FUZZY IMAGE PROCESSING AND RECOGNITION

INTRODUCTION

Pattern recognition and machine learning form a major area of research and development that encompasses the processing of pictorial and other non-numerical information obtained from interaction between science, technology, and society. A motivation for this spurt of activity in this field is the need for the people to communicate with computing machines in their natural mode of communication. Another important motivation is that scientists are also concerned with the idea of designing and making intelligent machines that can carry out certain tasks as we human beings do, the most salient outcome of which is the concept of future generation computing systems.

The ability to recognize a pattern is an essential requirement for sensory intelligent machines. Pattern recognition is a must component of the so-called "Intelligent Control Systems," which involve processing and fusion of data from different sensors and transducers. It is also a necessary function providing "failure detection," "verification," and "diagnosis task." Machine recognition of patterns can be viewed as a two fold task, consisting of learning the invariant and common properties of a set of samples characterizing a class, and of deciding that a new sample is a possible member of the class by noting that it has properties common to those of the set of samples. Therefore, the task of pattern recognition by a computer can be described as a transformation from the measurement space M to the feature space F and finally to the decision space D.

When the input pattern is a gray tone image, some processing tasks such as enhancement, filtering, noise reduction, segmentation, contour extraction, and skeleton extraction are performed in the measurement space to extract salient features from the image pattern, which is what is basically known as image processing.

The ultimate aim is to make its understanding, recognition, and interpretation from the processed information available from the image pattern. Such a complete image recognition/interpretation system is called a vision system, which may be viewed as consisting of three levels, namely, low level, mid level, and high level, corresponding to M, F, and D with an extent of overlapping among them.

In a pattern recognition or vision system, uncertainty can develop at any phase of the aforesaid tasks resulting from the incomplete or imprecise input information, the ambiguity/ vagueness in input image, the ill-defined and/or overlapping boundaries among the classes or regions, and the indefiniteness in defining/extracting features and relations among them. Any decision taken at a particular level will have an impact on all higher level activities. It is therefore required for a recognition system to have sufficient provision for representing these uncertainties involved at every stage, so that the ultimate output (results) of the system can be associated with the least uncertainty (and not be affected or biased very much by the earlier or lower level decisions).

UNCERTAINTIES IN A RECOGNITION SYSTEM AND RELEVANCE OF FUZZY SET THEORY

Some of the uncertainties that one encounters often while designing a pattern recognition or vision (1, 2) system will be explained in this section. Let us consider, first of all, the problem of processing and analyzing a gray tone image pattern. A gray tone image possesses some ambiguity within the pixels because of the possible multivalued levels of brightness. This pattern indeterminacy is because of inherent vagueness rather than randomness. The conventional approach to image analysis and recognition consists of segmenting (hard partitioning) the image space into meaningful regions, extracting its different features (e.g., edges, skeletons, centroid of an object), computing the various properties of and relationships among the regions, and interpreting and/or classifying the image. As the regions in an image are not always crisply defined, uncertainty can occur at every phase of the aforesaid tasks. Any decision taken at a particular level will have an impact on all higher level activities. Therefore, a recognition system (or vision system) should have sufficient provision for representing the uncertainties involved at every stage (i.e., in defining image regions, its features and relations among them, and in their matching) so that it retains as much as possible the information content of the original input image for making a decision at the highest level. The ultimate output (result) of the system will then be associated with least uncertainty (and, unlike conventional systems, it will not be biased or affected very much by the lower level decisions).

For example, consider the problem of object extraction from a scene. Now, the question is, "How can someone define exactly the target or object region in a scene when its boundary is ill-defined?" Any hard thresholding made for its extraction will propagate the associated uncertainty to the following stages, which might affect its feature analysis and recognition. Similar is the case with the tasks of contour extraction and skeleton extraction of a region.

From the aforesaid discussion, it becomes therefore convenient, natural, and appropriate to avoid committing ourselves to a specific (hard) decision (e.g., segmentation/thresholding, edge detection, and skeletonization) by allowing the segments or skeletons or contours to be fuzzy subsets of the image, with the subsets being characterized by the possibility (degree) of a pixel belonging to them. Prewitt (3) first suggested that the results of image segmentation should be fuzzy subsets rather than ordinary subsets.

Similarly, for describing and interpreting ill-defined structural information in a pattern, it is natural to define primitives (line, corner, curve, etc.) and relations among them using labels of fuzzy sets. For example, primitives that do not lend them-selves to precise definition may be defined in terms of arcs with varying grades of membership from 0 to 1 representing its belonging to more than one class. The production rules of a grammar may similarly be fuzzified to account for the fuzziness in physical relation among the primitives, thereby increasing the generative power of a grammar for syntactic recognition (4) of a pattern.

The incertitude in an image pattern may be explained in terms of grayness ambiguity, spatial (geometrical) ambiguity or both. Grayness ambiguity means "indefiniteness" in deciding a pixel as white or black. Spatial ambiguity refers to "indefiniteness" in shape and geometry (e.g., in defining centroid, sharp edge, perfect focusing, etc.) of a region. Another kind of uncertainty exists that my derive from the subjective judgment of an operator in defining the grades of membership of the object regions. This process is explained in the section on Flexibility in Membership Functions.

Let us now consider the problem of determining the boundary or shape of a class from its sampled points or prototypes. Various approaches (5–7) are described in the literature that attempt to provide an exact shape of the pattern class by determining the boundary such that it contains (passes through) some of the sample points, which need not be true. It is necessary to extend the boundaries to some extent to represent the possible uncovered portions by the sampled points. The extended portion should have lower possibility to be in the class than the portions explicitly highlighted by the sample points. The size of the extended regions should also decrease with the increase of the number of sample points, which leads one to define a multivalued or fuzzy (with continuum grade of belonging) boundary of a pattern class (8, 9).

Similarly, the uncertainty in classification or clustering of image points or patterns may develop from the overlapping nature of the various classes or image properties. This overlapping may result from fuzziness or randomness. In the conventional classification technique, it is usually assumed that a pattern may belong to only one class, which is not necessarily true. A pattern may have degrees of membership in more than one class. It is therefore necessary to convey this information while classifying a pattern or clustering a data set.

In the following section, we explain various fuzzy set theoretic tools for image analysis (which were developed based on the realization that many of the basic concepts in pattern analysis, for example the concept of an edge or a corner, do not lend themselves to precise definition).

IMAGE AMBIGUITY AND UNCERTAINTY MEASURES

An L level image X ($M \times N$) can be considered as an array of fuzzy singletons, each having a value of membership denoting its degree of possessing some property (e.g., brightness, darkness, edginess, blurredness, texture, etc.) In the notation of fuzzy sets, one may therefore write that

$$X = \{\mu_X(x_{mn}) : m = 1, 2, \ldots, M : n = 1, 2, \ldots, N\} \quad (1)$$

where $\mu_X(x_{mn})$ or μ_{mn} denotes the grade of possessing such a property μ by the (m, n)th pixel. This property μ of an image may be defined using global information, local information, or positional information, or a combination there of, depending on the problem. Again, the aforesaid information can be used in a number of ways (in their various functional forms), depending on individuals opinion and/or the problem to his hand, to define a requisite membership function for an image property. Basic principles and operations of image processing and pattern recognition in the light of fuzzy set theory are available in Reference 10.

Let us now explain the various image information measures (deriving from both fuzziness and randomness) and tools as well as their relevance to different operations for image processing and analysis. These measures are classified mainly in two groups, namely grayness ambiguity and spatial ambiguity.

GRAYNESS AMBIGUITY MEASURES

The definitions of some of the measures that were formulated to represent grayness ambiguity in an image X with dimension $M \times N$ and levels L (based on individual pixel as well as a collection of pixels) are listed below.

rth Order Fuzzy Entropy:

$$H^r(X) = (-1/k)\sum_i \{\mu(s_i^r)\log\{\mu(s_i^r)\} + \{1-\mu(s_i^r)\}\log\{1-\mu(s_i^r)\}\}, \quad i = 1, 2, \ldots, k \quad (2)$$

where s_i^r denotes the ith combination (sequence) of r pixels in X, k is the number of such sequences, and $\mu(s_i^r)$ denotes the degree to which the combination s_i^r, as a whole, possesses some image property μ.

Hybrid Entropy:

$$H_{hy}(X) = -P_w \log E_w - P_b \log E_b \quad (3)$$

with

$$\begin{aligned} E_w &= (1/MN)\sum_m\sum_n \mu_{mn}\exp(1-\mu_{mn}) \\ E_b &= (1/MN)\sum_m\sum_n (1-\mu_{mn})\exp(\mu_{mn}) \\ & m = 1, 2, \ldots, M, n = 1, 2, \ldots, N \end{aligned} \quad (4)$$

Here, μ_{mn} denotes the degree of "whiteness" of the (m,n)th pixel; P_w and P_b denote probability of occurrences of white ($\mu_{mn} = 1$) and black ($\mu_{mn} = 0$) pixels respectively; and E_w and E_b denote the average likeliness (possibility) of interpreting a pixel as white and black, respectively.

