SMOOTHING CIRCUITS

Most electronic circuits are designed to operate from a perfect constant-voltage direct-current (dc) supply. This is often shown on a schematic diagram as V_{CC}, V_{DD}, or $+12$ V, for instance. Ideally, the supply voltage would remain constant despite all disturbances: It would never show any spikes, ripples, or variations of any sort. It would not change in the face of variations at its input, and it would not be altered no matter how high the load current. In practice, such paragons of constancy do not exist, although expensive laboratory power supplies strive to approximate them.

When any electronic circuit is incorporated into end-use equipment, the available dc supply has imperfections that can interfere with the proper operation of the circuit. A realistic model of a practical dc voltage supply is shown in Fig. 1. It comprises an ideal dc voltage source V_{dc}, an alternating current (ac) voltage source $v_r(t)$ that represents superimposed ripple, and a generalized series impedance Z_s, which represents the supply's internal impedance and that of wires, PCB traces, and connections. In the ideal case, $v_r(t)$ and Z_s are both zero, but they take on nonzero values for a real supply. In a typical power supply, the dc source V_{dc} ultimately derives from a wall plug or similar electric utility line source with its own variations.

What is Smoothing?

It is possible to design an electronic circuit to withstand some variation at the dc supply. However, high-performance circuits such as audio amplifiers or precision sensors require a supply of the highest possible quality, and almost any design can benefit from a clean supply. The concept of a *smoothing circuit* is to add extra elements to Fig. 1 to provide an equivalent dc output that approaches the ideal. A smoothing circuit provides an interface between the nonideal supply and the intended electronic load. In power supply practice, it is usual to distinguish between a smoothing circuit, which has the primary function of eliminating effects of the ripple voltage $v_r(t)$ and the source impedance Z_s, and a *regulation circuit*, which has the primary function of enforcing a constant V_{dc}. However, many smoothing methods incorporate regulation functions.

Smoothing can take the form of a *passive* circuit, constructed mainly from energy storage elements, or an *active* circuit that corrects or cancels supply variation. Both of these major topics are discussed in this article. We begin with definitions, then consider fundamental issues of smoothing. Following this, we present passive smoothing methods in depth. Active smoothing methods are described after passive methods.

It is important to point out that an ideal dc voltage supply is not the only possibility for circuit power. In a few cases, an ideal dc current supply or a specific ideal ac supply is needed instead. The reference materials provide additional information about smoothing of current sources and ac supplies.

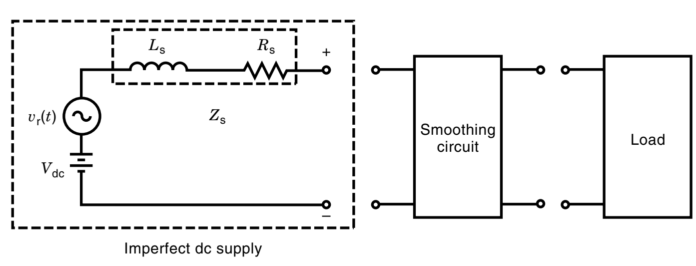

Figure 1. Model of an imperfect dc supply, smoothing circuit and load.

Definitions

The following definitions are in common use.

The *ripple factor r* provides a measure of the ripple voltage v_r relative to V_{dc}. It is usually defined in terms of peak-to-peak variation:

$$r = \frac{\text{Peak-to-peak supply voltage excursion}}{\text{Average voltage}} \tag{1}$$

If v_r is sinusoidal, with $v_r(t) = V_{pk}\cos(\omega_r t)$, then the ripple factor is $r = 2V_{pk}/V_{dc}$. An oscilloscope can be used in ac-coupled mode to measure the numerator of Eq. (1), and a dc voltmeter can measure the denominator. The ripple factor should be as small as possible.

A smoothing circuit should provide a lower value of r at its output than at its input. This is the basis for an important figure of merit.

Ripple attenuation is the ratio of the smoothing circuit's input ripple voltage to its output ripple voltage. It is often expressed in decibels:

$$\text{Ripple attenuation} = 20\log_{10}\left(\frac{\text{Peak-to-peak voltage excursion at input}}{\text{Peak-to-peak voltage excursion at output}}\right) \tag{2}$$

The ripple attenuation should be as large as possible.

When a smoothing circuit is not present, the output impedance of a power supply is the source impedance Z_s. Therefore, output impedance provides a second helpful figure of merit. The objective is to provide as low an impedance as possible over a wide frequency range.

The end-use application has certain important measures as well. The sensitivity of the load to supply variation is measured in terms of *power supply rejection.*

The *power supply rejection ratio* (PSRR) is a measure of the ultimate effect of supply variation at the point of end use. Given a final output voltage magnitude V_O and power supply ripple voltage magnitude V_r, the rejection ratio is given by

$$\text{Power supply rejection ratio} = -20\log_{10}(V_O/V_r) \tag{3}$$

Here V_O is that portion of the output voltage that is related to supply ripple (excluding signals, random noise, and other disturbances). The PSRR value should be as high as possible.

The frequency of the ripple waveform is important for smoothing circuit design.

The *ripple frequency* is defined as the fundamental frequency of the ripple waveform, whether or not the waveform is sinusoidal. In many cases, the ripple frequency is a multiple of the ac line frequency that provides the energy. In switching power supplies, the ripple frequency is usually much higher.

Example: Amplifier Load

Figure 2 shows an inverting amplifier supplied from a nonideal source, and will serve to illustrate the need for smoothing. In the case of a perfect voltage source with $v_r(t) = Z_s = 0$, the amplifier's intended output voltage is $V_O = -g_m R V_i$, where g_m is the transistor's transconductance. However, the supply ripple voltage feeds directly through R to the output, and it modifies the output voltage to $V_O = -g_m R V_i + V_r$. The circuit in Fig. 2 is poor in this respect. Its PSRR is unity, or 0 dB, and therefore it relies entirely on smoothing within the power supply for its function. Well-designed amplifiers, such as IC operational amplifiers, can have PSRR levels of 60 dB or greater.

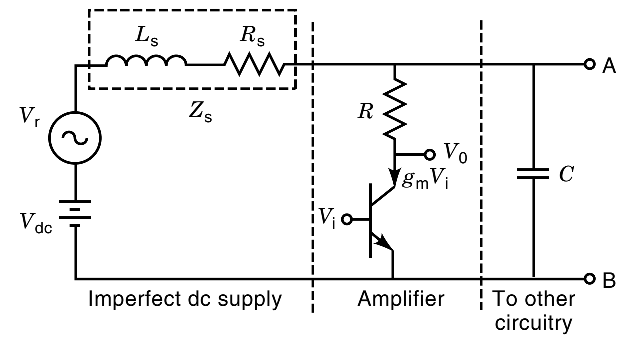

Figure 2. Imperfect dc supply feeding a simple amplifier and other circuitry.

Furthermore, the imperfect dc supply has impedance Z_s, which appears in series with R, so the output voltage becomes $V_O = -g_m(R + Z_s)V_i + V_r$. Since Z_s most likely depends on frequency, the amplifier's frequency response (the variation of V_O/V_i with frequency) is no longer flat. In addition, an alternating-current (ac) voltage develops across Z_s, equal to $g_m Z_s V_i$. If another circuit is powered from points A and B in Fig. 2, considered as dc supply rails, this extra ac voltage appears as superimposed ripple. Thus the supply impedance Z_s provides a mechanism by which signals in one circuit can couple into another. Under certain conditions, the cross-coupling can be strong enough to produce large-amplitude, self-sustaining oscillations.

A remedy for the impedance and cross-coupling effects is to connect a large capacitor between A and B. This is known as *decoupling* the circuits from each other. A "large capacitor" means one with impedance $|Z_C| = 1/(\omega C)$ that is much smaller than the impedances R and Z_s at all frequencies of interest. The idea is that the signal current in R should flow mostly through C rather than L_s. It will then develop a small voltage of approximately $g_m V_i/(\omega C)$ between points A and B. When R_s is small, a ripple current of approximately $V_r/(\omega L_s)$ will flow through L_s and C; this current will produce a ripple voltage $V_r/(\omega^2 L_s C)$ between A and B. The larger the value of C, the smaller the unwanted voltage $g_m V_i/(\omega C)$, and the better A and B approach ideal voltage rails. Higher values of L_s will also help reduce the ripple voltage between A and B once a capacitor is present. Rather than being an unwanted element, the source inductance becomes useful. In fact, extra inductance might be added in series to increase the smoothing effect.

This example shows the basis of a general passive smoothing circuit: L_s and C form a second-order *low-pass filter* between the ripple voltage source and the amplifier circuit. The filter passes dc unaffected. Well above the *cutoff frequency* of the filter at approximately the resonant frequency, $\omega_0 = 1/\sqrt{LC}$, ripple is attenuated by about 40 dB/decade.

SMOOTHING FROM AN ENERGY PERSPECTIVE

Before discussing practical smoothing techniques, we look at power and energy relations, which define fundamental physical limitations associated with smoothing.

