CHAOTIC SYSTEMS CONTROL

Almost all real physical, biological, and chemical as well as many other systems are inherently nonlinear. This is also the case with electrical and electronic circuits. Apart from systems designed to perform linear operations (usually in such cases they just operate in a small region in which they behave linearly) there exists an abundance of systems that are nonlinear by their principle of operation. Rectifiers, flip-flops, modulators and demodulators, memory cells, analog to digital (A/D) converters, and different types of sensors are good examples of such systems. In many cases the designed circuit, when implemented, performs in a very unexpected way, totally different from that for which it was designed. In most cases, engineers do not care about the origins and mechanisms of the malfunction; for them a circuit that does not perform as desired is of no use and has to be rejected or redesigned. Many of these unwanted phenomena, such as excess noise, false frequency lockings, squegging, and phase slipping have been found to be associated with bifurcations and chaotic behavior. Also many nonlinear phenomena in other science and engineering disciplines have a strong link with "electronic chaos." Examples are aperiodic electrocardiogram waveforms (reflecting fibrillations, arrythmias, or other types of heart malfunction), epileptic foci in electroencephalographic patterns, or other measurements taken by electronic means in plasma physics, lasers, fluid dynamics, nonlinear optics, semiconductors, and chemical or biological systems.

DEFINITION OF IS CHAOTIC BEHAVIOR

In this section we consider only deterministic systems (i.e., systems for which knowledge of the initial state at some initial time t_0, equations of evolution and input signals fully determine the state and outputs for any $t \geq t_0$).

Typically deterministic systems display three types of behavior of their solutions: they approach constant solutions, they converge toward periodic solutions, or they converge toward quasi-periodic solutions. These are the situations known to every practicing engineer.

Now it has been confirmed that almost every physical system can also display behaviors that cannot be classified in any of the above-mentioned three categories; the systems become aperiodic (chaotic) if their parameters, internal variables, or external stimulations are chosen in a specific way. How can we describe chaos except saying that it is the kind of behavior that is not constant, periodic, or quasi-periodic or convergent to any of the above? For the purpose of this article we consider some specific properties to qualify behavior as chaotic:

1. The solutions show sensitive dependence on initial conditions (trajectories are unstable in the Lyapunov sense) but remain bounded in space as time elapses (are stable in the Lagrange sense).
2. Trajectory moves over a strange attractor, a geometric invariant object that can possess fractal dimension. The trajectory passes arbitrarily close to any point of the attractor set—that is, there is a dense trajectory.
3. Chaotic behavior appears in the system as via a "route" to chaos that typically is associated with a sequence of bifurcations, qualitative changes of observed behavior when varying one or more of the parameters.

Sensitive dependence on initial conditions means that trajectories of a chaotic system starting from nearly identical initial conditions will eventually separate and become uncorrelated (but they will always remain bounded in space). Large variations in the observed long-term behavior due to very small changes of initial state are often referred to as "the butterfly effect" (increment of butterfly wings can change weather in a far away region of earth). Figure 1 gives an example of two trajectories starting from initial conditions differing by 0.001; after remaining close to each other for some period, they eventually separate. Sensitive dependence on initial conditions for a system is realized only with some finite accuracy ϵ. If two initial conditions are closer to each other than ϵ, then they are not distinguishable in measurements. The trajectories of a chaotic system starting from such initial conditions will, after a finite time, diverge and become uncorrelated. For any precision we use in measurements (experiments) the behavior of trajectories is not predictable—the solutions look virtually random despite being produced by a deterministic system. There is also another consequence of this property that may be appealing for control purposes: a very small stimulus in the form of tiny change of parameters can have a very large effect on the system's behavior.

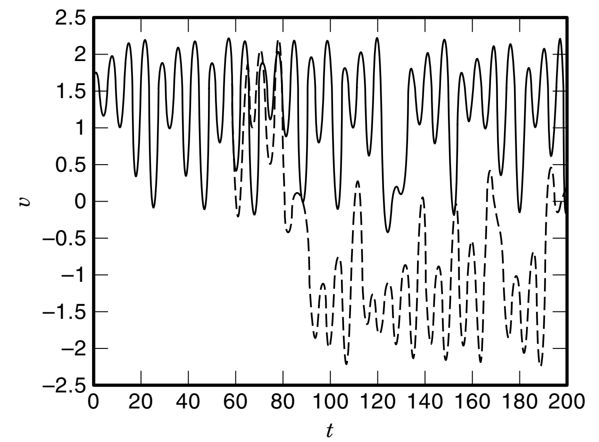


Figure 1. Illustration of the sensitive dependence on initial conditions—first fundamental property of chaotic systems. Two trajectories of Chua's oscillator starting from initial conditions with the difference of 0.001 in the first component for a short time stay close to each other but eventually separate resulting in waveforms of different shape.

The second property can be explained easily by Fig. 2. It is clear that the trajectory shown in this figure "fills" out some

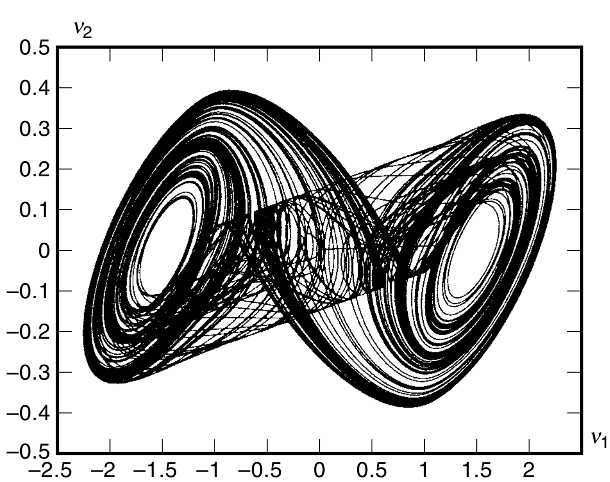


Figure 2. An example of a chaotic trajectory. Two-dimensional projection of the double scroll attractor observed in Chua's circuit is shown. The curve never closes itself, moves around in an unpredictable way, and densely fills some part of the space (here, the plane).

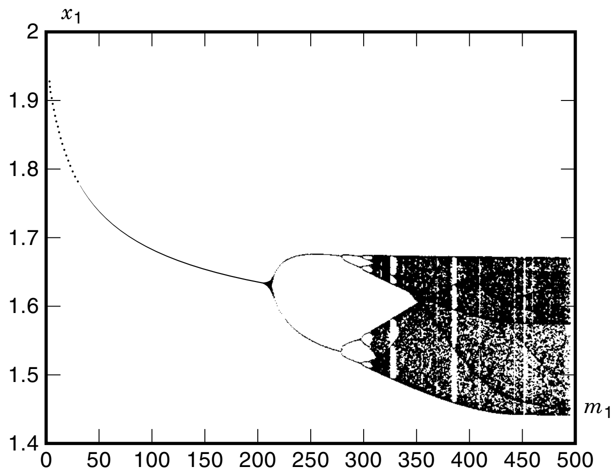


Figure 3. Bifurcation diagram for the *RC*-ladder chaos generator with slope m_1 chosen as bifurcation parameter. The diagram is obtained in such a way as for every chosen parameter value (abcissa) the long-term behavior of the chosen system variable is observed and coordinates of intersections of the orbit with a chosen plane are recorded and plotted. Thus for a chosen parameter value, the number of points plotted tells exactly what kind of behavior is observed. One point corresponds to a period-one orbit, two points to a period-two orbit, and a large number of points spread in an interval can be interpreted as chaotic behavior. Visible chaos appears via a "route" when the parameter is changed continuously—here, branching of the bifurcation tree can be interpreted as period doubling route to chaos. The diagram also confirms existence of a large variety of qualitatively different behaviors existing for suitably chosen values of parameter.

part of the space. If we arbitrarily choose a point within this region of space and a small ball of radius ϵ around it, the trajector will eventually pass through this ball after a finite time (which might be very long). As an example of the third property we give a typical bifurcation diagram obtained in numerical experiments (Fig. 3). By a suitable choice of parameter m_1 one can choose almost every type of periodic behavior apart from many chaotic states. There is an important fact often associated with bifurications: in many cases creation of new types of new trajectories that are observable in experiments (stable) via bifurcation is accompanied by creation of unstable orbits—invisible in experiments. Many of these unstable orbits persist also within the chaotic attractor. Many authors consider as fundamental the property of existence of a countable (infinite) number of unstable periodic orbits within an attractor.

Using proprietary numerical procedures it is possible to detect some of such orbits in numerical experiments (1). Figure 4 shows some of the periodic orbits uncovered from the double scroll attractor shown in Fig. 2.

The above-described fundamental properties of chaotic systems (their solutions) is the basis of the chaos control approaches described below.

WHAT CHAOS CONTROL MEANS

Chaos, so commonly encountered in physical systems, represents a rather peculiar type of behavior commonly considered as causing malfunctions, disastrous in most applications. It is obvious that an amplifier, a filter, an A/D converter, a phase-locked loop, or a digital filter generating chaotic responses is of no use—at least for its original purpose. Similarly, we would like to avoid situations where the heart does not pump blood properly (fibrillation or arrythmias) or epileptic attacks. Even more spectacular potential applications might be influencing rainfall and avoiding hurricanes and other atmospheric disasters believed to be associated with large-scale chaotic behavior.

