MINIMUM SHIFT KEYING

The term minimum shift keying (MSK) refers to a binary digital modulation format. This form of modulation was invented in the 1960s, and some variants of MSK have been considered for usage in large communications systems. In that context, the most widely known form of MSK is Gaussian MSK (GMSK), which has been formally selected and standardized in recent years for usage in the second generation of cellular systems, as well as in cordless telephony and PCS systems.

The special characteristics of an MSK signal are described below in nonmathematical terms. A brief history of the evolution of MSK follows, and the current trends in the field are discussed. The idea of modulation is introduced, and a discussion about some of the MSK concepts is presented. The detailed treatment of MSK and of its variants is then presented, along with some discussions about performance and applications. Advanced topics are covered, such as the applications to mobile communications channels (fading channels), and some generalizations of the MSK format and some recent demodulation and synchronization techniques are discussed.

MODULATION FRAMEWORK

The concept of *modulation* is briefly described here, along with its more specific forms. Then the MSK format is presented as a member of different modulation families. The action of *modulating* implies that a message signal, conveying some form of information, is used to modify another signal, in such a way that the message can be transmitted under a form more appropriate to the transmission medium. Modulating is therefore equivalent to transforming a signal into another one. In the world of wireless communications, the "transformed signal" is usually an electromagnetic wave which, in the absence of a modulating signal (the message), is transmitted through the air at a fixed amplitude and frequency. This unmodulated signal is often called the *carrier.* In analog telephony, the carrier is an electric current wave, while in a fiber optic network, it is a light wave.

When the carrier is modulated, some of its properties are changed according to the message, in such a way that these variations can be interpreted at the receiver as a version of the message. This implies that the action of modulating must be invertible through a corresponding *demodulation.* The carrier can be modulated in amplitude (AM), which implies that the power, or the strength, of the carrier is changed according to the message. The message may also modify the basic frequency of the carrier, which is known as frequency modulation (FM). Phase modulation (PM) of the carrier is another form of transformation, where the transmitted carrier changes its relative phase according to the message. FM and PM signals are intimately related since phase and frequency are related by a time-derivative operation.

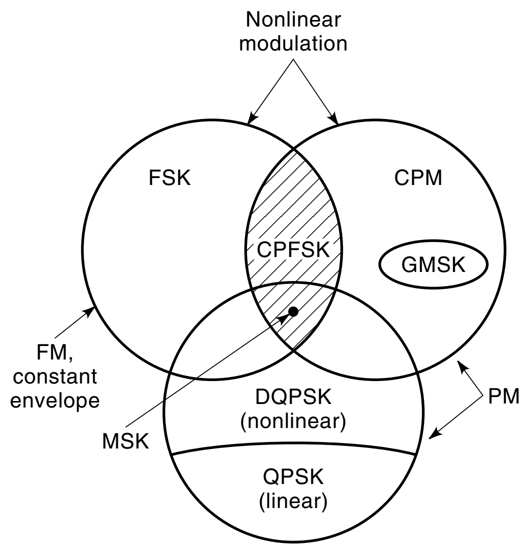

Figure 1. The relationship between some of the possible interpretations of an MSK signal. MSK is a form of CPFSK, and is equivalent to a form of differentially encoded QPSK scheme.

The MSK modulation format can be interpreted as a special case of many modulation characteristics. These interpretations are illustrated in Fig. 1. This view is restricted to digital signals, where a symbol, drawn from a finite set, is transmitted at a given time. The symbols are usually described by a finite number of binary digits (bits). The MSK format can be seen as a binary FM signal, also known as a binary *frequency shift keying* (FSK) signal, or as a form of differentially encoded quaternary PM signal, called *differential quaternary phase shift keying* (DQPSK). In FSK, each symbol (representing one bit or a group of bits) to be transmitted is associated to a given frequency, and the carrier is "keyed" according to the given symbol to transmit. In binary FSK there are two possible frequencies, one for each of the two polarities of interest (+1 or −1). Half of the difference between these two frequencies is referred to as the *frequency deviation.* A minimum shift keying signal can be interpreted as a binary FSK signal, in which the frequency deviation is equal to one quarter of the rate of bit transmission. This value is a minimum, in a sense described in a later section, which is the reason for the name *minimum shift keying.*

An MSK signal, when seen as an FSK signal, is characterized by the fact that the changes in the carrier frequency according to the different bits to transmit, is performed in a controlled manner, such that the phase of this carrier does not experience abrupt transitions at the symbols time boundaries. This form of signal is usually referred to as a *continuous phase FSK* signal (CPFSK). It can be generated by changing the frequency of a single frequency source, according to the symbols to transmit. Since the same source is used for the two symbols, the phase is kept continuous from one symbol transition to the other. The MSK signal, when interpreted as a CPFSK signal, is also a member of the more general class of *continuous phase modulated* (CPM) signals. In general, FSK and CPM signals are special cases of nonlinearly modulated signals, that is, the relationship between the modulating symbols and the modulated signal is nonlinear. (A linear relationship or system is defined as one satisfying both the superposition and the proportionality criteria.)

When the MSK signal is viewed as a DQPSK modulated signal, it is implied that groups of two bits are associated to one of the four phases of a carrier. This form of modulation is nonlinear, and can be generated by using a nonlinear differential encoder, followed by a linear QPSK modulator. Note that a linear modulation scheme is such that the modulating symbols are related to the modulated signal through a series of linear operations. The DQPSK interpretation has a very important consequence, in that it leads to a much more efficient demodulation technique than the one implied by the FSK interpretation.

An MSK signal is not modified in amplitude (since it is an FSK signal), which implies that it is a member of the "constant amplitude" family of signals. This characteristic is important in wireless applications, where nonlinear power amplifiers (class C) are used for the sake of cost reduction and better power efficiency (which helps to extend the battery life of the transceivers). The continuous phase of an MSK signal also has a great influence on the spectral occupancy of the modulated signal. Because the two modulating frequencies are switched in a controlled manner at the symbol transitions, the amount of high frequencies generated in these transitions is less than for an ordinary FSK signal. This gives a more compact frequency spectrum, thereby allowing the use of more channels in a given frequency band (higher spectrum efficiency).

EVOLUTION OF THE MSK FORMAT

It is fair to say that the first mathematical treatment of MSK was published by de Buda (1,2), although some related work can be found in Ref. 3. De Buda introduced the terminology *fast* FSK (FFSK), which represents the same format as MSK. The adjective "fast" is used because more bits per second can be transmitted in a given channel bandwidth, compared with binary phase shift keying (BPSK) (2).

Due to its constant amplitude, its compact frequency spectrum, its versatility in terms of demodulation, and its self-synchronization characteristics, the MSK format was considered for usage in some of the satellite systems of the 1970s and the 1980s. In spite of those good characteristics, MSK has not been used in operational satellite systems, mainly because these advantages have not been considered sufficient to justify the high cost of development of new modulators and demodulators, and the replacement of the QPSK systems already in operation in most systems (4).

Because of its continuous phase, MSK can be differentially detected (it is often called DMSK in this case), and it has been considered particularly attractive for land mobile communications channels, on both land-only and satellite-based systems. The advantage of DMSK over coherently detected MSK is that it can be demodulated with a much simpler receiver (a differential detector), which does not require the precise estimation of the carrier phase. This last point is important in mobile communications, since the mobile channel is such that the received signal is corrupted by time-varying reflections of the transmitted signal (so-called multipath fading). This creates amplitude and phase distortion on the received signal, and greatly complicates the precise carrier frequency and phase estimation.

Another important characteristic of a modulation scheme for mobile communications is the compactness of its power spectrum. In view of the fact that ordinary MSK does not meet the strict spectral requirements of mobile radios, some manipulations of its basic characteristics have been proposed. Some of these modifications are very effective when the MSK signal is generated by modulating a single frequency source [a voltage-controlled oscillator (VCO)]. In particular, the addition of a prefilter before the VCO removes some of the abrupt transitions in the data streams, and can produce a power spectrum with smoother characteristics than that of ordinary MSK. Depending on the form of the prefilter, the spectral occupancy of the resulting MSK-type signal may also be lower than that for straight MSK. Gaussian MSK, in which the prefilter impulse response follows a Gaussian function, was introduced by Murota and Hirade (5). This format has attracted much attention, and GMSK is the member of the MSK family most used in current communications systems. The Pan-European GSM system, the Digital European Cordless Telecommunications (DECT) scheme, as well as the DCS 1800 and PCS 1900 systems use this form of modulation (6).

As indicated before, the original MSK format was the source of abundant work, and many other variants and techniques have been proposed. The more important concepts have been briefly discussed in this introduction. Some others, requiring the help of some mathematics, will be mentioned in the following sections.

BASIC MATHEMATICAL THEORY OF MSK AND GMSK SIGNALS

In this section, minimum shift keying and Gaussian MSK signals are mathematically defined and characterized. In so doing, the concepts of complex baseband signals are utilized. A brief review of these basic concepts is given in Appendix 1. The MSK and GMSK signals are described below, in terms of their time and frequency characteristics. Their demodulation aspects over an additive white Gaussian noise (AWGN) channel are also covered. These developments over the static AWGN channel allow a basic understanding of the theory, and are a good introduction to the more advanced concepts.

Description of MSK Signals

As indicated above, an MSK signal can be viewed as a member of a number of modulation formats. In what follows, the nonlinear view of MSK is first developed, followed by its linear counterpart. These different characterizations of MSK are useful, not only from the point of view of the signal description, but also for the derivation and understanding of various demodulation structures. Some variants of MSK are discussed, among which GMSK is the most prominent.

MSK as a Nonlinear Modulation Scheme. The most general view of MSK is probably that of CPM. A general class of CPM signals can be defined, using the complex baseband representation (7,8)

$$v_{\text{CPM}}(t) = A \exp\left[j2\pi \sum_{n=-\infty}^{\infty} h_n I_n q(t - nT) + \phi_0\right] \quad (1)$$

where A is the constant amplitude of the carrier, $\{h_n\}$ is a sequence of modulation indices, $\{I_n\}$ is the sequence of symbols to transmit, $q(t)$ is the modulation phase response, T is the symbol interval, and ϕ_0 is an arbitrary initial phase. This initial phase is usually set equal to zero in the analysis. In general, the sequence $\{h_n\}$ varies in a cyclic manner through a set of indices. This produces the so-called *multi-h* type of CPM signals. For M-ary CPM signals, the symbol sequence $\{I_n\}$ is drawn from a set of M real symbols defined as $\{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ (M is normally a power of 2). The symbol rate is then $1/T$ symbols/s, and the corresponding bit rate $(\log_2 M)/T$ bit/s. The phase response $q(t)$ is represented as the integral of some frequency pulse $g(t)$,

$$q(t) = \int_{-\infty}^{t} g(\tau)\, d\tau \quad (2)$$

The function $g(t)$ is usually a smooth pulse shape over a finite time interval $0 \le t \le LT$, and zero outside. Its shape governs the smoothness of the carrier phase, over the sequence of transmitted symbols, and it is usually normalized such that $q(t) = 1/2$ when $t \to \infty$.

