation of current variation.

This calculation of sensitivity may be formalized if the bridge circuit represents a regular connection of two two-ports (bridge circuits with more than two subcircuits are very rare). We assume that the tuning branch is located in the first two-port.

Let us consider as an example the series connection of two two-ports [Fig. 5(a)]. As a first step we assume the condition of balance and then disconnect the indicator branch [Fig. 5(b)] and calculate the input current $I_{10}^{(1)}$. One can see that this current is equal to

$$I_{10}^{(1)} = \frac{E_g}{Z_g + Z_{11}^{(1)} + z_{11}^{(2)}} \tag{24}$$

The current I_{T0} in the tuning branch, Z_T, may be determined by considering the first two-port only [Fig. 5(c)] with the output open and the current $I_{10}^{(1)}$ applied to its input. For a linear two-port one can write that

$$I_{T0} = K_1 I_{10}^{(1)} \tag{25}$$

where K_1 is a transfer coefficient. Using the compensation theorem, one introduces in the tuning branch the compensating voltage

$$e_c = I_{T0}\delta Z \tag{26}$$

(other forms of the compensation theorem may also be used). For exact calculation of δI_o one has to preserve the variation δZ of the tuning impedance, as is shown in Fig. 5(d). Yet, to simplify the results (and assuming that δZ is small), this variation is usually omitted. The first two-port now becomes an autonomous active two-port. It may be represented as a passive two-port having the sources e_1 and e_2 connected in series with corresponding inputs. The source e_2 can be found as

$$e_2 = K_2 e_c \tag{27}$$

where K_2 is another transfer coefficient. Consider that the bridge circuit is nearly balanced, the current δI of the detector may be found from the "approximate" circuit shown in Fig. 5(e). The result will be

$$\delta I_0 \approx \frac{e_2}{Z_0 + z_{22}^{(1)} + z_{22}^{(2)}} \tag{28}$$

Finally, one can find that

$$S_i \approx \frac{K_1 K_2 E_g}{(Z_g + z_{11}^{(1)} + z_{11}^{(2)})(Z_0 + z_{22}^{(1)} + z_{22}^{(2)})} \tag{29}$$

The extension of this approach for other regular connections of two two-ports does not present any difficulty (5).

APPLICATION OF BRIDGE CIRCUITS FOR MEASUREMENT OF COMPONENT PARAMETERS

Bridges are commonly used circuits in measurements. They have high sensitivity and allow accurate measurements of resistance, capacitance, and inductance. Measurement accuracy is due to the null-balance method of output indication and to the fact that the bridge circuit configuration conveniently allows direct comparison of unknown components with precise standard units. Here we outline the basic ideas of such measurements. They are useful in laboratory environments and form the basis of commercial equipment designs (6).

Dc Bridges

The dc bridges are used for precise measurements of dc resistance. The conventional Wheatstone bridge is shown in Fig. 6(a). It consists of four arms R_1 to R_4, a zero-center galvanometer G, which serves as a balance detector, a protection shunt R_p, a battery V_B (1.5 to 9 V), and a set of switches S_1 to S_3. We assume that the resistor R_1 is a resistor whose value is unknown, the resistor R_4 is a standard resistor (usually a variable decade box providing, for example, 1 Ω steps from 1 to 11,100 Ω), and the resistors R_2 and R_3 are ratio-arm resistors that serve to provide multiplication of the standard resistance by convenient values, such as 100, 10, 1, 1/10, and 1/100.

The goal of the operating procedures is to achieve balance, which is indicated by a zero reading of the galvanometer with the switches S_1 and S_2 closed and the switch S_3 open. In the beginning of the procedure, S_3 is closed and S_1 and S_2 are open. To avoid transient galvanometer overloading, S_1 is closed first. Then S_2 is closed, and an approximate balance is achieved. Only then is switch S_3 opened, and the final balance is achieved. When balanced, the condition $R_1R_3 = R_2R_4$ is satisfied, and the unknown resistor value can be found as

$$R_1 = \frac{R_2}{R_3}R_4 \tag{30}$$

When the measurement procedure is finished the switches are returned to their initial state in reverse order (i.e., S_3 is closed first, then S_2 is opened, and, finally, S_1 is opened).

The main sources of measurement errors are the variance of ratio-arm resistors (the design characteristic), additional resistance introduced by poor contacts, resistance in the remote wiring of the unknown (the tactics used against these sources of errors are discussed in Ref. 7), changes in resistance of arms due to self-heating, spurious voltages introduced from the contact of dissimilar metals, and incorrect balance. The well-made bridge can be expected to measure from about 0.1 Ω to the low megohm range with approximately 1% accuracy, and for the range 10 Ω to 1 MΩ accuracies of 0.05% can be expected. A good practice is to make measurements on a series of extremely accurate and known resistors and to use the obtained errors as an error guide for measurements with the closest bridge constants. For measuring very high resistances, the galvanometer should be replaced by a high-impedance device. For measuring very low resistances, one has to use the double bridge described later.

In measurements of very low resistances, the resistance of the connector (yoke) between the unknown resistance R_1 and the standard resistance R_4 may seriously affect accuracy. The double (Kelvin) bridge [Fig. 6(b)] is devised to circumvent this difficulty. In this circuit the resistor R_6 represents the resistance of this connector, and R_5 and R_7 are small resistances of two additional arms. If one chooses

$$\frac{R_5}{R_7} = \frac{R_2}{R_3} \tag{31}$$

(this is easily done in practical designs; see Ref. 6), then, using Eq. (15), one can find that the relationship among R_1 to R_4 given by Eq. (30) will be preserved.

The Kelvin bridge allows one to measure the resistances in the range 1 Ω to 10 Ω with accuracy better than 0.1%, and in the range 0.1 Ω to 1 Ω better than 1%.