Correlation:

$$\begin{aligned} C(\mu_1, \mu_2) &= 1 - 4\left[\sum_m\sum_n \{\mu_{1mn} - \mu_{2mn}\}^2\right] \Big/ (X_1 + X_2) \\ &= 1 \quad \text{if} \quad X_1 + X_2 = 0 \end{aligned} \quad (5)$$

with

$$\begin{aligned} X_1 &= \sum_m\sum_n \{2\mu_{1mn} - 1\}^2 \\ X_2 &= \sum_m\sum_n \{2\mu_{2mn} - 1\}^2 \\ & m = 1, 2, \ldots, M : n = 1, 2, \ldots, N \end{aligned} \quad (6)$$

Here, μ_{1mn} and μ_{2mn} denote the degree of possessing the properties μ_1 and μ_2, respectively, by the (m, n)th pixel and $C(\mu_1, \mu_2)$ denotes the correlation between two such properties μ_1 and μ_2 (defined over the same domain).

These expressions (eqs. 2–6) are the versions extended to the 2-D image plane from those defined (ll,12) for a fuzzy set. $H^r(X)$ gives a measure of the average amount of difficulty in taking a decision whether any subset of pixels of size r possesses an image property. Note that no probabilistic concept is needed to define it. If $r = 1$, $H^r(X)$ reduces to (non-normalized) entropy as defined by De Luca and Termini (13). $H_{hy}(X)$, on the other hand, represents an amount of difficulty in deciding whether a pixel possesses a certain property μ_{mn} by making a prevision on its probability of occurrence it is assumed here that the fuzziness occurs because of the transformation of the complete white (0) and black pixels (1) through a degradation process, thereby modifying their values to lie in the intervals [0,0.5] and [0.5,1], respectively). Therefore, if μ_{mn} denotes the fuzzy set "object region", then the amount of ambiguity in deciding μ_{mn} a member of object region is conveyed by the term hybrid entropy depending on its probability of occurrence. In the absence of fuzziness (i.e., with exact defuzzification of the gray pixels to their respective black or white version), H_{hy} reduces to the two-state classical entropy of Shannon (14), the states being black and white. As a fuzzy set is a generalized version of an ordinary set, the entropy of a fuzzy set deserves to be a generalized version of classical entropy by taking into account not only the fuzziness of the set but also the underlying probability structure. In that respect, μ_{hy} can be regarded as a generalized entropy such that classical entropy becomes its special case when fuzziness is properly removed.

Note that equations (2) and (3) are defined using the concept of logarithmic gain function. Similar expressions using exponential gain function (i.e., defining the entropy of an n-state system) have been given by Pal and Pal (15–18).

$$H = \sum_i p_i e^{1-p_i}, \qquad i = 1, 2, \ldots, n \tag{7}$$

All these terms, which give an idea of "indefiniteness" or fuzziness of an image, may be regarded as the measures of average intrinsic information that is received when one has to make a decision (as in pattern analysis) to classify the ensembles of patterns described by a fuzzy set.

$H^r(X)$ has the following properties:

Pr 1: H^r attains a maximum if $\mu_i = 0.5$ for all i.

Pr 2: H_r attains a minimum if $\mu_i = 0$ or 1 for all i.

Pr 3: $H_r > H^{*r}$, where H^{*r} is the rth-order entropy of a sharpened version of the fuzzy set (or an image).

Pr 4: H_r is, in general, not equal to $\overline{H}^r$, where $\overline{H}^r$ is the rth-order entropy of the complement set.

Pr 5: $H^r \leq H^{r+1}$ when all $\mu_i \in$ [0.5,1].
$H^r \geq H^{r+1}$ when all $\mu_i \in$ [0,0.5].

The "sharpened" or "intensified" version of X is such that

$$\begin{aligned} &\mu_{x*}(x_{mn}) \geq \mu_x(x_{mn}) \quad \text{if} \quad \mu_x(x_{mn}) \geq 0.5 \\ \text{and} \\ &\mu_{x*}(x_{mn}) \leq \mu_x(X_{mn}) \quad \text{if} \quad \mu_x(x_{mn}) \leq 0.5 \end{aligned} \tag{8}$$

When $r = 1$, the property *Pr* 4 is valid only with the equal sign. The property *Pr* 5 (which does not occur for $r = 1$) implies that H^r is a monotonically non-increasing function of r for $\mu_i \in$ [0,0.5] and a monotonically nondecreasing function of r for $\mu_i \in$ [0.5,1] (when the "min" operator has been used to get the group membership value).

When all μ_i values are the same, $H^1(X) = H^2(X) = \ldots = H^r(X)$, which is because the difficulty in taking a decision regarding possession of a property on an individual is the same as that of a group selected therefrom. The value of H^r would, of course, be dependent on the μ_i values.

Again, the higher the similarity among singletons (supports), the quicker is the convergence to the limiting value of H^r. Based on this observation, an index of similarity of supports of a fuzzy set may be defined as $S = H^1/H^2$ (when $H^2 = 0$, H^1 is also zero and S is taken as 1). Obviously, when $\mu_i \in$ [0.5,1] and the min operator are used to assign the degree of possession of the property by a collection of supports, S will lie in [0, 1] as $H_r \leq H_{r+1}$. Similarly, when $\mu_i \in$ [0,0.5], S may be defined as H^2/H^1 so that S lies in [0, 1]. The higher the value of S, the more alike (similar) are the supports of the fuzzy set with respect to the fuzzy property μ. This index of similarity can therefore be regarded as a measure of the degree to which the members of a fuzzy set are alike. The details are available in Reference 19.

Therefore, the value of first order fuzzy entropy (H^1) can only indicate whether the fuzziness in a set is low or high. In addition, the value of $H^r, r > 1$ also enables one to infer whether the fuzzy set contains similar supports (or elements). The similarity index thus defined can be successfully used for measuring interclass and intraclass ambiguity (i.e., class homogeneity and contrast) in pattern recognition and image processing problems.

$H^1(X)$ is regarded as a measure of the average amount of information (about the gray levels of pixels) that has been lost by transforming the classic pattern (two-tone) into a fuzzy (gray) pattern X. Further details on this measure with respect to image processing problems are available in References 10 and 20–22. It is to be noted that $H^1(X)$ reduces to zero whenever μ_{mn} is made 0 or 1 for all (m, n), no matter whether the resulting defuzzification (or transforming process) is correct. In the following discussion, it will be clear how H_{hy} takes care of this situation.

Let us now discuss some of the properties of $H_{hy}(X)$. In the absence of fuzziness when MNP_b pixels become completely black ($\mu_{mn} = 0$) and MNP_W pixels become completely white ($\mu_{mn} = 1$), then $E_w = P_w$, $E_b = P_b$ and H_{hy} boils down to the two-state classical entropy

$$H_c = -P_w \log P_w - P_b \log P_b \tag{9}$$

the states being black and white. Thus H_{hy} reduces to H_c only when a proper defuzzification process is applied to detect (restore) the pixels. $|H_{hy} - H_c|$ can therefore be treated as an objective function for enhancement and noise reduction. The lower the difference, the less the fuzziness associated with the individual symbol and the higher the accuracy in classifying them as their original value (white or black). (This property is lacking with the $H_1(X)$ measure and the measure of Xie and Bedrosian (23), which always reduces to zero or some constant value irrespective of the defuzzincation process.) In other words, $|H_{hy} - H_c|$ represents an amount of information that was lost by transforming a two-tone image to a gray tone.

For a given P_w and P_b, ($P_w + P_b = 1, 0 \leq P_w, P_b, \leq 1$), of all possible defuzzifica-tions, the proper defuzzification of the image is the one for which H_{hy} is minimum.

$$\text{If } \mu_{mn} = 0.5 \quad \text{for all} \quad (m, n), \quad \text{then} \quad E_w = E_b \tag{10a}$$

and

$$H_{hy} = -\log[0.5\exp(0.5)] \quad (10b)$$

For example, H_{hy} takes a constant value and becomes independent of P_w and P_b, which is logical in the sense that the machine is unable to make a decision on the pixels because all μ_{mn} values are 0.5.

Spatial Ambiguity Measures Based on Fuzzy Geometry of Image

Many of the basic geometric properties of and relationships among regions has been generalized to fuzzy subsets. Such an extension, called fuzzy geometry (24–28), includes the topological concept of connectedness, adjacency and surroundedness, convexity, area, perimeter, compactness, height, width, length, breadth, index of area coverage, major axis, minor axis, diameter, extent, elongatedness, adjacency, and degree of adjacency. Some of these geometrical properties of a fuzzy digital image subset (characterized by piece-wise constant membership function $\mu X(x_{mn})$, or simply μ are listed below with illustrations. These properties may be viewed as providing measures of ambiguity in the geometry (spatial domain) of an image.