Minimum shift keying is defined as in Eq. (1), with a constant modulation index h, such that

$$\begin{aligned} h &= 1/2 \\ I_n &\in \{+1, -1\} \\ g(t) &= \begin{cases} 1/2T & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (3)$$

MSK is therefore a binary CPM modulation scheme, with the phase response given by

$$q_{\text{MSK}}(t) = \begin{cases} t/2T & \text{for } 0 \le t < T \\ 1/2 & \text{for } T \le t \end{cases}$$

The phase of the MSK signal is then (with $\phi_0 = 0$)

$$\phi_{\text{MSK}}(t) = \sum_{k=-\infty}^{n-1} I_k \frac{\pi}{2} + \frac{\pi}{2} I_n \left(\frac{t - nT}{T}\right) \quad nT \le t < (n+1)T \quad (4)$$

with $I_n \in \{+1, -1\}$. Equation (4) indicates that, at the end of every bit interval, the phase has been increased or decreased by $\pi/2$, and that within an interval, this phase increases or decreases linearly.

The linear phase increase from bit to bit is the main characteristic of an MSK signal. By differentiating the instantaneous phase, the instantaneous baseband frequency is obtained as

$$\begin{aligned} f_{\text{MSK}}(t) &= \frac{1}{2\pi} \frac{d\phi_{\text{MSK}}}{dt} \\ &= \frac{I_n}{4T} \text{ c/s} \end{aligned} \quad (5)$$

Equation (5) indicates that the MSK signal is also a *frequency modulated* signal, with its instantaneous frequency given by the carrier frequency plus or minus $\frac{1}{4}$ of the bit rate, depending on the transmitted bit. This is also the definition of a binary FSK signal, with a frequency deviation equal to one quarter of the bit rate. This value is the minimum frequency separation $(f_p - f_m)$ that makes the two MSK complex baseband signaling waveforms, $v_p(t) = \exp[j2\pi f_p t]$ and $v_m(t) = \exp[j2\pi f_m t]$, orthogonal over a symbol interval of length T (8). Because the phase of the transmitted signal is continuous, MSK is also a CPFSK format, as indicated in Fig. 1.

MSK as a Linear Modulation Scheme. The *offset quadrature PSK* (OQPSK) modulation scheme (9) is given mathematically by

$$v_{\text{OQPSK}}(t) = \sum_{n=-\infty}^{\infty} b_{2n} u(t - 2nT) + j b_{2n+1} u(t - 2nT - T) \quad (6)$$

where $b_n \in \{+1, -1\}$ and $u(t)$ is a time-limited pulse on the interval $[-T, T]$. Equation (6) represents a two-dimensional linear modulation scheme, in which the binary symbol stream is divided in two sets: the even symbols set $\{b_{2n}\}$, multiplying the time-shifted versions of the pulse in the real part of the complex signal, and the odd symbols set $\{b_{2n+1}\}$, multiplying the offset (by T) time-shifted pulse in the imaginary part. This form of modulation is linear, since the modulating symbols are linearly filtered with the pulse shape. In any symbol interval, the combination of the binary symbols b_{2n} and b_{2n+1} into a single complex quaternary symbol $b_n + jb_{n+1}$ produces a modulation scheme with four distinct symbols. If $u(t)$ is equal to one over $[-T, T]$, and zero otherwise, $v_{\text{OQPSK}}(t)$ takes on four distinct values: $\{1 + j, -1 + j, -1 - j, 1 - j\}$. The fact that the imaginary part is offset by T with respect to the real part implies that there cannot be a phase jump larger than $\pm\pi/2$ from one symbol to the other.

Consider a pulse shape of the form

$$u(t) = \cos\left(\frac{\pi t}{2T}\right) r_{2T}(t) \quad (7)$$

where $r_{2T}(t)$ is one over the interval $[-T, T]$, and zero otherwise. Multiply the sequence of modulating bits $\{b_n\}$ by the alternating sequence $\{e^{j\pi n/2}\}$, and filter with $u(t)$. This gives

$$\begin{aligned} v_{\cos}(t) &= \sum_{n=-\infty}^{\infty} b_n e^{j\pi n/2} \cos\left[\frac{\pi(t - nT)}{2T}\right] r_{2T}(t - nT) \\ &= \sum_{n=-\infty}^{\infty} b_{2n} \cos(n\pi) \cos\left[\frac{\pi(t - 2nT)}{2T}\right] r_{2T}(t - 2nT) \\ &\quad + j b_{2n+1} \cos(n\pi) \sin\left[\frac{\pi(t - 2nT)}{2T}\right] r_{2T}(t - 2nT - T) \end{aligned} \quad (8)$$

which is of the form of Eq. (6). The phase of $v_{\cos}(t)$ is given by

$$\phi(t) = b_n b_{n+1} \frac{\pi(t - nT)}{2T} + \sum_{k=-\infty}^{n-1} b_k b_{k+1} \frac{\pi}{2} \quad nT \le t < (n+1)T$$

Defining

$$c_n = b_n b_{n+1}$$

Equation (8) becomes

$$v_{\cos}(t) = \exp\left\{ j\left[\sum_{k=-\infty}^{n-1} c_k \frac{\pi}{2} + \frac{\pi}{2} c_n \left(\frac{t-nT}{T}\right)\right]\right\} \qquad (9)$$
$$nT \le t < (n+1)T$$

The phase of Eq. (9) follows the form of the MSK phase of Eq. (4), as long as the bits to transmit are differentially encoded as

$$b_n = I_n b_{n-1} \qquad (10)$$

Then $c_n = I_{n+1}$ and MSK can be seen as a *differentially encoded* linear modulation scheme. The linear relationship exists between $v_{\cos}(t)$ and the differentially encoded bits $\{b_n\}$, while the I_n's are nonlinearly related to $v_{\cos}(t)$. The pulse shaping can be interpreted as being performed by half sinusoids of length $2T$, as opposed to that of ordinary OQPSK, which is performed with a rectangular window (10). Note the subtle difference in MSK, where the pulse shape is multiplied by $\cos(n\pi)$ in Eq. (8). When MSK is considered as a form of differentially encoded linear modulation, it is often referred to as differential MSK (DMSK). Note here that the terminology used in the literature is not always consistent. The term MSK has been used by some authors to designate the format of Eq. (8), with and without the differential encoding of Eq. (10). Both forms have a constant envelope, and have the same spectral characteristics, but the use of the differential encoding is required for the resulting signal to be an FSK signal. In this article MSK always refers to the definition of Eqs. (1) and (3) [and therefore always implies differential encoding, if a linear modulator following Eq. (8) is used]. With this convention, the term DMSK is therefore redundant. The use of the term DMSK will therefore be limited to the cases where the receivers performs a *differential detection* of an *MSK* signal.

Time and Frequency Characteristics. In the complex plane, the baseband MSK signal of Eq. (9) evolves on a circle of constant amplitude (the radius of the circle determines the amplitude of the transmitted signal), moving by $\pm\pi/2$ rad from one binary symbol to the other. The counter clockwise rotation corresponds to $c_n = +1$ and the clockwise excursion corresponds to $c_n = -1$. By looking separately at the real and the imaginary parts of the baseband signal, the time representation of Fig. 2 is obtained. As indicated above, this representation is that of a differentially encoded OQPSK signal, with a sinusoidal pulse shape. The angular velocity of the complex signal is always equal to $\pm\pi/2T$ rad/s, corresponding to the two signaling frequencies of the MSK signal. The phase variations of a CPM signal are often displayed on a phase tree, which indicates the possible phase evolutions over time. The MSK phase tree is illustrated in Fig. 3.

The linear OQPSK interpretation given in Eq. (6) allows an easy frequency characterization of MSK. For a linear modulation scheme of this form, the power spectral density (PSD) is given by (8)

$$P_{\text{linear}}(f) = \frac{1}{T_s}|U(f)|^2 \Phi_{JJ}(f) \qquad (11)$$

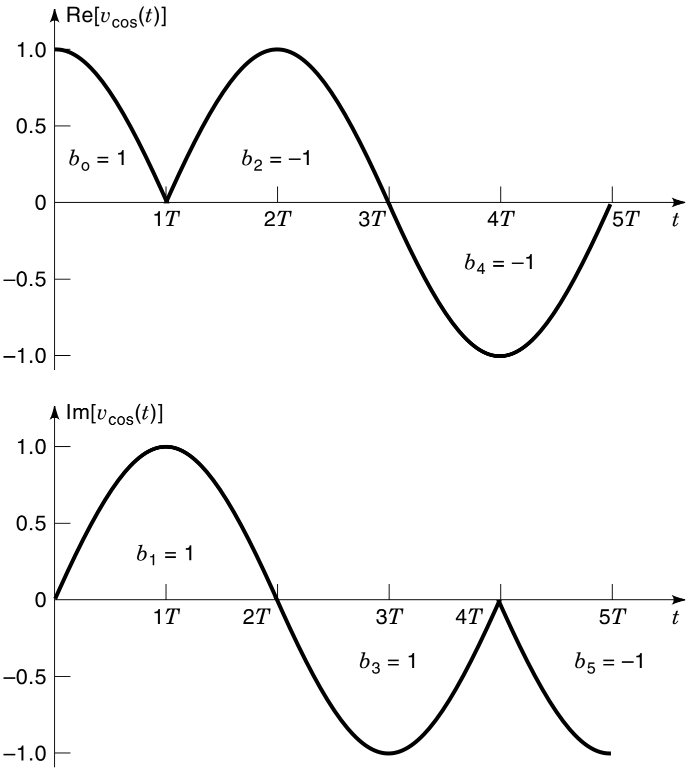

Figure 2. The OQPSK interpretation of an MSK signal, with a sinusoidal pulse shape, for the input sequence I_n = [1, 1, −1, −1, −1, 1]; n = 0, 1, . . ., 5, and b_{-1} = 1.

where T_s is the time duration of the complex symbols [in the case of Eq. (6), $T_s = 2T$], $U(f)$ is the Fourier transform of $u(t)$, and $\Phi_{JJ}(f)$ denotes the discrete Fourier transform of the symbol autocorrelation, that is,

$$\Phi_{JJ}(f) = \sum_{m=-\infty}^{\infty} \phi_{JJ}(m) e^{-j2\pi f m T_s}$$

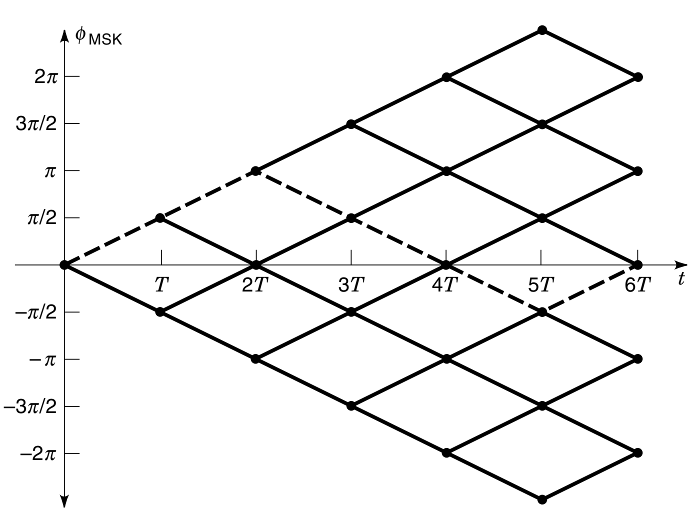

Figure 3. The phase tree of an MSK signal. The phase variations are always linear, with variations of $\pm\pi/2$ from one bit to the other. The dashed line corresponds to the input sequence I_n = [1, 1, −1, −1, −1, 1]; n = 0, 1, . . ., 5.

