MINIMIZATION

This article discusses optimization problems involved in real-time control of systems with a human operator in the loop. Such real-time systems involve multiple objective functions, and there are multiple optimal solutions. Often these objectives are conflicting; an example is planning for contingencies as well as the "base case" (current operating point). The solution to such multiobjective problems is typically a trade-off surface (known as a Pareto surface) whose axes are the various objective functions. The Pareto surface is to be presented to the human operator who will make the final decisions regarding control actions. Also, ideally the operator will be able to modify the parameters of the optimization process interactively to obtain desired results.

Mathematical optimization techniques such as nonlinear programming (NLP) have been used historically as the building blocks of real-time control systems. However, their inadequacies have been felt most acutely in the modeling of realistic control actions that do not fit well in the traditional optimization frameworks. The deficiencies are particularly notable in the following areas:

1. Modeling uncertainties in input data pertaining to system operation.
2. Modeling of "soft constraints."
3. Filtering and ranking control actions that the operator is expected to perform.
4. Modeling discrete control actions.
5. Modeling the "level of risk" that the operator is willing to take while planning for contingencies.

Let us consider these deficiencies in turn. Uncertainties in input data result from many sources. Chief among them are uncertainties introduced by inaccurate and/or imprecise sensory data. Such uncertainties are particularly evident in control systems designed for geographically distributed physical networks such as electric power grids and air-traffic control. Traditionally, such uncertainties have been handled through the use of probabilistic techniques, resulting in stochastic optimization models. Recently, fuzzy logic has emerged as a feasible alternative for modeling data uncertainties in optimization of physical systems (1).

Several decades ago, March and Simon (2) argued that human decision-makers usually "satisfice" rather than "optimize." Traditional optimization models treat most problem constraints as rigid constraints whose violations are impermissible. This is not satisfactory for two reasons. First, the uncertainties in the parameters of the underlying physical system naturally lead to situations where violations are tolerated for gains in the objective. Second, one of the popular ways of modeling multiple conflicting objectives is the "constraint method" which converts secondary objectives to constraints (with specified tolerances that are to be minimized) (3). Thus, such "soft constraints" need to be modeled explicitly in the optimization problem. Penalty factor techniques and fuzzy-logic-based methods have been shown to be effective for modeling such soft constraints (4,5).

Operators can perform only a limited number of control actions in time-critical situations. Hence, the solutions of optimization methods need to be such that a small number of actions with different priorities are suggested. Again, traditional optimization techniques are not completely suitable. While postprocessing of solutions using technologies such as expert systems is feasible, this will likely compromise the optimality of the solutions. It is preferable to start from a model that incorporates such filtering and prioritization.

Discrete control actions are often more effective than their continuous counterparts when quick changes are needed. However, such actions are typically avoided in many optimization models because they result in mixed-integer nonlinear programming (MINLP) models that are often very difficult to solve. This is unfortunate since the results of enhanced models can lead to more efficient operations. Operators may be forced to use such actions without the aid of optimization models. What is needed is a fresh look at integration of discrete actions in continuous models without necessarily leading to rigorous MINLP problems. Techniques such as fuzzy logic can provide help in this respect.

Discrete actions can introduce multiple minima that reflect practical solution alternatives. Finding the global minimum can be a daunting task and is the subject of an entire field of the relatively new field of global optimization. However, this possibility needs to be carefully considered since it can result in significantly better solutions.

Planning for contingencies constitutes an important aspect of control and operation of complex real-time systems. However, contingency planning has been difficult to tackle through traditional optimization models because of the inability of these models to account for the subjective "risk preferences" of the operators. Risk assessment and management is a fundamental component of contingency planning. The trade-off is between economics and "security" against contingencies.

The first two deficiencies have been addressed in the general fuzzy logic literature, and fuzzy techniques have been proposed for system operation with uncertain data and soft constraints. The last three deficiencies have not received as much attention, and they are still unresolved problems. In this article, we concentrate on the modeling issues pertaining to the last deficiency. We refer the reader to Ref. 6 for an elaborate discussion on the other two deficiencies (i.e., the third and the fourth deficiency).

PROBLEM FORMULATION

The conventional optimization problem for a control system can be formulated as:

$$\operatorname*{Min}_{Z} f(Z) \quad \text{s.t.} \quad G(Z) = 0, \qquad H(Z) \leq 0 \tag{1}$$

where

U is the vector of control variables
X is the vector of state variables
$Z = [u, X]^{\mathrm{T}}$ is the vector of all the decision variables

G is the set of system equations
H is the set of operation limits
$f(Z)$ is the objective function (usually the cost) which is to be minimized

To account for the contingencies that can occur in the system (i.e., for contingency constrained optimization), the problem can be formulated as the following decomposed multiobjective problem.