The most common goal of control for a chaotic system is suppression of oscillations of the "bad" kind and influencing the system in such a way that it will produce a prescribed, desired motion. The goals vary depending on a particular application. The most common goal is to convert chaotic motion into a stable periodic or constant one. It is not at all obvious how such a goal could be achieved, because one of the fundamental features of chaotic systems, the sensitive dependence on initial conditions, seems to contradict any stable system operation. Recently, several applications have been mentioned in the literature in which the desired state of system operation is chaotic. The control problems in such cases are defined as: converting unwanted chaotic behavior into another kind of chaotic motion with prescribed properties (this is the goal of chaos synchronization) or changing periodic behavior into chaotic motion (which might be the goal in the case of epileptic seizures). The last-mentioned type of control is often referred to as *anticontrol* of chaos.

Many chaotic systems display what is called multiple basins of attracton and fractal basin boundaries. This means that, depending on the initial conditions, trajectories can converge to different steady states. Trajectories in nonlinear systems may possess several different limit sets and thus exhibit a variety of steady-state behaviors depending on the initial condition, chaotic or otherwise. In many cases, the sets of initial states leading to a particular type of behavior are intertwined in a complicated way forming fractal structures. Thus we could consider elimination of multiple basins of attraction as another kind of control goal.

In some cases, chaos is the dynamic state in which we would like the system to operate. We can imagine that mixing of components in a chemical reactor would be much quicker in a chaotic state than in any other one, or that chaotic signals could be useful for hiding information. In such cases, however, we need a "wanted kind" of chaotic behavior with precisely prescribed features and/or we need techniques to switch between different kinds of behavior (chaos-order or chaos-chaos).

Considering the possibilities of influencing the dynamics of a chaotic circuit we can distinguish four basic approaches:

- variation of an existing accessible system parameter
- change in the system design-modification of its internal structure
- injection of an external signal(s)
- introduction of a controller (classical PI, PID, linear or nonlinear, neural, stochastic, etc.)

Because of the very rich dynamic phenomena encountered in typical chaotic systems, there are a large variety of approaches to controlling such systems. This article presents selected methods developed for controlling chaos in various aspects—starting from the most primitive concepts like

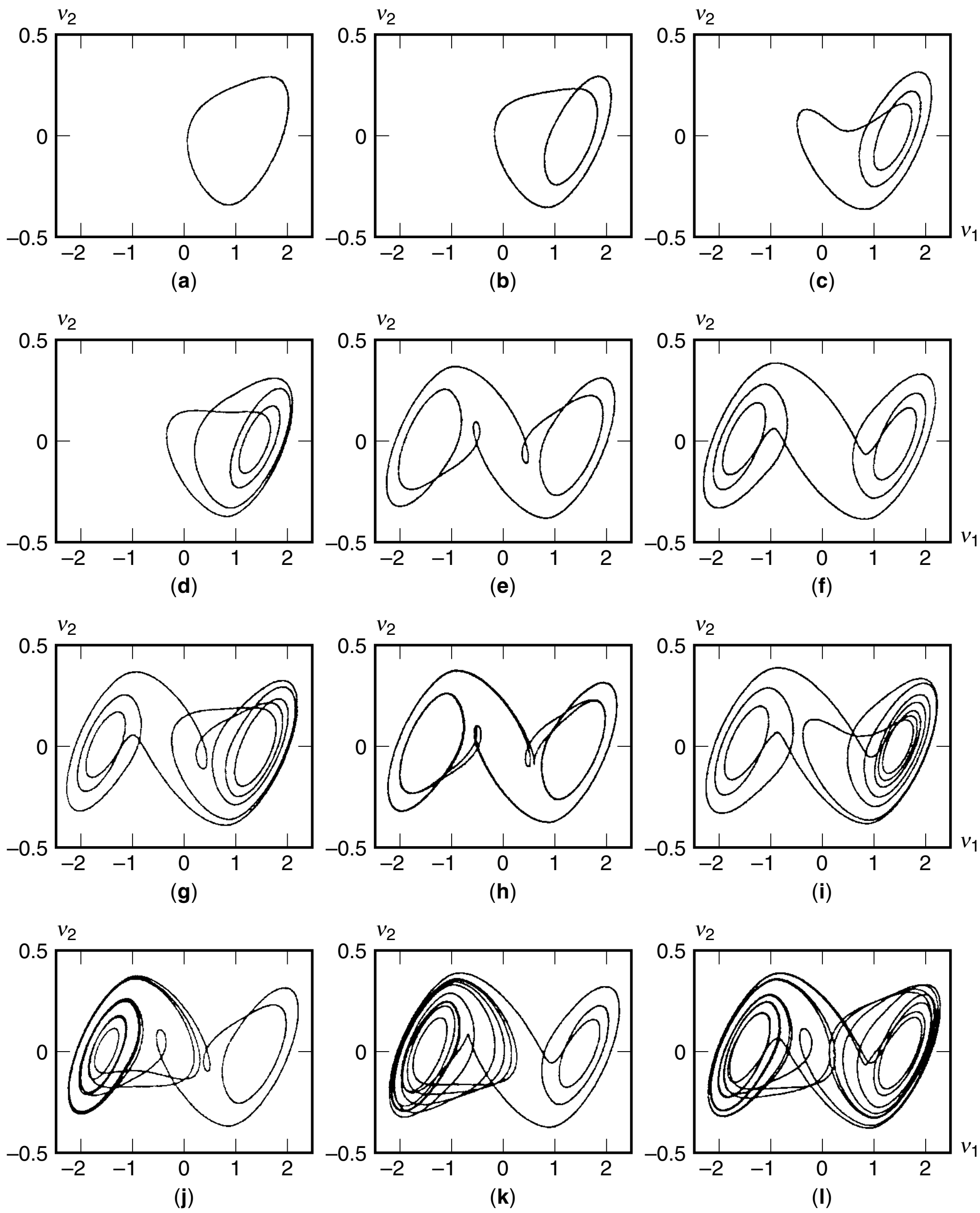


Figure 4. Second fundamental property of chaos. Within an attractor (visible in experiments and depicted in Fig. 2) an infinite but countable number of unstable periodic orbits exist. Such orbits are impossible to observe in experiments but can be detected using computer methods. In this picture some approximations to actual unstable periodic orbits are shown. These are uncovered using numerical calculations from time series measured for the double scroll attractor shown in Fig. 2. Notice the shape of the orbits—when superimposed these orbits reproduce the shape of the chaotic attractor.

parameter variation, through classical controller applications (open- and closed-loop control), to quite sophihsticated ones like stabilization of unstable periodic orbits embedded within a chaotic attractor.

GOALS OF CONTROL

As already mentioned, systems displaying chaotic behavior possess specific properties. Now we will exploit these properties when attacking the control problem. In what way does a chaotic system differ from any other object of control? How could its specific properties be advantageous for control?

The route to chaos via a sequence of bifurcations has two important implications for chaos control: first, it gives an insight into other accessible behaviors that can be obtained by changing parameters (this may be used for redesigning the system); second, stable and unstable orbits that are created or annihilated in bifurcations may still exist in the chaotic range and constitute potential goals for control.

Three fundamental properties of chaotic systems are of potential use for control purposes. For a long time the instability property (sensitive dependence on initial conditions) has been considered the main obstacle for control. How can one visualize successful control if the dynamics may change drastically with small changes of the initial conditions or parameters? How can one produce a prescribed kind of behavior if errors in initial conditions will be exponentially amplified?

This fundamental property does not, however, necessarily mean that control is impossible. It has been shown that despite the divergence of nearby starting trajectories, they can be convergent to another prescribed kind of trajectory—one simply has to employ a different notion of stability. In fact, we do not require that the nearby trajectories converge—the requirement is quite different—the trajectories should merely converge to some goal trajectory $g(t)$

$$\lim_{t\to\infty} |x(t) - g(t)| = 0 \tag{1}$$

Depending on a particular application $g(t)$ could be one of the solutions existing in the system or any external waveform we would like to impose. Extreme sensitivity may even be of prime importance as control signals are in such cases very small.

The second important property of chaotic systems that will be exploited is the existence of a countable infinity of unstable periodic orbits within the attractor, already considered earlier. These orbits, although invisible during experiments, constitute a dense set supporting the attractor. Indeed, the trajectory passes arbitrarily close to every such orbit. This invisible structure of unstable periodic orbits plays a crucial role in many methods of chaos control; with specific methods the chaotic trajectory can be perturbed in such a way that it will stay in the vicinity of a chosen unstable orbit from the dense set.

These fundamental properties of chaotic signals and systems offer some very interesting issues for control not available in other classes of systems (2,3). Namely,

- because of sensitive dependence on initial conditions it is possible to influence the dynamics of the systems using very small perturbations; moreover, the response of the system is very fast
- the existence of a countable infinity of unstable periodic orbits within the attractor offers extreme flexibility and a wide choice of possible goal behaviors for the same set of parameter values

SUPPRESSING CHAOTIC OSCILLATIONS BY CHANGING SYSTEM DESIGN

Effects of Large Parameter Changes

The simplest way of suppressing chaotic oscillations is to change the system parameters (system design) in such a way as to produce the desired kind of behavior. The influence of parameter variations on the asymptotic behavior of the system can be studied using a standard tool for analysis of chaotic systems—the bifurcation diagram. The typical bifurcation diagram reveals a variety of dynamic behaviors for appropriate choices of system parameters and tells us what parameter values should be chosen to obtain the desired behavior. In electronic circuits, changes in the dynamic behavior are obtained by changing the value of one of its passive elements (which means replacing one of the resistors, capacitors, or inductors). In Fig. 3 a sample bifurcation diagram reveals a variety of dynamic behaviors observed in the *RC* chaos generator (4) (when changing one of the slopes of the nonlinear element). Thus when the generator is operating in a chaotic range, one can tune (control) it using a potentiometer to obtain a desired periodic state existing and displayed in the bifurcation diagram.