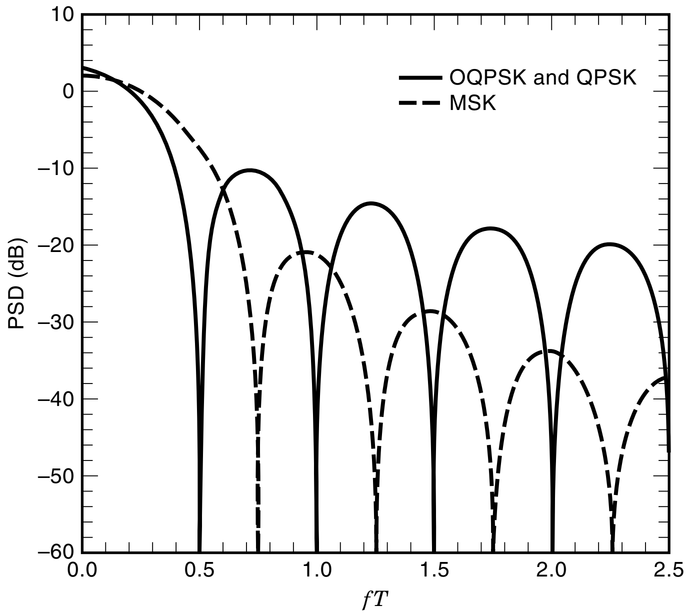

Figure 4. The power spectral density of QPSK, OQPSK, and MSK. The MSK main lobe is larger, but its side lobes fall off more rapidly.

where $\phi_{JJ}(m) = E[J_n^* J_{n+m}]$ is the autocorrelation function of the complex symbols $J_n = b_n + jb_{n+1}$ (8).

Assuming uncorrelated symbols, the PSD is uniquely related to the Fourier transform of the pulse shape [$\Phi_{JJ}(f) = 1$]. For OQPSK (and for ordinary QPSK), the pulse is rectangular over $[-T, T]$, and the PSD has a $\sin x/x$ form given by (8)

$$P_{\text{OQPSK}}(f) = 2T\left(\frac{\sin 2\pi fT}{2\pi fT}\right)^2 \tag{12}$$

For MSK, the pulse is given by Eq. (7), and Eq. (11) applies as long as the symbols are uncorrelated. The PSD is (8)

$$P_{\text{MSK}}(f) = \frac{16T}{\pi^2}\left(\frac{\cos 2\pi fT}{1-16f^2T^2}\right)^2 \tag{13}$$

Note that the expressions of Eqs. (12) and (13) are for unit amplitude waveforms, and that any value $A \neq 1$ implies a multiplication by A^2 of the PSD.

The PSDs of QPSK, OQPSK, and MSK are compared in Fig. 4. It is noted that MSK has a larger main lobe, but that its sidelobes are smaller and decrease faster than for QPSK and OQPSK. This difference is due to the smoother pulse shape in MSK. The compactness of the PSK power spectrum can be measured by the two-sided bandwidth $B_{99\%}$, containing 99% of the total power. For MSK, $B_{99\%} \approx 1.2/T$, while for QPSK and OQPSK, $B_{99\%} \approx 8/T$ (10). Another measure of spectral occupancy is the asymptotic rate of spectral decay. For QPSK and OQPSK, this spectral rolloff is proportional to $|f|^{-2}$, while for MSK it is $|f|^{-4}$ (11). These results indicate that MSK is more spectrally efficient in wideband applications, but that if narrowband channels are required, filtered versions of QPSK or OQPSK are preferable (because of their narrower main lobe). This fact is one of the reasons why filtered QPSK has often been selected over MSK in satellite systems.

Variants of MSK. Following the introduction of MSK, there has been a fair amount of research to find ways to increase the basic MSK spectral efficiency while retaining its constant envelope property. The general starting point for this research was the linear interpretation of Eq. (8), in which the pulse shape $u(t)$ was modified, but always limited to be nonzero over the interval $[-T, T]$. These modifications were referred to as *MSK-type signaling*. Amoroso has proposed a pulse shape given by $u(t) = \cos[(\pi t/2T) - \alpha \sin(2\pi t/T)]$ (12). The constant α is varied over [0, 0.5], in order to modify the fall-off rate of the spectral sidelobes. The case with $\alpha = 1/4$ was called sinusoidal frequency shift keying (SFSK), and has an asymptotic spectral decay of $|f|^{-8}$ (11). Recall that OQPSK and QPSK have a decay of $|f|^{-2}$, and that this value is $|f|^{-4}$ for MSK. Simon (13) analyzed a general form of pulses for MSK-type modulations, and proposed $u(t) = \cos[(\pi/4)(1 - \cos(\pi t/T)]$, for which the asymptotic spectral decay is $|f|^{-6}$. Rabzel and Pasupathy considered a general form of pulse, for which the asymptotic spectral decay is $|f|^{-(4M+4)}$, for M a positive integer (11). MSK and SFSK are special cases of this pulse, with $M = 0$ and $M = 1$, respectively. Bazin (14) studied another general MSK-type pulse shape, from which he proposed the format double sine FSK (DSFSK), with $u(t) = \cos[(\pi t/2T) - 1/3 \sin(2\pi t/T) + 1/24 \sin(4\pi t/T)]$. The asymptotic spectral decay of DSFSK is $|f|^{-12}$. Other authors have defined new families of MSK-type modulation formats, by optimizing the pulse shape with respect to the amount of interference between a given communications channel, and its adjacent neighbors (15–17). More recent results have been obtained in (18), where the pulse shape is allowed to produce a certain degree of amplitude modulation in the transmitted signal. This form of signal is more an OQPSK-type signal than an MSK-type or a CPM one, and its treatment is usually considered in the context of quadrature phase modulated signals.

The MSK format has also been generalized by allowing the modulating binary symbols of Eq. (8) to take on values other than ± 1. This form of modulation was called multi-amplitude MSK (MAMSK) (19). It departs strongly from the constant amplitude characteristics of MSK and its variants, and should be viewed more as a quadrature amplitude modulated (QAM) signal than as an MSK signal (8).

Because minimum shift keying is considered a member of the CPM family, its generalization through the nonlinear interpretation of Eq. (1) links it to a large amount of results and publications. A good reference presenting the most important aspects of CPM signals is Ref. 7. Gaussian MSK is a CPM signal, and is a natural extension of MSK.

MSK and Its CPM Relatives. As indicated before, MSK is a binary FSK format with continuous phase and a modulation index of 0.5. It is defined by Eqs. (1) and (3). Its frequency pulse $g(t)$ is therefore rectangular over the time interval [0, T]. By using nonbinary symbols and a corresponding rectangular pulse $g(t) = 1/2T_s$ over a symbol interval of length T_s, as well as other modulation indices, the class of CPFSK signals is defined. This form of signal can be conceptually generated by driving a controlled frequency source [usually called a voltage controlled oscillator (VCO)] with the modulating symbol stream. This is illustrated in Fig. 5. Note that the differential encoding of the modulating bits is not required here to produce a true MSK signal. A CPM signal, for which the frequency pulse $g(t)$ is nonzero over a single symbol inter-

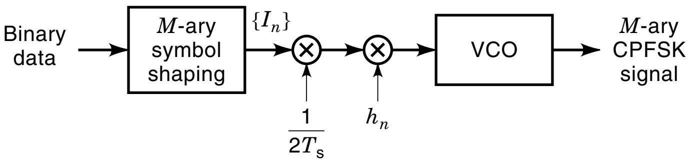

Figure 5. A conceptually simple method to generate CPFSK signals. For MSK, $h = 1/2$. GMSK can also be generated with this configuration, by replacing the single multiplication by $1/2T_s$ with a linear filter.

val only, is called a *full-response* CPM signal. CPFSK is therefore a member of this class, with a rectangular pulse shape. This concept can be generalized by allowing a frequency pulse with an arbitrary form, over an arbitrary number of symbol intervals. The use of more than one symbol interval (possibly an infinite number) defines the family of *partial response* CPM signals (irrespective of the pulse shape). This class of signals can be conceptually generated as in Fig. 5, where the single multiplication by $1/2T_s$ is replaced by a linear filter with impulse response $a(t)$. This response $a(t)$ is related to the frequency pulse $g(t)$ as

$$g(t) = \int_{-T_s/2}^{T_s/2} a(t-\tau)\,d\tau \tag{14}$$

which is the convolution of the premodulation filter $a(t)$ with a rectangular pulse over the interval $[-T_s/2, T_s/2]$. Note that this generalized conceptual way of generating continuous phase frequency modulated signals applies to all CPM signals, as defined in Eq. (1).

The use of a premodulation filter before the VCO has the effect of introducing *intersymbol interference* (ISI) in a controlled way. Encoding the transmitted symbols in such a way has been called correlative coding or partial response signaling (20–22). The term *generalized MSK* has also been used in this case by some authors (23). Correlative coding gives extra freedom for shaping the power spectral density of a modulated signal. When used with CPM signals it allows spectral manipulations while maintaining the constant amplitude of the transmitted signal. It may also produce benefits in the demodulation process, if the information about the controlled ISI is exploited in the receiver.

The work on partial response CPM schemes has been intense, and many new modulation formats were derived (24). Among those, the better known are tamed frequency modulation (TFM) (25), correlative phase shift keying (CORPSK) (26), and Gaussian MSK (GMSK) (5).

Gaussian MSK. GMSK, since its introduction, has enjoyed more and more popularity and has become the constant amplitude modulation scheme of choice in some of the most popular mobile communications systems (6). GMSK is a binary modulation scheme, with $h = 0.5$, and a premodulation filter having a Gaussian-shaped impulse response given by (6)

$$a_{\mathrm{GMSK}}(t) = \sqrt{\frac{2\pi}{\ln 2}}B\exp\left(-\frac{2\pi^2B^2t^2}{\ln 2}\right)$$

where B is the one-sided 3 dB bandwidth of the Gaussian filter. This bandwidth is a variable parameter that allows the system designer to modify the rate of spectral fall-off of the modulated signal. The product of B with the bit duration T is the parameter to modify in order to obtain different spectral characteristics. For MSK, $BT = \infty$ and, as BT decreases the sidelobes level fall off very rapidly. This is illustrated in Fig. 6. The corresponding frequency pulse $g(t)$, obtained from Eq. (14), is

$$g_{\mathrm{GMSK}}(t) = \frac{1}{2}\left\{-\mathrm{erf}\left[-\sqrt{\frac{2}{\ln 2}}\pi B(t-T/2)\right] + \mathrm{erf}\left[\sqrt{\frac{2}{\ln 2}}\pi B(t+T/2)\right]\right\} \tag{15}$$

where erf(x) is the error function, defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

Gaussian MSK, like MSK, has a modulation index equal to 0.5. But, unlike MSK, it is not a binary FSK signal, since it does not correspond to the transmission of two distinct carrier frequencies. Its phase is therefore not linear, and the phase changes from one bit to the other are not necessarily equal to multiples of $\pm\pi/2$. (This is a consequence of the intersymbol interference introduced by the Gaussian filter.) GMSK should therefore be seen as a binary digital FM signal.