Power and Energy

Let work or energy be denoted as a function $w(t)$. Instantaneous power is the rate of change of energy, $p(t) = dw/dt$. The first law of thermodynamics states that energy is conserved: Σw = constant, and after differentiating we obtain $\Sigma(dw/dt) = 0$. Therefore, the energy supplied to a system must match the energy output, plus any energy that is stored internally. In terms of power, we can identify a *power balance equation:*

$$\sum p_{\text{in}}(t) = \frac{d}{dt}\sum w_{\text{stored}} + \sum p_{\text{out}}(t) \tag{4}$$

That is, the total power flowing into any system equals the total power flowing out, plus the rate of change of internally stored energy.

In a typical electronic system, there are unwanted losses that constitute an output heat flow, and it is convenient to consider this heat output separately. Figure 3 shows a smoothing circuit that has instantaneous input power $p_{\mathrm{I}}(t)$, electrical output power $p_{\mathrm{O}}(t)$, internal energy storage $w(t)$, and heat output power (losses) $p_{\mathrm{H}}(t)$. The power balance equation for this circuit is

$$p_{\mathrm{I}}(t) = p_{\mathrm{O}}(t) + \frac{dw}{dt} + p_{\mathrm{H}}(t) \tag{5}$$

So, in general, the power entering a smoothing circuit at any instant undergoes a three-way split: Some power leaves via the electrical output, some increases the internal stored energy, and the remainder is dissipated as heat. This power balance equation identifies two techniques available for realizing smoothing circuits: energy storage and dissipation. Both will be considered shortly.

Let $p_{\mathrm{I}}(t) = P_{\mathrm{I}} + \tilde{p}_{\mathrm{I}}(t)$, where P_{I} is a constant (dc) term and $\tilde{p}_{\mathrm{I}}(t)$ is the ripple component (ac term), and similarly for the other variables. The power balance equation becomes

$$(P_{\mathrm{I}} + \tilde{p}_1) = (P_{\mathrm{O}} + \tilde{p}_{\mathrm{O}}) + \frac{d(W + \tilde{w})}{dt} + (P_{\mathrm{H}} + \tilde{p}_{\mathrm{H}}) \tag{6}$$

Since W is constant, an immediate simplification is that the dW/dt term is zero. Furthermore, we are interested in the long-term average power flows:

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^T p(t)\,dt \tag{7}$$

By definition, the ripple components have zero average, so the long-term result is

$$P_{\mathrm{I}} = P_{\mathrm{O}} + P_{\mathrm{H}} \tag{8}$$

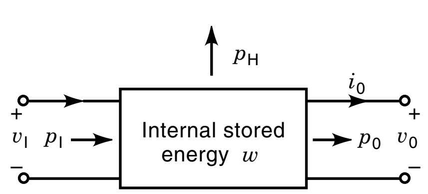

Figure 3. Power flows and energy in a smoothing circuit.

This average power balance equation leads to the definition of efficiency, $\eta = P_{\mathrm{O}}/P_{\mathrm{I}}$—that is, the ratio of average output power to average input power, usually expressed as a percentage. An important design aim of a smoothing circuit should be to attain a high efficiency—that is, approaching 100%.

If we take the full-power balance equation and subtract the average-power balance equation, we arrive at the ripple-power balance equation:

$$\tilde{p}_{\mathrm{I}} = \tilde{p}_{\mathrm{O}} + \frac{d\tilde{w}}{dt} + \tilde{p}_{\mathrm{H}} \tag{9}$$

The aim of smoothing is to make $\tilde{p}_{\mathrm{O}}$ close to zero. We can try to accommodate the input ripple power either by a change in stored energy, by dissipation, or by a combination of these.

Nondissipative Smoothing

Internal energy storage is used in smoothing circuits such as *LC* filters, which in theory can be 100% efficient. There are only two basic ways of storing energy in a circuit: electrically and magnetically. A capacitor provides electric storage based on its stored charge, $q = CV$. The derivative of this for constant capacitance provides the familiar property $i = C(dv/dt)$. At any point in time, the power into the device is $v(t)i(t)$, and the capacitive stored energy is

$$w_C(t) = \int_0^t v(t)i(t)\,dt = \int_0^t v(t)C\frac{dv(t)}{dt}\,dt = \int_0^v Cv\,dv = \frac{1}{2}Cv^2(t) \tag{10}$$

The other way of storing energy is in a magnetic field. An inductor provides magnetic flux linkage λ in its core proportional to the current i flowing in a coil around the core, with $\lambda = Li$. The time derivative of this for constant inductance, combined with Faraday's law by which $d\lambda/dt$ is equivalent to voltage, gives the conventional property $v = L(di/dt)$. The same procedure as for the capacitor yields the inductive stored energy:

$$w_L(t) = \frac{1}{2}Li^2(t) \tag{11}$$

Capacitance and inductance are electric *duals:* An expression concerning capacitance can be transformed into an expression about inductance by swapping the roles of v and i and then replacing C by L and q by λ. For a capacitor we can write $dv/dt = i/C$, so the larger the value of C, the smaller the rate of change of voltage—that is, the more constant the voltage remains. For an inductor we can write the dual expression $di/dt = v/L$. From an energy storage perspective, we deduce that capacitors act to maintain constant voltage; inductors act to maintain constant current.

With finite capacitance and inductance, true constancy cannot be achieved. The basis of nondissipative smoothing circuits is to alternate capacitors and inductors, progressively smoothing voltage and current in turn.

Figure 4(a) shows an input power with sinusoidal variation. Nondissipative smoothing circuits attempt to store ripple energy during the high parts of the cycle, and then they dispense this energy during low parts of the cycle to smooth the overall flow. The available power becomes P_{I}, the dashed line in Fig. 4(a).

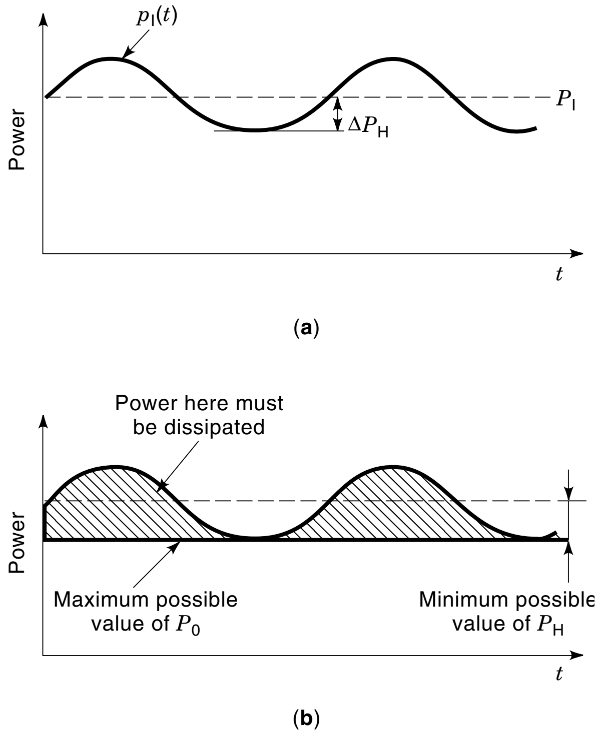

Figure 4. Hypothetical ripple power waveform. In (a), average power P_I is available with nondissipative smoothing. In (b), dissipative smoothing delivers a lower value.

Dissipative Smoothing

Dissipation is used in circuits such as linear voltage regulators and linear active smoothing filters. These devices are inherently less than 100% efficient. Their basic operation is to enforce constant output power by treating ripple as excess energy.

Recall the ripple-power balance equation,

$$\tilde{p}_I = \tilde{p}_O + \frac{d\tilde{w}}{dt} + \tilde{p}_H \tag{12}$$

If there is no stored energy, the term $d\tilde{w}/dt$ is zero. The objective of $\tilde{p}_O = 0$ can be achieved only if we make $\tilde{p}_H = \tilde{p}_I$—that is, if we convert all the input ripple power into heat power (loss). The lost energy is actually higher than this, and we should consider the implications for efficiency. Consider again an input power with ripple as in Fig. 4(a). Suppose $\tilde{p}_I(t)$ has a maximum downward excursion ΔP_H; this means that the total heat power $p_H(t)$ has a minimum value of $P_I - \Delta P_H$. By the second law of thermodynamics, this value must be positive: Heat power always flows *out* of the circuit (excluding heat engines). This forces us to set a positive value of P_H to account for correct heat flow. The implications can be seen in Fig. 4(b). Here the value of P_O is set as high as possible—to the minimum of the input power excursion—and all power in the shaded region must be thrown away as heat to meet the zero $\tilde{p}_O$ objective. Since P_H is positive in a dissipative smoothing circuit, the efficiency is less than 100%.

In a case with sinusoidal ripple power, such that $p_I(t) = P_I + p_i \cos(\omega t)$, the highest output power with $\tilde{p}_O = 0$ will be

$$P_O = P_I - p_i \tag{13}$$

The highest possible efficiency P_O/P_I gives

$$\eta \le 1 - \frac{p_i}{P_I} \tag{14}$$

Recall that nondissipative smoothing circuits cannot achieve zero ripple with finite component values. Although dissipative smoothing circuits can never achieve 100% efficiency, they *can* achieve zero ripple, in principle.