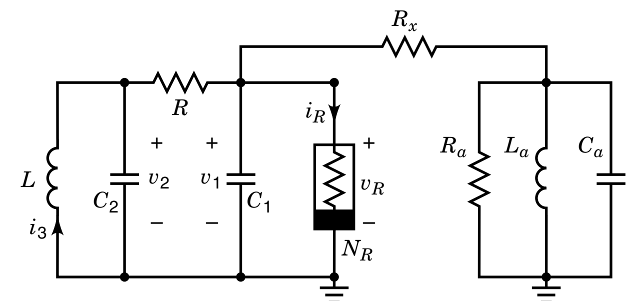


Figure 5. Chaos can be stabilized by adding a stabilizing subsystem to the chaotic one. As an example, a parallel *RLC* circuit is connected to the chaotic Chua's circuit and acts as a chaotic oscillation absorber.

This method, although intuitively simple, is hardly acceptable in practice; it requires large parameter variations (large energy control). This requirement cannot be met in many physical systems where the construction parameters are either fixed or can be changed over very small ranges. This method is also difficult to apply on the design stage as there are no simulation tools for electronic circuits allowing bifurcation analysis (e.g., SPICE has no such capability). On the other hand, programs offering such types of analysis require a description of the problem in closed mathematical form, such as differential or difference equations. Changes of parameters are even more difficult to introduce once the circuitry is fabricated or breadboarded, and if possible at all can be done only on a trial-and-error basis.

"Shock Absorber" Concept—Change in System Structure

This simple technique is being used in a variety of applications. The concept comes from mechanical engineering, where devices absorbing unwanted vibrations are commonly used (e.g., beds of machine-tools, shock absorbers in vehicle suspensions). The idea is to modify the original chaotic system design (add the "absorber" without major changes in the design or construction) in order to change its dynamics in such a way that a new stable orbit appears in a neighborhood of the original chaotic attractor. In an electronic system, the absorber can be as simple as an additional shunt capacitor or an *LC* tank circuit. Kapitaniak et al. (5) proposed such a "chaotic oscillation absorber" for Chua's circuit—it is a parallel *RLC* circuit coupled with the original Chua's circuit via a resistor (Fig. 5)—depending on its value the original chaotic behavior can be converted to a chosen stable oscillation. The equations describing dynamics of this modified system can be given in a dimensionless form:

$$\begin{aligned}
\dot{x} &= \alpha[y - x - g(x)] \\
\dot{y} &= x - y + z + \epsilon(y^1 - y) \\
\dot{z} &= -\beta y \\
\dot{y}' &= \alpha'[-\gamma' y' + z' + \epsilon(y - y')] \\
\dot{z}' &= -\beta' y'
\end{aligned} \tag{2}$$

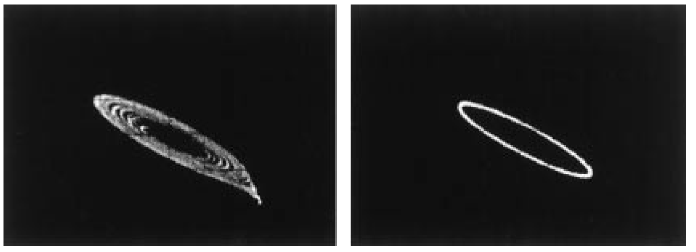


Figure 6. The "shock absorber" eliminates changes in the system behavior. For example, the spiral-type Chua's attractor can be quenched and a period-one orbit appears when parameters of the parallel *RLC* oscillation absorber, shown in Figure 5, are properly adjusted.

In terms of circuit equations, we have an additional set of two equations for the "absorber" (y^1, z^1) and a small term [$\epsilon(y^1 - y)$] through which the original equations of Chua's circuit are modified. Figure 6 shows the result of a laboratory experiment. Addition of a "shock absorber" in Chua's circuit changes chaotic behavior [Fig. 6(a)] to a periodic one [Fig. 6(b)].

EXTERNAL PERTURBATION TECHNIQUES

Several authors have demonstrated that a chaotic system can be forced to perform in a desired way by injecting external signals that are independent of the internal variables or structure of the system. Three types have been considered: (a) aperiodic signals ("resonant stimulation"), (b) periodic signals of small amplitude, and (c) external noise.

"Entrainment"—Open Loop Control

Aperiodic external driving is a classical control method and was one of the first methods introduced by Hübler (6,7) (resonant stimulation). A mathematical model of the considered experimental system is needed (e.g., in the form of a differential equation: $dx/dt = F(x)$, $x \in R^n$, where $F(x)$ is differentiable and a unique solution exists for every $t \geq 0$).

The goal of the control is to entrain the solution $x(t)$ to an arbitrarily chosen behavior $g(t)$:

$$\lim_{t\to\infty} |x(t) - g(t)| = 0 \quad (3)$$

Entrainment can be obtained by injecting the control signal:

$$\frac{dx}{dt} = F(x) + [\dot{g} - F(g)]1(t) \quad (4)$$

where $1(t)$ is 0 for $t < 0$ and 1 for $t > 0$. The entrainment method has the advantage that no feedback is required and no parameters are changed—thus the control signal can be computed in advance and no equipment for measuring the state of the system is needed. The goal does not depend on the system being considered, and in fact it could be any signal at all (except that solutions of the autonomous system since $\dot{g} - F(g) \equiv 0$ in this case, and there is no control signal). It should be noted, however, that this method has limited applicability since a good model of the system dynamics is necessary, and the set of initial statistics for which the system trajectories will be entrained is not known.

Weak Periodic Perturbation

Interesting results have been reported by Breiman and Goldhirsch (8), who studied the effects of adding a small periodic driving signal to a system behaving in a chaotic way. They discovered that external sinusoidal perturbation of small amplitude and appropriately chosen frequency can eliminate chaotic oscillations in a model of the dynamics of a Josephson junction and cause the system to operate in some stable periodic mode. Unfortunately, there is little theory behind this approach and the possible goal behaviors can be learned only by trial and error. Some hope for further understanding and applications can be based on using theoretical results known from the theory of synchronization.

Noise Injection

A noise signal of small amplitude injected in a suitable way into the circuit (system) offers potentially new possibilities for stabilization of chaos. The first observations date back to the work of Herzel (9). The effects of noise injection were also studied in an *RC*-ladder chaotic oscillator (10). In particular it has been observed that injection of noise of sufficiently high level can eliminate multiple domains of attraction. In the experiments with the *RC*-ladder chaos generator it has been found that the two main branches, representing two distinct, coexisting solutions, as shown in Fig. 3, will join together if white noise of high level is added. This approach, although promising, needs further investigation because there is little theory available to support experimental observations.

CONTROL ENGINEERING APPROACHES

Several investigators have tried to use known methods belonging to the "control engineer's toolkit." For example, PI and PID controllers for chaotic circuits, applications of stochastic control techniques, Lyapunov-type methods, robust controllers, and many other methodologies, including intelligent control and neural controllers, have been described in the literature. Chen and Dong (11) and Chapter 5 in Madan's book (12) give an excellent review of applications of such methods. In electronic circuits two schemes—linear feedback and time-delay feedback—seem to find the most successful applications.

Error Feedback Control

Several methods of chaos control have been developed that rely on the common principle that the control signal is some function ϕ of the difference between the actual system output $x(t)$ and the desired goal dynamics $g(t)$. This control signal could be an actual system parameter:

$$p(t) = \phi[x(t) - g(t)] \quad (5)$$

or an additive signal produced by a linear controller:

$$u(t) = K[x(t) - g(t)] \quad (6)$$

The control term is simply added to the system equations. One can readily see that, although mathematically simple, such an "addition" operation might pose serious problems in real applications. The block diagram of the control scheme is

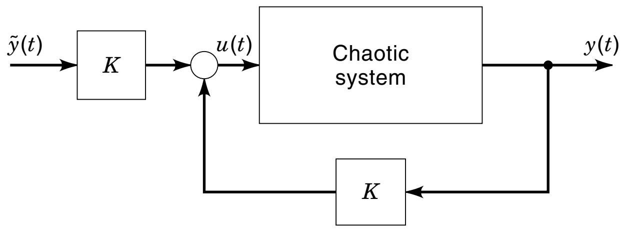


Figure 7. Standard control engineering methods can be used to stabilize chaotic systems, for example the linear feedback control scheme proposed by Chen and Dong, shown here.

shown in Fig. 7. Using error feedback, chaotic motion has been successfully converted into periodic motion both in discrete- and continuous-time systems. In particular, chaotic motions in Duffing's oscillator and Chua's circuit have been controlled (directed) toward fixed points or periodic orbits (11). The equations of the controlled circuit read:

$$\begin{aligned} \dot{x} &= \alpha[y - x - g(x)] \\ \dot{y} &= x - y + z - K_{22}(y - \tilde{y}) \\ \dot{z} &= -\beta y \end{aligned} \tag{7}$$

Thus we have a single term added to the original equations.

Figure 8 shows a double scroll Chua's attractor and large saddle-type unstable periodic orbit toward which the system has been controlled.

The important properties of the linear feedback chaos control method are that the controller has a very simple structure and that access to the system parameters is not required. The method is immune to small parameter variations but might be difficult to apply in real systems (interactions of many system variables are needed). The choice of the goal orbit poses the most important problem; usually the goal is chosen in multiple experiments or can be specified on the basis of model calculations.