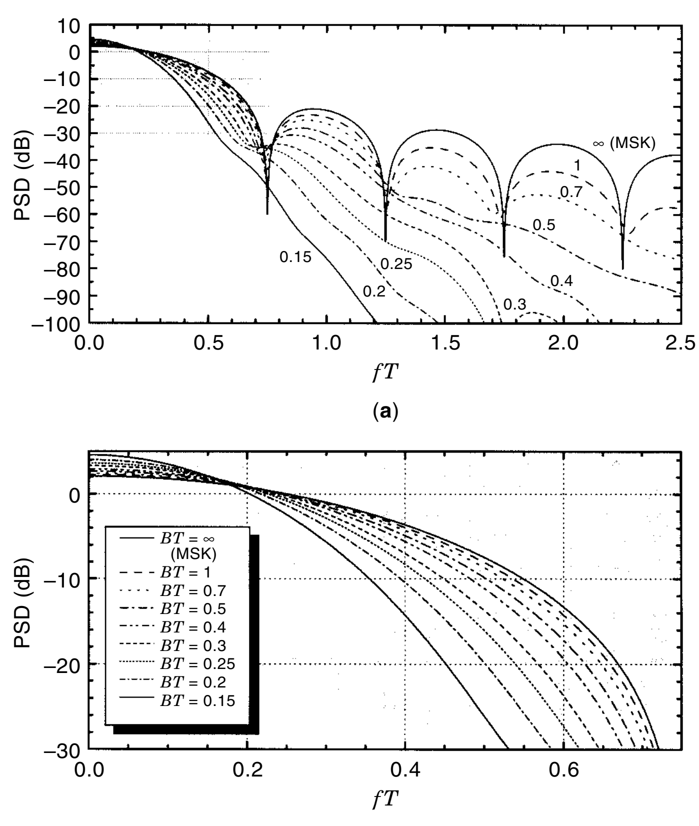

Figure 6. The one-sided power spectral density of GMSK for different BT products. (a) General view; (b) main lobe.

Demodulation of MSK and GMSK Signals over an AWGN Channel

In general, in order to demodulate in an optimum way [optimum in terms of minimum bit error rate (BER)], the receiver must use as much information as possible about the transmitted signal characteristics, and about the communications channel. The simplest channel to consider is the additive white Gaussian noise channel, in which the only impairment imposed on the transmitted signal is the addition of noncorrelated noise. This channel is also referred to as the *static* or the *nonfading* channel. The assumption that the transmission took place on this form of channel implies that there is no filtering action that introduces intersymbol interference. The channel simply attenuates the transmitted signal, delays it in time, and adds noise. Over an AWGN channel, a transmitted signal $v(t)$ is therefore received (in complex baseband format) (8) as

$$r(t) = Gv(t-d)e^{-j2\pi(f_c+\Delta f)d}e^{j2\pi\Delta ft}e^{j\theta(t)} + n(t) \qquad (16)$$

where G is the channel gain or loss, d is the time delay, f_c is the carrier frequency, Δf is the carrier frequency error, $\theta(t)$ is a time-varying phase error, and $n(t)$ is the complex baseband noise with power spectral density N_0. This model is acceptable in many communications scenarios, especially for fixed stations satellite communications (4).

Two forms of detection may be considered: (1) the coherent detection, in which it is assumed that any phase or frequency error encountered on the channel is totally compensated, and (2) the noncoherent detection, in which the frequency errors are removed, but the phase errors are assumed present and unknown. In order to accomplish a proper demodulation, the receiver must estimate as precisely as possible the symbol or bit transitions time (i.e., compensate for the delay d).

Coherent Demodulation. It is assumed here that $\Delta f = \theta(t) = 0$, that d has been compensated, and that $G = 1$. The received signal is then

$$r(t) = v(t) + n(t)$$

For an MSK signal, the two interpretations of Eqs. (1) and (8) lead to two different coherent demodulators, with different probabilities of bit errors. The binary FSK interpretation of Eq. (1) indicates that MSK is represented by two different signaling waveforms, given by

$$v(t) = \exp\left(\pm\frac{j\pi t}{2T}\right)$$

These two waveforms are orthogonal over a signaling interval of T seconds. Coherent detection for such a scheme is accomplished by cross-correlating the received waveform over [0, T], with the complex conjugate of the two signaling waveforms, and by making the binary decision in favor of the largest cross-correlation (8). This is equivalent to filtering the received signal with two filters, one with impulse response $h_1(t) = \exp[-j\pi(t-T)/2T]$, and the other with impulse response $h_{-1}(t) = \exp[j\pi(t-T)/2T]$, both nonzero over [0, T] only, and to make the binary decision in favor of the largest

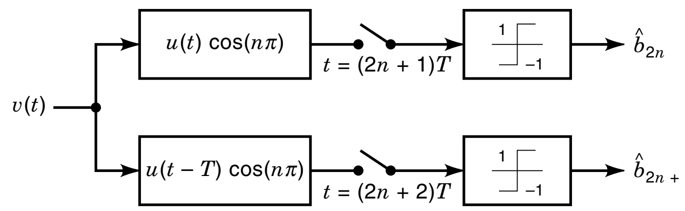

Figure 7. The coherent MSK receiver structure at complex baseband. This receiver requires differential decoding to recover the MSK-modulated bits.

filter output. The BER for such a matched filter detection (8) is given by

$$P_{Co} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \qquad (17)$$

where erfc(x) is the complementary error function, defined as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}\,dt$$

E_b is the energy per transmitted bit and N_0 is the white noise variance. The subscript "Co" refers to *Coherent* detection of *orthogonal* signals.

When the linear interpretation of Eq. (8) is considered, it is noticed that the complex waveform $v(t)$ is composed of two filtered BPSK signals, one in the real component, and one in the imaginary part. The real part is modulated by a binary stream $\{b_{2n}\}$, independent from the modulating stream $\{b_{2n+1}\}$ in the imaginary component. In addition, the complex baseband noise $n(t)$ has independent real and imaginary components (8). The transmission of the MSK waveform over the AWGN channel can be interpreted as the transmission of two independent BPSK waveforms with pulse shape $u(t)$ given by Eq. (7), over two independent AWGN channels. The demodulation of the two modulating bit streams can then be accomplished independently in each of the quadrature channels. The optimum demodulator for such an OQPSK form of signal consists in a quadrature demodulator, in which the real and the imaginary parts of the received signal are independently matched filtered with $u(t)$ and $u(t-T)$, respectively (with alternating signs, due to the $\cos(n\pi)$ factor) (9). The form of this demodulator is illustrated in Fig. 7. The two demodulated bit streams $\{\hat{b}_{2n}\}$ and $\{\hat{b}_{2n+1}\}$ both have a bit error rate equal to that of ordinary BPSK, with a bit rate of $1/2T$ bit/s. The probability of bit error is then (8)

$$P_{Ca} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad (18)$$

Here, the subscript "Ca" refers to *Coherent* detection of *antipodal* signals. The term antipodal is used to indicate that the two signaling waveforms in the BPSK quadrature channels are such that one is equal to the other multiplied by -1, that is, $u(t-2nT)$ is transmitted when $b_{2n}\cos n\pi = 1$, and $-u(t-2nT)$ is transmitted when $b_{2n}\cos n\pi = -1$. The probability of Eq. (18) applies to the decoded bits $\{\hat{b}_n\}$. The bits $\{b_n\}$ correspond to the *differentially encoded* source bits $\{I_n\}$, as in-

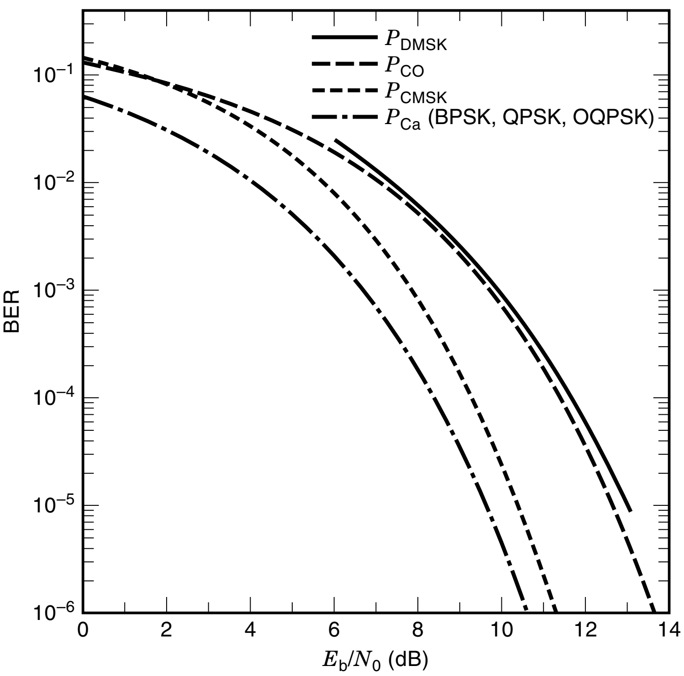

Figure 8. The probability of error for coherent and noncoherent demodulation of MSK over an AWGN channel. P_{DMSK} is obtained from Ref. 40.

dicated in Eq. (10), which implies that the demodulated bit stream $\{\hat{b}_n\}$ must be *differentially decoded* as

$$\hat{I}_n = \hat{b}_n \hat{b}_{n-1} \tag{19}$$

This differential decoding tends to propagate the bit errors and therefore to worsen the performance of the coherent receiver. The ultimate BER for coherent demodulation of MSK [including the differential decoding of Eq. (19)] is

$$P_{\text{CMSK}} = 2P_{\text{Ca}}(1 - P_{\text{Ca}}) \tag{20}$$

where P_{Ca} is given in Eq. (18). Note that this BER expression also applies to BPSK, QPSK, and OQPSK, when the source bit stream is differentially encoded prior to modulation. The error rates of Eqs. (17), (18), and (20) are illustrated in Fig. 8. It is noted that the difference in E_b/N_0 between MSK and BPSK tends toward zero as E_b/N_0 increases. Also, the linear view of MSK allows a coherent demodulation that is close to 3 dB better than the coherent demodulation used when the signal is considered as a binary FSK signal. (The difference between P_{Co} and P_{CMSK} is 3 dB when E_b/N_0 tends toward infinity.) It is then clear that if coherent demodulation is feasible, then the quadrature receiver of Fig. 7 should always be used with MSK. Note also that if the FSK property is not required, the linear modulation of Eq. (8) should be used without differential encoding (resulting in a signal that is not strictly an MSK signal), which would avoid the penalty induced by differential decoding [due to Eq. (20)].

Coherent Demodulation of GMSK. For a general CPM scheme using an infinitely long pulse shape, a coherent demodulator which minimizes the probability of error must observe the received signal over the entire time axis. The usual detection criterion is the maximization of the symbol sequence likelihood, called maximum likelihood sequence estimation (MLSE) (27,28). For CPM schemes with a rational modulation index and a time-limited pulse shape, the MLSE detector can be implemented as a bank of matched filters sampled at every symbol interval, followed by the Viterbi algorithm (7). The performance of such a detector is usually expressed in terms of asymptotic signal-to-noise ratio (SNR) and minimum Euclidean distance between the possible transmitted sequences (27). The BER performance for MLSE of GMSK, for large E_b/N_0, is given by (5)

$$P_{\text{GMSK}} = \frac{1}{2}\,\text{erfc}\left(\frac{d_{\min}}{2\sqrt{N_0}}\right) \quad \text{for} \quad E_b/N_0 \to \infty \tag{21}$$

where d_{min} is the minimum Euclidean distance between any two transmitted sequences. This minimum distance varies with the BT product. It is equal to $\sqrt{2E_b}$ for $BT \approx 0.1$ and converges to $2\sqrt{E_b}$ for BT approaching infinity (5). This last case corresponds to coherent detection of MSK (before differential decoding). For BT products larger than 0.1, the asymptotic MLSE performance with GMSK is therefore within 3 dB from that of MSK. For $BT = 0.3$, the asymptotic loss is approximately 0.5 dB, and for $BT = 0.5$, it is approximately 0.25 dB (5).