Waveform Considerations and Frequency-Domain Analysis

The implications of ripple factor, regulation, and many measures of smoothing performance depend on the nature of the ripple waveform. Ripple waveforms in conventional power supplies can be sinusoidal or triangular, or they can take the form of narrow spikes. We can consider ripple as a periodic signal with a particular spectrum and then apply frequency-domain analysis. Formally, we represent a ripple signal $v_r(t)$ by its Fourier series,

$$v_r(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t) \tag{15}$$

where the radian frequency ω is related to the frequency f and the time period T of the signal as $\omega = 2\pi f = 2\pi/T$. The time function $a_1 \cos(\omega t) + b_1 \sin(\omega t)$ is defined as the *fundamental* of ripple, while terms at higher values of n are *harmonics.* Because most smoothing filters have a low-pass nature, they attenuate the harmonics more than the fundamental. For design purposes, it is often sufficient to consider the gain at the fundamental ripple frequency, $\omega = \omega_r$. Consider input and output voltage phasors taken at this frequency. We will express the gain in decibels:

$$\text{Gain (dB)} = 20 \log_{10} \left| \frac{V_O(j\omega_r)}{V_I(j\omega_r)} \right| \tag{16}$$

For arbitrary complex frequency s, the transfer function can be written as

$$A(s) = \frac{V_O(s)}{V_I(s)} \tag{17}$$

PASSIVE SMOOTHING FILTERS

Let us now approach the design of passive filters for smoothing. In principle, smoothing filters might be designed like the low-pass filters employed for signal processing (1,2). But unlike signal filters, where existing source and load impedances can be augmented with resistance to the desired value, smoothing filters must avoid resistance wherever possible in the interest of efficiency. Therefore they generally have ill-defined source and load impedances, and standard low-pass filter tabulations are inapplicable except in special circumstances.

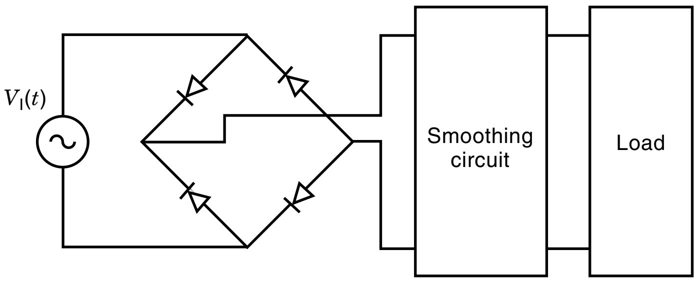

Figure 5. Diode bridge with output smoothing filter and load.

To illustrate the point, consider a smoothing filter fed from a diode-bridge rectifier, as shown in Fig. 5. The rectifier's effective output impedance is not clear, and in fact it can vary considerably, even within a cycle. Even the basic circuit action is unclear, since the timing of diode turn-on and turn-off will depend on the smoothing circuit and the load. Conventional circuit theory is difficult to apply in such circumstances.

In almost any practical smoothing application, the dynamic characteristics of the load are unknown or poorly defined. Given a dc load specified as drawing current I at voltage V, one might assume a resistive load, with $R = V/I$. In reality, this may not be the case: At one extreme, the load could approximate a constant current of I (a linear voltage regulator tends to behave this way); at the other it might approach a constant voltage of V (a battery to be charged is one common case). In between, the load could be resistive, capacitive, or inductive, with the possibility of nonlinear behavior or time variation. If information about the source and load impedances is available to the filter designer, it should be utilized. Otherwise, assumptions must be made, with an attempt toward worst-case analysis.

RC Smoothing

The simplest type of passive smoothing filter is the single-section RC low-pass filter of Fig. 6(a). This is widely used for decoupling purposes at low power levels, where its limited efficiency is not a concern. For this filter, worst-case design can

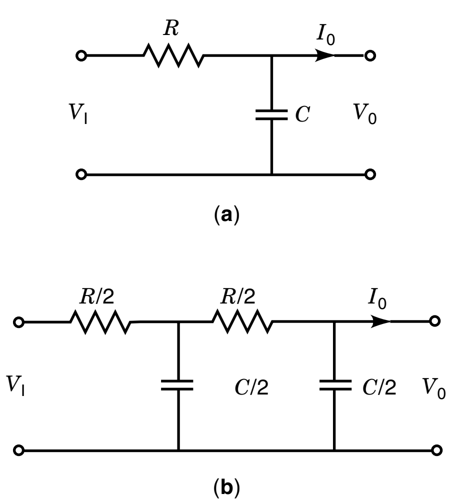

Figure 6. *RC* smoothing filters, single section and multisection.

consider the situation with an input voltage source and a current-source load. When fed from a voltage source v_I and delivering an output current I_O, the filter output voltage is $v_I - I_O R$, and its efficiency is

$$\eta = 1 - \frac{I_O R}{V_I} \tag{18}$$

The value of R should be chosen to give an acceptable efficiency. The voltage gain transfer function is

$$A(s) = \frac{1}{1 + sCR} \tag{19}$$

The gain at the ripple radian frequency ω_r is

$$\begin{aligned}\text{Gain (dB)} &= -20\log_{10}\sqrt{1 + (\omega_r CR)^2} \\ &\approx -20\log_{10}(\omega_r CR) \qquad \text{if } \omega_r CR \gg 1\end{aligned} \tag{20}$$

The one-section filter has a corner frequency at $\omega_0 = 1/(RC)$, and the gain falls asymptotically (i.e., for $\omega >> \omega_0$) at 20 dB/decade, or 6 dB/octave. Thus, with R selected on efficiency grounds, the value of C is chosen to give the desired attenuation of the fundamental ripple frequency.

Two or more RC networks can be cascaded to form a multisection filter, as in Fig. 6(b). A two-section filter, with each section comprising $R/2$ and $C/2$, provides

$$A(s) = \frac{1}{1 + \frac{3}{4}sRC + \frac{1}{16}(sRC)^2} \tag{21}$$

The gain is

$$\begin{aligned}\text{Gain (dB)} &= -20\log_{10}\sqrt{1 + \tfrac{7}{16}(\omega_r RC)^2 + \tfrac{1}{256}(\omega_r RC)^4} \\ &\approx 24 - 40\log_{10}(\omega_r RC) \qquad \text{if } \omega_r RC \gg 1\end{aligned} \tag{22}$$

The corner radian frequency is $\omega_0 = 4/(RC)$, and the gain falls asymptotically at 40 dB/decade or 12 dB/octave. A three-section filter, with $R/3$ and $C/3$ in each section, has

$$A(s) = \frac{1}{1 + \frac{2}{3}sRC + \frac{5}{81}(sRC)^2 + \frac{1}{729}(sRC)^3} \tag{23}$$

and gain of

$$\begin{aligned}&\text{Gain (dB)} \\ &= -20\log_{10}\sqrt{1 + \tfrac{26}{81}(\omega_r RC)^2 + \tfrac{13}{6561}(\omega_r RC)^4 + \tfrac{1}{531{,}441}(\omega_r RC)^6} \\ &\approx 57 - 60\log_{10}(\omega_r RC) \qquad \text{if } \omega_r RC \gg 1\end{aligned} \tag{24}$$

The corner frequency is now $\omega_0 = 9/(RC)$, and the gain falls at 60 dB/decade or 18 dB/octave. It is rare to encounter more than three sections in practice. The efficiency of these multisection filters depends only on the total resistance R, so it is the same as for a single section.

We can represent the gain conveniently with a Bode plot, which shows gain in decibels as a function of frequency (on a log scale in hertz) over the range of interest. The Bode plots in Fig. 7 represent RC filters with one to three sections.

Given a total resistance R and a total capacitance C, what is the best number of sections to use? For n sections, the corner frequency is proportional to n^2, while the slope of the high frequency asymptote is $-20n$ dB/decade. A two-section filter gives greater attenuation than one section if $\omega_r RC > 12.0$; otherwise it is more effective to use one section. Similarly, three sections are better than two if $\omega_r RC > 32.9$ (with analysis based on the fundamental). When deciding on the number of sections, practical factors should also be taken into account, such as availability of suitable components, their size, cost, PCB area occupied, and the effect upon reliability.

As an example, let us consider the design of a filter with the following specifications: $V_I = 12$ V, $I_O = 10$ mA, $\eta = 98\%$, $f_r = \omega_r/(2\pi) = 120$ Hz, gain ≤ -30 dB. From the efficiency formula, Eq. (18), we find $R = 36\ \Omega$. Using the approximate gain formulae, we obtain the following values:

n	$\omega_r RC$	C	Practical Component Values
1	31.62	1165 μF	36 Ω, 1200 μF
2	22.39	825 μF	2 × 18 Ω, 2 × 470 μF
3	28.18	1038 μF	3 × 12 Ω, 3 × 330 μF

The two-section filter might be considered the best because it has the lowest value of $\omega_r RC$ and therefore the lowest total capacitance. But a single-section filter is simpler, and it might be the preferred design solution in practice.

There are more sophisticated RC smoothing circuits (3,4), including the *parallel T* notch network (4,5). These were used in the past when low ripple was essential. Today, active smoothing methods are a better alternative to these rather sensitive circuits.