Base-Case Subproblem

$$\begin{aligned} &\min_{Z_0} f(Z_0) \\ &\min_{Z_0} \|U_0 - U_k^*\|^2, \qquad k = 1, 2, \ldots, N \\ &\text{s.t.} \quad G_0(Z_0) = 0, \qquad H_0(Z_0) \le 0 \end{aligned} \tag{2}$$

where

$Z_0 = [U_0, X_0]^T$ is the base-case decision vector
U_k^* is the latest value of the kth subproblem control variables
G_0 is the set of system equations for the base case
H_0 is the set of operating limits for the base case
N is the number of contingencies

***N* Contingency Subproblems**

$$\min_{Z_k} \|U_0^* - U_k\|^2 \quad \text{s.t.} \quad G_k(Z_k) = 0, \qquad H_k(Z_k) \le 0 \tag{3}$$

where

U_0^* is the latest value of the base case control variables
$Z_k = [U_k, X_k]^T$ is the decision vector for the kth subproblem
G_k is the set of system equations for the kth subproblem
H_k is the set of operating limits for the kth subproblem

A FUZZY MODEL

Let $f(Z_0) \le C_0 + \delta_c$ represent an imprecise upper bound on the maximum permissible operating cost, where C_0 is the optimal cost obtained by solving the general constrained optimization problem without contingency constraints, and δ_c is the "tolerance" parameter that is a measure of the fuzziness in this constraint. So the fuzzy goal is to keep $f(Z_0)$ "as close to" C_0 as possible, but no greater than $C_0 + \delta_c$.

Let μ_c be the membership function that represents the extent to which a given $f(Z_0)$ satisfies the fuzzy goal. Such a membership function can take any value in [0, 1]. The higher its value, the greater the degree of satisfaction of the fuzzy goal by the given $f(Z_0)$. If we assume that the operator's satisfaction decreases linearly with deviation from C_0, we can use the following mathematical formulation for this (linear) membership function:

$$\mu_c(f(Z_0)) = \begin{cases} 1 & \text{if } f(Z_0) \le C_0 \\ \dfrac{(C_0 + \delta_c - f(Z_0))}{\delta_c} & \text{if } C_0 < f(Z_0) < C_0 + \delta_c \\ 0 & \text{if } f(Z_0) \ge C_0 + \delta_c \end{cases} \tag{4}$$

Let η_k represent either of the functions $\|U_0^* - U_k\|^2$ or $\|U_k^* - U_0\|^2$. Let Δ be the vector of the ramp limits (the increase or decrease rate) for the control variables. Then the fuzzy ramping constraints can be formulated as $\eta_k < \|\Delta\|^2 + \delta_\Delta$, where δ_Δ is a parameter that specifies the tolerance on the violation of this constraint. The corresponding fuzzy goal is to keep η_k "as close to" $\|\delta_\Delta\|^2$ as possible but no greater than $\|\Delta\|^2 + \delta_\Delta$. We can define a linear membership function $\mu_\Delta(\eta_k)$, as follows:

$$\mu_\Delta(\eta_k) = \begin{cases} 1 & \text{if } \eta_k \le \|\Delta\|^2 \\ \dfrac{(\|\Delta\|^2 + \delta_\Delta - \eta_k)}{\delta_c} & \text{if } \|\Delta\|^2 < \eta_k < \|\Delta\|^2 + \delta_\Delta \\ 0 & \text{if } \eta_k \ge \|\Delta\|^2 + \delta_\Delta \end{cases} \tag{5}$$

Both the membership functions are displayed in Fig. 1.

Let $\eta_k(U_0)$ represent $\|U_k^* - U_0\|^2$, and let $\eta_k(U_k)$ represent $\|U_0^* - U_k\|$. Our fuzzy formulation of the decomposition presented in the previous section is the following:

Base-Case Subproblem

$$\begin{aligned} &\max_{Z_0} \min\{\mu_c(f(Z_0)),\ \mu_\Delta(\eta_k(U_0)),\ k = 1, \ldots, N\} \\ &\qquad\qquad \text{s.t.} \quad G_0(Z_0) = 0, \qquad H_0(Z_0) \le 0 \end{aligned} \tag{6}$$

In Eq. (6), the Min operator is used to represent the intersection of the $N + 1$ fuzzy sets corresponding to the two membership functions. The resulting Max–Min formulation aims to find an operating point that maximizes the degree of satisfaction of the least satisfied fuzzy relation, for a given set of N postcontingency operating points. This is one way to seek a compromise between the $N + 1$ fuzzy relations.

***N* Contingency Subproblems**

$$\begin{aligned} &\max_{Z_k} \mu_\Delta(\eta_k(U_0)) \\ &\text{s.t.} \quad G_k(Z_k) = 0, \qquad H_k(Z_k) \le 0 \end{aligned} \tag{7}$$

In solving each contingency subproblem, we seek to find a postcontingency operating point that is "closest" (per the Euclidean norm) to the given base case, in control space. This is tantamount to maximizing the degree of satisfaction of the corresponding membership function.