Because the complexity of the Viterbi algorithm grows exponentially with the length of the phase pulse, a large amount of work was performed toward the search of suboptimum coherent demodulators approaching the performance of the full-MLSE detector. Several general types of such demodulators were proposed (29). Two of these types implement reduced-complexity Viterbi detectors. In one type, the Viterbi algorithm employed in the receiver is for a simpler CPM scheme than the one transmitted. Since the number of matched filters and the number of states used in the Viterbi algorithm are related to the length of the pulse shape of Eq. (2), a simpler phase tree (Fig. 3) can be defined, by assuming a pulse shape in the receiver that differs from the transmitter pulse in length and/or in shape. This kind of reduced-state algorithm has been shown to induce losses of less than one dB, at error rates lower than 10^{-1} (30). In another type of reduced-complexity MLSE receiver, the phase tree used in the Viterbi algorithm is not different from the one corresponding to the transmitted signal, but its search is limited to certain sections only. This kind of *limited search* algorithm has been studied extensively in the areas of decoding of convolutional codes, MLSE for intersymbol interference channels, and general trellis decoding. In the detection of CPM signals, this reduced-search form of algorithm has been shown to induce very small losses (31).

Other forms of coherent receivers with lower complexity have been proposed. For GMSK, the preferred structure of suboptimum coherent demodulator is that of the quadrature receiver of Fig. 7, with different forms of filter impulse response (5). (When the modulation index is equal to 0.5, this structure may also be applied to other CPM schemes.) By choosing the impulse response $u(t)$ such that the probability of error is minimized, this GMSK receiver has a performance that is within 0.5 dB from the asymptotic BER given by Eq. (21) (32). Note that this so-called *parallel MSK-type* receiver, in which the bit decisions are made alternatively in one of the two parallel branches, has a *serial* version, in which the filters in the two branches are different. The output of these branches is summed, and the decisions are made from sampling a single signal stream, at intervals nT. This form of serial MSK-type receiver was first proposed for MSK, and has been naturally extended for CPM signals with a modulation index of 0.5 (33,34). In the absence of synchronization errors, the performance of the parallel and serial receivers is identical. The serial approach has the advantage of lower complexity and lower sensitivity to synchronization errors (33). This serial approach can also be used in the transmitter, as a form of filtered BPSK.

Noncoherent Demodulation. It is assumed that for the model of Eq. (16), $\Delta f = 0$, d has been compensated and $G = 1$. In deriving the optimum noncoherent demodulation scheme, it is usually assumed that the phase error $\theta(t)$ is a time-invariant random variable, uniformly distributed over $[0, 2\pi]$ (8). For a general CPM modulation scheme, the optimum symbol-by-symbol noncoherent receiver observes the whole transmitted signal, and makes a decision in a given symbol interval by maximizing the likelihood probability of this symbol, given all the possible transmitted sequences containing this particular symbol (35). This receiver is fairly complex, comprising a bank of matched filters for which the output signals are processed by computing the squared modulus and by applying a Bessel function of order zero (although, due to the monotonicity of the Bessel function with its argument, it is sufficient to use this argument only) (8). The performance of such an optimum noncoherent receiver approaches the performance of the coherent receiver, at least for a large E_b/N_0 (35–37). This behavior is related to the phase memory of the CPM schemes, which allows a performance improvement with larger observation times. This is also true for noncoherent detection of MSK, even if it was established before that the optimum observation interval for coherent detection is two-bit long. More recent work on noncoherent detection is based on maximum likelihood detection over a time-limited block of transmitted symbols. This work applies to full-response (38), as well as to partial-response CPM (39), and leads to the use of multiple levels of differential detectors.

Differential detection over a single symbol interval is, for some of the CPM schemes, an attractive form of noncoherent demodulation. It has the advantage of being simple to implement, and can approach, in some cases, the performance of the optimum noncoherent technique. Consider the MSK phase of Eq. (4), sampled at the symbol intervals $t = (n + 1)T$. The result is

$$\phi_{\text{MSK}}(n+1) = \sum_{k=-\infty}^{n-1} I_k \frac{\pi}{2} + \frac{\pi}{2} I_n$$

Computing the phase difference between intervals $n + 1$ and n gives

$$\begin{aligned}\Delta\phi_{\text{MSK}}(n+1) &= \phi_{\text{MSK}}(n+1) - \phi_{\text{MSK}}(n) \\ &= \frac{\pi}{2} I_n\end{aligned}$$

This equation indicates that, because of the phase accumulation property of MSK, all the information about the nth transmitted symbol is contained in the phase difference $\Delta\phi_{\text{MSK}}(n + 1)$. A simple method to obtain this information is to compute the cross-product between the complex numbers, obtained from the complex baseband-received signal by sampling at $t = (n + 1)T$ and $t = nT$. For MSK, this gives

$$\begin{aligned}e^{j\phi_{\text{MSK}}(n)} \times e^{j\phi_{\text{MSK}}(n+1)} &= \sin\left(\frac{\pi}{2} I_n\right) \\ &= I_n\end{aligned}$$

This equation indicates that, for MSK on an AWGN channel, it is possible to recover the transmitted symbols by simply differentially detecting the received signal. When differential detection is used with MSK, the term DMSK is usually employed. Note that the received MSK signal must be filtered before the detection, in order to reduce the effects of the wide-band AWGN. For MSK, there does not exist a single receive filter that both maximizes the signal-to-noise ratio at its output (the matched filter) and introduces no intersymbol interference. This fact is responsible for the increased BER of DMSK, compared with coherent MSK. (Differential detection also tends to cause error propagation.) The ease of implementation of the differential detection technique is therefore compensated by a lower performance. The BER of DMSK is usually inferior to that of BPSK by more than 3 dB. A simulated DMSK performance curve is given in Fig. 8 (40). This performance is obtained by using, as a receive filter, a phase-equalized fourth-order Butterworth filter with a bandwidth to bit rate ratio of 1.1. This filter was found to give very good performance for differential detection of MSK (41).

Differential detection of CPM signals has been extensively studied for general schemes (42,43) and, specifically, for binary schemes with a modulation index $h = 0.5$. It applies to GMSK, although the inherent intersymbol interference of this partial-response scheme tends to decrease the performance of the detector (in addition to any degradation induced by the intersymbol interference due to the receive filter) (44). In order to reduce some of the intersymbol interference effects, two-bit differential detection has been proposed (44,45). By differentially encoding the bits to transmit before they are modulated by the GMSK modulator the phase difference over two bits becomes proportional to the transmitted bits, with a larger immunity to noise (a larger decision interval in the so-called eye diagram at the detector output).

The use of multiple symbol differential detection has been generalized, especially by combining the output of a bank of such detectors, each one computing the differential phase over a larger observation window. For a full-response CPM signal [$g(t)$ in Eq. (2) is nonzero over only a single symbol interval $L = 1$], observed over k symbol intervals, the phase difference is

$$\Delta\phi_k(n+1)|_{L=1} = \pi h \sum_{i=0}^{k-1} I_{n-i}$$

This equation indicates that, by differentially detecting over multiple symbols, it is possible to take advantage of the phase memory of the CPM scheme, and possibly to increase the performance of a single symbol differential detector. This fact has been recognized for some time, and error control feedback decoding principles (46) have been applied on the binary decisions at the output of a bank of two differential detectors (47), and later for a bank of three detectors (48). This technique was often called *nonredundant error correction,* since it uses the detector's hard decisions as if they were produced by a convolutional encoder, which allows "error correction" at the output of the ordinary one-bit differential detector. This technique can be viewed as a particular case of noncoherent maximum likelihood detection over a block of transmitted symbols (49). As the number of detectors increases (i.e., as the length of the observed block grows) the performance improves, and tends toward that of differentially decoded coherent MSK, given in Eq. (20) (50).

For partial response signals, inherent intersymbol interference is present at the output of the differential detectors (in addition to any channel-induced ISI). As with the full-response case, the outputs of different levels of differential detection can be combined with different forms of decision feedback equalization in order to cancel some of this interference. This method has been studied for GMSK (51,52), as well as for other forms of partial-response CPM schemes (53).

Differential detection over an interval Δt of less than one symbol has been proposed (54). In this technique, by using the phase difference $\Delta\phi(t) = \phi(t + \Delta t) - \phi(t)$ for $\Delta t < T$, the channel dispersive effects (on a mobile multipath fading channel) can be avoided, in part. This results in a BER performance that is not as degraded as the conventional one-bit differential detector on a multipath mobile communications channel (55).

Another form of noncoherent detection, called *frequency discriminator detection,* has been studied extensively for CPM signals and other narrowband frequency modulated signals (56). A frequency discriminator transforms the frequency modulation of an FM signal into an equivalent amplitude modulation. The output of a perfect frequency discriminator is given by

$$p(t) = \frac{a}{2\pi}\frac{d\phi(t)}{dt} \tag{22}$$

where a is a linear gain. Note that because the time derivative of Eq. (22) can be expressed as

$$\frac{d\phi(t)}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\phi(t)}{\Delta t}$$

the output of a differential phase detector can be seen as an approximation of a frequency discriminator, followed by an integration function (in practice, a low-pass filter). Because of this link, frequency discriminator detection of FSK can enjoy, like fractional-bit differential detection, a reduction of the multipath effects on the BER performance (57). This form of detection applies to GMSK, and like differential detection, has been considered in conjunction with decision feedback and maximum likelihood decoding (58,59).

ADVANCED TOPICS

The use of the MSK format (particularly GMSK) in current systems and standards is mostly restricted to mobile wireless communications, in which the transmitted signal is corrupted by multipath fading. This channel is inherently more problematic than the static AWGN channel in terms of demodulation and synchronization of the received signal. Constant or quasiconstant amplitude modulation schemes have been considered, and sometimes adopted, for wireless communications applications. The main reason for this interest lies in the fact that constant envelope signals allow the use, in the mobile stations, of inexpensive and less power-hungry nonlinear power amplifiers. (The amplitude nonlinearity does not affect adversely the transmitted signal.) The phase accumulation inherent to the MSK format allows noncoherent differential detection or frequency discriminator detection in the receiver, which is generally found to be easier to implement than coherent demodulation on wireless fading channels (because the fading gain variations often do not allow a good carrier phase estimation at the receiver).

Demodulation of MSK and GMSK Signals over Fading Channels

The static AWGN channel model of Eq. (16) is not adequate to characterize the mobile wireless channel. In general wireless transmissions, the signal reaching the receiver is the sum of a number of reflected versions of the transmitted signal, each one affected by different time-varying delays and gains. If the transmitted signal is $v(t)$, a widely accepted mathematical model for the received signal is (6)

$$r(t) = \sum_{i=1}^{N} G_i(t)v(t-d_i) + n(t) \tag{23}$$

where $G_i(t)$ and d_i are, respectively, the gain and the delay of the ith reflected path, and $n(t)$ is the complex baseband noise. The gain $G_i(t)$ is represented with a time-varying complex Rayleigh random variable, that is, its real and imaginary components are independent zero-mean Gaussian random processes, with a maximum frequency content equal to the Doppler spread of the channel (8).