Compactness (24):

$$\mathrm{comp}(u) = \frac{a(u)}{[p(u)]^2} \quad (11)$$

where

$$a(\mu) = \sum \mu \quad (12a)$$

and

$$p(\mu) = \sum_{i,j,k} |\mu(i) - \mu(j)||A(i,j,k)| \quad (12b)$$

Here, $a(\mu)$ denotes area of μ, and $p(\mu)$, the perimeter of μ, is just the weighted sum of the lengths of the arcs $A(i,j,k)$ (24) along which the region $\mu(i)$ and $\mu(j)$ meet, weighted by the absolute difference of these values. Physically, compactness means the fraction of maximum area (that can be encircled by the perimeter) actually occupied by the object. In the non-fuzzy case, the value of compactness is maximum for a circle and is equal to $1/4\pi$. In the case of the fuzzy disc, where the membership value is only dependent on its distance from the center, this compactness value is $\geq 1/4\pi$. Of all possible fuzzy discs, compactness is therefore minimum for its crisp version.

Height and Width (24):

$$h(\mu) = \sum_n \max_m \mu_{mn} \quad (13)$$

and

$$w(\mu) = \sum_m \max_n \mu_{mn} \quad (14)$$

So, height/width of a digital picture is the sum of the maximum membership values of each row/column.

Length and Breadth (26, 27):

$$l(\mu) = \max_m \left(\sum_n \mu_{mn}\right) \quad (15)$$

and

$$b(\mu) = \max_n \left(\sum_m \mu_{mn}\right) \quad (16)$$

The length/breadth of an image fuzzy subset gives its longest expansion in the column/row direction. If μ is crisp, $\mu_{mn} = 0$ or l, then length/breadth is the maximum number of pixels in a column/row. Comparing equations 17 and 18 with 15 and 16, we notice that the length/breadth takes the summation of the entries in a column/row first and then maximizes over different columns/rows, whereas the height/width maximizes first the entries in a column/row and then sums over different columns/rows.

Index of Area Coverage (26, 27):

$$\mathrm{IOAC}(\mu) = \frac{a(\mu)}{l(\mu)b(\mu)} \quad (17)$$

In the non-fuzzy case, the *10AC* has a value of 1 for a rectangle (placed along the axes of measurement). For a circle, this value is $\pi r^2/(2r * 2r) = \pi/4$. *10AC* of a fuzzy image represents the fraction (which may be improper also) of the maximum area (that can be covered by the length and breadth of the image) actually covered by the image.

Again, note the following relationships.

$$l(X)/h(X) \leq 1 \quad (18a)$$

and

$$b(X)/w(X) \leq 1 \quad (18b)$$

When equality holds for equation (20), the object is either vertically or horizontally oriented. Similarly, major axis, minor axis, center of gravity, and density are also defined in Reference 27.

Degree of Adjacency (27):

The degree to which two crisp regions S and T of an image are adjacent is defined as

$$a(S,T) = \sum_{p \in BP(S)} \frac{1}{1 + |\mu(p) - r(q)|} \times \frac{1}{1 + d(p)} \quad (19)$$

Here, *d(p)* is the shortest distance between p and q, q is a border pixel *(BP)* of T, and p is a border pixel of S. The other symbols have the same meaning as in the previous discussion.

The degree of adjacency of two regions is maximum (= 1) only when they are physically adjacent (i.e., $d(p) = 0$) and their membership values are also equal [i.e., $\mu(p) = r(q)$]. If two regions are physically adjacent, then their degree of adjacency is determined only by the difference of their membership values. Similarly, if the membership values of two regions are equal, their degree of adjacency is determined by their physical distance only. The readers may note the difference between equation (22) and the adjacency definition given in Reference 24.

FLEXIBILITY IN MEMBERSHIP FUNCTIONS

As the theory of fuzzy sets is a generalization of the classic set theory, it has greater flexibility to capture faithfully the various aspects of incompleteness or imperfection (i.e., deficiencies) in information of a situation. The flexibility of fuzzy set theory is associated with the elasticity property of the concept of its membership function. The grade of membership is a measure of the compatibility of an object with the concept represented by a fuzzy set. The higher the value of membership, the less the amount (or extent) to which the concept represented by a set needs to be stretched to fit an object.

As the grade of membership is both subjective and dependent on context, some difficulty of adjudging the membership value still remains. In other words, the problem is how to assess the membership of an element to a set, which is an issue where opinions vary, giving rise to uncertainties. Two operators, namely "Bound Functions" (29) and "Spectral Fuzzy Sets" (30), have been defined to analyze the flexibility and uncertainty in membership function evaluation. These operators are explained below along with their significance in image analysis and pattern recognition problems.

Consider, for example, a "bright image," which may be considered as a fuzzy set. It is represented by an S-type function that is a nondecreasing function of gray value. Now, the question is, "can any such nondecreasing function be taken to represent the above fuzzy set?" Intuitively, the answer is "no." Bounds for such an S-type membership function μ have been reported (29) based on the properties of fuzzy correlation (11). The correlation measure between two membership functions μ_1 and μ_2 relates the variation in their functional values.

The significance of the bound functions in selecting an S-type function μ for the image segmentation problem has been reported in detail in Reference 31. It has been shown that, for detecting a minimum in the valley region of a histogram, the window length w of the function μ: [0, w] $\rightarrow$ [0,1] should be less than the distance between two peaks around that valley region. The ability to make the fuzzy set theoretic approach flexible and robust will be demonstrated further in the next section.

The concept of spectral fuzzy sets is used where, instead of a single unique membership function, a set of functions reflecting various opinions on membership elements is available so that each membership grade is attached to one of these functions. By giving due respect to all the opinions available for further processing, it reduces the difficulty (ambiguity) in selecting a single function. A spectral fuzzy subset F having n supports is characterized by a set or a band (spectrum) of r membership functions (reflecting r opinions) and may be represented as

$$F = \cup_j[\cup_i \mu_F^i(x_j)/x_j], \qquad x_j \in \Psi, \\ i = 1, 2, \ldots, r, \quad j = 1, 2, \ldots, n \tag{20}$$

where r, the number of membership functions, may be called the cardinality of the opinion set. $\mu_F^i(x_j)$ denotes the degree of belonging of X_j to the set F according to the ith membership function. The various properties and operations related to it have been defined by Pal and Das Gupta (30). The incertitude or ambiguity associated with this set is two-fold, namely ambiguity in assessing a membership value to an element (d_1) and ambiguity in deciding whether an element can be considered to be a member of the set (d_2).

The (dis)similarity between the concept of spectral fuzzy sets and those of the other tool such as probabilistic fuzzy set, interval-valued fuzzy set, fuzzy set, of type 2, or ultra fuzzy set (32–36) (which have also considered the difficulty in settling a definite degree of fuzziness or ambiguity), has been explained in Reference 30.

The concept has been found to be significantly useful (30) in segmentation of ill-defined regions where the selection of a particular threshold becomes questionable as far as its certainty is concerned. In other words, questions may develop like, "where is the boundary?" or "what is the certainty that a level 1, say, is a boundary between object and background?" The opinions on these queries may vary from individual to individual because of the differences in opinion in assigning membership values to the various levels. In handling this uncertainty, the algorithm gives due respect to various opinions on membership of gray levels for object region, minimizes the image ambiguity $d(= d_1 + d_2)$ over the resulting band of membership functions, and then makes a soft decision by providing a set of thresholds (instead of a single one) along with their certainty values. A hard (crisp) decision obviously corresponds to one with maximum d value (i.e. the level at which opinions differ most). The problems of edge detection and skeleton extraction (where incertitude occurs from ill-defined regions and various opinions on membership values) and any expert system-type application (where differences in experts' opinions leads to an uncertainty) may also be similarly handled within this framework.

SOME EXAMPLES OF FUZZY IMAGE PROCESSING OPERATIONS

Let us now describe some algorithms to show how the aforesaid information measures and geometrical properties can be incorporated in handling uncertainties in various operations (e.g., gray level thresholding, enhancement, contour detection and skeletonization by avoiding hard decisions, and providing output in both fuzzy and nonfuzzy (as a special case) versions). is to be noted that these low level operations (particularly image segmentation and object extraction) play a major role in an image recognition system. As mentioned earlier, any error made in this process might propagate to feature extraction and classification.

Enhancement in Property Domain

The objective of enhancement techniques is to process a given image so that the result is more suitable than the original for a specific application. The term "specific" is, of course, problem-oriented. The techniques used here are based on the modification of pixels in the fuzzy property domain of an image (10, 20, 2l).