Ac Bridges

The Wheatstone bridge circuit may also be used for impedance measurements at audio/radio frequencies. The battery is replaced by a generator of sinusoidal voltage, and a sensitive headset or oscilloscope is used as the balance detector [Fig. 7(a)]. The arms now may include reactive components, and when the condition of balance, as in Eq. (1), is satisfied, one can find the component Z_1 from the equality

$$Z_1 = \frac{Z_2}{Z_3}Z_4 \tag{32}$$

Introducing $Z_i = R_i + X_i$ (i = 1, 2, 3, 4) in Eq. (32), one obtains

$$R_1 + jX_1 = \frac{R_2 + jX_2}{R_3 + jX_3}(R_4 + jX_4) = A + jB \tag{33}$$

Hence, to measure two quantities R_1 and X_1, one can use six parameters to achieve the balance. This results in an enormous number of different bridges (8) adapted for particular circumstances.

The selection of configurations to be used in a wider range of applications (6) is dictated by two factors. First, in general, attainment of balance is a progressive operation requiring back and forth in-turn improvements in resistive and reactive balances. For rapid balancing it is desirable that the adjustment of resistive part A be independent of the adjustment made in the reactance part jB. This cannot always be done.

In the region of balance the detector voltage is

$$\delta V_0 = K(Z_1Z_3 - Z_2Z_4) \tag{34}$$

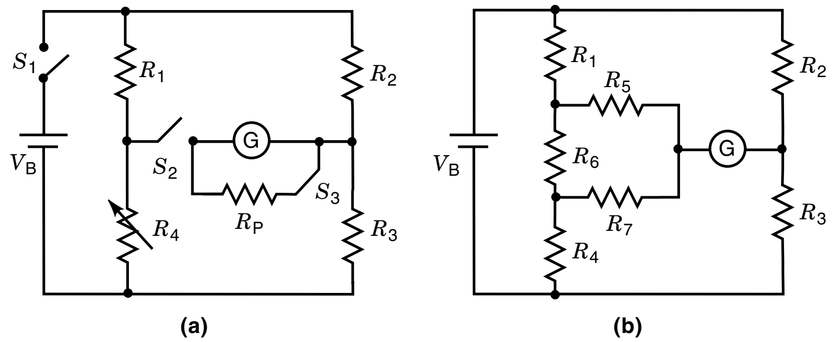

Figure 6. (a) Wheatstone and (b) Kelvin bridges.

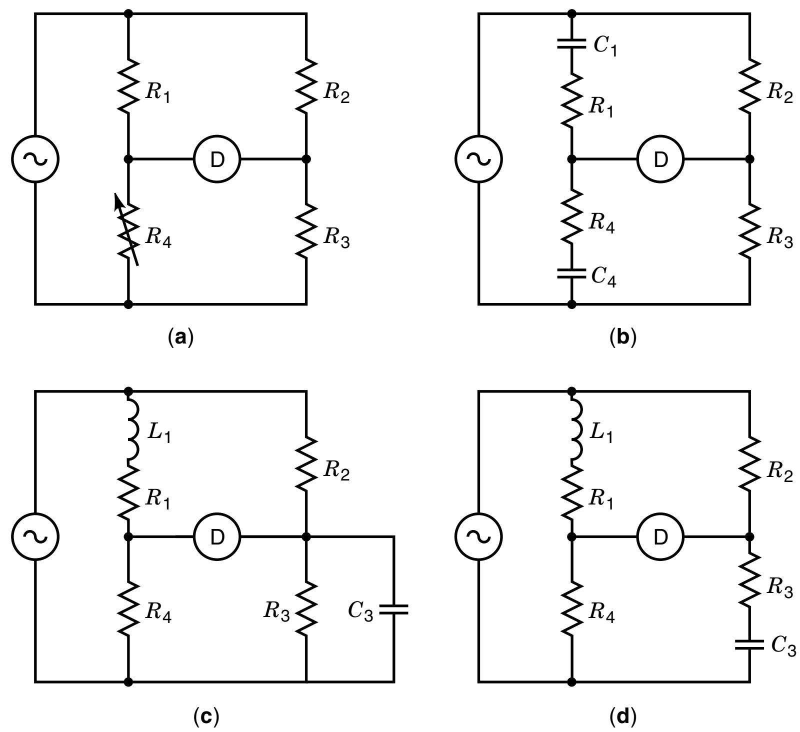

Figure 7. Ac bridges: (a) Wheatstone bridge; (b) ratio-arm capacitive bridge; (c) Maxwell inductance bridge; (d) Hay inductance bridge.

where K may be assumed constant (9). In general, the most rapid convergence to balance is obtained when the phase angle between the selected pair of adjustable components in Eq. (34) is $\pi/2$ and least rapid when the angle tends to zero. For example, for the bridge of Fig. 7(b) the balance equations are

$$R_1 = \frac{R_2}{R_3} R_4 \quad C_1 = \frac{R_3}{R_2} C_4 \tag{35}$$

If R_4 and C_4 can be adjusted, rapid balancing is obtained. If R_4 and R_2 are chosen for adjustment, the convergence can be very slow (9).

The second important factor is that a standard capacitance more nearly approaches the ideal no-loss reactance than does the best wire-wound coil type of inductance. Hence, it is desirable to measure an inductance in terms of capacitance. This can be obtained in the Maxwell bridge [Fig. 7(c)]. The balance equations for the Maxwell bridge are

$$L_1 = R_2 R_4 C_3 \quad R_1 = \frac{R_2}{R_3} R_4 \tag{36}$$

The Maxwell bridge is mainly applied for measuring coils of low Q-factors. Indeed, $Q_1 = \omega L_1/R_1 = \omega C_3 R_3$, and a coil with $Q_1 > 10$ may require very high values of R_3. This limitation is removed in the Hay bridge [Fig. 7(d)]. The balance equations for the Hay bridge are

$$R_1 = \frac{R_2}{R_3} R_4 \frac{1}{Q_1^2 + 1} \quad L_1 = R_2 R_4 C_3 \frac{1}{Q_1^2 + 1} \tag{37}$$

where $Q_1 = \omega L_1/R_1 = 1/(\omega C_3 R_3)$.

One can see that a disadvantage of the last two circuits is the interaction between reactive and resistive balance, yet the desire to have a constant standard capacitor prevails, and four basic configurations shown in Fig. 7 are used in general-purpose universal impedance bridges (6).