The performance of CPM signals has been investigated for different channels represented by special cases of Eq. (23). The land mobile channel, in which N is usually assumed to vary from 1 to 3, and the satellite mobile channel, in which G_1 is equal to a constant and $N = 2$, have been extensively studied. The relative propagation delays in the satellite mobile systems are usually small, and the delays in the model of Eq. (23) are usually set equal to zero ($d_1 = d_2 = 0$). The simplest model used in land mobile communications is that of the flat fading Rayleigh channel, obtained from Eq. (23) with $N = 1$. This channel is purely multiplicative (except for the additive white noise) and the time-varying gain is the same for all frequencies. The time variations of this gain are generally classified as *slow* and *fast.* The former is usually related to a gain with a time constant much larger than one symbol interval, while the latter refers to the case where the gain variations are much faster than the symbol rate. When N is larger than 1 in Eq. (23), time dispersion usually appears and induces intersymbol interference. This channel is referred to as *frequency selective.*

Coherent Demodulation. Because of the difficulty in accurately estimating the carrier phase on a fading channel, and the possibility of using efficient noncoherent methods, coherent detection of CPM signals has received less attention than noncoherent detection in wireless communications. In slow flat fading conditions, the coherent quadrature receiver of Fig. 7 has been analyzed in Refs. 5, 29, and 32. It is shown that for most CPM cases, the asymptotically optimum filter (i.e., when $E_b/N_0 \to \infty$) for the AWGN channel can be used without severe degradation. In slow fading, the MSK modulation scheme is less affected than other CPM signals, since the quadrature receiver is then optimal in AWGN conditions. When cochannel interference is present, most MSK-type modulation perform similarly, while if adjacent channel interference is dominant, the smoother modulation schemes (like GMSK) offer an improved performance because of the lower side lobes (60).

In slow frequency selective fading conditions, the coherent quadrature receiver for GMSK has been specifically studied, and it is found that, with a *BT* product of 0.25, its performance is, in general, slightly better than that of MSK (61).

Noncoherent Demodulation. In fading conditions, differential detection and frequency discriminator detection are the most popular choices. Depending on the fading rate, both methods, when used with MSK and GMSK signals, may induce important losses, compared with coherent detection of MSK on an AWGN channel. At high E_b/N_0 an irreducible BER is observed. This error floor increases with the fading rate, and is due to the random phase imposed on the transmitted signal by the fading process. The choice between differential detection and frequency discrimination is usually dictated by the system design. If the stability of the carrier frequency is poor at the receiver, frequency discrimination is usually preferred.

Differential detection with one-bit delay has been analyzed in fast Rayleigh fading conditions (62). It is found that the performance of GMSK is worse than that of MSK, mainly because of the built-in ISI present in the former. The loss is small, however, for *BT* products of 0.4 or larger. The two-bit differential detector is analyzed in Ref. 63, and it has a performance worse than the one-bit detector in fast fading conditions. The discriminator detection approach for MSK-type signals over land mobile channels has received attention in Ref. 64. As with the differential detector, the discriminator introduces a loss of a few decibels, compared with the coherent detection of MSK on an AWGN channel. As in the detection of GMSK over the AWGN channel, some forms of decision feedback equalization, following the differential or discriminator detector, have been studied (65,66). This equalization technique results in gains up to 3 dB on the AWGN channel, compared with ordinary differential detection, but in little improvements on fading channels.

Differential detection with one- and two-bit delay has also been investigated for satellite mobile channels. Slow fading conditions have been considered in Ref. 45, while the fast fading case has been treated in Refs. 52 and 67, along with decision feedback. As with the land mobile case, the use of decision feedback improves the receiver's performance for very slow fading channels, and there is little advantage in using two-bit differential detection instead of one-bit detection on a fast fading channel. The discriminator detector has also been studied for satellite applications in Ref. 58, with and without decision feedback.

Maximum Likelihood Sequence Estimation. Maximum likelihood sequence estimation of CPM signals transmitted over Rayleigh flat fading channels is discussed in Ref. 68. The resulting receiver is based on linear prediction theory, and implements a bank of FIR filters and square operations, followed by a Viterbi algorithm. The filters actually perform a linear prediction of the fading channel, and remove the channel effects before the Viterbi algorithm. The result is a noncoherent receiver that is superior to one-bit differential detection in fast fading conditions, and that theoretically does not have an irreducible BER. (A good practical implementation would result in a very small error floor.) The price to pay for such superior performance is larger computational complexity.

Symbol Timing and Carrier Phase Recovery of MSK and GMSK Signals

The synchronization aspects of CPM signals are generally studied as a joint data and synchronization estimation problem (29,69). The solution to this problem can be formulated in terms of a maximum likelihood procedure, with a known or unknown data sequence. However, the complexity of this method is such that other practical solutions have to be considered for real-life applications. The typical practical techniques decouple the estimation problem, and perform a separate estimation of the received signal symbol transitions, and of its carrier frequency and phase errors. (For noncoherent detection, phase synchronization is not necessary.)

The phase information of a general CPM signal is given in the argument of Eq. (1). Assuming a constant modulation index with a rational value of the form $h = p/q$, it can be shown that if the CPM signal is raised to the power q, the result is another CPM signal, with an integer modulation index. A key property of this new signal is that its power spectral density exhibits discrete spectral components, with frequencies separated by the symbol rate (29). The receiver can make use of these discrete tones by filtering them, and deriving the symbol transitions and carrier phase errors from the result. For MSK-type signals, with $h = 1/2$, such a synchronizer can be used by squaring the received signal (2,70). The new signal has a unitary modulation index, and the spectral lines are at the carrier frequency f_c, plus and minus $n/(2T)$, with $n = 1, 2, \ldots$. This is also true for CPM signals with $h = 1/2$, like GMSK.

Figure 9 illustrates the block diagram of the basic approach for MSK. The filters following the squaring device eliminate some of the noise coming from the modulation itself, as well as from the signal and noise cross-products. The multiplicative nonlinearity results in the sum and difference of the tones at the input. The tones at $4f_c$ and at $1/T$ are then isolated, with proper band-pass filters. The clock phase of the receiver can be adjusted with the output $T(t)$, while the carrier phase is adjusted with $C(t)$. Note that a phase ambiguity problem exists with the signal $C(t)$ and has to be resolved (29). The approach of Fig. 9 can be implemented at RF or IF frequency, as well as at baseband. A generalized version of the symbol timing estimation portion of this algorithm has been investigated in (71), for the case of fast flat fading channels. Good results were obtained for modulation schemes having a negligible amount of ISI. Other algorithms based on nonlinear processing have been studied for MSK-type signals (72–74).

The problem of estimating the carrier frequency errors for MSK signals has been addressed in (74). Since MSK is a differentially encoded linear phase modulated signal, several of the techniques used for BPSK or QPSK are also applicable for MSK-type signals, especially in the discrete-time domain.

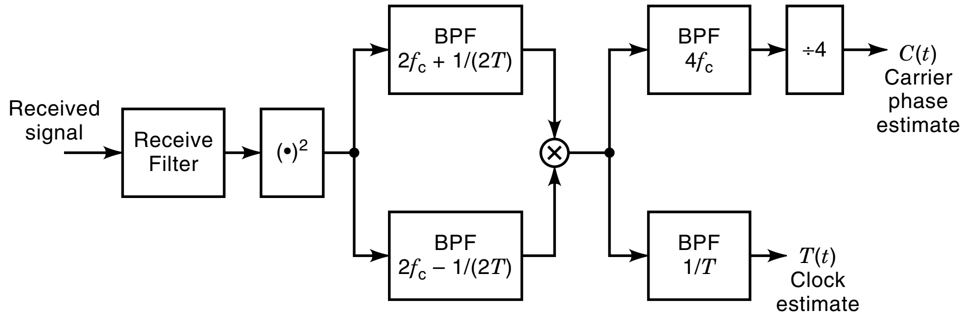

Figure 9. A possible structure for the clock and phase synchronization of MSK.

Some Generalizations and Recent Developments

The basic MSK signal format has been generalized somewhat by allowing the use of a premodulation filter, as in GMSK. Some other forms of generalizations have been studied, such as the use of M-ary modulating symbols and the use of multiple modulation indices.

***M*-ary Modulation.** The binary CPM and MSK-type schemes can be generalized to use M-ary symbols. An example of this scheme is M-ary GMSK using k-bit symbols (75), for which $h = 1/M$ with $M = 2^k$. The signal generation is the same as for binary GMSK, except that the input of the premodulation filter is a sequence of M-ary symbols. The general impact of such a generalization is to increase the amount of ISI in the noncoherent receiver, or to increase the number of states in the MLSE receiver. The spectral efficiency of such a modulation scheme is generally increased. Another generalization is multiple amplitude MSK (MAMSK), in which two or more MSK signals, with different complex envelope amplitudes, are added together (19,76). The continuous phase property of the MAMSK signal is preserved, but the constant envelope feature is lost. These M-ary modulation schemes are often studied in conjunction with trellis coding, to achieve better bit error performances without bandwidth expansion (77).

Multi-*h* Modulation. The information-carrying phase function in multi-h continuous phase modulation is given by the argument of Eq. (1), where h_n is the modulation index in the nth symbol interval. In multi-h signals h_n is taken from a finite set of values and is repeated in sequence with a period equal to the number of elements in the set. The modulation indices are often chosen from a set of rational numbers, with a common integer denominator q such that

$$h_i = \frac{p_i}{q} \quad i = 1, 2, 3, \ldots, K$$

where the p_i's are K integers smaller than q. The objective in using a multi-h set is to increase the minimum distance in the phase trellis (which is related to the phase tree) between possible transmitted symbol sequences, such that the immunity to sequence errors is increased (29,78).

Coherent detection is usually accomplished with the MLSE technique. The Viterbi algorithm is preceded by a bank of matched filters, cyclically switched interval by interval, to allow the reuse of the modulation indices. The complexity is therefore increased, with respect to a single-h modulation, because of the bank switching and because of a larger number of states in the phase trellis. This increased complexity has limited the use of the multi-h format to CPM signals with a rectangular frequency pulse $g(t)$.

Simplified receivers have been developed with small degradation relative to MLSE (79,80). The combination of multi-h CPM with error-correcting codes has also been investigated, seeking ways to increase the minimum distance in the phase trellis (78). Finally, multi-T realizations of multi-h phase codes, in which the symbol duration changes cyclically, have been investigated to simplify some system design aspects (81).

Spread Spectrum with the MSK Family. CPM signals can be used in spread spectrum applications, for the sake of reducing the bandwidth of the spread signal and allowing the use of efficient nonlinear power amplifiers. The transmitted signal is obtained by spreading the modulating data stream with a larger bandwidth pseudorandom binary signal [direct sequence (DS) spread spectrum], and by modulating the resulting signal with a CPM scheme. The use of the MSK and GMSK ($BT = 0.3$) modulation formats have been studied for the mobile radio channel (82,83).