To solve this fuzzy model using standard optimization methods, we need to convert Eq. (4) to an equivalent "crisp"

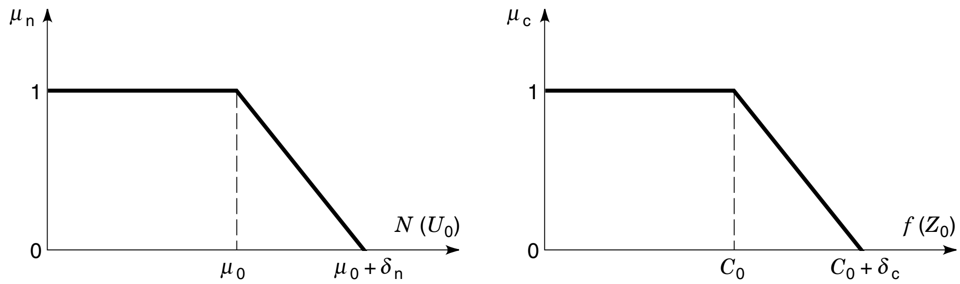

Figure 1. Membership functions for our fuzzy goals.

formulation. To do this, we introduce $N + 1$ (one for each subproblem) membership variables, β_k, as follows:

Base-Case Subproblem

$$
\begin{aligned}
&\max_{Z_0,\beta_0} \beta_0 \\
&\text{s.t.}\quad f(Z_0) + \delta_c\beta_0 \le C_0 + \delta_c \\
&\qquad \eta_k(U_0) + \delta_\Delta\beta_0 \le \|\Delta\|^2 + \delta_\Delta, \qquad k = 1, \ldots, N \\
&\qquad G_0(Z_0) = 0, \qquad H_0(Z_0) \le 0, \qquad 0 \le \beta_0 \le 1
\end{aligned}
\tag{8}
$$

***N* Contingency Subproblems**

$$
\begin{aligned}
&\max_{Z_k,\beta_k} \beta_k \\
&\text{s.t.}\quad \eta_k(U_k) + \delta_\Delta\beta_k \le \|\Delta\|^2 + \delta_\Delta\|^2 + \delta_\Delta, \qquad k = 1, \ldots, N \\
&\qquad G_k(Z_k) = 0, \qquad H_k(Z_k) \le 0, \qquad 0 \le \beta_k \le 1
\end{aligned}
\tag{9}
$$

OBTAINING FUZZY TOLERANCE PARAMETERS

In this section, we describe a procedure for obtaining the parameters, δ_c and δ_Δ, used in the model presented above. This procedure is based on the work of Zimmermann (1).

1. Solve the base case in Eq. (2) for the first objective (ignoring the second one) to get U_0^* and $f(U_0^*)$. $C_0 = f(U_0^*)$.
2. For each contingency k, using U_0^* obtained from 1:
   i. Solve Eq. (3) to get U_k^*.
   ii. Solve the second objective in Eq. (2) subject to the constraints to get U_{0k}^*.
   iii. Calculate $f(U_{0k}^*)$.

Then, $C_0 + \delta_c = \text{Max}\{f(U_{0k}^*), k = 1, \ldots, N\}$ and $\|\Delta\|^2 + \delta_\Delta = \text{Max}\{\|U_k^* - U_0^*\|^2, k = 1, \ldots, N\}$.

The idea is to set the upper bound on the base-case cost equal to the maximum deviation from the optimal cost that is necessary to minimize the correction time of any of the contingencies. Similarly, the upper bound on the correction time is equal to the maximum of the correction times of any of the contingencies, corresponding to the least-cost base-case point. Δ is determined from the maximum correction times specified by the operator.

We would also expect to query the operator regarding our choice of a linear function to reflect the rate of decrease in the degrees of satisfaction. This can be done by picking a couple of points within the bounds and asking the operator for the change in the satisfaction compared with one of the bounds. If a linear function is not found to be appropriate, it can be replaced by one of the other functions reported in the fuzzy logic literature such as hyperbolic or exponential (7). Once the bounds are known, this is relatively easy to do.

THE ALGORITHM

Let $\Delta\beta_k$ represent the change in β_k between two successive iterations. Let ϵ represent a "termination" parameter. Then the algorithm for our fuzzy approach is as follows:

Step 1. Given $\|\Delta\|^2$, use the above procedure to get δ_c, δ_Δ, and U_0^* corresponding to C_0.

Step 2. Given U_k^*, solve the base-case subproblem in Eq. (8) to obtain Z_0^* and β_0.

Step 3. Given U_0^*, solve the N contingency subproblems in Eq. (9) to obtain Z_k^* and β_k for $k = 1, \ldots, N$.

Step 4. If $\|\Delta\beta_i\| < \epsilon$, for all $i = 0, \ldots, N$, stop; else go to step 2.

The use of the above algorithm for a real-world example (an electric power system control problem) is described in Ref. 8.

CONCLUSION

This article discusses a method for real-time optimization problems. The models described here have been applied to control of electric power networks and are discussed in detail in Refs. 8–10. We believe that modeling deficiencies in many such optimization problems can be fruitfully addressed using fuzzy logic.

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