It is impossible here to give even a brief survey of specialized bridges; yet four configurations deserve to be mentioned. Figure 8(a) shows the bridge with the voltage source and detector interchanged. This allows one to apply a polarizing voltage and measure the parameters of electrolytic capacitors. The battery that supplies this voltage must be shunted by a bypass capacitor, C_B. Figure 8(b) shows a configuration (the Owen bridge) adapted for incremental inductance measurements. A filter reactor L_{RF} inserted in the bridge measurement circuit minimizes the effect of core-induced harmonics in determining the balance point (R_2 and C_3 are used for balance).

Figure 8(c) shows the Shering bridge, which is also used for measuring the capacitance and dissipation factor of the capacitors—especially at high voltages. The lower part of this bridge (resistors R_4 and R_3 and capacitor C_3) may be maintained at a relatively low potential, and the adjustment to the variable elements can therefore be made safely. The balance equations are

$$C_1 = C_2 \frac{R_3}{R_4} \quad R_1 = R_4 \frac{C_3}{C_2} \tag{38}$$

Other useful configurations of ac bridges with a wide range of application (bridges for measuring mutual inductances) can be found in Refs. 6 and 9. Some improvements of the measuring techniques (the Wagner ground) are described well in Ref. 9.

As a consequence in the development of transformers with very tight magnetic coupling, the ratio arms of some bridges may be replaced by transformer coils. A transformer can also be used as a current comparator. An example of a circuit using these two properties of transformers is shown in Fig. 8(d) (9). Here the generator is connected to the primary winding of voltage transformer T_1, and the secondary windings of T_1 are tapped to provide adjustable sections of N_1 and N_2 turns, respectively. The primary windings of the current transformer T_2 are also tapped to provide sections with adjustable turns n_1 and n_2. The secondary of T_2 is connected to a detector. Let $Y_1 = G_1 + jB_1$ be the unknown admittance, and $Y_2 = G_2 + jB_2$ be a suitable comparison standard. Balance may be achieved by any suitable combination of adjustments of Y_2 and tap positions. The balanced condition corresponds to zero net flux in the primary of T_2. Hence, the condition of balance is

$$n_1 I_1 = n_2 I_2 \tag{39}$$

If the resistance and the flux leakage in the primary windings of T_2 can be neglected and the core flux is zero, the external ends of the current transformer have the same potential as the ground line. The voltages V_1 and V_2 then appear across Y_1 and Y_2, respectively, so that $I_1 = Y_1 V_1$ and $I_2 = Y_2 V_2$. In addition, the ratio of the induced voltages in the secondary of T_1 is $V_2/V_1 = N_2/N_1$. Substituting these simple relationships in Eq. (39) and separating real and imaginary parts, one obtains

$$G_1 = \frac{n_2 N_2}{n_1 N_1} G_2 \quad B_1 = \frac{n_2 N_2}{n_1 N_1} B_2 \tag{40}$$

Hence, using suitable combinations of the tappings, a wide range of multiplying factors are available. For a given set of standards, this provides a much wider range of measurements than does the conventional ac Wheatstone bridge. This bridge also allows independent balancing of the conductive and susceptive components (9).

The degree of accuracy obtained in bridge impedance measurements can be considerably enhanced by adopting substitutional methods of measurement. In these methods, the bridge is first balanced with the unknown impedance connected in series or in parallel with a standard component in one of the bridge arms and then rebalanced with the unknown either short- or open-circuited. The unknown can then be determined in terms of the changes made in the adjustable elements, and the accuracy depends on the difference between the two sets of balance values obtained. Residual errors, such as stray capacitance and stray magnetic coupling, and any uncertainty in the absolute values of the fixed bridge components are virtually eliminated. These effects are nearly the same whether or not the unknown is in the circuit.

APPLICATION OF BRIDGE CIRCUITS IN TRANSDUCERS

Bridge circuits are frequently used to configure transducers—that is, the circuits providing information about physical variables (temperature, force, pressure, etc.) capable of changing the value of one or more components of the bridge. In transducers, one measures the voltage occurring at the detector (or a current through the detector). The problems that occur in this case can be demonstrated using the circuit shown in Fig. 9(a).

In this circuit the resistors R_1, R_2, R_3 are constant and the resistor $R_3 = R_0(1 + x)$ is a linear function of a dimensionless variable x. One can assume that the detector resistance R_0 is very high, and then find the voltage V_0 at the detector terminals. One then has

$$V_0 = V\frac{px + (p - mn)}{(n + p)(m + 1 + x)} \tag{41}$$

where $m = R_2/R_0$, $n = R_4/R_0$. When the variable $x = 0$, the circuit should be balanced; this requires that the condition $p = mn$ be satisfied. The voltage at the detector then becomes