Current Mobile Wireless Systems and Standards

For systems using single carrier per channel (SCPC) frequency division multiplex (FDM), the MSK modulation format is not narrowband enough to meet practical capacity requirements. The GMSK scheme has better spectral characteristics, with a limited performance degradation. For this reason, GMSK has been adopted for application in some of the major land mobile cellular systems across the globe. Such systems include the European Global System for Mobile Communications (GSM), using the 900 MHz band, the Digital Communications System 1800 (DCS 1800), also in Europe in the 1800 MHz band, and the North American Personal Communication Service 1900 (PCS 1900), in the 1900 MHz band. GMSK has also been selected in the second generation of British Cordless Telephones (CT-2), with a common air interface (CAI), and in the Digital European Cordless Telephone (DECT) system. The MSK family of signals is also being used in *wide-area wireless data systems,* for high mobility, wide-ranging, and low-data-rate digital communications. Systems like the Cellular Digital Packet Data (CDPD), the Mobitex RAM Mobile data network, and the Metricom Microcellular Data Network (MDN) all employ a GMSK modulation format (84,85).

GSM, DCS 1800, and PCS 1900 Systems. The European GSM system uses GMSK, with a premodulation filter having a BT product of 0.3. The binary data rate going into the transmitter pulse shaping filter is 270.833 kbit/s, on both the forward (base station to mobile) and the reverse (mobile to base station) links. Eight digital voice channels share this bit stream, using Time Division Multiple Access (TDMA). The radio frequency signal band is 890 MHz to 915 MHz for the reverse link, and 935 MHz to 960 MHz for the forward channel, with a channel spacing of 200 kHz. To meet the spectral requirement of the system, the power level of the modulated signal, 400 kHz away from the carrier frequency, must be at least 60 dB below the value at the carrier frequency. The demodulation technique is not specified and is left up to the manufacturer. Equalization is usually required, in order to cope with the transmitted signal ISI and frequency selective fading. Except for some differences, especially at the cell deployment level, the DCS 1800 and PCS 1900 systems are essentially L-band upconverted versions of the GSM norm. The signal definitions are the same as those of the GSM standard.

Cordless Telephones. The cordless system CT2/CAI was the first popular digital cordless telephone standard in operation. Its spectral location is in the band 864.1 MHz to 868.1 MHz, with a channel spacing of 100 kHz. The transmitted modulation scheme is Gaussian FSK (GFSK), with a *BT* product usually equal to 0.3. The channel data rate is 72 kbit/s, with a modulation index between 0.4 and 0.7. The peak frequency deviation range of the transmitted signal is limited to the interval between 14.4 kHz and 25.2 kHz. The North American version of CT2/CAI, known as the Personal Communications Interface (PCI), also specifies the same modulation format. The DECT standard is used in the band 1880 MHz to 1900 MHz, with a 1.728 MHz channel spacing and a 1152 kbit/s channel data rate. The *BT* product is usually 0.5, with a modulation index of 0.35 to 0.7. The cordless telephone receivers usually implement noncoherent demodulation (85,86).

Wide-Area Wireless Data Systems. The mobile data networks are systems used to complement the cellular systems, by transmitting low bit rate data. The Mobitex RAM Mobile data network, for trunked radio networks, is used in several countries in Europe and in North America. The mobile transmits in the range 896 MHz to 901 MHz, and the base station in the range 935 MHz to 940 MHz. The bit rate is 8 kbit/s, in a 12.5 kHz channel, using the GMSK modulation format. Noncoherent detection is performed in the receiver. Another system is the Cellular Digital Packet Data (CDPD) service, used as an overlay to the existing analog cellular telephone network. It takes advantage of the idle time in the analog system to transmit packets of data, at a rate of 19.2 kbit/s. The operation is done transparently, in the 30 kHz channel. The GMSK modulation format is used, with a *BT* product of 0.5. The base to mobile transmission is in the range 869 MHz to 894 MHz, and the mobile to base transmission is in the 824 MHz to 849 MHz range (84).

Current Trends

The topic of minimum shift keying and CPM signals is a fairly mature one, in which most fundamental characteristics have probably been discovered. Despite this fact, a regular flow of results is still being generated, especially in the area of mobile communications. Some work is still being accomplished in the area of signal shaping and spectrum efficiency of the MSK-type signals. The same is true for the detection and demodulation of these signals, for fading channels, and channels facing adjacent and cochannel interference. Trellis-coded GMSK signals have been studied recently for AWGN channels, and are likely to be considered for fading channels. Maximum *a posteriori* (MAP) detection of CPM signals and iterative processing have also received attention recently. Research is currently being done, considering MSK and GMSK, in the area of satellite on-board processing and band-limited nonlinear amplification. Broadband indoor and outdoor wireless applications, using direct-sequence spread spectrum with MSK and GMSK, continue to generate an increasing amount of interest. GMSK is also currently being studied for application in a number of wireless data services, such as High Performance Radio LAN (HYPERLAN), simulcast paging, microcellular networks, cellular digital packet data, and asynchronous transfer mode (ATM) for indoor communications.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to the Directorate of Mobile and Personal Communications of the Canadian Communications Research Centre, for the resources made available during the completion of this project.

APPENDIX 1. COMPLEX BASEBAND REPRESENTATION OF MODULATED SIGNALS

In this appendix, the basic theory of complex baseband representation of signals, also referred to as “complex envelope representation,” is briefly discussed. Chapter 3 of Ref. 8 is an example of a more complete treatment of the topic. The complex baseband representation serves the purpose of mathematically expressing a real band-pass signal in a format where the carrier or reference frequency is zero, and the amplitude and phase information is readily available. This representation is an extension of the familiar two-dimensional phasor (vector) representation of sinusoidal signals.

Consider a real band-pass signal with a narrow bandwidth concentrated around a reference or carrier frequency f_c. This real signal can be mathematically expressed as

$$s(t) = a(t)\cos[2\pi f_c t + \phi(t)] \tag{24}$$

where $a(t)$ is the signal amplitude and $\phi(t)$ is the time-varying phase. The modulation of the signal is contained in its amplitude and its phase. (The frequency modulation is equal to the time derivative of the phase.) The signals $a(t)$ and $\phi(t)$ show all the modulation information carried by $s(t)$, and must be preserved in the complex baseband representation. Equation (24) can also be expressed as

$$\begin{aligned} s(t) &= a(t)\cos[\phi(t)]\cos[2\pi f_c t] - a(t)\sin[\phi(t)]\sin[2\pi f_c t] \\ &= \mathrm{Re}\{[x(t) + jy(t)]e^{j2\pi f_c t}\} \end{aligned}$$

where

$$\begin{aligned} x(t) &= a(t)\cos[\phi(t)] \\ y(t) &= a(t)\sin[\phi(t)] \end{aligned}$$

The signal

$$u(t) = x(t) + jy(t)$$
$$= a(t)e^{j\phi(t)}$$

is the complex baseband equivalent of $s(t)$ and contains all the amplitude and phase information of the real band-pass signal.

Similarly, the impulse response $h(t)$ of a band-pass system is related to the complex baseband impulse response $c(t)$ as

$$h(t) = 2\text{Re}[c(t)e^{j2\pi f_c t}]$$

The 2 factor is included to make the power at the output of the complex baseband filter equal to that at the output of the band-pass system.

BIBLIOGRAPHY

1. R. de Buda, The fast FSK modulation system, *Int. Conf. Commun.,* Montreal, June 1971, pp. 41.25–41.27.
2. R. de Buda, Coherent demodulation of frequency-shift keying with low deviation ratio, *IEEE Trans. Commun.,* **COM-20**: 429–435, 1972.
3. M. L. Doelz and E. H. Heald, Collin Radio Co., *Minimum-shift data communication system,* US Patent 2 977 417, March 28, 1961.
4. G. Maral and M. Bousquet, *Satellite Communications Systems,* 2nd ed., Chichester: Wiley, 1993.
5. K. Murota and K. Hirade, GMSK modulation for digital mobile radio telephony, *IEEE Trans. Commun.,* **COM-29**: 1044–1050, 1981.
6. T. S. Rappaport, *Wireless Communications: Principles & Practice,* Upper Saddle River, NJ: Prentice-Hall, 1996.
7. C. E. Sundberg, Continuous phase modulation, *IEEE Commun. Magazine,* **24** (4): 25–38, 1986.
8. J. G. Proakis, *Digital Communications,* 2nd ed., New York: McGraw-Hill, 1989.
9. S. Pasupathy, Minimum shift keying: A spectrally efficient modulation, *IEEE Commun. Mag.,* **17** (4): 14–22, 1979.
10. S. A. Gronemeyer and A. L. McBride, MSK and offset QPSK modulation, *IEEE Trans. Commun.,* **COM-24**: 809–820, 1976.
11. M. Rabzel and S. Pasupathy, Spectral shaping in minimum shift keying (MSK)-type signals, *IEEE Trans. Commun.,* **COM-26**: 189–195, 1978.
12. F. Amoroso, Pulse and spectrum manipulation in the minimum (frequency) shift keying (MSK) format, *IEEE Trans. Commun.,* **COM-24**: 381–384, 1976.
13. M. K. Simon, A generalization of minimum-shift-keying (MSK)-type signaling based upon input data symbol pulse shaping, *IEEE Trans. Commun.,* **COM-24**: 845–856, 1976.
14. B. Bazin, A class of MSK baseband pulse formats with sharp spectral roll-off, *IEEE Trans. Commun.,* **COM-27**: 826–829, 1979.
15. I. Kalet, A look at crosstalk in quadrature-carrier modulation systems, *IEEE Trans. Commun.,* **COM-25**: 884–892, 1977.
16. I. Kalet and B. E. White, Suboptimal continuous shift keyed (CSK) demodulation of the efficient implementation of low crosstalk data communication, *IEEE Trans. Commun.,* **COM-25**: 1037–1041, 1977.
17. R. E. Eaves and S. M. Wheatley, Optimization of quadrature-carrier modulation for low crosstalk and close packing of users, *IEEE Trans. Commun.,* **COM-27**: 176–185, 1979.
18. A. J. Vigil, M. A. Belkerdid, and D. C. Malocha, Results of optimal discrete pulse shaping for MSK-type signals, *IEEE Trans. Commun.,* **44**: 769–771, 1996.
19. W. J. Weber, P. H. Stanton, and J. T. Sumida, A bandwidth compressive modulation system using multi-amplitude minimum shift keying (MAMSK), *IEEE Trans. Commun.,* **COM-26**: 543–551, 1978.
20. A. Lender, Correlative level coding for binary data transmission, *IEEE Spectrum,* **14**: 104–115, 1966.
21. S. Pasupathy, Correlative coding: A bandwidth-efficient signaling scheme, *IEEE Commun. Mag.,* **15** (4): 4–11, 1977.
22. P. Kabal and S. Pasupathy, Partial-response signaling, *IEEE Trans. Commun.,* **COM-23**: 921–934, 1975.
23. P. Galko and S. Pasupathy, On a class of generalized MSK, *Int. Conf. Commun.,* Denver, June 1981, pp. 2.4.1–2.4.5.
24. T. Aulin, G. Lindell, and C. E. Sundberg, Selecting smoothing pulses for partial-response digital FM, *IEE Proc. Pt. F.,* **128** (4): 237–244, 1981.
25. F. de Jager and C. B. Dekker, Tamed frequency modulation, a novel method to achieve spectrum economy in digital transmission, *IEEE Trans. Commun.,* **COM-26**: 534–542, 1978.
26. D. Muilwijk, Correlative phase shift keying—A class of constant envelope modulation techniques, *IEEE Trans. Commun.,* **COM-29**: 226–236, 1981.
27. T. Aulin and C. E. Sundberg, Continuous phase modulation—Part I: Full response signaling, *IEEE Trans. Commun.,* **COM-29**: 196–209, 1981.
28. T. Aulin, N. Rydbeck, and C. E. Sundberg, Continuous phase modulation—Part II: Partial response signaling, *IEEE Trans. Commun.,* **COM-29**: 210–225, 1981.
29. J. B. Anderson, T. Aulin, and C. E. Sundberg, *Digital Phase Modulation,* New York: Plenum, 1986.
30. A. Svensson, C. E. Sundberg, and T. Aulin, A class of reduced-complexity Viterbi detectors for partial response continuous phase modulation, *IEEE Trans. Commun.,* **COM-32**: 1079–1087, 1984.
31. S. J. Simmons and P. H. Wittke, Low complexity decoders for constant envelope digital modulations, *IEEE Trans. Commun.,* **COM-29**: 1273–1280, 1983.
32. A. Svensson and C. E. Sundberg, Optimum MSK-type receivers for CPM on Gaussian and Rayleigh fading channels, *IEE Proc. Pt. F,* **131** (5): 480–490, 1984.
33. F. Amoroso and J. A. Kivett, Simplified MSK signaling techniques, *IEEE Trans. Commun.,* **COM-25**: 433–441, 1977.
34. A. Svensson and C. E. Sundberg, Serial MSK-type detection of partial response continuous phase modulation, *IEEE Trans. Commun.,* **COM-33**: 44–52, 1985.
35. T. Aulin and C. E. Sundberg, Partially coherent detection of digital full response continuous phase modulated signals, *IEEE Trans. Commun.,* **COM-30**: 1096–1117, 1982.
36. A. Svensson, C. E. Sundberg, and T. Aulin, Partial response continuous phase modulation with partially coherent detection, *Int. Conf. Commun.,* Boston, June 1983, pp. A6.5.1–A6.5.9.
37. W. P. Osborne and M. B. Luntz, Coherent and noncoherent detection of CPFSK, *IEEE Trans. Commun.,* **COM-22**: 1023–1036, 1974.
38. M. K. Simon and D. Divsalar, Maximum-likelihood block detection of noncoherent continuous phase modulation, *IEEE Trans. Commun.,* **41**: 90–98, 1993.
39. A. Abrardo, G. Benelli, and G. R. Cau, Multiple-symbol differential detection of GMSK for mobile communications, *IEEE Trans. Veh. Technol.,* **VT-44**: 379–389, 1995.
40. D. Boudreau, 2400 BPS TMS32010 MODEM implementation for mobile satellite applications, *13th Biennial Symp. Commun.,*