The contrast intensification operator on a fuzzy set A generates another fuzzy set $A' = \mathrm{INT(A)}$ in which the fuzziness is reduced by increasing the values of $\mu_A(x_{mn})$ that are above 0.5 and decreasing those that are below it. Define this INT operator by a transformation T_1 of the membership function μ_{mn} as

$$\begin{aligned} T_1(\mu_{mn}) &= T_1'(\mu_{mn}) = 2\mu_{mn}^2, 0 \le \mu_{mn} \le 0.5 \\ &= T_1''(\mu_{mn}) = 1 - 2(1-\mu_{mn})^2, 0.5 \le \mu_{mn} \\ & m = 1, 2, \ldots M, \ n = 1, 2, \ldots N \end{aligned} \quad (21)$$

In general, each μ_{mn} in X (Eq. 1) may be modified to μ'_{mn} to enhance the image X in the property domain by a transformation function T_r where

$$\begin{aligned} \mu'_{mn} &= T_r(\mu_{mn}) = T_r'(\mu_{mn}), 0 \le \mu_{mn} \le 0.5 \\ &= T_r''(\mu_{mn}), 0.5 \le \mu_{mn} \le 1 \\ r &= 1, 2, \ldots \end{aligned} \quad (22)$$

The transformation function T_r is defined as successive applications of T_1 by the recursive relationship (20)

$$T_s(\mu_{mn}) = T_1\{T_{s-1}(\mu_{mn})\}, \qquad s = 1, 2, \ldots \quad (23)$$

and $T_1(P_{mn})$ represents the operator INT denned in equation (24).

As r increases, the enhancement function (curve) in $\mu_{mn} - \mu'_{mn}$ plane tends to be steeper because of the successive application of INT. In the limiting case, as $r \to \infty$, T_r produces a two-level (binary) image. It is to be noted here that, corresponding to a particular operation of T', one can use any of the multiple operations of T'', and vice versa, to attain a desired amount of enhancement. Similarly, some other enhancement functions can be used independently instead of those used in equation (24).

The membership plane μ_{mn} for enhancing contrast around a cross-over point may be obtained from References 11 and 20.

$$\mu_{mn} = G(x_{mn}) = [1 + (|\hat{x} - x_{mn}|/F_d)^{F_e}]^{-1} \quad (24)$$

where the position of cross-over points bandwidth, and hence the symmetry of the curve, are determined by the fuzzifiers F_e and F_d. When ${}^a x = x_{\max}$ (maximum level in X), μ_{mn} represents an S-type function. When ${}^a x =$ any arbitrary level l,μ_{mn} represents a π-type function.

After enhancement in the fuzzy property domain, the enhanced spatial domain x'_{mn} may be obtained from

$$x'_{mn} = G^{-1}(\mu'_{mn}), \qquad \alpha \le \mu'_{mn} \le 1 \quad (25)$$

where α is the value of μ_{mn} when $x_{mn} = 0$.

Note that the aforesaid method provides a basic module of fuzzy enhancement. In practice, one may use it with other smoothing, noise cleaning, or enhancement operations Tor resulting in desired outputs. An extension of this concept to enhance the contrast among various ill-defined regions using multiple applications of π and $(1 - \pi)$ functions has been described in References 21 and 37 for edge detection of X-ray images. The edge detection operators involve *max* and *min* operations. Reference 38 demonstrates, in this regard, an attempt to use a relaxation (iterative) algorithm for fast image enhancement using various orders of S functions; convergence has also been analyzed.

Fuzzy image enhancement technique has been applied by Krell et al. (39) for enhancing the quality of images taken by electronic postal imaging device needed by clinicians to verify the shape and the location of "therapy beam" with respect to the patients anatomy. Lukac et al. (40) performed cDNA microarray image processing using fuzzy vector filtering framework. Various other fuzzy enhancement operators have been developed to reduce degradation in images (41–48). Reference 49 uses a fuzzy regularization approach to carry out blind image deconvolution. Recently, fuzzy techniques have also been used in impulse noise detection and reduction (50). Furthermore, the concept fuzzy transformation has been developed for low level image processing applications (51).

Optimum Enhancement Operator Selection

When an image is processed for visual interpretation, it is ultimately up to the viewers to judge its quality for a specific application and how well a particular method works. The process of evaluation of image quality therefore becomes subjective, which makes the definition of a well-processed image an elusive standard for comparison of algorithm performance. Again, it is customary to have an iterative process with human interaction to select an appropriate operator for obtaining the desired processed output. For example, consider the case of contrast enhancement using a nonlinear functional mapping. Not every kind of nonlinear function will produce a desired (meaningful) enhanced version. The questions that automatically develop are "Given an arbitrary image, which type of nonlinear functional form will be best suited without prior knowledge on image statistics (e.g., in remote applications like space autonomous operations where frequent human interaction is not possible) for highlighting its object?" and "Knowing the enhancement function, how can one quantify the enhancement quality for obtaining the optimal one?" Regarding the first question, even if the image statistics are given, it is possible only to estimate approximately the function required for enhancement and the selection of the exact functional form still needs human interaction in an iterative process. The second question, on the other hand, needs individual judgment, which makes the optimum decision subjective.

The method of optimization of the fuzzy geometrical properties and entropy has been found (52) to be successful, when applied on a set of different images, in providing quantitative indices to avoid such human iterative interaction in selecting an appropriate nonlinear function and to make the task of subjective evaluation objective.

Threshold Selection (Fuzzy Segmentation)

Given an L level image X of dimension $M \times N$ with minimum and maximum gray values l_{min} and l_{max}, respectively, the algorithm for its fuzzy segmentation into object and background may be described as follows:

Step 1: Construct the membership plane using the standard S function as

$$\mu_{mn} = \mu(l) = S(l: a, b, c) \quad (26)$$

or

$$\mu_{mn} = \mu(l) = 1 - S(l: a, b, c) \quad (27)$$

(depending on whether the object regions possess higher or lower gray values) with cross-over point b and band width $\Delta b = b - a = c - b$.

Step 2: Compute the parameter $I(X)$ where $I(X)$ represents either grayness ambiguity or spatial ambiguity, as stated earlier, or both.

Step 3: Vary b between l_{min} and l_{max} and select those b for which $I(X)$ has local minima or maxima depending on $I(X)$. (Maxima correspond to the correlation measure only.) Among the local minima/maxima, let the global one have a cross-over point at s.

The level s, therefore, denotes the cross-over point of the fuzzy image plane μ_{mn}, which has minimum grayness and/or geometrical ambiguity. The μ_{mn} plane then can be viewed as a fuzzy segmented version of the image X. For the purpose of nonfuzzy segmentation, we can take s as the threshold (or boundary) for classifying or segmenting an image into object and background.

Faster methods of computation of the fuzzy parameters are explained in Reference 27. Note that $w = 2\Delta b$ is the length of the window (such that $[0, w] \rightarrow [0,1]$). that was shifted over the entire dynamic range. As w decreases, the possibility of detecting some undesirable thresholds (spurious minima) increases because of the smaller value of Δb. On the other hand, an increase in w results in a higher value of fuzziness and thus leads toward the possibility of losing some of the weak minima.

The criteria regarding the selection of membership functions and the length of window (i.e., w) have been reported in References 29 and 31 assuming continuous functions for both histogram and membership function. It is shown that μ should satisfy the bound criteria derived based on the correlation flexibility in membership functions (section). Another way of handling this uncertainty using spectral fuzzy sets for providing a soft decision is explained in Referece 30.

Let us now describe another way of extracting an object by minimizing higher order entropy (Eq. 2) of both object and background regions using an inverse π function as shown by the solid line in Fig. 1. Unlike the previous algorithm, the membership function does not need any parameter selection to control the output.

Suppose s is the assumed threshold so that the gray level ranges $[1, s]$ and $[s + 1, L]$ denote, respectively, the object and background of the image X. The inverse π-type function to obtain μ_{mn} values of X is generated by taking the union of $S[x; s - (L - s), s, L]$ and $1 - S(x; 1, s, (s + s + s - 1)]$, where S denotes the standard S function. The resulting function as shown by the solid line makes μ lie in [0.5,1]. As the ambiguity (difficulty) in deciding a level as a member of the object or the background is maximum for the boundary level s, it has been assigned a membership value of 0.5. Ambiguity decreases as the gray value moves away from s on either side. The μmn thus obtained denotes the degree of belonging of a pixel x_{mn} to either object or background. As is not necessarily the mid point of the entire gray scale, the membership function may not be a symmetric one.

Therefore, the task of object extraction is to:

Step 1: Compute the rth-order fuzzy entropy of the object H_O^r and the background H_B^r considering only the spatially adjacent sequences of pixels present within the object and background, respectively. Use the "min" operator to get the membership value of a sequence of pixels.

Step 2: Compute the total rth-order fuzzy entropy of the partitioned image as $H_s^r = H_O^r + H_B^r$.

Step 3: Minimize H_s^r with respect to s to get the threshold for object background classification.

Referring back to the section on Grayness Ambiguity Measures, it is seen that H^2 reflects the homogeneity among the supports in a set in a better way than H^1 does. The higher the value of r, the stronger is the validity of this fact. Thus, considering the problem of object–background classification, the improper selection of the threshold is more strongly reflected by H_r than H^{r-1}.