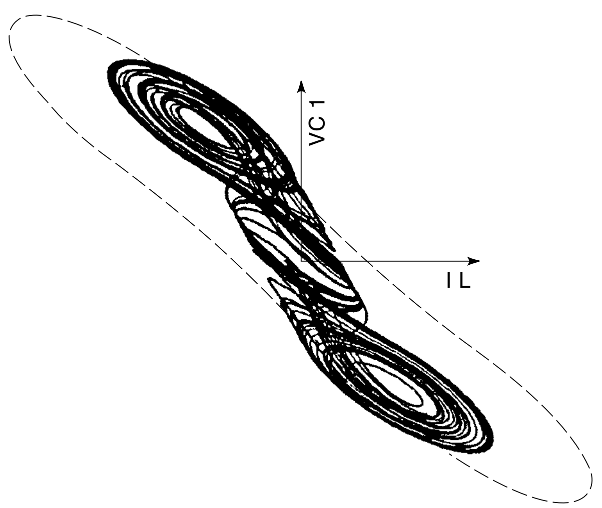


Figure 8. Linear feedback method in many cases enables stabilization of a simple orbit which is a solution of the system. For example, the double scroll (chaotic) attractor and a saddle type unstable periodic orbit coexist in Chua's circuit. This periodic orbit can be stabilized using linear feedback.

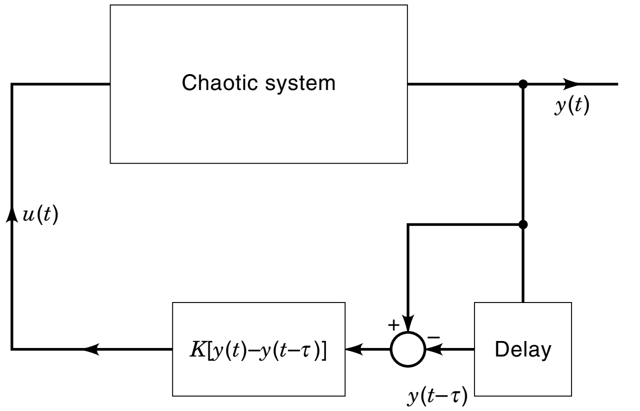


Figure 9. Block diagram of the delay feedback control scheme proposed by Pyragas. Injection of signal proportional to the difference between the original output and its delayed copy can stabilize operation of a chaotic system when the time delay and gain in the feedback loop are chosen appropriately.

STABILIZING UNSTABLE PERIODIC ORBITS

Time-Delay Feedback Control (Pyragas Method)

An interesting method has been proposed by Pyragas (13). The control signal applied to the system is proportional to the difference between the output and a delayed copy of the same output:

$$\frac{dx}{dt} = F[x(t)] + K[y(t) - y(t - \tau)] \tag{8}$$

Tuning the delay τ one can approach many of the periods of the unstable periodic orbits embedded within the chaotic attractor. In such a situation, the control signal approaches 0. A block diagram of the control scheme is shown in Fig. 9. Depending on the delay constant τ and the linear factor K, various kinds of periodic behaviors can be observed in the chaotic system. In the case of Chua's circuit we were able, for example, to convert chaotic motion into a periodic one, as shown in Fig. 10.

Pyragas obtained very promising results in the control of many different chaotic systems, and despite the lack of mathematical rigor, this method is being successfully used in several applications.

An interesting application of this technique is described by Mayer-Kress et al. (14). Pyragas's control scheme has been used for tuning chaotic Chua's circuits to generate musical

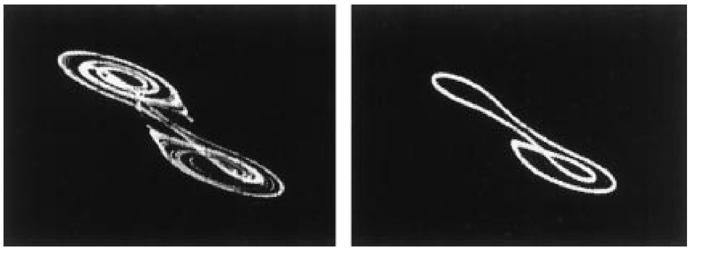


Figure 10. The double scroll attractor can be eliminated and the behavior converted to one of the periodic orbits in experiments in the delayed feedback control of Chua's circuit.

tones and signals. More recently Celka (15) used Pyragas's method to control a real electrooptical system.

The positive features of the delay feedback control method are that no external signals are injected and no access to system parameters is required. Any of the unstable periodic orbits can be stabilized provided that delay is chosen in an appropriate way. The control action is immune to small parameter variations. In real electronic systems, the required variable delay element is readily available (for example, analog delay lines are available as off-the-shelf components). The primary drawback of the method is that there is no a priori knowledge of the goal (the goal is arrived at by trial and error).

Ott–Grebogi–Yorke Local Linearization Approach

Ott, Grebogi, and Yorke (16,17) in 1990 proposed a feedback method to stabilize any chosen unstable periodic orbit within the countable set of unstable periodic orbits existing in the chaotic attractor. To visualize best how the method works, let us assume that the dynamics of the system are described by a k-dimensional map:$x_{n+1} = F(x_n, p)$, $x_i \in R^k$. This map, in the case of continuous-time systems, can be constructed (e.g., by introducing a transversal surface of section for system trajectories, p is some accessible system parameter that can be changed in some small neighborhood of its nominal value p^*). To explain the method we will concentrate now on stabilization of a period-one orbit. Let $x_F = F(x_F, p^*)$ be the chosen fixed point (period one) of the map around which we would like to stabilize the system. Assume further that the position of this orbit changes smoothly with p parameter changes (i.e., p^* is not a bifurcation value) and there are small changes in the local system behavior for small variatons of p. In a small vicinity of this fixed point we can assume with good accuracy that the dynamics are linear and can be expressed approximately by:

$$x_{n+1} - x_0 = A(x_n - x_0) + g(p_n - p^*) \tag{9}$$

The elements of the matrix $A = \partial F/\partial x\ (x_F, p^*)$ and vector $g = \partial F/\partial p\ (x_F, p^*)$ can be calculated using the measured chaotic time series and analyzing its behavior in the neighborhood of the fixed point. Further, the eigenvalues λ_s, λ_u and eigenvectors e_s, e_u of this matrix can be found

$$Ae_u = \lambda_u e_u \quad \text{and} \quad Ae_s = \lambda_s e_s \tag{10}$$

where the subscripts "u" and "s" correspond to unstable and stable directions respectively. These eigenvectors determine the stable and unstable directions in the small neighborhood of the fixed point (Fig. 11).

$$A = [e_u \quad e_s] \begin{bmatrix} \lambda_u & 0 \\ 0 & \lambda_s \end{bmatrix} [e_u \quad e_s]^{-1} \tag{11}$$

Let us denote by f_s, f_u the contravariant eigenvectors [$f_s^T e_s = f_u^T e_u = 1$, $f_s^T e_u = f_u^T e_s = 0$; see Fig. 11(c)]. Thus

$$A = [e_u \quad e_s] \begin{bmatrix} \lambda_u & 0 \\ 0 & \lambda_s \end{bmatrix} \begin{bmatrix} f_u^T \\ f_s^T \end{bmatrix} = \lambda_u e_u f_u^T + \lambda_s e_s f_s^T \tag{12}$$

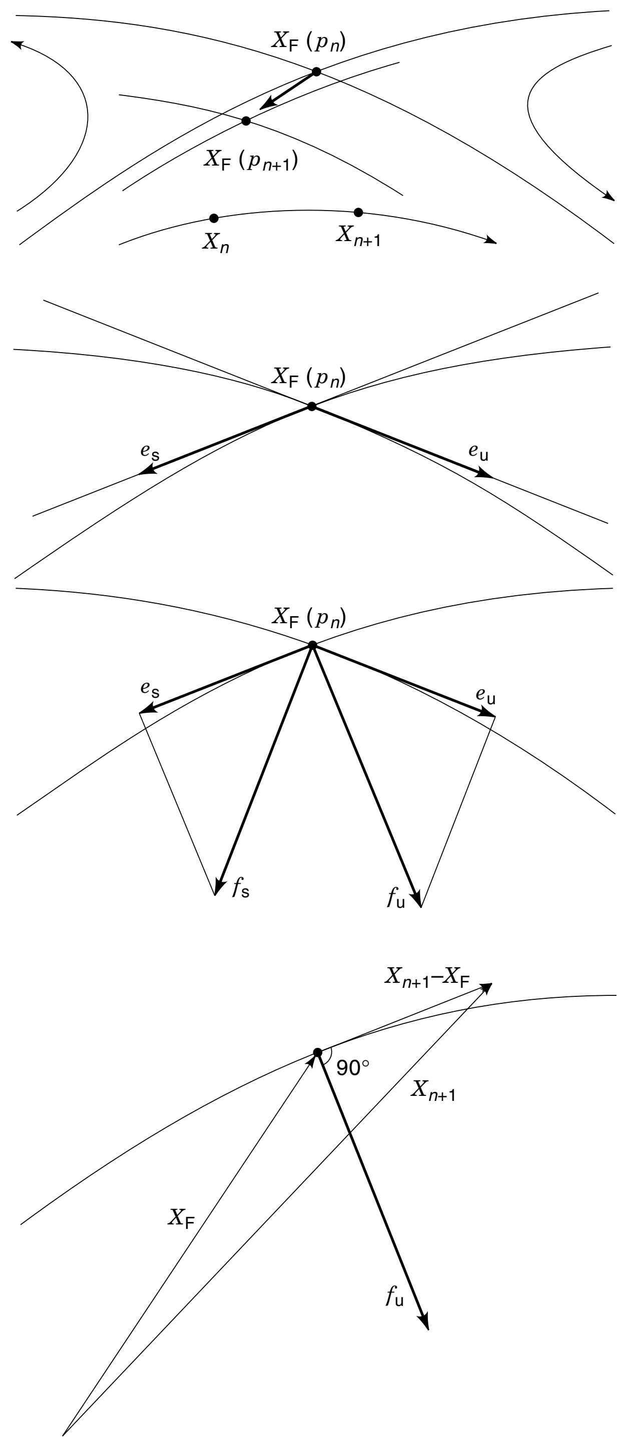


Figure 11. Explanation of the linearization technique used by the Ott–Grebogi–Yorke chaos stabilization method. (a) Parameter change causes displacement of the fixed point. In a small neighborhood of the fixed point the behavior of trajectories and displacement of the fixed point can be considered as linear. (b) Stable and unstable eigenvectors of the linearization matrix A. (c) New contravariant basis vectors. (d) Action of the control—the trajectory is forced to move onto the stable manifold of the fixed point.

