INTEGRATING CIRCUITS

Since inductors tend to be bulky and expensive (except for applications at frequencies higher than approximately 1 GHz), integrating circuits generally consist of a current signal which is integrated onto a capacitor to form a voltage signal. Cascading one integrating circuit to another requires that the voltage signal be converted to a current signal. An integrating circuit can be realized using an opamp, resistor, and capacitor. The input voltage is converted to an input current which is directly proportional to the input voltage via Ohm's Law. This current is integrated onto the capacitor. The output voltage is produced at the output of the opamp.

The major applications of integrating circuits are in filters, slow analog-to-digital converters, and image sensor circuits. Integrating circuits are the basic building blocks used to synthesize frequency selective circuits, or filters. The complexity of the filter function, in terms of the number of poles, determines the number of integrating circuits that must be included in the circuit. A high precision but low speed technique for analog-to-digital conversion employs an integrating circuit, comparator, and a counter or timer. The integrating capacitor is charged for a specified amount of time by a current which is proportional to the input signal. The capacitor is discharged by a fixed amount of current. The length of time required to fully discharge the capacitor determines the value of the input signal. Another major application of integrating circuits is in image sensing circuits. Incident light is converted to a photo-current which is integrated on a storage capacitor for a specified length of time. The final voltage on the capacitor is directly proportional to the photocurrent.

Performance criteria for integrating circuits are cost and dynamic range. In integrated circuits, cost is a nondecreasing function of the silicon area occupied by the circuit and the power consumed by the circuit. We measure dynamic range as the ratio of the largest to the smallest signal level that the circuit can handle. A tradeoff exists between these performance criteria; that is, the higher the dynamic range required, the higher the cost of the integrator.

INTEGRATING CIRCUITS

There are several means of realizing the mathematical function of integration using only resistors, transistors, amplifiers, and capacitors. The function we want to synthesize is of the form:

$$v_O(t) = A\int_0^t v_I(t)\,dt + B \tag{1}$$

The Inverting Integrator

The inverting integrator, also known as the Miller integrator, is shown in Fig. 1(a). It is an inverting amplifier, where the feedback element is the capacitor C. The input voltage, $v_I(t)$, is converted to a current $i_I(t) = v_I(t)/R$, since a virtual ground exists at the negative input terminal of the opamp. This current is integrated on the capacitor, forming an output voltage according to the relation,

$$v_O(t) = -\frac{1}{RC}\int_0^t v_I(t)\,dt - v_C(0) \tag{2}$$

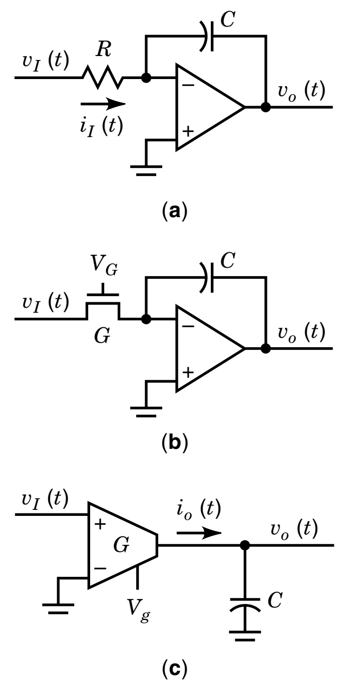

Figure 1. Integrating circuits: (a) the inverting integrator, (b) the MOSFET-C integrator, and (c) the transconductance-C integrator.

where $v_C(0)$ is the initial voltage stored on the capacitor. Note that even a small dc input voltage will eventually result in an output voltage which is huge. A theoretically infinite output voltage is prevented by the finite dc voltage supplies of the opamp.

The integrator time constant of the Miller integrator is RC, which has units of seconds.

The MOSFET-C Integrator

The MOSFET-C integrator is shown in Fig. 1(b). It is popular in integrated circuit design, where the amplifier, capacitor, and resistance are fabricated on the same substrate. An MOS transistor operating in the triode region acts like a voltage-controlled resistor, where the nominal conductance $G = 1/R$ has units of $1/\Omega$. Using the same analysis as for the Miller integrator, we find that the integrating time constant is C/G. The two main advantages of the MOSFET-C integrator over the inverting integrator are: (1) an MOS transistor generally occupies less silicon area than an equivalent resistor, and (2) the conductance is tunable via the gate voltage V_G. The latter property is particularly important in integrated circuit design, where the tolerance on capacitors is approximately 10 to 50%.