The methods of object extraction (or segmentation) described above are all based on gray level thresholding. Another way of doing this task is by pixel classification. The details on this technique, using *fuzzy c-means, fuzzy isodata, fuzzy dynamic clustering, and fuzzy relaxation*, are available in References (2, 10, and 53–60). The fuzzy c-means (FCM) algorithm is a well-known clustering algorithm used for pixel classification. Here, we describe it in brief.

Fuzzy segmentation results in fuzzy partitions of $X = \{x_1, x_2, \ldots, x_n\}$, where X denotes a set of n unlabeled column vectors in R_P (i.e., each element of X is a p-dimensional feature vector). A fuzzy c-partition (c is an integer, $1 \leq c \leq n$) is the matrix $U = [\mu_{ik}], i = 1, 2, \ldots, c,\ k = 1, \ldots, n$ that satisfies the following constraints:

$$\mu_{ik} \in [0, 1], \quad \sum_{i=1}^{c} \mu_{ik} = 1, \quad \text{and} \quad 0 < \sum_{k=1}^{n} \mu_{ik} < n \qquad \text{for all } i, k$$

Here, the kth column of U represents membership values of x_k to the c fuzzy subsets and $\mu_{ik} = \mu_i(x_k)$ denotes the grade of membership of x_k in the ith fuzzy subset.

The FCM algorithm searches the local minimum of the following objective function:

$$J_m(U, V) = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^m \|x_k - v_i\|_A^2, \qquad 1 \leq m \leq \infty$$

where U isafuzzy c-partition of X, $\|\cdot\|_A$ is any inner product norm, $V = \{v_1, v_2, \ldots, v_c\}$ is a set of cluster centers, $v_i \in R^p$, and $m \in [1, \infty]$ is the weighting exponent on each fuzzy membership. For $m > 1$ and $x_k \neq v_i$ for all i, k, it has been shown that $J_m(U, V)$ may be minimized only if

$$\mu_{ik} = 1 \Big/ \sum_{j=1}^{c} \left(\frac{\|x_k - v_i\|_A^2}{\|x_k - v_j\|_A^2} \right)^{2/(m-1)}$$

and

$$v_i = \frac{\sum_{k=1}^{n} (\mu_{ik})^m x_k}{\sum_{k=1}^{n} (\mu_{ik})^m}, \qquad 1 \leq i \leq c$$

The FCM algorithm, when Euclidian distance norm is considered, can only be used for hyperspherical clusters with approximately equal dimensions. To cope with clusters having large variability in cluster shapes, densities, and the number of data points in each cluster, Gustafson and Kessel (61) used the scaled Mahalanobis distance in the FCM algorithms. By the use of a distance measure derived from maximum likelihood estimation methods, Gath and Geva (62) obtained an algorithm that is eifective even when the clusters are ellipsoidal in shape and unequal in dimension.

As the value of c (i.e., the number of clusters) is not always known, several cluster validity criteria have been suggested in the literature to find the optimum number of clusters. These criteria include partition coefficient, classification entropy, properties coefficient, total within-class distance of clusters, total fuzzy hyper volume of clusters, and partition density of clusters (60–63).

Generalizing the FCM algorithm further, Dave (64) proposed the fuzzy c shells (FCS) algorithm to search for clusters that are hyper ellipsoidal shells. One of its advanced versions is believed to be better than Hough transformation (in terms of memory and speed of computation) when used for ellipse detection. It is also shown (64) that the use of fuzzy memberships improves the ability to attain global optima compared with the use of hard membership. For the same purpose, Krishnapuram et al. (65) proposed another algorithm that is claimed to be less time consuming than that of Dave.

For further information, readers may consult References 66–69. The article in Reference 66 describes a modified version of the FCM, which incorporates supervised training data. The article of Cannon et al. (67) describes an approach that reduces the computation required for the FCM, by using look up tables, by a factor of six. Another simplified form of FCM in this line is mentioned in Reference 68. The authors in Reference (69) have proposed a new heuristic fuzzy clustering technique and have referred to it as the Fuzzy J-Means (FJM).

Soft decision making has been used to develop many other segmentation algorithms for various applications such as document image processing, ultrasound image processing, satellite image analysis, MR image analysis, and remote sensing (70–78). Algorithms for applications such as classification of MR brain images (79) and microcalcification detection (80) have been succesfully implemented.using fuzzy techniques. Before leaving this section, we mention the work in Reference (81), which defines the concept of fuzzy objects and describes algorithms for their extraction.

Contour Detection

Edge detection is also an image segmentation technique where the contours/boundaries of various regions are extracted based on the detection of discontinuity in grayness. Here we present a method for fuzzy edge detection using an edginess measure based on H^1 (Eq. 2), which denotes an amount of difficulty in deciding whether a pixel can be called an edge (19). Let N^3_{xy} be a 3×3 neighborhood of a pixel at (x,y). The edge–entropy H^E_{xy} of the pixel *(x,y)*, giving a measure of edginess at *(x,y)*, may be computed as follows. For every pixel *(x,y)*, compute the average, maximum, and minimum values of gray levels over N^3_{xy}. Let us denote the average, maximum, and minimum values by *Avg*, *Max*, and *Min*, respectively. Now define the following parameters.

$$D = \max\{\text{Max} - \text{Avg}, \text{Avg} - \text{Min}\} \tag{28}$$

$$B = \text{Avg} \tag{29}$$

$$A = B - D \tag{30}$$

$$C = B + D \tag{31}$$

A π-type membership function (Fig. 2) is then used to compute μ_{xy} for all $(x, y) \in N^3_{xy}$ such that $\mu(A) = \mu(C) = 0.5$ and $\mu(B) = 1$. It is to be noted that $\mu_{xy} \geq 0.5$. Such a μ_{xy}, therefore, characterizes a fuzzy set "pixel intensity close to its average value," averaged over $N^3_{x,y}$. When all pixel values over $N^3_{x,y}$ are either equal or close to each other (i.e., they are within the same region), such a transformation will make all $\mu_{xy} = 1$ or close to 1. In other words, if no edge exists, pixel values will be close to each other and the μ values will be close to one (1); thus resulting in a low value of H^1. On the other hand, if an edge does exist (dissimilarity in gray values over $N^3_{x,y}$), then the μ values will be more away from unity; thus resulting in a high value of H^1. Therefore, the entropy H^1 over $N^3_{x,y}$ can be viewed as a measure of edginess ($H^E_{x,y}$) at the point (x,y). The higher the value of $H^E_{x,y}$, the stronger the edge intensity and the easier its detection. Such an entropy plane will represent the fuzzy edge detected version of the image.

The proposed entropic measure is less sensitive to noise because of the use of a dynamic membership function based on a local neighborhood. The method is also not sensitive to the direction of edges. Other edginess measures and algorithms based on fuzzy set theory are available in References 10, 21, and 37.

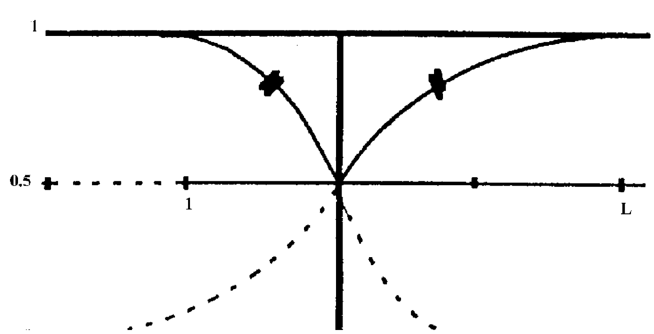

Figure 1. Inverse π function (solid line) for computing object and background entropy.

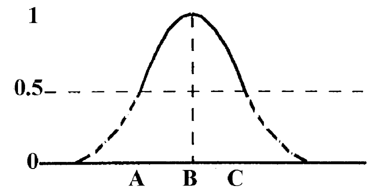

Figure 2. π Function for computing edge entropy.

Fuzzy Skeleton Extraction

Let us now explain two methods for extracting the fuzzy skeleton (skeleton having an ill-defined boundary) of an object from a gray tone image without getting involved into its (questionable) hard thresholding. The first one is based on minimization of the parameter *1OAC* (Eq. 19) or compactness (Eq. 12) with respect to α-cuts (α-cut of a fuzzy set A comprises all elements of X whose membership value is greater than or equal to α, $0 < \alpha \leq 1$) over a fuzzy "core line" (or skeleton) plane. The membership value of a pixel to the core line plane depends on its property of possessing maximum intensity, and property of occupying vertically and horizontally middle positions from the ϵ-edges (pixels beyond which the membership value in the fuzzy segmented image becomes less than or equal to ϵ, $\epsilon > 0$) of the object (82). If a nonfuzzy (or crisp) single-pixel-width skeleton is deserved, it can be obtained by a contour tracing algorithm (83) that takes into account the direction of contour. Note that the original image cannot be reconstructed, like the other conventional techniques of gray skeleton extraction (2–85) from the fuzzy skeleton obtained here.