This implies that f_u^T is a left eigenvector of A with the same eigenvalue e_u:

$$f_u^T A = f_u^T(\lambda_u e_u f_u^T + \lambda_s e_s f_s^T) = \lambda_u f_u^T \tag{13}$$

The control idea (16–18) now is to monitor the system behavior until it comes close to the desired fixed point (we assume that the system is ergodic and the trajectory fills the attractor densely; thus eventually it will pass arbitrarily close to any chosen point within the attractor) and then change p by a small amount so the next state x_{n+1} should fall on the stable manifold of x_0 [i.e., choose p_n such that $f_u^T(x_{n+1} - x_F) = 0$]:

$$p_n = -\left(\frac{\lambda_u}{f_u^T g}\right) f_u^T(x_n - x_F) + p^* \tag{14}$$

which can be expressed as a local linear feedback action:

$$p_{n+1} = p_n + C f_u^T[x_n - x_F(p_n)] \tag{15}$$

The actuation of the value of the control signal to be applied at the next iterate is porportional to the distance of the system state from the desired fixed point $[x_n - x_F(p_n)]$ projected onto the perpendicular unstable direction f_u. The constant C depends on the magnitude of the unstable eigenvalue λ_u and the shift g of the attractor position with respect to the change of the system parameter projected onto the unstable direction f_u. The Ott–Grebogi–Yorke (OGY) technique has the notable advantage of not requiring analytical models of the system dynamics and is well-suited for experimental systems. One can use either the full information from the process of the delay coordinate embedding technique using single variable experimental time series [see Dressler and Nitsche (19)]. The procedure can also be extended to higher-period orbits. Any accessible variable (controllable) system parameter can be used for applying perturbation, and the control signals are very small. The method also has several limitations. Its application in multiattractor systems is problematic. It is sensitive to noise, and the transients before achieving control might be very long in many cases. We have carried out an extensive study of application of the OGY technique to controlling chaos in Chua's circuit (12). Using an application-specific software package (20), we were able to find some of the unstable periodic orbits embedded in the double scroll Chua's chaotic attractor and use them as control goals.

Figure 12 shows the time evolution of the voltages when attempting to stabilize unstable period-one orbit in Chua's circuit. Before control is achieved, the trajectories exhibit chaotic transients before entering the close neighborhood of the chosen orbit.

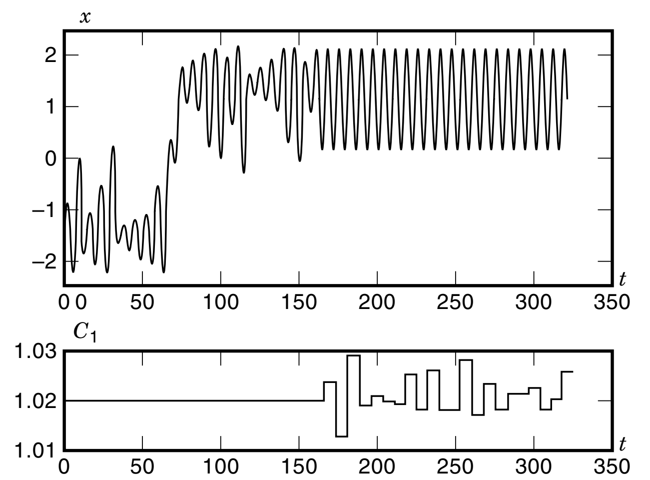


Figure 12. Typical results of stabilization of a period-one orbit in Chua's circuit using the OGY method. Time-waveform of voltage across the C_1 capacitor and variations of the control signal are shown.

Sampled Input Waveform Method

A very simple, robust, and effective method of chaos control in terms of stabilization of an unstable periodic orbit has been proposed (21). A sampled version of the output signal, corresponding to a chosen unstable periodic trajectory uncovered from a measured time series, is applied to the chaotic system causing the system to follow this desired orbit. In real systems, this sampled version of the unstable periodic orbit can be programmed into a programmable waveform generator and used as the forcing signal.

The block diagram of this control scheme is shown in Fig. 13. For controlling chaos in Chua's circuit (compare the circuit diagram shown as the left-side subcircuit in Fig. 5) we try to force the system with a sampled version of a signal $\hat{V}_1(t)$ $[(\hat{V}_1(t) = C^T\hat{x}(t)]$. Forcing the system with a continuous signal $\hat{V}_1(t)$ will force the system to exhibit a solution $x(t)$, which tends asymptotically toward $\hat{x}(t)$. This is obvious since forcing $V_1(t)$ will instantaneously force the current through the piecewise linear resistance to a "desired" value $i_R(t)$. The remaining subcircuit (R, L, C_2), which is an RLC stable circuit, will then exhibit a voltage $V_2(t)$ and a current $i_3(t)$, which will asymptotically converge towards $\hat{V}_2(t)$ and $\hat{i}_3(t)$.

The sampled input control method is very attractive as the goal of the control can be specified using analysis of the output time-series of the system; access to system parameters is not required. The control technique is immune to parameter variations, noise, scaling, and quantization. Instead of a controller, we need a generator to synthesize the goal signal. Signal sampling reduces the memory requirements for the gener-

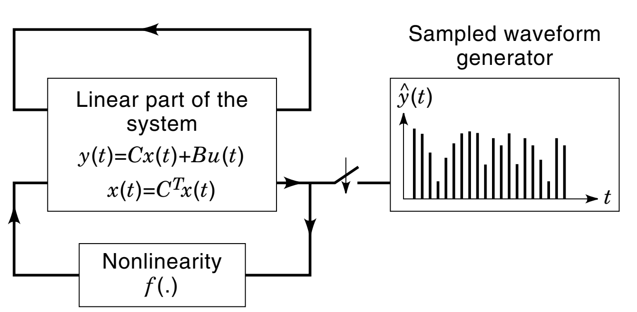
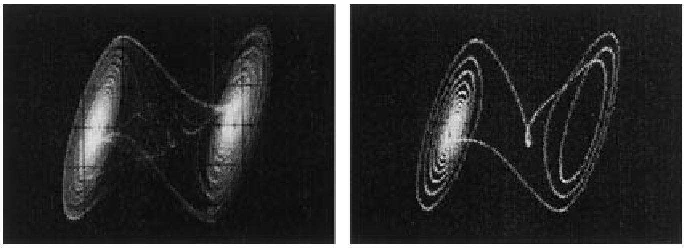


Figure 13. Block diagram of the sampled input chaos control system. A sampled version of a periodic signal corresponding to an unstable orbit uncovered from measured output is used to force the chaotic system which here has a special structure. This structure consists of a stable linear part and a scalar, static nonlinearity in the feedback path. Forcing signal is applied to the input of the nonlinearity.



(a) (b)

Figure 14. Using the sampled input forcing the double scroll attractor (a) observed in the experimental system can be converted into a long periodic orbit (b) stabilized during laboratory experiments.

ator. Figure 14 shows the chaotic attractor and two sample orbits controlled within the chaos range.

CHAOS CONTROL BY OCCASIONAL PROPORTIONAL FEEDBACK

In real applications, a "one-dimensional" version of the OGY method—the occasional proportional feedback (OPF) method—has proved to be most efficient. To explain the action of the OPF method let us consider a return map as shown in Fig. 15. For present consideration we take an approximate one-dimensional map obtained for the *RC*-ladder chaos generator (4). For nominal parameter values the position of the graph of the map is as shown by the rightmost curve; all periodic points are unstable. In particular, the point P is an unstable equilibrium. Looking at the system operation starting from point v_n, at the next iteration (the next passage of the trajectory through the Poincaré plane) one would obtain v_{n+1}. We would like to direct the trajectories toward the fixed point P. This can be achieved by changing a chosen system parameter such that the graph of the return map moves to a new position as marked on the diagram, thus forcing the next iteration to fall at v^*_{n+1}; after this is done the perturbation can be removed and activated again if necessary.

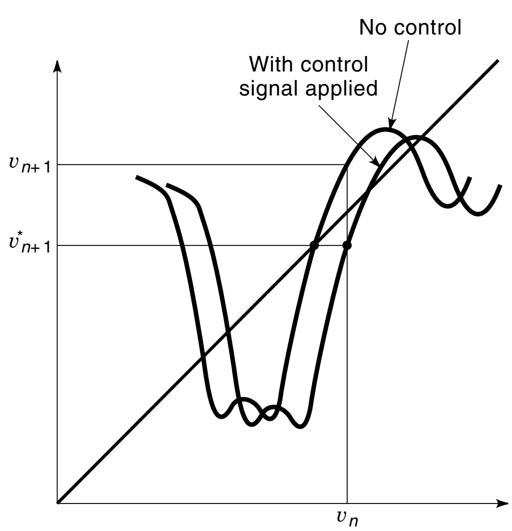


Figure 15. Explanation of the action of the occasional proportional feedback method using a graph of the first return map. Variation of an accessible system parameter causes displacement of the graph—when the control signal is chosen appropriately this displacement can be such that from a given coordinate the next iterate will fall exactly onto the unstable fixed point.