The Transconductance-C Integrator

The transconductance-C integrator is shown in Fig. 1(c). It consists of a transconductance amplifier which converts a differential input voltage to an output current via the relation

$$i_O = G(v_+ - v_-) \tag{3}$$

In Fig. 1(c), we note that $v_+ = v_I(t)$, and $v_- = 0$ V. Thus, the output current is equal to the input voltage times the conductance. The current $i_O(t)$ is integrated on the capacitor, produc-

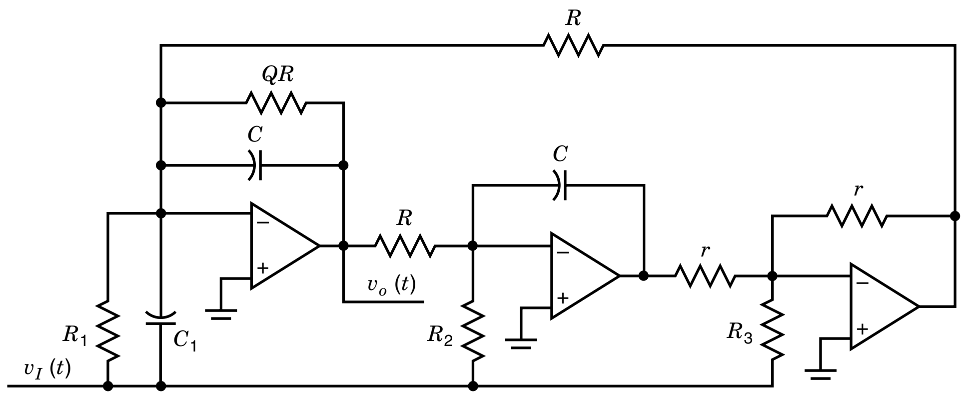

Figure 2. The feedforward Tow–Thomas two-integrator-loop biquadratic circuit.

ing the output voltage, $v_O(t)$. The integrator time constant is the same as that for the MOSFET-C integrator, namely, C/G. The transconductance-C integrator is also used in integrated circuit design. Compared to the MOSFET-C integrator, the transconductance-C integrator has simpler circuits and is generally capable of operating at higher speeds. On the other hand, the MOSFET-C integrator has a higher maximum dynamic range at a given cost than the transconductance-C integrator.

Other Integrator Circuits

The switched-capacitor integrator employs a capacitor and at least two MOS transistors operating as switches to emulate the resistance R. A global clock signal, operating at a frequency much higher than the bandwidth of the input signal, turns the switches on and off at a rate which is inversely proportional to the effective resistance seen by the input. The switched-capacitor integrator is common in integrated circuit design. Tuning the integrator time constant is straightforward since it depends on the clock frequency and the ratio of capacitors. In integrated circuit design, the tolerance on the ratio of capacitors can be lower than 1%. The cost of this design strategy is increased circuit complexity, power consumption, and noise due to the global clock signal.

If the input signal is a current, integration is easily achieved using a single capacitor. During the reset phase of a capacitor integrator, the capacitor voltage is set to a known initial voltage, typically either 0 V or V_{DD}. During the integration phase, the input current is integrated on the capacitor to produce an output voltage which is proportional to the input current. This output voltage is typically buffered and scaled before it is sent off chip.

MAJOR APPLICATIONS OF INTEGRATOR CIRCUITS

Three major applications of integrator circuits are in filters, dual-slope analog-to-digital converters, and image sensor circuits.

The Biquadratic Filter

The two-integrator-loop biquadratic circuit, or biquad, is used to synthesize an arbitrary second-order filter function in s, where $s = j\omega$, and ω is the frequency in radians/s. The feedforward Tow-Thomas biquad is drawn in Fig. 2. It consists of two inverting integrators and an inverting amplifier. Its transfer function is

$$\frac{V_O(s)}{V_I(s)} = -\frac{s^2\dfrac{C_1}{C} + s\dfrac{1}{C}\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) + \dfrac{1}{C^2RR_2}}{s^2 + s\dfrac{1}{QCR} + \dfrac{1}{C^2R^2}} \tag{4}$$

By a suitable choice of circuit components, Eq. (4) can be configured as a second-order low-pass, bandpass, high-pass, notch, or allpass filter. Fourth and higher-order filter functions are constructed by cascading two or more biquads together.