LC Smoothing

Single-stage and two-stage LC smoothing filters are shown in Fig. 8. For these filters, the efficiency is 100% in principle, although in practice it will be limited by parasitic resistances within the components. For the single-stage circuit, the voltage gain transfer function is

$$A(s) = \frac{1}{1 + s^2 LC} \tag{25}$$

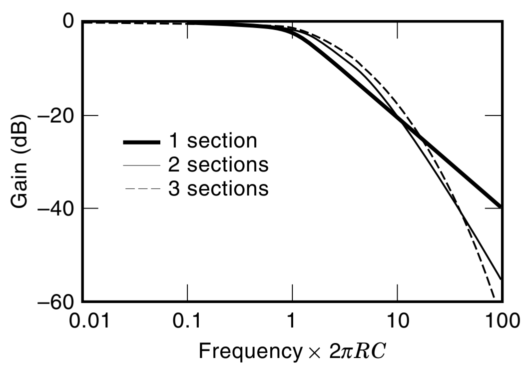

Figure 7. Frequency response for multisection RC filters.

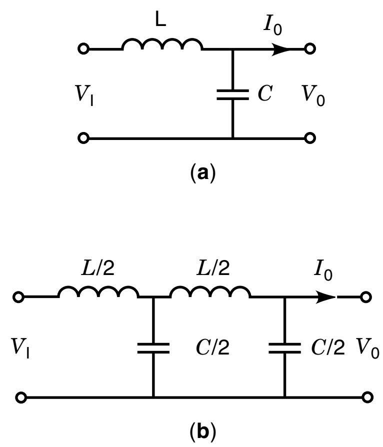

Figure 8. LC smoothing filters, single section and multisection.

The gain is

$$\text{Gain (dB)} = -20\log_{10}|1 - \omega_r^2 LC| \tag{26}$$

The LC product is associated with a resonant frequency $\omega_0 = 1/\sqrt{LC}$. With this in mind,

$$\begin{aligned}\text{Gain (dB)} &= -20\log_{10}|1 - (\omega_r/\omega_0)^2| \\ &\approx -40\log_{10}(\omega_r/\omega_0) \qquad \text{if } \omega_r/\omega_0 \gg 1\end{aligned} \tag{27}$$

It is essential that the resonant frequency be significantly lower than the ripple frequency; otherwise, the filter could actually increase the ripple.

When two LC networks are cascaded to form a multisection filter, additional resonances are created, and it becomes even more important to ensure that $1/\sqrt{LC}$ is well below the ripple frequency. The two-section filter with component values $L/2$ and $C/2$ has the transfer function

$$A(s) = \frac{1}{1 + \frac{3}{4}s^2 LC + \frac{1}{16}(s^2 LC)^2} \tag{28}$$

The gain is

$$\text{Gain (dB)} = -20\log_{10}|1 - \tfrac{3}{4}\omega_r^2 LC + \tfrac{1}{16}(\omega_r^2 LC)^2| \tag{29}$$

With $\omega_0 = 1/\sqrt{LC}$ once again, the gain becomes

$$\begin{aligned}\text{Gain (dB)} &= -20\log_{10}|1 - \tfrac{3}{4}(\omega_r/\omega_0)^2 + \tfrac{1}{16}(\omega_r/\omega_0)^4| \\ &\approx 24 - 80\log_{10}(\omega_r/\omega_0) \qquad \text{if } \omega_r/\omega_0 \gg 1\end{aligned} \tag{30}$$

The filtering effect as frequency increases is much larger than for a two-section RC smoother. However, the frequency behavior is more complicated, having two resonant peaks. A Bode diagram for one-, two-, and three-section LC filters with no load is shown in Fig. 9. For the single-section filter, the ripple frequency must be at least $\sqrt{2}$ times the resonant frequency to ensure some reduction. Frequencies more than about five times the resonant value are strongly attenuated.

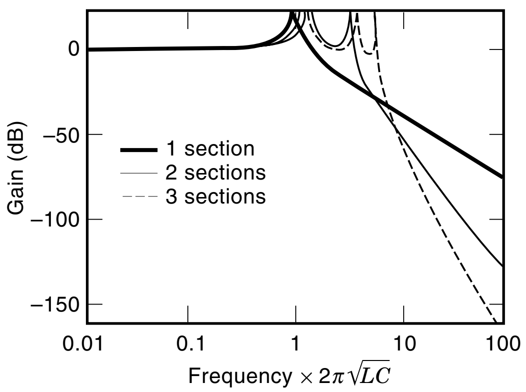

Figure 9. Frequency response for multisection *LC* filters.

For the two-section filter, the ripple will increase without bound if the ripple frequency is 1.23 or 3.24 times the resonant frequency $\omega_0 = 1/\sqrt{LC}$, but again the filter is effective if the ripple value is at least five times the resonant value. The two-section filter gives better results than the single-section filter provided that $\omega_r\sqrt{LC} > 5.2$. The three-section filter has peaks near $1.34\omega_0$, $3.74\omega_0$, and $5.41\omega_0$, and it is better than the two-section filter only if $\omega_r\sqrt{LC} > 8.8$. The resonance problems make even the three-section filter rarely used for smoothing except at extreme power levels (several kilowatts or more).

The gain parameters for Fig. 9 depend only on the *LC* product, but do not give guidance on the selection of *L* and *C*. One general rule is to choose $\omega_r\sqrt{LC} > 5$. A second requirement can be generated based on the impedance needs of an ideal dc supply: The output impedance should be much lower than the load impedance. This implies that the impedance of the capacitor across the output terminals should be much lower than the effective load impedance, Z_L. The single-section *LC* filter thus requires

$$\frac{1}{\omega_r C} \ll |Z_L| \quad \text{or} \quad C \gg \frac{1}{\omega_r |Z_L|} \tag{31}$$

In the preceding *RC* example, the load draws 10 mA from a 12 V source. The effective load impedance is 12 V/10 mA = 1200 Ω. For a single-stage *LC* filter with ripple frequency of 120 Hz, this requires $C >> 1.1\ \mu$F. A value of 100 μF will be suitable. With the resonance requirement $\omega_r\sqrt{LC} > 5$, we find $L > 0.44$ H.

The actual performance of *LC* smoothing circuits is closely linked to the quality of their components. Any real inductor or capacitor has its own internal *equivalent series resistance* (ESR). In smoothing circuits, ESR values are often not much different from the intended filter component impedance levels. For example, ESR in the output capacitor of an *LC* network can limit the ability to provide low output impedance. Discussion of the nature of ESR and its effect on filters can be found in Ref. 2.

LC Blocking and Traps: Resonant Smoothing

Since the ripple frequency is well-defined in many systems, there are certain circumstances in which resonance can be used to advantage. Consider the series blocking circuit of Fig. 10. This combination blocks all flow at $\omega_0 = 1/\sqrt{LC}$. The transfer function is

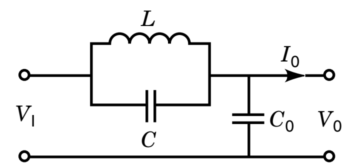

Figure 10. *LC* filter with blocking pair.

$$A(s) = \frac{1 + s^2LC}{1 + s^2L(C + C_O)} \tag{32}$$

At high frequency, $A(s) \approx C/(C + C_O)$, so the design requires $C_O >> C$ to give useful high-frequency attenuation. The unwanted high-gain resonance then occurs at a relatively low frequency $1/\sqrt{L(C + C_O)}$. If $LC = 1/\omega_r^2$ and $C_O = 10C$, this combination will give excellent ripple attenuation at ω_r, and more than 20 dB of attenuation at higher frequencies. As in the basic *LC* filters, the value of C_O is chosen to provide a low output impedance.

Consider again the previous power supply example with 1200 Ω load impedance. With a blocking filter, it is likely that the largest remaining ripple component will appear at $3f_r$, or 360 Hz. An output capacitor value of 5 μF will make the impedance sufficiently low at this frequency. This suggests a capacitor value of 0.5 μF for the blocking pair. The inductor is selected based on $1/\sqrt{LC} = 240\pi$ rad/s, giving $L = 3.5$ H. Blocking filters are most useful when specific high frequencies are involved, rather than power line frequencies. A similar design procedure to block 20 kHz ripple in a switching power supply will lead to a smaller inductor.

Figure 11 shows a shunt trap filter. In this case, the load will not see any ripple at the single frequency $\omega_0 = 1/\sqrt{LC}$ except owing to component ESR values. The transfer function is

$$A(s) = \frac{1 + s^2LC}{1 + s^2(L + L_I)C} \tag{33}$$

At very high frequency, $A(s) \approx L/(L + L_I)$, so the circuit should have $L_I >> L$ to provide good high-frequency attenuation. This circuit acts as the dual of the blocking circuit. The impedance of the inductor L_I should be much higher than that of the load to prevent flow of unwanted frequencies. Like blockers, traps are used to eliminate specific high frequencies. In high-power supplies, traps are often used to eliminate particular strong harmonics rather than for broad ripple smoothing. Additional discussion of tuned traps can be found in Ref. 6.

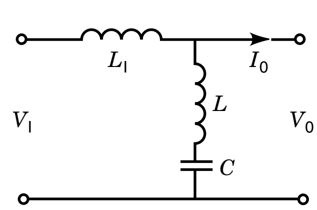

Figure 11. *LC* filter with a trap.