In mathematical terms we can compute the control signal using only one variable, for example ξ_1:

$$p(\xi) = p_0 + c(\xi_1 - \xi_{F1}) \tag{16}$$

This method has been successfully implemented in a continuous-time analog electronic circuit and used in a variety of applications ranging from stabilization of chaos in laboratory circuits (22–24) to stabilization of chaotic behavior in lasers (25–27). The OPF method may be applied to any real chaotic system (also higher-dimensional ones) where the output can be measured electronically and the control signal can be applied via a single electrical variable. The signal processing is analog and therefore is fast and efficient. Processing in this case means detecting the position of a one-dimensional projection of a Poincaré section (map), which can be accomplished by the window comparator, taking the input waveform. The comparator gives a logical high when the input waveform is inside the window. A logical AND operation is performed on this signal and on the delayed output from the external frequency generator. This logical signal drives the timing block that triggers the sample-and-hold and then the analog gate. The output from the gate, which represents the error signal at the sampling instant, is then amplified and applied to the interface circuit that transforms the control pulse into a perturbation of the system. The frequency, delay, control pulse width, window position, width, and gain are all adjustable. The interface circuit used depends on the chaotic system under control.

One of the major advantages of Hunt's controller over OGY is that the control law depends on only one variable and does not require any complicated calculations in order to generate the required control signal. The disadvantage of the OPF method is that there is no systematic method for finding the embedded unstable orbits (unlike OGY). The accessible goal trajectories must be determined by trial and error. The applicability of the control strategy is limited to systems in which the goal is suppression of chaos without more strict requirements.

IMPROVED ELECTRONIC CHAOS CONTROLLER

Recently, in collaboration with colleagues from University College, Dublin, we have proposed an improved electronic chaos controller that uses Hunt's method without the need for an external synchronizing oscillator. Hunt's OPF controller used the peaks of one of the system variables to generate the 1^D map. Hunt then used a window around a fixed level to set the region where control was applied. In order to find the peaks, Hunt's scheme used a synchronizing generator. In our modified controller (28,29), we simply take the derivative of the input signal and generate a pulse when it passes through zero. We use this pulse instead of Hunt's external driving oscillator as the "synch" pulse for our Poincaré map. This obviates the need for the external generator and so makes the controller simpler and cheaper to build.

The variable level window comparator is implemented using a window comparator around zero and a variable level shift. Two comparators and three logic gates form the window around zero. The synchronizing generator used in Hunt's controller is replaced by an inverting differentiator and a comparator. A rising edge in the comparator's output corresponds to a peak in the input waveform. We use the rising edge of the comparator's output to trigger a monostable flip-flop. The falling edge of this monostable's pulse triggers another monostable, giving a delay. We use the monostable's output pulse to indicate that the input waveform peaked at a previous fixed time. If this pulse arrives when the output from the window comparator is high then a monostable is triggered. The output of this monostable triggers a sample-and-hold on its rising edge that samples the error voltage; on its falling edge, it triggers another monostable. This final monostable generates a pulse that opens the analog gate for a specific time (the control pulse width). The control pulse is then applied to the interface circuit, which amplifies the control signal and converts it into a perturbation of one of the system parameters, as required.

We tested our controller using a chaotic Colpitts oscillator (30) and laboratory implementation of Chua's circuit. Implementation of a laboratory Chua's circuit together with interface circuit to connect the controller is shown in Fig. 16. Figure 17 shows an example of stabilization of a period-four orbit (found by trial-and-error search) using the improved chaos controller. In Fig. 18 we show oscilloscope traces for the goal trajectory and the control signal (bottom trace). It is interesting to note the impulsive action of the controller.

CHAOS-TO-CHAOS CONTROL

Synchronization of a given system solution with an externally supplied chaotic signal can be considered a particular type of control problem. The goal of the control scheme is to track (follow) the desired (input) chaotic trajectory. In particular, the input signal might come from an identical copy of the considered system, the only difference being the initial conditions. It is only very recently that such a control problem has been recognized in control engineering. The linear coupling technique and the linear feedback approach to controlling chaos can be applied for obtaining any chosen goal—regardless of whether it is chaotic, periodic, or constant in time. For a review of the chaos synchronization concepts and applications we refer the reader to Ogorzalek (31).

One can also envisage controlling a chaotic system toward chaotic targets that are not solutions of the system itself (goals might be chaotic trajectories originating from different systems). An impressive example of this kind of control/influence could be in generating Lorenz-like behavior in Chua's circuit (32). We believe that this kind of chaotic synchronization—control to a chaotic goal—could lead to new developments and possibly new applications of chaotic systems.

CONTROL OF SPATIOTEMPORAL CHAOTIC SYSTEMS

Chaos control becomes much more complicated in the case of large coupled and possibly very high-dimensional systems (such as neural networks), spatiotemporal systems (governed by partial differential equations or time-delay equations), because there exists a very rich repertoire of spatiotemporal behaviors depending on parameters of the system, architecture of interconnections, and external signals applied to it. It is believed that chaos control concepts in spatiotemporal systems might give explanations for the functioning of the brain. In controlling spatiotemporal systems we should consider first of all the goals we would like to achieve—they may be different in this case from the goals considered so far (stabilization of periodic orbits or anticontrol toward a desired chaotic waveform). In particular one can consider:

1. Formation of specific spatial or spatiotemporal patterns; influence on the spatial patterns might be needed, for example, in models of crystal growth, memory patterns, creation of waves with prescribed characteristics, and so on.
2. Stabilization of wanted behavior; this kind of operation might be required, for example, in the case of associative memory.
3. Synchronization/desynchronization; in some cases it might be desirable to obtain a coherent operation of the whole spatial structure or a part of the cells only. One can also envisage "anticontrol" desynchronization, as in the case of epileptic foci and recovery of normal brain functioning.
4. Efficient switching between attractors; we should envisage this kind of goal in the models of brain functions: change of concentration on various objects is linked with attractor switchings.
5. Removal of a specific type of behavior (e.g., spiral waves; this is a medical application such as defibrillation).
6. Cluster stabilization; in this kind of approach only a small spatial cluster in the multidimensional medium is to be stabilized while all the surrounding medium has to operate in a chaotic mode.

There is also more flexibility in applying control signals—they might be applied at the borders, at every cell, at specific locations in space, and so on. Also, connections between the cells in the network might be varied in some cases.

Coupled Map Lattices

A coupled map lattices (CML) system is a good target to study the control of spatiotemporal chaos because of existence of very rich spatiotemporal chaotic behavior in the control-free CML (33). In controlling a one-dimensional CML, stabilizing the system from spatiotemporal chaos not only to homogeneous stationary states but also to periodic states both in space and time has been demonstrated already (34). The idea of pinnings (putting some local control) plays a very important role in stabilizing spatiotemporal chaos. One advantage of the pinnings is to avoid the overflow in numerical simulation. Moreover, Hu and Qu have reported that a lower pinning density shows better control performance than a higher one in numerical experiments (34). Further analysis is needed of the relationship between the pinning density and control performance (34,35).

An important application of controlling CML is to suppress or skip very long transient chaotic (sometimes called "supertransient") waveforms (34). Such phenomena are often observed in CML systems, and sometimes one cannot see the