Queens Univ., Kingston, Ontario, Canada, June 1986, pp. B.3.1–B.3.4.

41. S. Crozier, B. Mazur, and R. Matyas, Performance evaluation of differential detection of MSK, *Global Commun. Conf. GLOBECOM,* Miami, November 1982, pp. A5.1.1–A5.1.5.
42. T. Aulin and C. E. Sundberg, On differential detection of partial response continuous phase modulated signals, *Int. Conf. Commun.,* Denver, June 1981, pp. 56.1.1–56.1.6.
43. A. Svensson, On an improved differential detector for continuous phase modulation, *Int. Conf. Commun.,* Toronto, June 1986, pp. 31.3.1–31.3.7.
44. S. Ogose and K. Murota, Differentially encoded GMSK with 2-bit differential detection, *Trans. IECE Jpn.,* **J64-B** (4): 248–254, 1981.
45. M. K. Simon and C. C. Wang, Differential detection of Gaussian MSK in a mobile radio environment, *IEEE Trans. Veh. Technol.,* **VT-33**: 307–320, 1984.
46. A .J. Viterbi and J. K. Omura, *Principles of Digital Communication and Coding,* New York: McGraw-Hill, 1979.
47. T. Masamura et al., Differential detection of MSK with nonredundant error correction, *IEEE Trans. Commun.,* **COM-27**: 912–918, 1979.
48. T. Masamura, Intersymbol interference reduction for differential MSK by nonredundant error correction, *IEEE Trans. Veh. Technol.,* **39**: 27–36, 1990.
49. D. Boudreau, A unified view of noncoherent detection and differential detection of phase modulated signals, *3rd Canadian Workshop Info. Theory Appl.,* Rockland, Canada, May/June 1993, pp. 125–135, New York: Springer-Verlag, 1994.
50. D. Boudreau, *The Application of Feedback Decoding in the Differential Detection of Continuous Phase Modulated Signals,* M. Eng. Thesis, Carleton Univ., Ottawa, Canada, 1987.
51. A. Yongaçoğlu, D. Makrakis, and K. Feher, Differential detection of GMSK using decision feedback, *IEEE Trans. Commun.,* **36**: 641–649, 1988.
52. I. Korn, GMSK with differential phase detection in the satellite mobile channel, *IEEE Trans. Commun.,* **38**: 1980–1986, 1990.
53. D. Makrakis and K. Feher, Multiple differential detection of continuous phase modulation signals, *IEEE Trans. Veh. Technol.,* **42**: 186–196, 1993.
54. S. Ariyavisitakul et al., Fractional-bit differential detection of MSK: A scheme to avoid outages due to frequency-selective fading, *IEEE Trans. Veh. Technol.,* **VT-36**: 36–42, 1987.
55. I. Crohn and E. Bonek, Suppression of the irreducible errors in a frequency-selective fading channel by fractional-bit differential detection, *IEEE Trans. Veh. Technol.,* **43**: 1039–1048, 1994.
56. M. K. Simon and C. C. Wang, Differential versus limiter–discriminator detection of narrowband FM, *IEEE Trans. Commun.,* **COM-31**: 1227–1234, 1983.
57. R. Petrovic and A. F. Molisch, Reduction of multipath effects for FSK with frequency-discriminator detection, Int. Symp. Personal, Indoor Mobile Radio Commun. PIMRC, Helsinki, September 1997, pp. 943–948.
58. I. Korn, GMSK with limiter discriminator detection in satellite mobile channel, *IEEE Trans. Commun.,* **39**: 94–101, 1991.
59. J. P. Fonseka, Noncoherent detection with Viterbi decoding for GMSK signals, *IEE Proc. Commun.,* **143** (6): 373–379, 1996.
60. R. B. Rhodes Jr., S. G. Wilson, and A .B. Svensson, MSK-type reception of continuous phase modulation: Cochannel and adjacent channel interference, *IEEE Trans. Commun.,* **COM-35**: 185–193, 1987.
61. C. W. Lee, Y. M. Chung, and S. U. Lee, Bit errror probabilities of CPM signals in frequency-selective fading channels, *IEE Proc., part I,* **138** (5): 465–472, 1991.
62. P. Varshney, J. E. Salt, and S. Kumar, BER analysis of GMSK with differential detection in a land mobile channel, *IEEE Trans. Veh. Technol.,* **42**: 683–689, 1993.
63. S. M. Elnoubi, Analysis of GMSK with two-bit differential detection in land mobile radio channels, *IEEE Trans. Commun.,* **COM-35**: 237–240, 1987.
64. P. Varshney and S. Kumar, Performance of GMSK in a land mobile radio channel, *IEEE Trans. Veh. Technol.,* **40**: 607–613, 1991.
65. F. Adachi and K. Ohno, Performance analysis of GMSK frequency detection with decision feedback equalisation in digital land mobile radio, *IEE Proc. Pt. F,* **35** (3): 199–207, 1988.
66. S. S. Shin and P. T. Mathiopoulos, Differentially detected GMSK signals in CCI channels for mobile cellular telecommunication systems, *IEEE Trans. Veh. Technol.,* **42**: 289–293, 1993.
67. W. S. Smith and P. H. Wittke, Differential detection of GMSK in Rician fading, *IEEE Trans. Commun.,* **42**: 216–220, 1994.
68. J. H. Lodge and M. L. Moher, Maximum likelihood sequence estimation of CPM signals transmitted over Rayleigh flat-fading channels, *IEEE Trans. Commun.,* **38**: 787–794, 1990.
69. M. Morelli, U. Mengali, and G. M. Vitetta, Joint phase and timing recovery with CPM signals, *IEEE Trans. Commun.,* **45**: 867–876, 1997.
70. A. N. D'Andrea, U. Mengalli, and R. Reggiannini, Carrier phase and clock recovery for continuous phase modulated signals, *IEEE Trans. Commun.,* **COM-35**: 1095–1101, 1987.
71. F. Patenaude, J. H. Lodge, and P. A. Galko, Symbol timing tracking for continuous phase modulation over fast flat-fading channels, *IEEE Trans. Veh. Technol.,* **40**: 615–626, 1991.
72. U. Lambrette and H. Meyr, Two timing recovery algorithms for MSK, *Int. Conf. Commun.,* New Orleans, June 1994, pp. 1155–1159.
73. A. N. D'Andrea, U. Mengalli, and R. Reggiannini, A digital approach to clock recovery in generalized minimum shift keying, *IEEE Trans. Veh. Technol.,* **39**: 227–234, 1990.
74. R. Mehlan, Y. E. Chen, and H. Meyr, A fully digital feedforward MSK demodulator with joint frequency offset and symbol timing estimation for burst mobile radio, *IEEE Trans. Veh. Technol.,* **42**: 434–443, 1993.
75. B. A. Cochran, Development and application of M-ary Gaussian minimum shift keying modulation, *Int. Conf. Commun.,* Seattle, June 1995, pp. 1805–1810.
76. I. Altunbas and Ü. Aygölü, Multiple multiamplitude minimum shift keying trellis codes for AWGN and fading mobile satellite channels, *Int. Conf. Universal Personal Commun.,* San Diego, September 1994, pp. 199–203.
77. J. B. Anderson and C. E. Sundberg, Advances in constant envelope coded modulation, *IEEE Commun. Magazine,* **29** (12): 36–45, 1991.
78. I. Sasase and S. Mori, Multi-h phase-coded modulation, *IEEE Commun. Mag.,* **29** (12): 46–56, 1991.
79. A. Premji and D. P. Taylor, Receiver structures for multi-h signaling formats, *IEEE Trans. Commun.,* **COM-35**: 439–451, 1987.
80. A. Premji and D. P. Taylor, A practical receiver structure for multi-h CPM signals, *IEEE Trans. Commun.,* **COM-35**: 901–908, 1987.
81. P. Szulakiewicz, M-ary linear phase multi-T phase codes, *IEEE Trans. Commun.,* **37**: 197–199, 1989.
82. J. S. Lehnert, Serial MSK spread-spectrum multiple-access communications, *IEEE Trans. Commun.,* **40**: 1119–1127, 1992.
83. F. Giannetti, M. Luise, and R. Reggiannini, Continuous-phase modulations for CDMA radio communications: Modem architecture and performance, *Euro. Trans. Telecommun.,* **7** (3): 225–233, 1996.

84. K. Pahlavan and A. H. Levesque, Wireless data communications, *Proc. IEEE,* **82**: 1398–1430, 1994.

85. J. E. Padgett, C. G. Günther, and T. Hattori, Overview of wireless personal communications, *IEEE Commun. Magazine,* **33** (1): 28–41, 1995.

86. W. H. W. Tuttlebee, *Cordless Telecommunications Worldwide,* London: Springer-Verlag, 1997.

DANIEL BOUDREAU
FRANÇOIS PATENAUDE
Communications Research Centre

HISTORY OF MINE ELECTRICAL SYSTEMS