The second method is based on fuzzy medial axis transformation *(FMAT)* (28) using the concept of fuzzy disks. A fuzzy disk with center P is a fuzzy set in which membership depends only on the distance from P. For any fuzzy set f, a maximal fuzzy disk $gP^f \leq f$ exists centered at every point P, and f is the *sup* of the gP^f's. (Moreover, if f is fuzzy convex, so is every gP^f, but not conversely.) Let us call a set S_f of points f-sufficient if every $gP^f \leq gQ^f$ for some set of Q in S_f; evidently f is then the *sup* of the gQ^f's. In particular, in a digital image, the set of Q's at which gf is a (non-strict) local maximum is f-sufficient. This set is called the fuzzy medial axis of f, and the set of gQ^f's is called the fuzzy medial axis transformation *(FMAT)* of f. These definitions reduce to the standard one if f is a crisp set.

For a gray tone image X (denoting the non-normalized fuzzy "bright image" plane), the *FMAT* algorithm computes, first of all, various fuzzy disks centered at the pixels and then retains a few (as small as possible) of them, as designated by gQ's, so that their union can represent the entire image X. That is, the pixel value at any point t can be obtained from a union operation, as t has membership value equal to its own gray value (i.e., equal to its non-normalized membership value to the bright image plane) in one of those retained disks.

Note that the above representation is redundant (i.e., some more disks can further be deleted without affecting the reconstruction). The redundancy in pixels (fuzzy disks) from the fuzzy medial axis output can be reduced by considering the criterion $gP^f(t) \leq \sup gQ_i^f(t), i = 1, 2, \ldots$ instead of $gP^f(t) \leq gQ^f(t)$. In other words, eliminate many other $gP^{f\prime}$s for which there exists a set of $gQ^{f\prime}$s whose sup is greater than or equal to gP^f.

Let *RFMAT* denote the *FMAT* after reducing its redundancy. The fuzzy medial axis is seen to provide a good skeleton of the darker (higher intensity) pixels in an image apart from its exact representation. *FMAT* of an image can be considered as its core (prototype) version for the purpose of image matching. It is to be mentioned here that such a representation may not be economical in a practical situation. The details on this feature and the possible approximation to make it practically feasible are available in Reference (86)

Note that the membership values of the disks contain the information of image statistics. For example, if the image is smooth, the disk will not have abrupt change in its values. On the other hand, it will have abrupt change in case the image has salt and pepper noise or edginess. The concept of fuzzy *MAT* can therefore be used as spatial filtering (both high pass and low pass) of an image by manipulating the disk values to the extent desired and then putting them back while reconstructing the processed image. A gray-scale thinning algorithm is described in References 60 and 87 based on the concept of fuzzy connectedness between two pixels; the dark regions can be thinned without ever being explicitly segmented.

SOME APPLICATIONS

Here we provide a few applications of the methodologies and tools described before.

Motion Frame Analysis and Scene Abstraction

With rapid advancements in multimedia technology, it is increasingly common to have time-varied data like video as computer data types. Existing database systems do not have the capability of search within such information. It is a difficult problem to automatically determine one scene from another because no precise markers exist that identify where they begin and end. Moreover, divisions of scenes can be subjective, especially if transitions are subtle. One way to estimate scene transitions is to approximate the change of information between each of two successive frames by computing the distance between their discriminatory properties.

A solution is provided in Reference 88 to the problem of scene estimation/abstraction of motion video data in the fuzzy set theoretic framework. Using various fuzzy geometrical and information measures (see image Ambiguity and uncertainty measures section) as image features, an algorithm is developed to compute the change of information in each of two successive frames to classify scenes/frames.

Frame similarity is measured in terms of weighted distance in fuzzy feature space. This categorization process of raw input visual data can be used to establish structure for correlation. The investigation not only attempts to determine the discrimination ability of the fuzziness measures for classifying scenes, but also enhances the capability of nonlinear, frame-accurate access to video data for applications such as video editing and visual document archival retrieval systems in multimedia environments. Such an investigation is carried out in NASA Johnson Space Center, Texas (88).

A set of digitized videos of previous space shuttle missions obtained from NASA/JSC was used (Fig. 3). The scenes were named payload deployment, onboard astronaut, remote manipulator arm, and mission control room. Experiments were conducted for various combinations of uncertainty, orientation, and shape measures. As an illustration, Fig. 4 shows a result when entropy, compactness, length/height was considered as a feature set for computing distance between two successive frames. Here the abscissa represents the total number of frame distances in the sampled time series, and the ordinate is the compound distance value between two successive images. Each scene consists of six frames. Therefore, a change of scene occurs at every sixth index on the abscissa. The scene separation is denoted with vertical grid lines. The effectiveness of the aforesaid fuzzy geometrical parameter is also demonstrated (89) for recognizing overlapping fingerprints with a multilayer perceptron.

In the last decade, substantial advancement has occurred in video and motion analysis using fuzzy sets. Recently, a new video shot boundary detection technique using fuzzy logic has been proposed in Reference 90. The authors in Reference 91 used fuzzy C-planes clustering to propose a motion estimation technique, which is an important block in most of the video processing systems. Other applications such as traffic handling (92) in video processing have also been implemented using fuzzy techniques.

Handwritten Character Recognition

Handwritten characters, like all patterns of human origin, are examples of ill-defined patterns. Hence, the recognition of handwritten characters is a very promising field for the application of pattern recognition techniques using the fuzzy approach. It has been claimed that the concept of vagueness underlying fuzzy theory is more appropriate for describing the inherent variability of such systems than the probabilistic concept of randomness. An important application of handwriting recognition is to build efficient man-machine interface for communicating with the computer by human beings. Several attempts have been made for handwritten character recognition in different languages. Here we mention a pioneering contribution of Kickert and Koppelaar (93), the subsequent developments based on their work, and then an attempt made for fuzzy feature description in this context.

The 26 capital letters of the English alphabet constituting the set

$$L = \{H_k | k = 1, 2, \ldots, 26\} = \{A, B, C, \ldots, Y, Z\}$$

are seen to be composed of the elements of the following set of "ideal" elements (93)

$$V_T = \{a_i | i = 1, 2, \ldots, 7\} = \{/, |, \backslash, -, (,), \in\}$$

where $\in$ is a null segment whose use will be explained shortly. Also, a set P exits of 11 ordered recognition routines capable of recognizing the "ideal" segments. Each element of P can be considered as a portion of a context-free grammar having productions of the form

$$A \rightarrow a_i B \quad \text{or} \quad A \rightarrow a_i, \qquad \text{where } a_i \in V_T,\ A, B \in V_n^*$$

with V_n being the non-terminal elements of the grammar.

Each of the 11 recognition routines is applied sequentially to any unknown pattern S to be recognized as one of the members of L. Each routine attempts to recognize a given segment in a given structural context. If successful, the application of the rules in P results in a parsing of S as a vector of segments $S = (x_1, x_2, \ldots, x_n)$, where $x_i \in V_T$.

Each letter, then, is defined by its vector of segments. Let us assume that the vectors are padded out with null segments $\in$ so that all letters are defined by vectors of equal length. Each letter, therefore, can be defined as follows:

$$H_k = (bk_1, \ldots, bk_n), \qquad k = 1, 2, \ldots, 26$$

where

$$\begin{aligned} bk_j &= a_i \qquad \text{for some } i, i = 1, 2, \ldots, 7 \\ &= j\text{th segment of the } k\text{th letter} \end{aligned}$$

The element of fuzziness is introduced by associating with each segment $a_i \in V_T$ a fuzzy set on the actual pattern space. With each a_i is associated a fuzzy membership function μ_{ai} so that, given a segment x_i of a pattern S, $\mu_{a\,j}(x_i)$ is a measure of the degree to which the segment x_i corresponds to the ideal segment a_j.