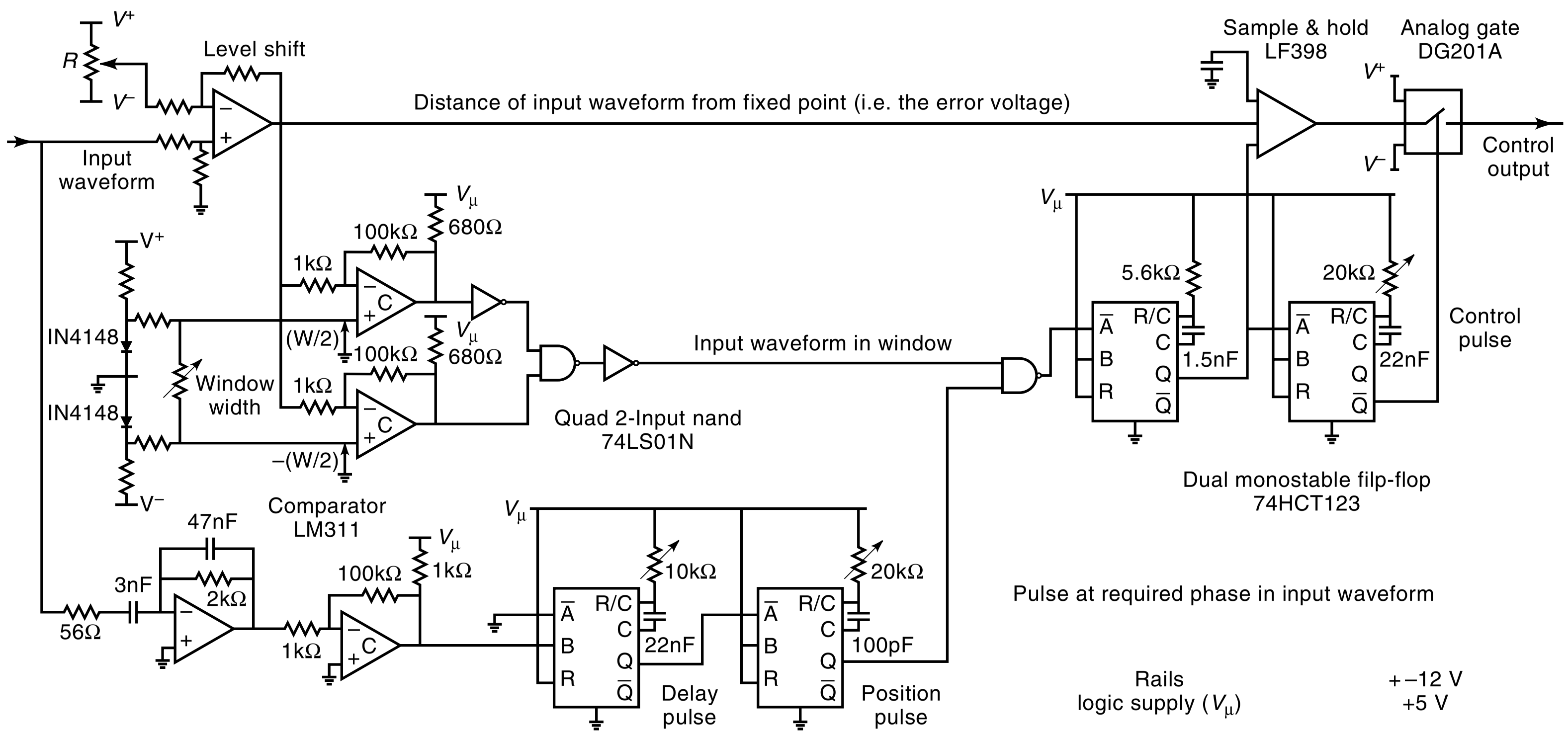


Figure 16. Improved analog chaos occasional proportional feedback controller without external synchronization.

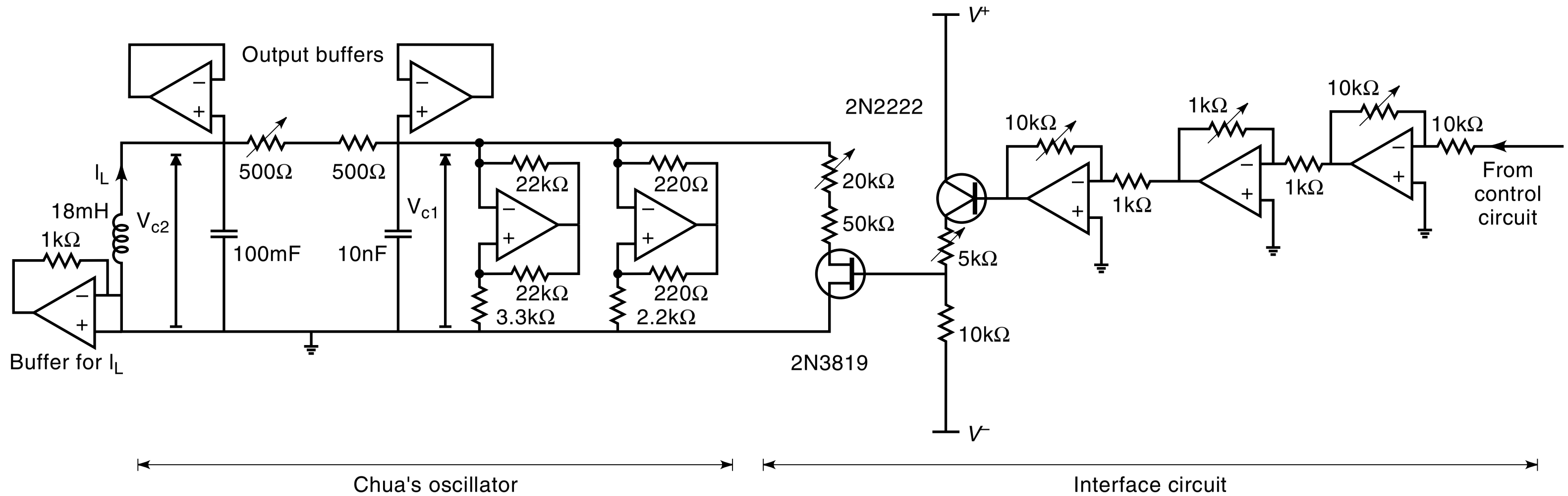


Figure 17. Circuit diagram for the implementation of Chua's circuit and the interface circuit. The interface circuit is specific for the considered chaotic system. Controller circuitry, as shown in Figure 16, is universal.

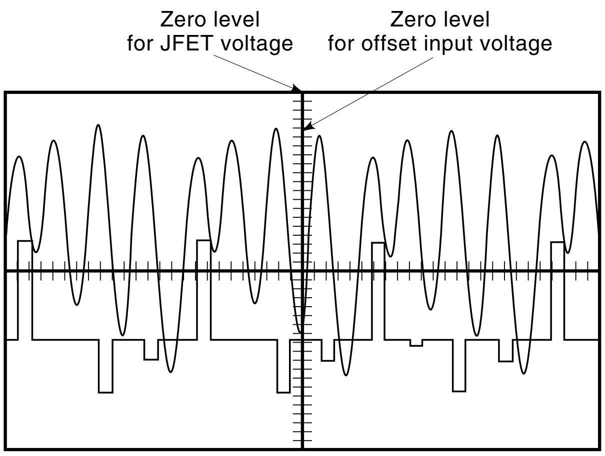


Figure 18. Oscilloscope traces of period-four solution stabilized in Chua's circuit and controlling signal produced by the improved chaos controller.

steady state for millions or more of iterations in numerical experiments. However, how to determine the desired (target) state of control in suppressing or skipping such transient chaos is still an open problem.

Spatial and Temporal Modulation of Extended Systems

The effects of global spatial and temporal modulation on pattern-forming systems have been widely studied. Global modulation means here that control signals are applied to every cell throughout the network. Examples of effects of this type of stimulation/control include pattern instability under periodic spatial forcing, spatial disorder induced in an autowave medium (Belousov–Zhabotinsky reaction), continuous variation of the wavelength of a pattern, or transitions between structures with incommensurate wavelengths [see Perez-Meñuzuri et al. (36) for a good list of references]. This global control method remains purely empirical.

Introducing Disorder to Tame Chaos

Interesting observations have been made recently by Braiman et al. (37). Based on earlier observations that noise injection can remove chaos in low-dimensional systems, they proposed to introduce uncorrelated differences between chaotic oscillators coupled in a large array. They identified two mechanisms by which disorder can stabilize chaos. The first requires small disorder and relies on disturbance of the system "position" in a very high-dimensional parameter space, resulting in change of the observed attractor. The second mechanism requires large perturbations; removing some of the oscillators in the array from their initial chaotic regime can possibly trigger the whole array into orderly behavior. The experiments of Braiman and others suggest that spatial disorder might be one of control mechanisms of pattern formation and self-organization.

Turing Patterns: Defect Removal

Perez-Meñuzuri et al. (36) studied creation of Turing patterns in arrays of discretely coupled dynamic systems. They discovered spontaneous creation of hexagonal or rhombic patterns when systems parameters were adjusted in some specific way. In many cases the observed patterns were not perfectly homogeneous (symmetrical). It turned out from several experiments that the defect can be removed by external side-wall stimulation—boundary control. These experiments demonstrate a potential principle for influencing crystal growth to obtain perfect structures. The control strategy applied in this case is a local one—only boundaries of the network are being excited (in contrast to global modulation).

Control of the Model Cortex

Babloyantz et al. (38) considered applications of feedback control of the Pyragas type to include control mechanisms in a model cortex. They studied a model in which all cells have linear dynamics but the connections are nonlinear of the sigmoid type. A single stabilizable periodic orbit that corresponds to bulk oscillations of the network has been found. Neurological data suggest that synchronized states in the brain are triggered when external stimuli are applied. Based on the simulation experiments, the authors proposed the following theory for attentiveness: it results from momentary (short time scale) control of chaotic activity observed in the cerebral cortex. Since the number of neural cells in the cortex is in the range of 10^{11}, the number of different stabilizable spatiotemporal patterns must be enormous and we can easily imagine that each stimulus can stabilize its corresponding characteristic state. Attentiveness, concentration, and recognition of patterns as well as wakefulness and sleep could be explained in terms of chaos control processes.

Controlling Autowaves: Spatial Memory

A particular type of pattern formation and self-organization in arrays of chaotic systems is autowaves (39). Development of autowaves in an array of chaotic oscillators can be controlled in several ways. First, adjustment of coupling between the oscillators gives a global control mechanism for dynamic phenomena. Second, when the network is operating in an autowave regime, one can observe the memory effect (39): The position of external stimulation controls the form of the observed spatial pattern. Finally, noise injection can destroy or quench patterns, introducing disorder.

Control of Ventricular Fibrillation: Quenching of Spiral Waves

Creation of spiral waves in heart tissue is now believed to be the principal cause of many arrythmias and heart disorders, including often-fatal ventricular fibrillation. Avoiding situations leading to spiral and scroll waves and eventually quenching such developing waves are of paramount importance in cardiology. Biktashev and Holden (40) proposed a feedback version of the resonant drift phenomenon (i.e., directed motion of the autowave vortex by applying an external signal) to remove the unwanted phenomena. Simulation studies confirm that amplitudes of signals needed for defibrillation using the proposed method are substantially less than those of conventional single-shock techniques used currently in medical practice.