Dual-slope Analog-to-digital Converter

The dual-slope analog-to-digital converter (ADC) is a high-precision but low speed ADC method. A block diagram is given in Fig. 3(a). The dual-slope ADC consists of an inverting integrator, a comparator, a high-speed counter, and control logic. The charge on the capacitor is initially set to zero. During the integration phase, the input voltage $v_I(t)$ is converted to an input current that is integrated on the capacitor C for a

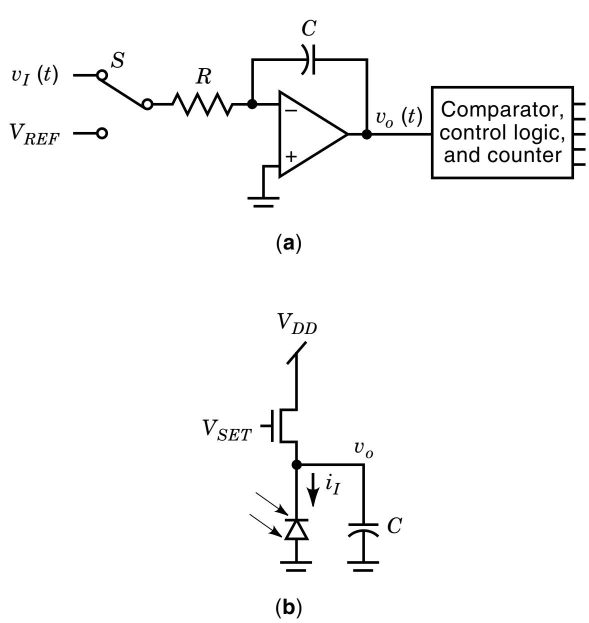

Figure 3. Two applications of integrating circuits: (a) a dual-slope analog-to-digital converter, and (b) a CMOS image sensor circuit.

fixed length of time. During the discharge phase, current derived from a known reference voltage is used to discharge the capacitor. The length of time, as measured in counts, required to discharge the capacitor to 0 V is proportional to the input voltage.

CMOS Image Sensor Circuits

CMOS image sensor circuits receive as input a photocurrent which is proportional to the intensity of the incoming light. The photocurrent is generated at a reverse-biased *pn* junction in the silicon. A simplified diagram is shown in Fig. 3(b). A description of its operation is as follows. The integrating capacitor is initially set to V_{DD}. During the integrating time, the photocurrent discharges the capacitor. The difference between V_{DD} and the final voltage is proportional to the mean value of the photocurrent.

PERFORMANCE CRITERIA

Two performance criteria that we consider are its cost, as measured in its area and power dissipation, and dynamic range. Other performance criteria may be relevant, such as maximum bandwidth or minimum supply voltage.

Cost

In integrated circuit design, the cost of an integrator is a nondecreasing function of the area and power consumption. In this analysis, we assume that the technology is fixed; that is, the designer is unable to change all the parameters of the devices, except their size.

Area. The silicon area of an integrator is generally dominated by the area of the capacitor. Define the capacitance per unit area of a capacitor in a given fabrication technology as C_{OX}, with units of F/m^2. Then, the total area of the integrator can be written as

$$A = \zeta \frac{C}{C_{OX}} \tag{5}$$

where ζ is an area factor that takes into account the portion of silicon area used by the amplifier and conductor. This area factor is always greater than unity.

Power. Static power consumption is the power supply voltage times the quiescent, or bias, current of the amplifier. Let us denote the total amplifier bias current as I_B, the positive supply voltage as V_{DD}, and the negative supply voltage as $-V_{DD}$. Then the static power consumption is

$$P_S = 2V_{DD}I_B \tag{6}$$

Dynamic power consumption results primarily from the charging and discharging of the integrating capacitance. In most designs, dynamic power consumption is much larger than static power consumption. The maximum dynamic power consumption occurs when the capacitor is fully charged and discharged with each cycle of the input signal. Let the input signal have amplitude V_{DD} and frequency f_S. Then, the maximum dynamic power consumption is

$$P_D = 4f_S V_{DD}^2 C \tag{7}$$

The factor 4 takes into account positive charging to V_{DD} and negative charging to $-V_{DD}$. Total power consumption is the sum, $P_S + P_D$.

Dynamic Range

Dynamic range (DR) is defined as the maximum input level the circuit can handle divided by the noise level. By level, we mean the mean-square value of the input or output voltage. Dynamic range is generally expressed in dB.

Noise. Sources of noise in the integrator are found in the conductance and the amplifier. Thermal noise associated with each element gives rise to a power spectral density that has a flat frequency power spectrum with one-sided density

$$S_{TH} = 4kTR\xi \tag{8}$$

where k is Boltzmann's constant, T is the absolute temperature, and ξ the combined noise factors of the conductance and amplifier. In general, ξ is greater than unity.