$$V_0 = V\frac{mx}{(m + 1)(m + 1 + x)} \tag{42}$$

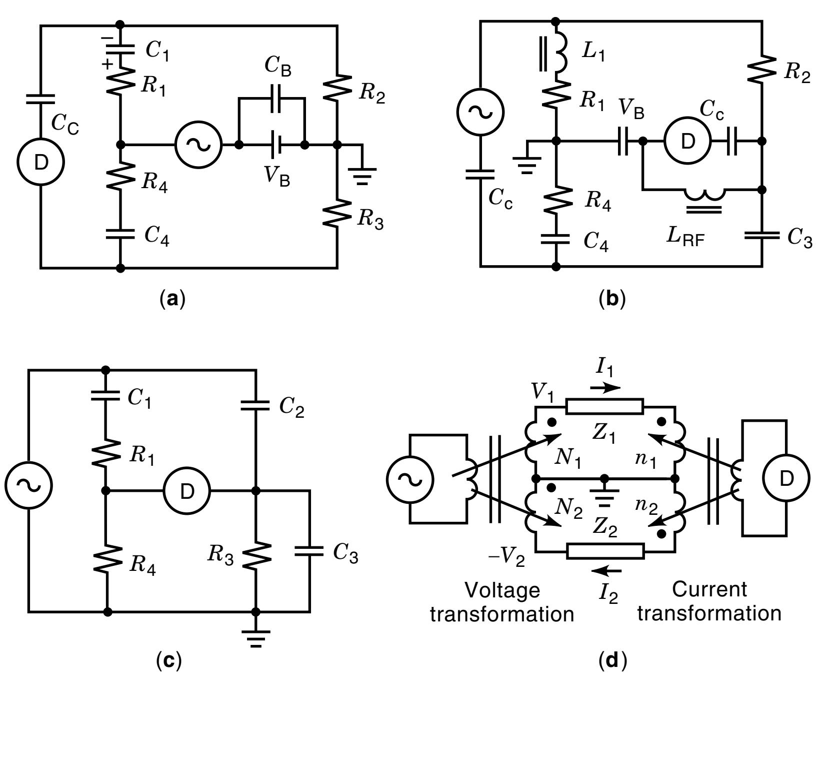

Figure 8. Some special ac bridges: (a) electrolytic capacitor bridge; (b) Owen increment inductance bridge; (d) transformer bridge.

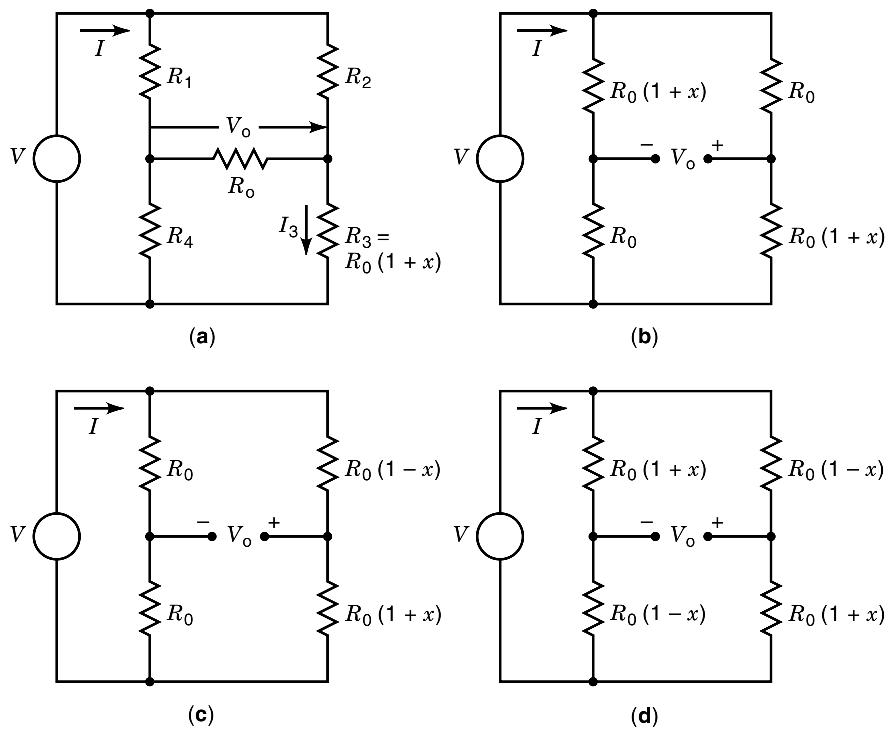

Figure 9. Resistive transducer bridges: (a) with one variable resistor; (b) with two variable resistors; (c) with push-pull variable resistors; (d) with four variable resistors.

One can see that V_0 is a nonlinear function of x. The desired ideal response would be

$$V_{0i} = V\frac{mx}{(m+1)^2} \tag{43}$$

The relative error due to nonlinearity, ϵ_n, may be calculated as

$$\epsilon_n = \frac{V_{0i} - V_0}{V_{0i}} = \frac{x}{m+1+x} \approx \frac{x}{m+1} \tag{44}$$

The reduction of ϵ_n is a frequent requirement in transducer applications. In the case being considered this can be achieved by increasing m and restricting the range of x. This means that one is trying to establish a constant current through R_3 (assuming that the voltage V is constant) and is using the bridge measurements for reasonably small x.

Another important parameter of the circuit of Fig. 9(a) is its sensitivity. Resistor R_3 may be considered as the "tuning" element of the bridge; in this case its variation for small x is $\delta x = R_0 x$, and in the vicinity of balance one can take $\delta V_0 = V_0$. Then the voltage sensitivity is

$$S_v = \frac{V_0}{R_0 x} = \frac{V}{R_0}\frac{m}{(m+1)(m+1+x)} \tag{45}$$

Its maximum value

$$S_{vmax} = \frac{V}{R_0}\frac{1}{(m+1)^2} \tag{46}$$

is achieved when $m \approx 1$. This result shows that in this particular case the condition of maximum sensitivity conflicts with minimization of nonlinearity error.

However, this situation is not always inevitable. If the current I in the circuit of Fig. 9(a) is constant the detector voltage will be

$$\begin{aligned} V_0 &= \mathrm{IR}_0\frac{mnx}{(m+1)(n+1)+x} \\ &\approx \mathrm{IR}_0\frac{mnx}{(m+1)(n+1)}\left[1 - \frac{x}{(m+1)(n+1)}\right] \end{aligned} \tag{47}$$

The nonlinearity error is

$$\epsilon_n = \frac{x}{(m+1)(n+1)} \tag{48}$$

This error is decreasing for increasing m and n. The sensitivity in this case is

$$S_v = I\frac{mnx}{(m+1)(n+1)+x} \tag{49}$$

and its maximum value, achievable for $m = \infty$, $n = \infty$, is

$$S_{vmax} = I \tag{50}$$

Hence, in this case there is no contradiction between optimization of sensitivity and reduction of nonlinearity. In the passive circuit, though, the condition of constant current I can be achieved only approximately.

The results of analysis for the bridge with one variable resistor may be represented by Table 1. It allows one to conclude (7) that—to reduce the nonlinearity error—one has to restrict the measuring range, work with reduced sensitivity, or consider current source realization in order to use it as a power supply for the bridge or the variable resistor.