The recognition procedure is now simply explained. The sequence of recognition rules is executed, evaluating all possible parsings of the input pattern. For each letter H_k for which a parse can be made, the result is a sequence $(x_i, x_2, \ldots, x_n)$ of segments. The membership of S in H_k is the intersection in the sense of fuzzy sets of the memberships of the segments x_i

$$\mu H_k(S) = \min[\mu bk_1(x_1), \ldots, \mu bk_n(x_n)]$$

Finally, the pattern is recognized as letter H_m if

$$\mu H_m(x) = \max_k \mu H_k(S)$$

This approach was criticized by Stallings (94), who developed a Bayesian hypothesis-testing scheme for the same problem. Given a pattern S, hypothesis H_k is that the writer intended letter H_k Associated with each decision is a cost C_{ij}, which is the cost of choosing H_i when H_j is true. The parsing of the pattern is performed as before. Only a probability is associated with each segment for a given letter. Regarding unknown densities, stallings (94) suggests the use of maximum likelihood tests. As both membership function and probability density functions are maps into the interval [0,1], the only difference is the use of min/max operators, where, the author argues, the "min" operator loses a lot of information and is drastically affected by one low value. The author claims that a though frequentistic

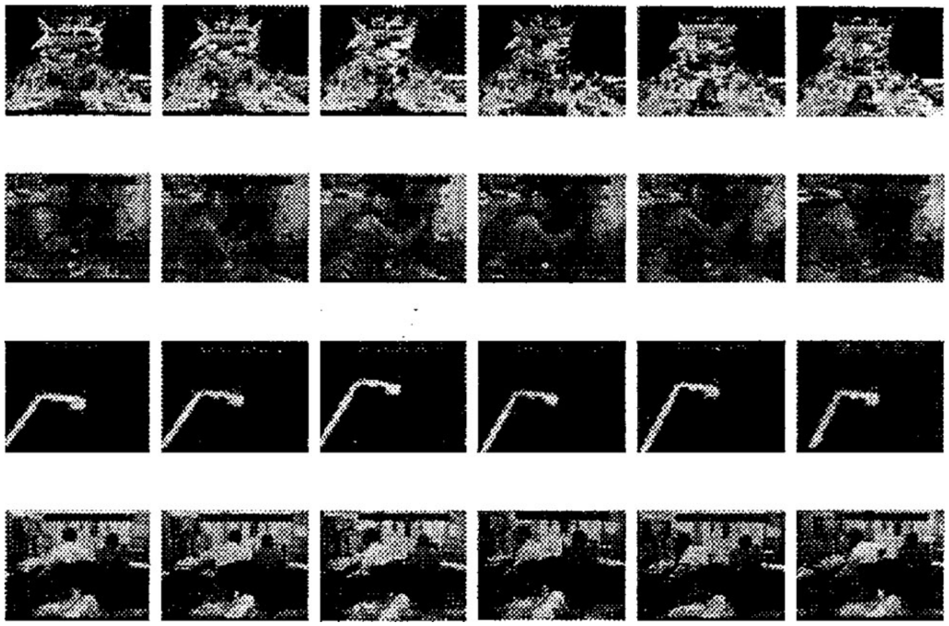

Figure 3. A payload deployment sequence of four scenes as input data.

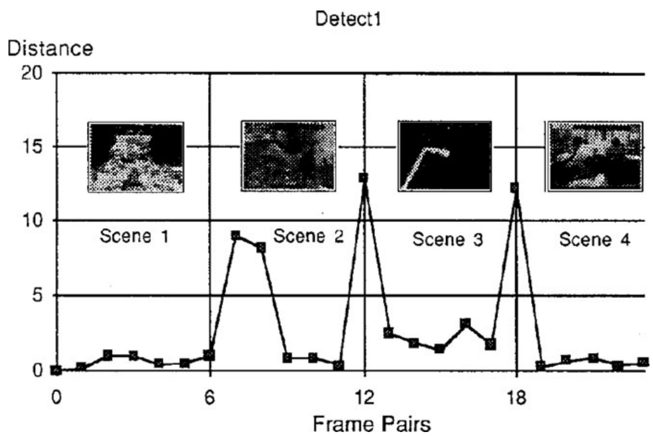

Figure 4. Distances between successive frames with feature set (entropy, compactness, length/height).

probabil-ity is not appropriate in dealing with pattern variability, subjective probability is perfectly suitable and more intuitively obvious than "grade of membership."

In a rejoinder (95), it is argued that fuzzy set theory is more flexible than is assumed in Reference 94, where all arguments are directed against a particular case (93). Recalling the idea of collectives (from property sets), where the arithmetic average replaces "min," there remains little difference between the schemes in References 93 and 94. In a reply, Stalling insisted that the Bayesian approach is superior because offers a convenient way for assignment of costs to errors and gains to correct answers. For the recognition of handwritten English capital letters, the readers may also refer to the work described in Reference 96.

Existing computational recognition methods use feature extraction to assign a pattern to a prototype class. Therefore, the recognition ability depends on the selection procedure. To handle the inherent uncertainties/imprecision in handwritten characters, Malaviya and Peters (97) have introduced fuzziness factor in the definition of selected pattern features. The fuzzy features are confined to their meaningfulness with the help of a multistage feature aggregation, which can be combined in a set of linguistic rules that form the fuzzy rule-base for handwritten information recognition. Note that the concept of introducing fuzziness in the definition and extraction of features and in their relations is not new. A detailed discussion is available in Reference 61 and 98 by Pal and others, for extraction of primitives for X-ray identification and character recognition in terms of gentle, fair, and sharp curves. A similar interpretation of the shape parameters of triangle, rectangle, and quadrangle in terms of membership for "approximate isosceles triangles," "approximate equilateral triangles," "approximate right triangle," and so on has also been made (99) for their classification in a color image. However, the work in Reference (97) is significant from the point that it has described many global, positional, and geometral features to account for the variabilities in patterns, which are supported with experimental results.

To represent the uncertainty in physical relations among the primitives, the production rules of a formal grammar are fuzzified to account for the fuzziness in relation among the primitives, thereby increasing the generative power of a grammar. Such a grammar is called fuzzy grammar (100–102).

It has been observed (98) that the incorporation of the element of fuzziness in defining "sharp," "fair," and "gentle" curves in the grammars enables one to work with a much smaller number of primitives. By introducing fuzziness in the physical relations among the primitives, it was also possible to use the same set of production rules and nonterminals at each stage, which is expected to reduce, to some extent, the time required for parsing in the sense that parsing needs to be done only once at each stage, unlike the case of the non-fuzzy approach, where each string has to be parsed more than once, in general, at each stage. However, this merit has to be balanced against the fact that the fuzzy grammars are not as simple as the corresponding nonfuzzy grammars.

In recent times, the use of fuzzy theory in various kinds of recognition tasks has increased significantly. Complex fuzzy systems have been designed to recognize gestures (103) and describe relative positions in images (104). The authors in Reference (105) have extended the application of fuzzy logic to recognize olfactory (smell) signals.

Detecting Man-Made Objects from Remote Sensing Images

In a remotely sensed image, the regions (objects) are usually ill-defined because of both grayness and spatial ambiguities. Moreover, the gray value assigned to a particular pixel of a remotely sensed image is the average reflectance of different types of ground covers present in the corresponding pixel area ($36.25m$–$36.25m$ for the Indian Remote Sensing (IRS) imagery). Therefore, a pixel may represent more than one class with a varying degree of belonging.

A multivalued recognition system (6,7) formulated based on the concept of fuzzy sets has been used for detecting curved structure from IRS images (108). The system is capable of handling various imprecise inputs and in providing multiple class choices corresponding to any input. Depending on the geometric complexity (8, 9) and the relative positions of the pattern classes in the feature space, the entire feature space is decomposed into some overlapping regions. The system uses Zadeh's compositional rule of inference (109) to recognize the samples. The recognition system is initially applied on an IRS image to classify (based on the spectral knowledge of the image) its pixels into six classes corresponding to six land cover types, namely *pond water, turbid water, concrete structure, habitation, vegetation, and open space*. The green and infrared band information, being sensitive than other band images to discriminate various land cover types, are used for the classification.

The clustered images are then processed for detecting the narrow concrete structure curves. These curves include basically the roads and railway tracks. The width of such attributes has an upper bound, which was considered there to be three pixels for practical reasons. So all the pixels lying on the concrete structure curves with width not more than three pixels were initially considered as the candidate set for the narrow curves. As a result of the low pixel resolutions ($36.25m$ — $36.25m$ for IRS imagery) of the remotely sensed images, all existing portions of such real curve segments may not be reflected as concrete structures and, as a result, the candidate pixel set may constitute some broken curve segments. To identify the curves in a better extent, a traversal through the candidate pixels was used. Before the traversing process, one also needs to thin the candidate curve patterns so that a unique traversal can be made through the existing curve segments with candidate pixels. Thus, the total procedure to find the narrow concrete structure curves consists of three parts: 1) selecting the candidate pixels for such curves, 2) thinning the candidate curve patterns, and 3) traversing the thinned patterns to make some obvious connections between different isolated curve segments. The multiple choices provided by the classifier in making a decision are used to a great extent in the traversal algorithm. Some of the movements are governed by only the second and combined choices.

After the traversal, the noisy curve segments (i.e., with insignificant lengths) are discarded from the curve patterns. The residual curve segments represent the skeleton version of the curve patterns. To complete the curve pattern, the concrete structure pixels lying in the eight neighboring positions corresponding to the pixels on the above obtained narrow curve patterns are now put back. This resultant image represents the narrow concrete structure curves corresponding to an image frame (108).

The results are found to agree well with the ground truths. The classification accuracy of the recognition system (107, 108) is not only found to be good, but also its ability of providing multiple choices in making decisions is found to be very effective in detecting the road-like structures from IRS images.

Content-Based Image Retrieval (CBIR)

In the last few years, researchers have witnessed an upsurge of interest in content-based image retrieval (CBIR), which is a process of selecting similar images from a collection by measuring similarities between the extracted features from images themselves. Real-life images are inherently fuzzy because of several uncertainties developing in the imaging process. Moreover, measuring visual similarities between images highly depends on subjectivity of human perception of image content. As a result, fuzzy image processing for extracting visual features finds an important place in image pretrieval applications. Let us explain here, in brief, an investigation carried out in Reference 110 on image retrieval is based on fuzzy geometrical features. Here a fuzzy edge map is extracted for each image. Using the edge map, a fuzzy compactness vector is computed that is subsequently used for measuring the similarity between the query and the database image.