Boundary and Defect-Induced Control in a Network of Chua's Circuits

An extensive simulation study has been carried out to discover the possibilities of controlling pattern formation in CNN (cellular neural network) arrays composed of chaotic Chua's circuits. The open-loop control strategy has been applied at the edge cells only. Thus by the number of cells excited the formation of wavefronts and their shape can easily be modified. Furthermore, it has been found out that the introduction of defects in the network could serve as a means of inducing spiral wave formation with the "tips" positioned at some prescribed locations.

Chaotic Neural Networks

Aihara (41,42) has proposed a neural network model composed of simple mathematical neurons, which are described by difference equations, and exhibit chaotic dynamics. Chaos control in such chaotic neural networks may be useful to improve the performance of the associative memory and to solve optimization problems. Control of a simple chaotic neural network has been reported (43). It has also reported that chaotic neural networks that have global or nearest-neighbor coupling can be controlled by a modified exponential control method (44). However, these results are not sufficient for the applications of controlling chaos mentioned previously because these results are only on the networks with homogeneous synaptic weights (couplings). In order to apply controlling chaos to the networks for associative memory and solving optimization problems, development of control methods for large-scale chaotic neural networks with inhomogeneous synaptic weights is needed.

ELECTRONIC CHAOS CONTROLLERS

The widespread interest in chaos control is due to its extremely interesting and important possible applications. These applications range from biomedical ones (e.g., defibrillation or blocking of epileptic seizures), through solid-state physics, lasers, aircraft wing vibrations and even weather control, just to name a few attempts made so far. Looking at the possible applications alone it becomes obvious that chaos control techniques and their possible implementations will greatly depend on the nature of the process under consideration. From the control implementation perspective, real systems exhibiting chaotic behavior show many differences. The main ones are (45):

- speed of the phenomenon (frequency spectrum of the signals)
- amplitudes of the signals
- existence of corrupting noises, their spectrum and amplitudes
- accessibility of the signals to measurement
- accessibility of the control (tuning) parameters
- acceptable levels of control signals

In most cases, electronic equipment will play a crucial role. In some applications, like the biomedical ones, we would possibly need implantable devices. In looking for an implementation of a particular chaos controller, we must first look at these system-induced limitations. How can we measure and process signals from the system? Are there any sensors available? Are there any accessible system variables and parameters that could be used for the control task? How do we choose the ones that offer the best performance for achieving control? What devices can be used to apply the control signals? Can we make off-line computations? At what speed do we need to compute and apply the control signals? What is the lowest acceptable precision of computation? Can we achieve control in real time?

A slow system like a bouncing magneto-elastic ribbon (with eigenfrequencies below 1 Hz) is certainly not as demanding as a telecommunications channel (possibly running at GHz) or a laser for control.

In electronic implementations, one must look at several closely linked areas: sensors (for measurements of signals from a chaotic process), electronic implementation of the controllers, computer algorithms (if computers are involved in the control process), and actuators (introducing control signals into the system). External to the implementation (but directly involved in the control process and usually fixed using the measured signals) is determining the goal of the control.

Despite the many methods that have been developed and described in the literature (3,11,46), most are still only of academic interest because of the lack of success in implementation. A control method cannot be accepted as successful if computer simulation experiments are not followed by further laboratory tests and physical implementations. Only very few results of such tests are known; among the exceptions are: the control of a green-light laser (27), the control of a magnetoelastic ribbon (47), and a few other examples.

Implementation Problems for the OGY Method

When implementing the OGY method for a real-world application one must perform the following series of elementary operations (45):

1. Data acquisition—measurement of a (usually scalar) signal from the chaotic system under consideration. This operation should be performed in such a way as not to disturb the existing dynamics. For further computerized processing, measured signals must be sampled and digitized (A/D conversion).
2. Selection of appropriate control parameter
3. Finding unstable periodic orbits using experimental data (measured time series) and fixing the goal of control
4. Finding parameters and variables necessary for control
5. Application of the control signal to the system; this step requires continuous measurement of system dynamics in order to determine the moment at which to apply the control signal (i.e., the moment when the actual trajectory passes in a small vicinity of the chosen periodic orbit) and immediate reaction of the controller (application of the control pulse) in such an event.

In computer experiments, it has been confirmed that all these steps of OGY can be carried out successfully in a great variety of systems, achieving stabilization of even long-period orbits.

There are several problems that arise during the attempt to build an experimental setup. Though variables and parameters can be calculated off-line, one must consider that the signals measured from the system are usually corrupted because of noise and several nonlinear operations associated with the A/D conversion (possibly rounding, truncation, finite word-length, overflow correction, etc.). Use of corrupted signal values and the introduction of additional errors by computer algorithms and linearization used for the control calculation may result in a general failure of the method. Additionally, there are time delays in the feedback loop (e.g., waiting for the reaction of the computer, interrupts generated when sending and receiving data.)

Effects of Calculation Precision. To test the effects of the precision of calculations in (45) the case of calculating control parameters to stabilize a fixed point in the Lozi map [see (45)] was considered. A partial answer to the question of how the A/D conversion accuracy and the resulting calculations of limited precision affect the possibilities for control has been found. In the tests the quality of computations alone, without looking at other problems like time delays in the control loop, was taken into account.

To compare the results of digital manipulations, first the interesting parameters were computed using analytical formulas. Next the same parameters were calculated using different word-length and different implementations of the arithmetic operations (overflow rules, rounding, or truncation, etc.).

Comparing the results of computations, it was found that an accuracy of two to three decimal digits is possible to achieve and the calculations are precise enough to ensure proper functioning of the OGY algorithm in the case of the Lozi system. To have some safety margin and robustness in the algorithm, the acceptable A/D accuracy cannot be lower than 12 bits and probably it would be best to apply 16-bit conversion. This kind of accuracy is nowadays easily available using general purpose A/D converters even at speeds in the MHz range. Implementing the algorithms, one must consider the cost of implementation—with growing precision and speed requirements, the cost grows exponentially. This issue might be a great limitation when it comes to integrated circuit (IC) implementations.

Approximate Procedures for Finding Periodic Orbits. Another possible source of problems in the control procedure is errors introduced by algorithms for finding periodic orbits (goals of the control). Using experimental data one can only find approximations to unstable periodic orbits (48,49).

In control applications we used the procedure introduced by Lathrop and Kostelich for recovering unstable periodic orbits from an experimental time series. The results obtained using this procedure strongly depend on the choice of accuracy ϵ and the length of the measured time series. Further, they depend on the choice of norm and the number of state variables analyzed. Also, the stopping criterion ($\|x_{m+k} - x_m\| < \epsilon$) in the case of discretely sampled continuous-time systems is not precise enough. This means that one can never be sure of how many orbits have been found or whether all orbits of a given period have been recovered. As this step is typically carried out off-line, it does not significantly affect the whole control procedure. It has been found in experiments that when the tolerances chosen for detection of unstable orbits were too large, the actual trajectory stabilized during control showed greater variations and the control signal had to be applied at every iteration to compensate for inaccuracies. Clearly, making the tolerance large can cause failure of control.

Effects of Time Delays. Several elements in the control loop may introduce time delays that can be detrimental to the functioning of the OGY method (45). Although all calculations may be done off-line, two steps are of paramount importance:

- detection of the moment when the trajectory passes the chosen Poincaré section
- determination of the moment at which the control signal should be applied (close neighborhood of chosen orbit)

When these two steps are carried out by a computer with a data acquisition card, at least a few interrupts (and therefore a time delay) must be generated in order to detect the Poincaré section, to decide it is in the right neighborhood, and to send the correct control signal.

Most experiments with OGY control of electronic circuits have been able to achieve control when the systems were running in the 10 Hz to 100 Hz range. We found out that for higher-frequency systems time delays become a crucial point in the whole procedure. The failure of control was mainly due to the late arrival of the control pulse. The system was being controlled at a wrong point in state space where the formulas used for calculations were probably no longer valid; trajectory was already far away from the section plane when the control pulse arrived.

CONCLUSIONS

The control problems existing in the domain of chaotic systems are neither fully identified nor solved completely. Because of the extreme richness of these phenomena, especially in higher-order systems, every month new papers appear describing new problems and proposing new solutions. Among the many unanswered questions these seem to be the most interesting: How can the methods already developed be used in real applications? What are the limitations of these techniques in terms of convergence, initial conditions, and so on? What are the limitations in terms of system complexity and possibilities of implementation? Are these methods useful in biology or medicine? Can we use the "butterfly effect" to tame and influence large-scale systems?

New application areas have opened up thanks to these new developments in various aspects of controlling chaos. These include neural signal processing (50,51), biology and medicine [Nicolis (52), Garfinkel et al. (53), Schiff et al. (54)], and many others. We can expect in the near future a breakthrough in the treatment of cardiac dysfunction thanks to the new generation of defibrillators and pacemakers functioning on the chaos-control principle. There is great hope also that chaos-control mechanisms will give us insight into one of the greatest mysteries—the workings of the human brain.

There is one more control problem associated in a way with chaos control, although not directly. Sensitive dependence on initial conditions, the key property of chaotic systems, offers yet another fantastic control possibility called "targeting" [Kostelich et al. (55), Shinbrot et al. (56)]. A desired point in the phase space is reached by piecing together in a controlled way fragments of chaotic trajectories. This method has already been applied successfully for directing satellites to desired positions using infinitesimal amounts of fuel [see Farquhar et al. (57)].

Finally, we stress that almost all chaotic systems known to date have strong links with electronic circuits; variables are sensed in an electric or electronic way; identification, modeling, and control are carried out using electric analogs; electronic equipment and electronic computers and usually sensors, transducers, and actuators are also electric by principle of operation. This guarantees an infinite wealth of opportunities for researchers and engineers.

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