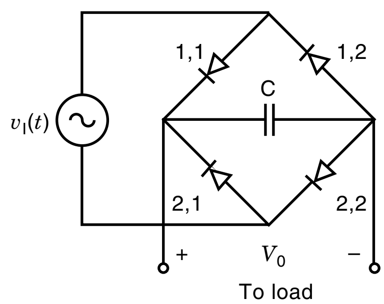

Figure 12. Full-bridge rectifier with capacitive filter.

Capacitive Smoothing for Rectifiers

A rectifier can be used with a purely capacitive smoothing filter. With a resistive load, the structure becomes an RC combination of the filter element and the load. This arrangement, shown in Fig. 12, is sometimes called the *classical rectifier* circuit. It is very common for filtering rectifiers below 50 W, and it is used widely at higher power ratings as well.

In the classical rectifier, the shape of the ripple combines a sinusoidal portion during times when the diodes conduct, and an exponential decay during times when the diodes do not conduct. The nature of the waveform supports useful approximations to the shape without resorting only to the fundamental frequency (6). The circuit output waveforms are given in Fig. 13. The waveform $|v_I(t)|$ is shown as a dotted line for reference. When a given diode pair is on, the output is connected directly, and $v_O = |v_I|$. When the diodes are off, the output is unconnected, and v_O decays exponentially according to the RC time constant: assuming a resistive load, R. Consider the time of the input voltage peak, and assume that the diodes labelled 1,1 and 2,2 are on. In this arrangement, the output voltage matches the input apart from the diode forward drops (which are assumed to be small for the moment) and the input current is $i_I = i_C + i_R$. The arrangement will continue until the diode current drops to zero and the devices turn off. This time t_{off} occurs shortly after the voltage peak, and the time of the peak is a good approximation to the turn-off point.

Once the diodes are off, the output decays exponentially from its initial value. The initial voltage will be $v_I(t_{off})$, and the time constant τ will be the RC product, so

$$\text{Diodes off:} \qquad v_O(t) = V_{pk} \sin \omega_I t_{off} e^{-(t-t_{off})/\tau} \tag{34}$$

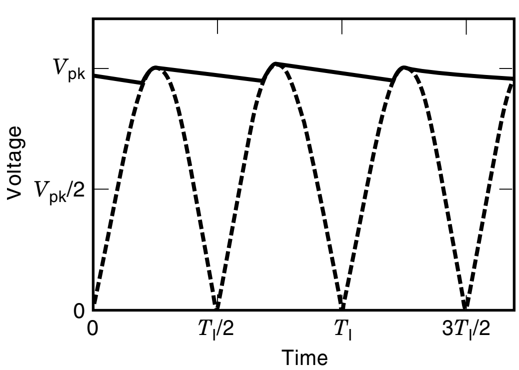

Figure 13. Output voltage of classical rectifier.

This decay will continue as long as the diodes are reverse-biased, that is, $v_O > |v_I|$. When the full-wave voltage increases again during the next half-cycle, two diodes will turn on at time t_{on} as the full-wave value crosses the decaying output. The output voltage maximum is the peak input V_{pk}, while the minimum output occurs at the moment of diode turn-on, t_{on}. Thus the peak-to-peak output ripple is $V_{pk} - |v_I(t_{on})|$. To guarantee small ripple, the output voltage should decay very little while the diodes are off. This means $\tau >> t_{on} - t_{off}$ to keep the ripple low. For small ratios of $(t_{on} - t_{off})/\tau$, the exponential can be represented accurately by the linear term from its Taylor series expansion,

$$e^x \approx 1 + x, \qquad x \ll 1 \tag{35}$$

The time difference $t_{on} - t_{off}$ cannot be more than half the period of $v_I(t)$, so the low ripple requirement can be expressed as $RC >> T_I/2$ if T_I is the input period.

The details of the output voltage waveform and the target of having low ripple lead to several reasonable assumptions that can be made for the smoothing filter, leading in turn to a simplified design framework:

1. The turn-off time occurs close to the voltage peak. It can be assumed to coincide with the peak input voltage, and the output voltage at that moment will be V_{pk}.
2. The voltage decay while the diodes are off is nearly linear, as in Eq. (35). Thus after t_{off}, the voltage falls linearly as $V_{pk}(1 - t/RC)$, with t being measured from the voltage peak.
3. Since the total time of voltage decay never exceeds half the period (for the full-wave rectifier case), the voltage will not be less than $V_{pk}[1 - T_I/(2RC)]$. The peak-to-peak ripple ΔV_O will be no more than $V_{pk}T_I/(2RC)$.
4. The diodes are on just briefly during each half-cycle. The input current flows as a high spike during this interval. Given a turn-on time t_{on}, the peak input current is approximately $C(dv/dt) = \omega C V_{pk} \cos(\omega_I t_{on})$.

All these simplifications require $RC >> T_I/2$, the usual case to produce low ripple.

These simplifications are equivalent to assuming that the output waveform is a sawtooth, with a peak value of V_{pk} and a trough value of $V_{pk}[1 - T_I/(2RC)]$. Therefore, the ripple is also a sawtooth, with a peak-to-peak value of $\Delta V_O = V_{pk}T_I/(2RC)$. This shape yields a simple design equation. Given an output load current I_O (approximately equal to V_{pk}/R if the load is resistive), a desired ripple voltage ΔV_O, and input frequency $f_I = 1/T_I$, we have

$$\Delta V_O = \frac{I_O}{2 f_I C}, \quad \text{or} \quad C = \frac{I_O}{2 f_I \Delta V_O} \tag{36}$$

The capacitance is selected based on the highest allowed load current. Notice that if a half-wave rectifier substitutes for the bridge, the basic operation of the circuit does not change, except that the maximum decay time is T_I instead of $T_I/2$. The factors of 2 in Eq. (36) will not be present.

The sawtooth waveform can be filtered further by connecting an LC or RC passive circuit after the smoothing capacitor. Sawtooth ripple with a peak-to-peak value of ΔV_O corresponds

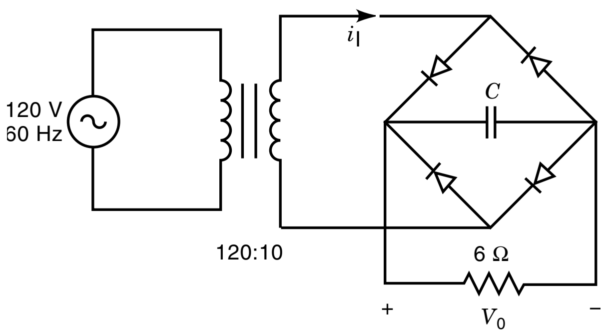

Figure 14. Circuit to provide 12 V output from a smoothed rectifier.

to a ripple waveform fundamental of

$$\frac{\Delta V_O}{\pi}\sin \omega_r t \tag{37}$$

and this $\Delta V_O/\pi$ amplitude provides a good basis for design of further filtering.

The capacitor handles a substantial ripple current, and this should be considered when choosing the component. Once the peak input current $I_{pk} = \omega C V_{pk}\cos(\omega_I t_{on})$ is determined, the rms current in the capacitor can be estimated. The capacitor current will be a series of approximately sawtooth spikes of height I_{pk} and a narrow width $T_I - t_{on}$, and this waveform has an rms value I_C given by

$$I_C \approx \sqrt{\frac{I_{pk}^3}{3\pi \omega_I C V_{pk}}} \tag{38}$$

This expression can be used to determine the ripple current rating requirement of the capacitor.

Capacitive Smoothing Example

Let us illustrate capacitive smoothing by choosing a suitable capacitor for a classical rectifier. The rectifier is to supply 12 V ± 3% to a 24 W load, based on a 120 V, 60 Hz input source. The arrangement needed to solve this problem is shown in Fig. 14.

A transformer will be needed to provide the proper step-down ratio for this design. For completeness, let us include a typical 1 V diode on-state forward drop. The load should draw 24 W/12 V = 2 A, and therefore it is modeled with a 6 Ω resistor. When a given diode pair is on, the output will be two forward drops less than the input waveform, or $|v_I| - 2$ V. We need a peak output voltage close to 12 V. Therefore, the peak value of voltage out of the transformer should be about 14 V. The root mean square (rms) value of v_I is almost exactly 10 V for this peak level. Let us choose a standard 120 V to 10 V transformer for the circuit on this basis.

To meet the ±3% ripple requirement, the output peak-to-peak ripple should be less than 0.72 V. The capacitance should be

$$C = \frac{I_O}{2 f_I \Delta V_O} = \frac{2\text{A}}{2(60\,\text{Hz})(0.72\,\text{V})} = 23\,\text{mF} \tag{39}$$

The approximate methods overestimate the ripple slightly, since the time of the exponential decay is less than $T_I/2$, so a standard 22 mF capacitor will meet the requirements. The waveforms that result from these choices are shown in Fig. 15. Notice that the current into the rectifier bridge does indeed flow as a series of spikes—with a peak value of about 36 A. The rms capacitor current is about 7 A.