An increase of sensitivity with a simultaneous decrease of nonlinearity can also be achieved by using two variable resis-

Table 1. Properties of the Bridge with One Variable Resistor

Supply Condition	Sensitivity	Nonlinear Error, ϵ_n	Maximal Sensitivity	Parameters Required	Approximate Conditions
	$S_v = \frac{V_o}{xR_0}$				
V constant	$\frac{V}{R_0}\frac{m}{(m+1)(m+1+x)}$	$\frac{x}{m+1}$	$\frac{V}{R_0}\frac{1}{(m+1)^2}$	$m^2 = 1 + x$, $q = \infty$	$R_2 = R_3$
I constant	$I\frac{mn}{mn+m+n+1+x}$	$\frac{x}{mn+m+n+1}$	I	$m = \infty$, $n = \infty$, $q = \infty$	$R_2 \gg R_0$, $R_4 \gg R_0$
I_3 constant	$I_3\frac{m}{m+1}$	absent	I_3	$m = \infty$, $q = \infty$	$R_2 \gg R_0$
	$S_i = \frac{I_o}{xR_0}$				
V constant	$\frac{V}{R_0^2}\frac{m}{(m+1)\alpha + (m^2+\alpha)x}$	$\frac{x(m^2+\alpha)}{\alpha(m+1)}$	$\frac{V}{R_0}\frac{(1-m)}{(1+m)}$	$q = m^2(q+1)$, $n = 0$ small x	$R_4 \ll R_0$
I constant	$\frac{I}{R_0}\frac{mn}{(n+1)\alpha + [m(n+1)+q]x}$	$\frac{x[m(n+1)+q]}{(n+1)\alpha}$	$\frac{I}{R_0}\frac{1}{(n+1)}$	$q = n^2 - 1$, $m = \infty$ small x	$R_2 \gg R_0$
I_3 constant	$\frac{I}{R_0}\frac{m}{(m+1)q}$	absent	$\frac{I_3}{R_0}\frac{1}{q}$	$m = \infty$ large q	$R_2 \gg R_0$, $R_m \gg R_0$

Note: $m = R_2/R_0$, $n = R_4/R_0$, $p = R_1/R_0$, $q = R_o/R_0$; $R_3 = R_0(1 + x)$; $\alpha = q(m + 1) + m(n + 1)$; balance requirement $p = mn$.

tors in opposite arms [Fig. 9(b)] or by using resistors undergoing opposite variations in adjacent arms [Fig. 9(c)] or by using variable resistors in all four arms [Fig. 9(d)]. Table 2 (7) summarizes and compares the results for the output voltage in all such bridges for the case in which all resistors have the same initial value of R_0. Again, one can see that the bridges powered by a current source (which leads to active circuits) have more choices for linearization.

The realization (10) of the current source for the powering bridge usually involves [Fig. 10(a)] a second voltage source (denoted here as Zener diode voltage, V_R) and an operational amplifier. The bridge current is $I = V_R/R_R$.

Switching to active circuits, some other alternatives should be considered. The circuit of Fig. 10(b), which provides a linearly dependent output voltage

$$V_0 = -V\frac{x}{2} \tag{51}$$

can be considered as one alternative. The circuit of Fig. 10(b) requires the bridge to have five accessible terminals. The circuit of Fig. 10(c), with two operational amplifiers, can also be considered. It provides a linearly dependent output voltage

$$V_0 = V\frac{R_G}{R_0}x \tag{52}$$

and amplifies the bridge output signal.

Capacitive and inductive transducers can be used in a variety of ac bridge circuits. Here we discuss only the so-called Blumlein bridge circuit. It has particular advantages for use with variable capacitive transducers (11) and is used frequently with inductive transducers as well. The circuit is shown in Fig. 11(a). Let the detector impedance be $Z_0 = \infty$. Two variable impedances (sensor arms), $Z + \delta Z$ and $Z - \delta Z$, operate in a push-pull fashion. The ratio arms represent a

Table 2. Output Voltage for Bridges with Variable Resistors and Supplied by a Constant Voltage or Current

R_1	R_2	R_3	R_4	Constant V	Constant I
R_0	R_0	$R_0(1+x)$	R_0	$V\frac{x}{2(2+x)}$	$IR_0\frac{x}{4+x}$
$R_0(1+x)$	R_0	$R_0(1+x)$	R_0	$V\frac{x}{(2+x)}$	$IR_0\frac{x}{2}$
R_0	R_0	$R_0(1+x)$	$R_0(1-x)$	$V\frac{2x}{4-x^2}$	$IR_0\frac{x}{2}$
R_0	$R_0(1-x)$	$R_0(1+x)$	R_0	$V\frac{x}{2}$	$IR_0\frac{x}{2}$
$R_0(1-x)$	R_0	$R_0(1+x)$	R_0	$-V\frac{x^2}{4-x^2}$	$-IR_0\frac{x^2}{2}$
$R_0(1+x)$	$R_0(1-x)$	$R_0(1+x)$	$R_0(1-x)$	Vx	IR_0x