Another major source of noise for integrators is $1/f$, or flicker, noise. Its power spectrum is proportional to the inverse of the frequency, hence, its name. Flicker noise becomes dominant at low frequencies.

We now configure the integrator as a single-pole lowpass filter by placing a conductance of value $1/R$ in parallel with the capacitor. Its transfer function is given by

$$\frac{V_O(s)}{V_I(s)} = \frac{1}{1 + s/RC} \tag{9}$$

Considering thermal noise only, the noise level at the output is found by integrating the product of the power spectrum in Eq. (8) and the square magnitude of the lowpass filter in Eq. (9) over all positive frequencies. In order to account for the presence of two conductors, the noise level is found to be

$$\overline{V_N^2} = 2\frac{kT}{C}\xi \tag{10}$$

The expression Eq. (10) shows that the noise level does not depend on the value of the resistor, but primarily on the value of the capacitory.

Distortion. The sources of distortion in the integrator are: (1) the capacitor, (2) the amplifier, and (3) the conductance. Linear capacitors are available in CMOS fabrication processes suitable for analog and mixed-mode circuits. Linear capacitors are needed to obtain a high dynamic range integrator. In addition, an amplifier which operates over the full voltage supply range is necessary to achieve low distortion for large amplitude input signals. If we suppose that the conductance is implemented using an MOS transistor operating in the triode region, there will be some range of input voltages over which the effective resistance is nearly constant. One measure of distortion that is mathematically tractable is the maximum deviation of the effective resistance, expressed as a percent of the nominal value. Then, the linear range is defined as the continuous set of input voltages over which the maximum deviation of the effective resistance is less than $d\%$ of the nominal value, where $d\%$ is the amount of distortion. Other distortion measures are possible, such as mean-square error; however, they can be much more complex to compute, as they may require knowledge of the input signal and the circuit topology.

The highest achievable linear range is limited by the voltage supplies. As such, the maximum input and output amplitude range for a sinusoidal signal is V_{DD}. For an input signal of maximum amplitude, the level is $V_{DD}{}^2/2$. As a result, the highest achievable dynamic range of the low-pass filter is

$$DR = \frac{\overline{V_I^2}}{V_N^2} = \frac{CV_{DD}^2}{4kT\xi} \tag{11}$$

Cost Versus Dynamic Range

Here, we relate the cost of the integrator to its dynamic range. If we only consider dynamic power dissipation as found in Eq. (7), we see that

$$DR = \frac{P_D/f_s}{16kT\xi} \tag{12}$$

Both the numerator and the denominator in Eq. (12) have the units of energy. Thus, the upper limit on the dynamic range of the low-pass filter is directly proportional to the amount of energy dissipated in the integrator. For example, in order to achieve a dynamic range of 60 dB, we must dissipate at least $16 \times 10^6\ kT\xi$ J per cycle of the input signal.

We can rearrange Eq. (7) to solve for frequency of the input signal, as in

$$f_S = \frac{P_D}{4V_{DD}^2 C} \tag{13}$$

Thus, for fixed power supply voltage and power consumption, the input frequency is constrained by the size of the capacitor. The lower the frequency of the input, the larger the area of the integrating capacitor. Values of integrated capacitors range from as low as 10 fF to as high as 1 nF; however, the area penalty of the largest value is not amenable to mixed analog/digital circuits.

Differential Signaling

Differential signaling is a technique used primarily in integrated circuits to increase the maximum amplitude range of the input and output signals by a factor of two. Differential signaling has two other major benefits: even-order distortion in the amplifier, conductor, and capacitor is cancelled, as are common-mode sources of noise. The cost of using differential signaling is increased circuit complexity, area, and static power consumption due to common-mode feedback circuitry. The MOSFET-C integrator is unable to take full advantage of differential signaling since the capacitor must have a virtual ground at one node. Thus, its maximum dynamic range is given by Eq. (11). On the other hand, the transconductance-C integrator can employ a differential signal across the capacitor. Notwithstanding, the transconductance-C integrator cannot operate at the maximum input amplitude. It is concluded, then, that the MOSFET-C integrator can approach the dynamic range maximum in Eq. (11), whereas the best transconductance-C integrator is at least several dB lower.

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INTEGRATION. See CALCULUS.

INTEGRATION OF DATABASES. See DATABASE DESIGN.