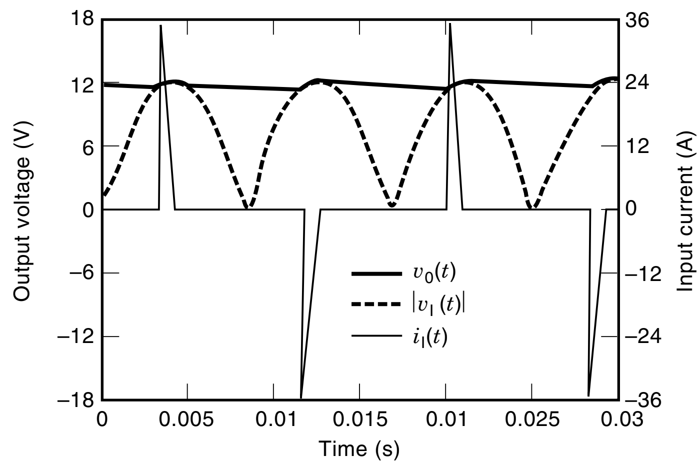

Figure 15. Output voltage and input current for rectifier example.

The assumptions used for simplified rectifier design yield excellent end results, compared with more precise calculations. For example, precise calculation shows that a 22 mF capacitor yields 0.66 V ripple instead of 0.72 V. The linear ripple assumption is a conservative approximation: It always overestimates the actual ripple when exponentials are involved.

Current Ripple Issues

The narrow input current spikes in Fig. 15 beg the question of whether smoothing will be needed for the input current. The high current will produce significant voltage drops in any circuit or device impedances, and it raises issues of coupling through an imperfect ac source. To improve the input current waveform, inductance can be placed in series with the rectifier input. With added inductance, the moment of diode turn-off will be delayed by the inductor's energy storage action.

When the diodes are on, the new circuit is a rectified sine wave driving an inductor in series with the parallel RC load. To analyze the situation, it is convenient to use the fundamental of the ripple voltage expected to be imposed on the inductor-capacitor-load combination. For large inductor values, this ripple will not exceed that of the rectified sinusoid. For small inductor values, this ripple is approximately the sawtooth waveform with the capacitor alone. The function $|V_{pk}\sin(\omega_I t)|$ has a fundamental component of amplitude $4V_{pk}/3\pi$, while the sawtooth has the fundamental amplitude given in Eq. (37). Figure 16 shows an equivalent circuit based on the fundamental of the imposed ripple voltage.

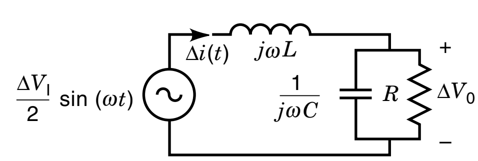

Figure 16. Fundamental equivalent circuit for ripple evaluation in a rectifier.

From the circuit in Fig. 16, we can compute the current drawn from the fundamental source, and then we can use a current divider computation to find the ripple imposed on the load resistance. The result for the peak-to-peak output ripple voltage as a function of the input peak-to-peak ripple voltage ΔV_I (estimated based on the fundamental) is

$$\Delta V_O = \frac{\Delta V_I}{(1 + j\omega L/R - \omega^2 LC)} \tag{40}$$

To make sure the inductor gives a useful effect, it is important that $\omega_0 = 1/\sqrt{LC}$ be significantly less than the ripple frequency ω_r.

Consider again the example 12 V supply described previously. The ripple frequency is 240π rad/s. Inductor values up to about 100 μH have little effect, or could even increase the ripple, owing to resonance. An inductance of 200 μH yields a value of $|\Delta V_O/\Delta V_I| = 0.67$. What value of imposed input ripple should we use? The value $\omega_r^2 LC$ in the denominator of Eq. (40) is 2.5, which is only a little larger than 1. The input ripple will be nearly that of the capacitor alone, and the inductor would be expected to reduce ripple by about 30%. Simulation results were computed for the complete rectifier, and they showed a reduction by 28% to 0.042 V. At the same time, the peak value of i_I dropped from almost 36 A to only 8.2 A.

Figure 17 shows the current i_I for no inductance, for 200 μH, and for a 2 mH inductor. The current waveforms show how the turn-off delay brings down the peak value and makes the current smoother. One important aspect is that the turn-off delay decreases the peak voltage at the output. For example, the 2 mH case provides a 9 V output instead of the required 12 V. For a larger inductor, the actual output voltage would be the average of the rectified sine wave, equal to $2V_{pk}/\pi$. In the limit of $L \to \infty$ the current i_I becomes a square wave with a peak value equal to the load current. More complete discussion of designs of LC filters for rectifier smoothing can be found in Refs. 3, 4, and 6.

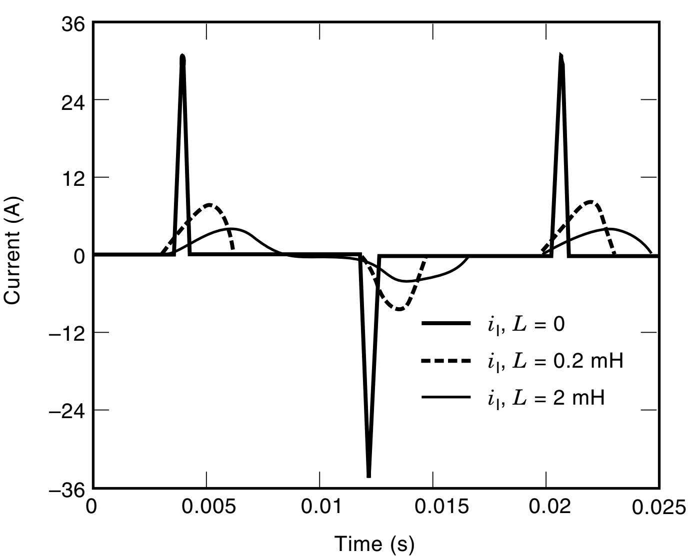

Figure 17. Rectifier input current when inductance is added.

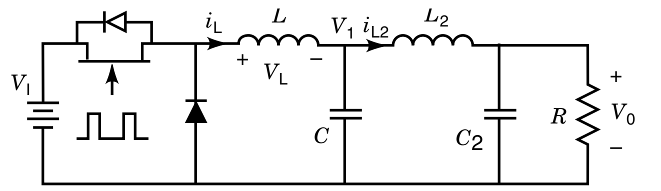

Figure 18. A dc–dc buck converter with two-stage LC output filter.

Example: Smoothing for dc–dc Converters

A dc–dc converter application will help illustrate LC smoothing design and a simplified approach. A typical switching dc–dc converter, with a two-stage LC output filter, is shown in Fig. 18. The semiconductor devices act to impose a voltage square wave at the filter input terminals.

The key to a simplified approach is this: At each node within the filter, it is desired that the ripple be very low. This simple concept means, for instance, that the voltage V_1 in Fig. 18 should be nearly constant. If that is true, the inductor L is exposed to a square voltage waveform. The various waveforms are given in Fig. 19. Since the inductor voltage $v_L = L(di/dt)$, the inductor current in Fig. 19(b) can be determined from

$$i_L(t) = \int \frac{v_L(t)}{L}\, dt \tag{41}$$

The integral of a square wave is a triangle wave. This supports a linear ripple assumption for further analysis.

With linear ripple, the inductance L becomes relatively easy to select. Consider a case in which an inductor is exposed to a square wave of amplitude V_{pk}, a frequency of f, and a pulse width of DT, as illustrated in Fig. 19(a). In the step-down circuit of Fig. 18, this would lead to an average output voltage of DV_{pk}. While the voltage is high, the inductor is exposed to $(1 - D)V_{pk} = L(di_L/dt)$. Since the ripple is linear, this can be written $v_L = L(\Delta i_L/\Delta t)$, with $\Delta t = DT$. Now, the inductance can be chosen to meet a specific current ripple requirement,

$$L = \frac{v_L DT}{\Delta i_L} \quad \text{and} \quad \Delta i_L = \frac{v_L DT}{L} \tag{42}$$

This simple but powerful expression leads to a quick selection of values once the circuit requirements are established.

A special value of inductance from Eq. (42) is the one that sets current ripple to ±100%. This is the minimum inductance that will maintain current flow $i_L > 0$ and is termed the *critical inductance*, L_{crit}. Setting a specific ripple level is equivalent to setting the ratio L/L_{crit}. For example, if the current ripple is to be ±10%, the inductance should be ten times the critical value, and so on.