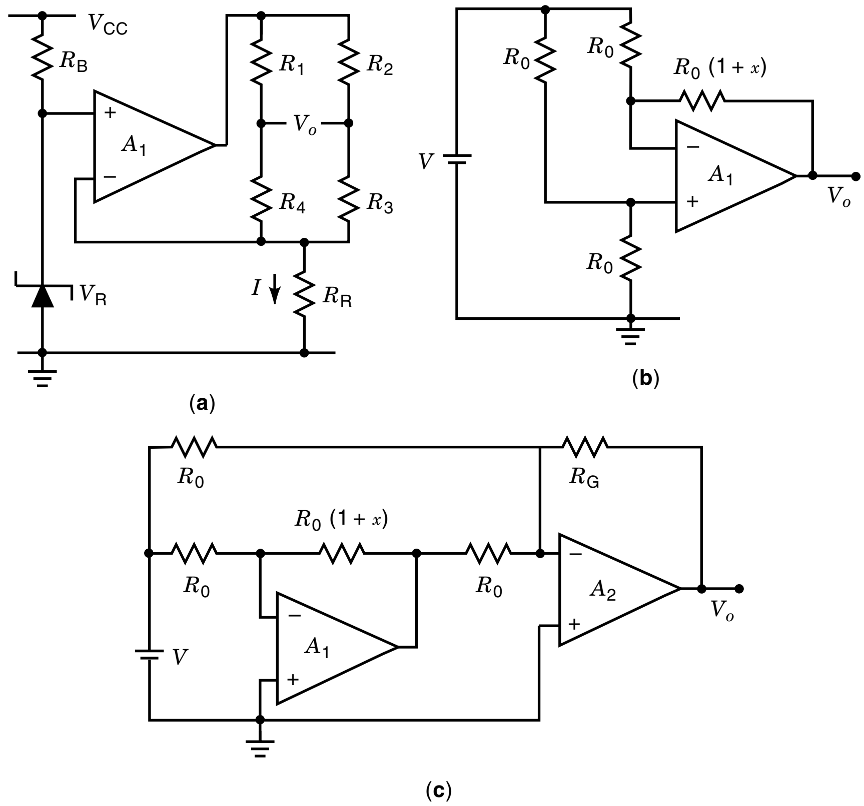

Figure 10. (a) Current source for bridge powering and linearized active bridge circuits: (b) with one amplifier and (c) with two amplifiers.

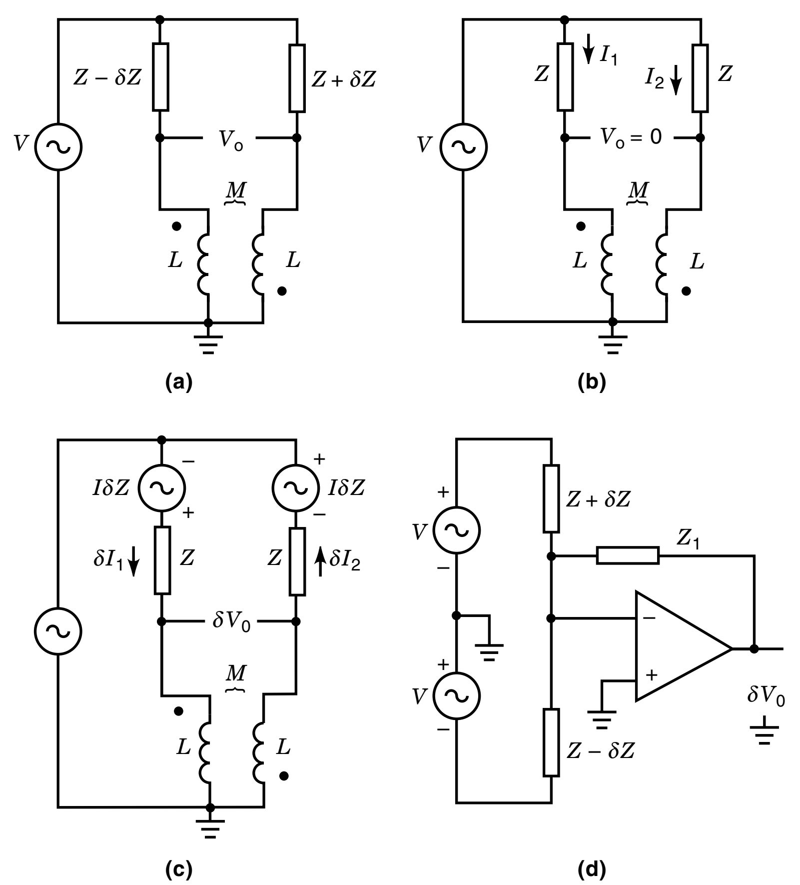

Figure 11. (a) Blumlein bridge and (b) the circuits for calculation of branch currents and (c) current variations; (d) pseudobridge circuit.

transformer with two tightly coupled coils (i.e., $L = M$). The bridge is fed by a sinusoidal voltage V.

Analysis of steady state can be done in two stages. When the sensor arms have equal impedances Z [Fig. 11(b)], one finds that

$$I_1 = I_2 = I = \frac{V}{Z} \tag{53}$$

Indeed, in this condition the magnetic flux in the transformer is zero and the detector voltage and the voltage at each transformer coil are zero. The variations of the impedances may be represented, in accordance with the compensation theorem (see the first section of this article), by two voltage sources. The circuit [Fig. 11(c)] that includes only these two sources can be used for calculation of current variations. Writing two loop equations for this circuit, one finds that

$$\delta I_1 = \delta I_2 = \delta I = \frac{I\delta Z}{Z + 2j\omega L} \tag{54}$$

The variation of the detector voltage is

$$\delta V_0 = 2I\delta Z - 2Z\delta I = 2V\frac{\delta Z}{Z}\left(\frac{2j\omega L}{Z + 2j\omega L}\right) \tag{55}$$

This result can be used for evaluation of the Blumlein bridge sensitivity. For a capacitive sensor, $Z = 1/i\omega C$. Then $\delta Z/Z = -\delta C/C$, and one obtains

$$\delta V_0 = 2V\frac{\delta C}{C}\left(\frac{2\omega^2 LC}{1 - 2\omega^2 LC}\right) \tag{56}$$

Hence, the sensitivity of the Blumlein bridge with capacitive sensor arms is a function of frequency. For a stable result one must choose the parameters so that $2\omega^2 CL \gg 1$.

For an inductive sensor $Z = i\omega l$. Then $\delta Z/Z = \delta l/l$ and

$$\delta V_0 = 2V\frac{\delta l}{l}\left(\frac{2L}{1 + 2L}\right) \tag{57}$$

This analysis demonstrates that in the Blumlein bridge one essentially has comparison of currents at zero potential of the transformer arms. Hence, the capacitive parasitics at the detector terminals are not important. Using a third output transformer coil (as was the case for the transformer bridge), one can realize a very sensitive capacitive sensor.

The idea of current comparison is more directly used in the "pseudobridge" circuit [Fig. 11(d)], where the difference in currents of the sensor arms is entering the virtual ground and produces the output signal

$$\delta V_0 \approx 2V\frac{Z_1}{Z}\frac{\delta Z}{Z} \tag{58}$$

Pseudobridges are mostly used with capacitive sensors (12).

BRIDGE CIRCUITS IN NETWORK SYNTHESIS

Let us return to the lattice form of the bridge [Fig. 1(b)] and consider the impedances Z_1 to Z_4 as a coupling two-port of the transmission system [Fig. 2(b)]. One then finds that the transfer function of this system is

$$T(s) = \frac{V_0}{E_g} = \frac{z_{21}Z_0}{Z_g Z_0 + Z_g z_{22} + Z_0 z_{11} + |z|} \tag{59}$$

where

$$z_{11} = \frac{(Z_1 + Z_4)(Z_2 + Z_3)}{Z_1 + Z_2 + Z_3 + Z_4} \qquad z_{22} = \frac{(Z_1 + Z_2)(Z_3 + Z_4)}{Z_1 + Z_2 + Z_3 + Z_4}$$

$$z_{12} = z_{21} = \frac{Z_2 Z_4 - Z_1 Z_3}{Z_1 + Z_2 + Z_3 + Z_4} \qquad |z| = z_{11}z_{22} - z_{12}^2$$

One of the main problems of network synthesis is the realization of a transfer function with prescribed zeros. The zeros of transmission (which are the zeros of the transfer function) can now be interpreted as the frequencies at which the bridge is balanced. This result, obtained for the simple bridge, is valid for all bridge configurations. Hence, the synthesis of simple rejection filters (such as the twin-T bridge, or T bridge), the transfer function of which includes two complex-conjugate zeros, can be simplified if the balance condition is used directly for the choice of filter elements.

The control of transmission zeros location becomes especially simple if the lattice is symmetric. For $Z_2 = Z_4 = Z_a$ and $Z_1 = Z_3 = Z_b$, the transmission zeros occur at those values of s for which the two branch impedances have equal values. This can be arranged to occur for any value of s; hence, the locations of the transmission zeros of a lattice are unrestricted and may occur anywhere in the s-plane. For example, if $Z_1 = Z_3 = R_0$ and $Z_2 = Z_4 = R + Ls + 1/Cs$, the transmission zeros are given by the zeros of a polynomial

$$LCs^2 + (R - R_0)Cs + 1 = 0 \tag{60}$$

and are located in the left-half s-plane for $R > R_0$, on the $j\omega$-axis for $R = R_0$, and in the right-half s-plane for $R < R_0$. If $L = 0$, one can obtain a zero on the positive real axis.

It can be proved (13) that every symmetric, passive, reciprocal, lumped, and time-invariant two-port has a physically realizable lattice equivalent. Thus, the lattice is the "most general" symmetric two-port. The lattice has an important role in the modern network synthesis (15) and, in the past, was a useful tool in the general image parameter theory of filter design (16).

BRIDGE CIRCUITS IN ELECTRONICS

In this section we describe oscillators, the operation of which can only be fully understood if the bridge balanced condition is considered.

Figure 12 shows the Wien bridge [Fig. 12(a)], twin-T bridge [Fig. 12(b)], and Meachem bridge [Fig. 12(c)] sinusoidal oscillators. The steady-state operation of all three oscillators requires that, at a certain frequency, the condition

$$AT_B(j\omega) = 1 \tag{61}$$

be satisfied. Here, $T_B(j\omega)$ is the transfer function of the corresponding bridge calculated at $s = j\omega$. The transfer functions of the Wien bridge and twin-T bridge should be designed so

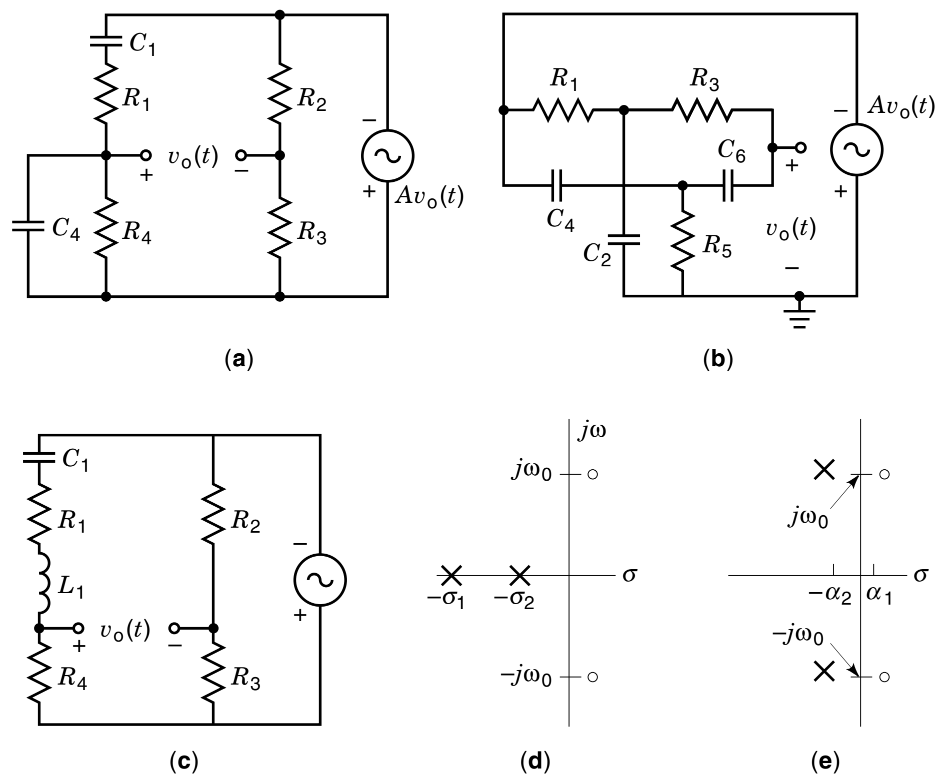

Figure 12. Sinusoidal oscillators: (a) Wien bridge; (b) twin-T bridge; (c) Meachem bridge; and pole-zero diagrams of bridge transfer functions: (d) Wien and twin-T bridge; (e) Meachem bridge.