Now consider the capacitor C that follows the inductor. The desire for low ripple means that the current in inductor L_2 should be almost constant. The current in the capacitor C will be very nearly the triangular ripple current flowing in L. Since $i_C = C(dv_C/dt)$, we have

$$v_C = \int \frac{i_C(t)}{C}\, dt \tag{43}$$

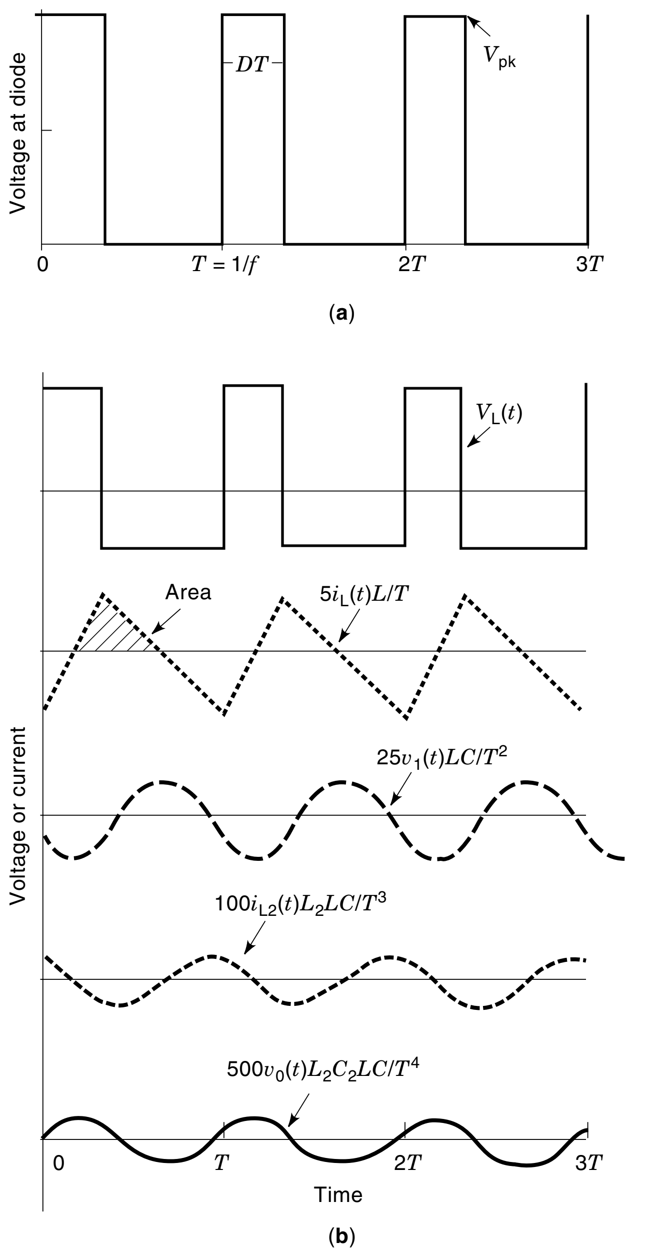

Figure 19. Current and voltage waveforms at points within the LC filter of Fig. 18. (a) Diode voltage. (b) Voltages and currents in the smoothing filter.

The integral of a triangle is a piecewise-quadratic waveform. Of more immediate concern is the effect on voltage ripple, as shown in Fig. 19(b). When the capacitor current is positive, the voltage will be increasing. The total amount of the voltage increase, Δv_1, will be proportional to the shaded area under the triangle, and

$$\Delta v_1 = \frac{1}{C}\frac{1}{2}\frac{T}{2}\frac{\Delta i_L}{2} = \frac{\Delta i_L T}{8C} \tag{44}$$

With the ripple current value from Eq. (42), this means that the ripple on voltage V_1 is

$$\Delta v_1 = \frac{v_L D T^2}{8LC} \tag{45}$$

To provide low relative ripple $\Delta v_1/v_L$, the resonant radian frequency $1/\sqrt{LC}$ must be well below the square-wave radian frequency $2\pi/T$. This is easy to see by requiring $\Delta v_1/v_L << 1$ in Eq. (45). Then

$$\frac{DT^2}{8LC} \ll 1, \qquad \frac{1}{\sqrt{LC}} \ll f\sqrt{\frac{8}{D}} \tag{46}$$

The next inductor L_2 should provide almost constant output v_O, so it is exposed to the piecewise quadratic ripple voltage from capacitor C. Analysis of this waveform is more complicated, but it can be approximated well as by a sine wave with peak value $\Delta v_1/2$. The fundamental should provide a good basis for further stages. Then the approximate sinusoidal waveform appears entirely across L_2. The ripple current in L_2 is

$$i_{L2} = \frac{1}{L_2}\int \frac{\Delta v_1}{2}\sin(\omega_r t)\,dt = \frac{\Delta v_1}{2L_2\omega_r}\cos(\omega_r t) \tag{47}$$

Since $\omega_r = 2\pi/T$, the peak-to-peak current ripple in L_2 is

$$\Delta i_{L2} = \frac{\Delta v_1 T}{2\pi L_2} = \frac{v_L D T^3}{16\pi L_2 LC} \tag{48}$$

By a similar process, the final output voltage ripple is

$$\Delta v_O = \frac{\Delta i_{L2} T}{2\pi C_2} = \frac{v_L D T^4}{32\pi^2 L_2 C_2 LC} \tag{49}$$

Since these relationships are based on ideal results for each part, the assumption here is that the ripple is reduced significantly at each stage. This requires $1/\sqrt{L_2C_2} < 2\pi/T$, and so on.

Actually, it is unusual in the context of dc–dc conversion to reduce the ripple substantially in each LC filter stage. More typically, the first stage performs the primary ripple reduction, while the second stage uses much smaller components to filter out the effects of ESR in C.

ACTIVE SMOOTHING

In active smoothing, circuits that resemble amplifiers are used in addition to storage elements for the smoothing process. Both dissipative and nondissipative approaches exist, but dissipative methods are the most common. The energy arguments require dissipative active smoothers to deliver output power below the minimum instantaneous input power. For this reason, most dissipative active methods combine smoothing and regulation. Voltage regulators are covered in a separate article, so just a short introduction is provided here.

Series-Pass Smoothing

Figure 20 shows a simple *series-pass* circuit for smoothing and regulation. In the arrangement shown, a constant low-

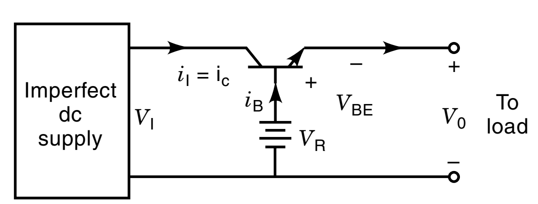

Figure 20. Series-pass active smoothing and regulating circuit.

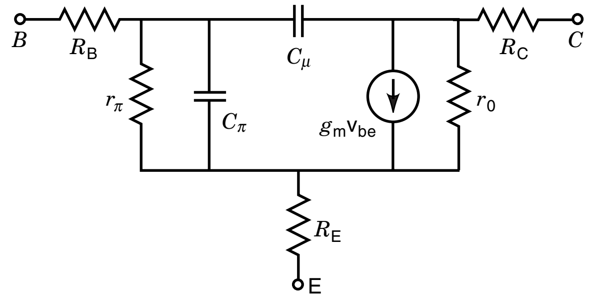

Figure 21. Hybrid-π transistor model.

power reference voltage V_R is produced and is then used to drive the base of a *pass transistor*. In its active region, the transistor exhibits a nearly constant base–emitter voltage drop, and the current i_C equals βi_B. The emitter voltage will be $V_R - V_{BE}$, which is nearly constant. If β is high, the base current will be low, and the voltage V_{BE} will change little as a function of load.

From a smoothing perspective, the interesting aspect of the series-pass circuit is that the output voltage is independent of the input voltage, provided that the input voltage is high enough for proper biasing of the transistor. This is exactly representative of the dissipative smoothing concept. For example, if the output is intended to be 5 V and if the transistor voltage V_{CE} should be at least 2 V for proper bias in the active region, then any input voltage higher than 7 V will support constant 5 V output. The input voltage can vary arbitrarily, but the output will stay constant as long as it never dips below 7 V.

The efficiency of a series-pass circuit is easy to compute. If β is very large, the emitter current $I_E = I_O$ will equal the collector input current $I_C = I_I$. The efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{V_O I_O}{V_I I_I} = \frac{V_O}{V_I} \qquad (50)$$

High efficiency demands that the input and output voltages be as close as possible.

A real series-pass circuit will still exhibit a certain level of ripple. The basis for this can be seen in the small-signal hybrid-π model of a bipolar transistor (7), shown in Fig. 21. In the case of fixed voltage at the base terminal and voltage with ripple at the collector terminal, the stray elements r_0 and C_μ both provide a path by which ripple current can reach the output. In a good-quality transistor, r_0 can be made very high, so internal capacitive coupling to the base terminal is the most important leakage path. In practical applications, it is common to provide a passive smoothing filter stage at the series-pass element input to help remove high-frequency harmonics. The output of the regulator is provided with capacitive decoupling to prevent loop problems such as those described in the Introduction. With these additions, a series-pass smoother can reduce ripple to just a few millivolts even with several volts of input ripple. That is, ripple reductions on the order of 60 dB or more can be achieved. Discussions of integrated circuits that implement series-pass circuits can be found in Refs. 8 and 9.

Shunt Smoothing

Series smoothing makes use of the ability of a bipolar transistor to establish an emitter current that depends on the base input rather than on the input source. Shunt smoothing is a dual of this in certain respects: It makes use of the ability of certain elements to establish a voltage that is independent of the source. In Fig. 22, a basic form and a typical implementation of a shunt regulator are shown. The imperfect dc supply provides energy flow and current i_I through the resistance R_I. The fixed voltage delivers the desired output current I_O. The fixed element makes I_O independent of i_I, and smoothing is accomplished.