that

$$T_B(s) = K\frac{(s^2 - \alpha_1 s + \omega_0^2)}{(s+\sigma_1)(s+\sigma_2)} \tag{62}$$

has two complex-conjugate zeros located in the right half of the s-plane in the vicinity of the points $\pm j\omega_0$ [the result of Eq. (62) assumes that, for the twin-T bridge, the real zero and real pole are cancelled]. For the Wien bridge, $\omega_0 = \sqrt{(R_1C_1R_4C_4)}$, $\alpha_1 = [(R_2R_4 - R_1R_3)/(R_1R_3R_4C_4)] - 1/(R_1C_3)$. For the twin-T bridge the elements providing desirable zeros location should be chosen using the balance condition of Eq. (13). The transfer function of the Meachem bridge should be

$$T_B(s) = K\frac{(s^2 - \alpha_1 s + \omega_0^2)}{(s + \alpha_2 s + \omega_0^2)} \tag{63}$$

Here, $\omega_0 = \sqrt{(L_1C_1)}$, $\alpha_1 = (R_2R_4 - R_1R_3)/(2L_1R_3)$, and $\alpha_2 = (R_1 + R_4)/(2L_1)$. In all cases, ω_0 is the desirable oscillation frequency.

In the vicinity of the points $\pm j\omega_0$, the transfer function of the bridge will be

$$T_B(j\omega_0) = |T_B(j\omega_0)|e^{\phi(j\omega_0)} \tag{64}$$

and the condition of Eq. (61) can be rewritten as

$$A|T_B(j\omega_0)| = 1 \qquad \phi(j\omega_0) = 0 \text{ or } 180^\circ \tag{65}$$

The first condition in Eq. (65) gives the required amplifier gain, and the second condition gives the required sign of gain.

A very important oscillator parameter (16) is the indirect frequency stability, S_ω. It is calculated as

$$S_\omega = \omega_0 \frac{d\phi}{d\omega}\bigg|_{\omega=\omega_0} \tag{66}$$

In the vicinity of ω_0, only the nearest zeros and poles are important, and this stability will be

$$S_\omega \approx -2Q_z \tag{67}$$

for the Wien-bridge and twin-T oscillators. Here $Q_z = \omega_0/(2\alpha_1)$. For the Meachem bridge oscillator, one has

$$S_\omega \approx -2Q_z - 2Q_p \tag{68}$$

where $Q_p = \omega_0/(2\alpha_2)$. One can see that the achieved indirect frequency stability is determined by the chosen bridge imbalance [the reactance branch in Meachem bridge oscillator is usually a crystal, and the location of poles in $T_B(\omega)$ is determined by the crystal parameters]. The connection between bridge imbalance and design for frequency stability is well known for the Wien bridge and Meachem bridge oscillators (16), however, it is still not clearly understood in the twin-T bridge oscillator design (17).

The application of the bridge circuits to design of nonsinusoidal oscillators is less known. Using switches in a two-operational amplifier multivibrator, one can obtain control of the oscillation frequency by detuning a resistive bridge (Fig. 13).

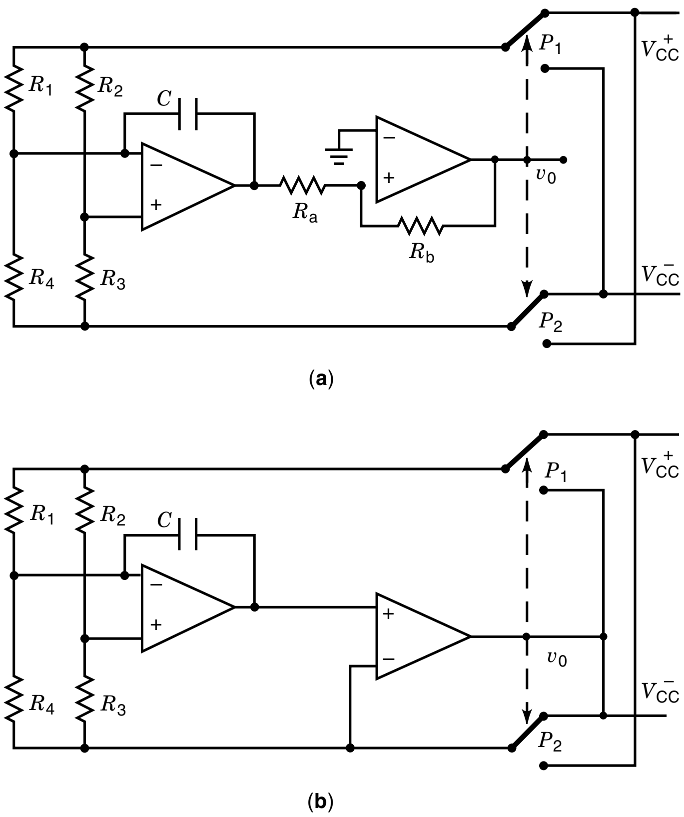

Figure 13. Bridge-controlled multivibrators: (a) with Schmitt trigger; (b) with comparator.

For normal circuit operation, the bridge should be unbalanced. The oscillation frequency of this circuit is

$$f_0 = \frac{1}{2C}\left(\frac{R_2}{R_1} - \frac{R_3}{R_4}\right) \frac{(V_{CC}^+ - V_{CC}^-)R_b}{[R_b(R_3 - R_2)(V_{CC}^+ - V_{CC}^-) + R_a(R_2 + R_3)(V_0^+ - V_0^-)]} \quad (69)$$

The use of a comparator allows one to eliminate the feedback resistances of the Schmitt trigger [Fig. 13(b)]. For this circuit, the oscillation frequency is

$$f_0 = \frac{1}{4CR_3}\left(\frac{R_2}{R_1} - \frac{R_3}{R_4}\right) \quad (70)$$

Both circuits are used as bridge-to-frequency converters in two-wire transducers (18).

CONCLUSION

Bridge circuits form a specialized, yet a wide-ranging, group of circuits that find application in measurement techniques, transducers, network synthesis, and electronics.

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