The simple circuit of Fig. 22(a) is actually very common in battery-powered devices. When a charger is connected, it is only necessary to make sure that the average value of i_I is greater than I_O to provide both regulation and charging. With the charger is disconnected, the battery continues to maintain operation of the load. In this case, the storage action of the battery means that the instantaneous value of i_I is unimportant; an unfiltered rectifier can be used as the imperfect dc supply, for example. The actual level of output ripple depends on the quality of the battery as a dc source.

The Zener diode circuit of Fig. 22(b) is common for use in generation of reference voltages, and it is also widely used for low-power sensor supply requirements and similar applications. Since the diode does not have any energy storage capability, this smoother requires that the instantaneous value of i_I must always be greater than the desired I_O. If the output current is not well-defined, a worst-case design must estimate the highest possible output current as the basis for the minimum i_I.

In a shunt regulator, there is power loss in the resistor: the square of the rms value of i_I times the input resistance R_I. There is also additional power in the fixed voltage element: the difference $i_I - I_O$ times V_O. The power in the voltage element is lost in the Zener circuit, or it serves as charging power in the battery circuit. The output power P_O is $V_O I_O$. If we could select a value of input current to exactly match the output current (the best-case situation with minimum input

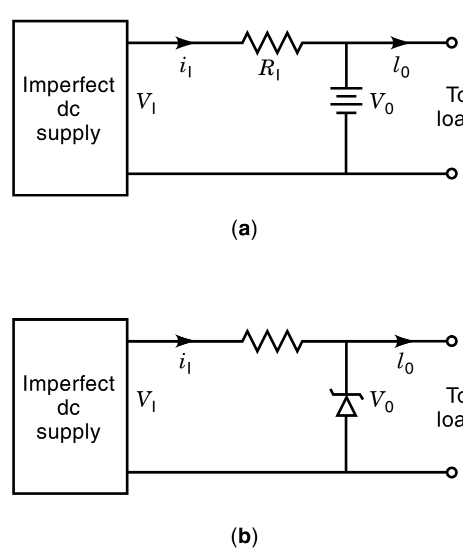

Figure 22. Shunt smoothing alternatives.

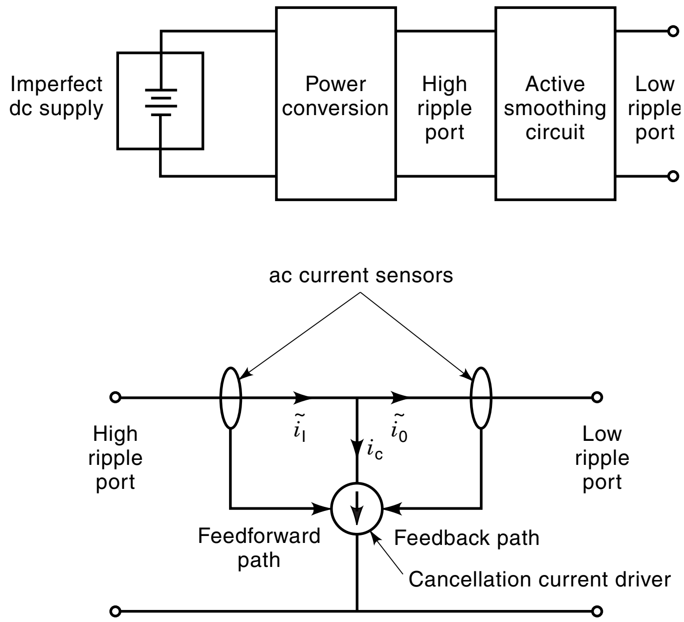

Figure 23. A general active smoothing system and its generic implementation.

current and minimum loss), the loss power P_L would be $I_O^2R_I$. Since the input current is $(V_I - V_O)/R_I$, the efficiency would be

$$\eta = \frac{P_O}{P_O + I_O^2 R_I} = \frac{V_O I_O}{V_O I_O + I_O\left(\dfrac{V_I - V_O}{R_I}\right)R_I} = \frac{V_O}{V_I} \tag{51}$$

Any realistic design must have higher input current. For this reason, a shunt smoother is always less efficient than a series smoother for given input and output voltages. It is a very simple circuit, however, which explains its wide use, especially at low power levels.

Summary of Linear Smoothing

Series and shunt smoothers are termed linear active circuits because they function as amplifiers. The output is a linear function of the selected reference value, and it is independent of the input. Large voltage ratios are troublesome for the linear smoothers. For example, if either type is used for 48 V to 5 V conversion, the best possible efficiency is only about 10% even if ripple is not an issue.

Both series and shunt smoothers benefit if the input source ripple is relatively low, provided that this is used to advantage to keep the voltage ratio low as well. For example, a series smoother with an input voltage of 7 V ± 0.1 V and an output voltage of 5 V can produce an output with less than 1 mV of ripple with efficiency of about 70% if the series-pass element can work with bias levels down to 1.9 V. A shunt smoother can function with even lower bias. With a fixed load level of 100 mA, an input voltage of 5.5 V ± 0.1 V, and an output voltage of 5.00 V, a shunt regulator with efficiency up to about 85% can be designed.

Cancellation Smoothers

In cancellation smoothing (10,11), methods similar to series and shunt circuits are used, but handle only the ripple itself. This avoids the dc dissipation inherent in both of the linear methods, since the dc input to output voltage ratio becomes irrelevant. The general principle of cancellation smoothing can be observed in Fig. 23, which shows a shunt-type *current injection* cancellation smoother. In such a system, the input ripple current $\tilde{i}_I$ is sensed, and the controller creates a cancelling current i_C as close as possible to $-\tilde{i}_I$. The practical version in Fig. 24 shows an input filter inductor and a transformer-based amplifier coupler. With these additions, the current amplifier handles none of the dc current or power, and there is very little dissipation.

Cancellation smoothing can work either on feedforward or feedback principles (or with a combination of these). In feedforward cancellation, the input ripple current is sensed, and an amplifier with good bandwidth and a gain of exactly −1 is needed. The ability of such a system to produce smooth output depends on the accuracy of the gain. For example, if the actual gain is −0.99 instead of −1.00, then the output ripple will be 0.01 times that at the input—a reduction of 40 dB. It is not trivial to produce precise gain over a wide frequency

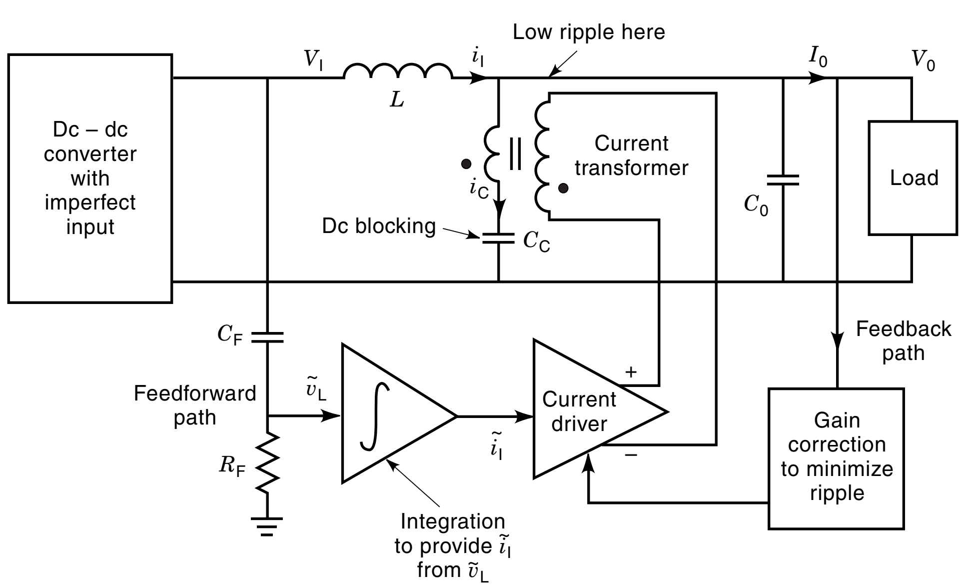

Figure 24. Implementing a feedforward ripple current canceler.

range, but even 10% gain error can provide a useful 20 dB of current ripple reduction.

In feedback cancellation, the operating principle is to measure and amplify the output ripple signal $\tilde{i}_O$ and use this to develop a correction current equal to the gain k times the output ripple. Feedback cancellation has the advantage of correcting ripple caused by noise at the output as well as ripple derived from the imperfect dc supply. High gain is required. For example, if $k = 100$, the output ripple ideally is a factor of 100 lower than the input ripple, and the ripple is reduced by 40 dB. Gain of $k = 1000$ reduces current ripple by up to 60 dB, and so on. This is too simplistic, however. The sensing device for $\tilde{i}_O$ has an associated time lag. If the gain is too high, this time lag will lead to instability. Feedback cancellation is extremely effective in systems with low ripple frequencies (below about 10 kHz), since a brief time lag in the sensor will not have much effect at low speeds.

Cancellation methods, often combining feedforward and feedback techniques, have been used in power supplies at 50 Hz and 60 Hz (12,13), spacecraft systems switching at up to 100 kHz (11), and a variety of dc–dc conversion systems (10,14).

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