The process involves extracting the possible edge candidates using the concept of Top and Bottom of the intensity surface of a smoothened image. The extracted edge candidates are assigned gradient membership value $\mu_m(P)$

within (0.0 to 1.0) computed from the pixel contrast ratio over a fixed window. The selected points are categorized as weak, medium, and strong edge pixels based on their gradient membership value $\mu_m(P)$. Multilevel thresholding is performed by using $(\alpha - cut)$ to segregate the edge pixels. Fuzzy edge maps $s_{n\alpha}$ consisting of different types of edge pixels are obtained from the candidate set s_n by varying $\mu_m(P)$, from which the connected subsets $s\prime_{n\alpha}$ as shown in Fig. 5(b) and (c) are obtained. Fuzzy compactness value is computed from the fuzzy edge map $s\prime_{n\alpha}$, obtained at different $(\alpha - cut)$ to index an image of the database.

$$s_{n\alpha} = \{(P \in s_n : \mu_m(P) \geq \alpha)\} \tag{32}$$

where $0.5 \geq \alpha \geq 1$. This geometrical feature is invariant to rotation translation and scaling by definition. It physically means the maximum area that can be encircled by the perimeter. The similarity between the feature vectors of two images are computed by the widely used Euclidean distance metric. The retrieval results are shown in Fig. 6. From the experimental results of Fig. 6, it is seen that images are retrieved with fairly satisfactory precision.

Some other significant work on image retrieval are available in Reference (111–113). The authors in Reference (111) propose an image retrieval system using texture similarity, whereas the authors in Reference (112) present a novel information fusion approach for use in content-based image retrieval. Retrieval of color images has been investigated in Reference (113). Recently, similarity-based online feature selection was applied to bridge the gap between high level semantic concepts and low level visual features in content-based image retrieval (114). Note that all these methods mentioned above use fuzzy theory to handle various kinds of ambiguities.

Segmentation of Brain Magnetic Resonance Image

Image segmentation is an indispensable process in the visualization of human tissues, particularly during clinical analysis of magnetic resonance (MR) images. A robust segmentation technique based on fuzzy set theory for brain MR images is proposed in Reference (115).

The method proposed in Reference (115) is based on a fuzzy measure to threshold the image histogram. The image is thresholded based on a criterion of similarity between gray levels. The second-order fuzzy correlation is used for assessing such a concept. The local information of the given image is extracted through a modified co-occurrence matrix. The technique proposed here consists of two linguistic variables bright, dark modeled by two fuzzy subsets and a fuzzy region on the gray level histogram. Each of the gray levels of the fuzzy region is assigned to both defined subsets one by one and the second-order fuzzy correlation using modified co-occurrence matrix is calculated.

First, let us define two linguistic variables dark, bright modeled by two fuzzy subsets of X, denoted by A and B, respectively. The fuzzy subsets A and B are associated with the histogram intervals $[x_{min}, x_p]$ and $[x_{max}, x_q]$, respectively, where x_p and x_g are the final and initial gray-level limits for these subsets, and x_{min} and x_{max} are the lowest and highest gray levels of the image, respectively.

Next, we calculate $C_A(x_{\min} : x_p)$ and $C_B(x_p : x_{\max})$, where $C_A(x_{\min} : x_p)$ is the second-order fuzzy correlation of fuzzy subset A and its two-tone version and $C_B(x_p : x_{\max})$ is the second-order fuzzy correlation of fuzzy subset B and its two-tone version using modified co-occurrence matrix. The second-order fuzzy correlation can be expressed in the following way:

$$C(\mu_1, \mu_2) = 1 - \frac{4\sum_{i=1}^{L}\sum_{j=1}^{L}[\mu_1(i,\ j) - \mu_2(i,\ j)]^2 t_{i\,j}}{Y_1 + Y_2} \tag{33}$$

where t_{ij} is the frequency of occurrence of the gray level i followed by j; that is, $T = [t_{ij}]$ is the modified co-occurrence matrix, which is given by

$$t_{i\,j} = \sum_{a \in X, b \in ag} \frac{\delta}{(1 + |\Delta|^2)} \tag{34}$$

where

$$b \in ag = \{(m, n-1), (m, n+1), (m+1, n), (m-1, n), (m-1, n-1), (m-1, n+1), (m+1, n-1), (m+1, n+1)\}$$

$$\Delta = \frac{1}{4}\max\{|x_{m-1,n} + x_{m-1,n+1} + x_{m,n} + x_{m,n+1} - x_{m+1,n} - x_{m+1,n+1} - x_{m+2,n} - x_{m+2,n+1}|,\ |x_{m,n-1} + x_{m,n} + x_{m+1,n-1} + x_{m+1,n} - x_{m,n+1} - x_{m,n+2} - x_{m+1,n+1} - x_{m+1,n+2}|\}$$

$$\delta = \begin{cases} 1 & \text{if gray level value of } a \text{ is } i \text{ and that of } b \text{ is } j \\ 0 & \text{otherwise} \end{cases}$$

and

$$Y_k = \sum_{i=1}^{L}\sum_{j=1}^{L}[2\mu_k(i,\ j) - 1]^2 t_{i\,j};\ k = 1, 2$$

To calculate correlation between a gray-tone image and its two-tone version, μ_2 is considered as the nearest two-tone version of μ_1. That is,

$$\mu_2(x) = \begin{cases} 0 & \text{if } \mu_2(x) \leq 0.5 \\ 1 & \text{otherwise} \end{cases} \tag{35}$$

As the key of the proposed method is the comparison of fuzzy correlations, we have to normalize those measures, which is done by computing a normalizing factor α according to the following relation:

$$\alpha = \frac{C_A(x_{\min} : x_p)}{C_B(x_q : x_{\max})} \tag{36}$$

To obtain the segmented version of the gray-level histogram, we add to each of the subsets A and B a gray-level x_i picked up from the fuzzy region and form two fuzzy subsets A' and B' that are associated with the histogram intervals $[x_{min}, x_i]$ and $[x_i, x_{max}]$, where $x_p < x_i < x_q$. Then we calculate $C_{A'}(x_{\min} : x_i)$ and $C_{B'}(x_i : x_{\max})$. The ambiguity of the gray value of x_i is calculated as follows:

$$\mathcal{A}(x_i) = 1 - \frac{|C_{A'}(x_{\min} : x_i) - \alpha \cdot C_{B'}(x_i : x_{\max})|}{(1 + \alpha)} \tag{37}$$

Finally, applying this procedure for all gray levels of the fuzzy region, we calculate the ambiguity of each gray level.

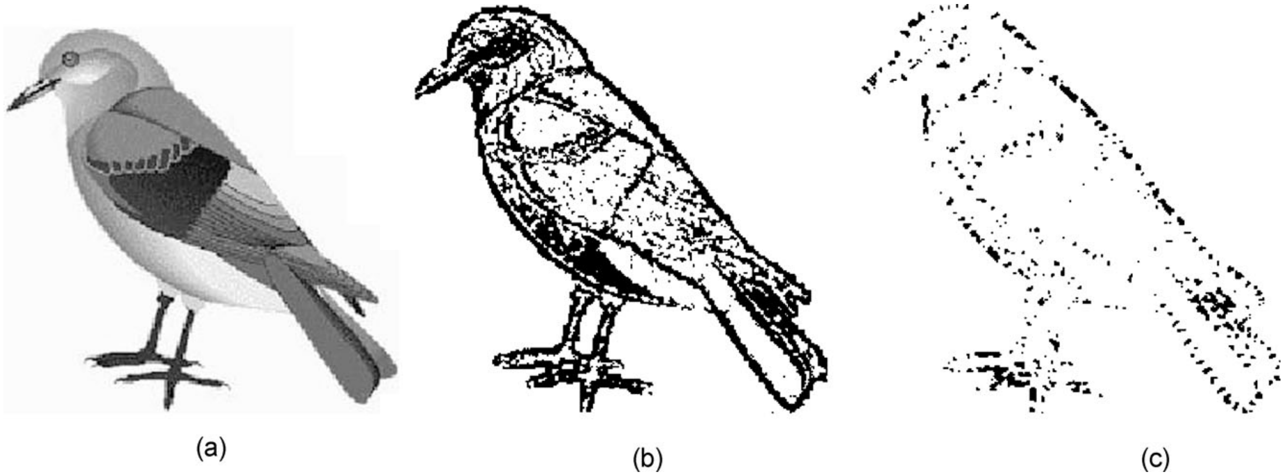

Figure 5. (**a**) Original image. Fuzzy edge map for candidates with (**b**) $\mu_m P \geq 0.6$ (**c**) $\mu_m P \geq 0.8$.

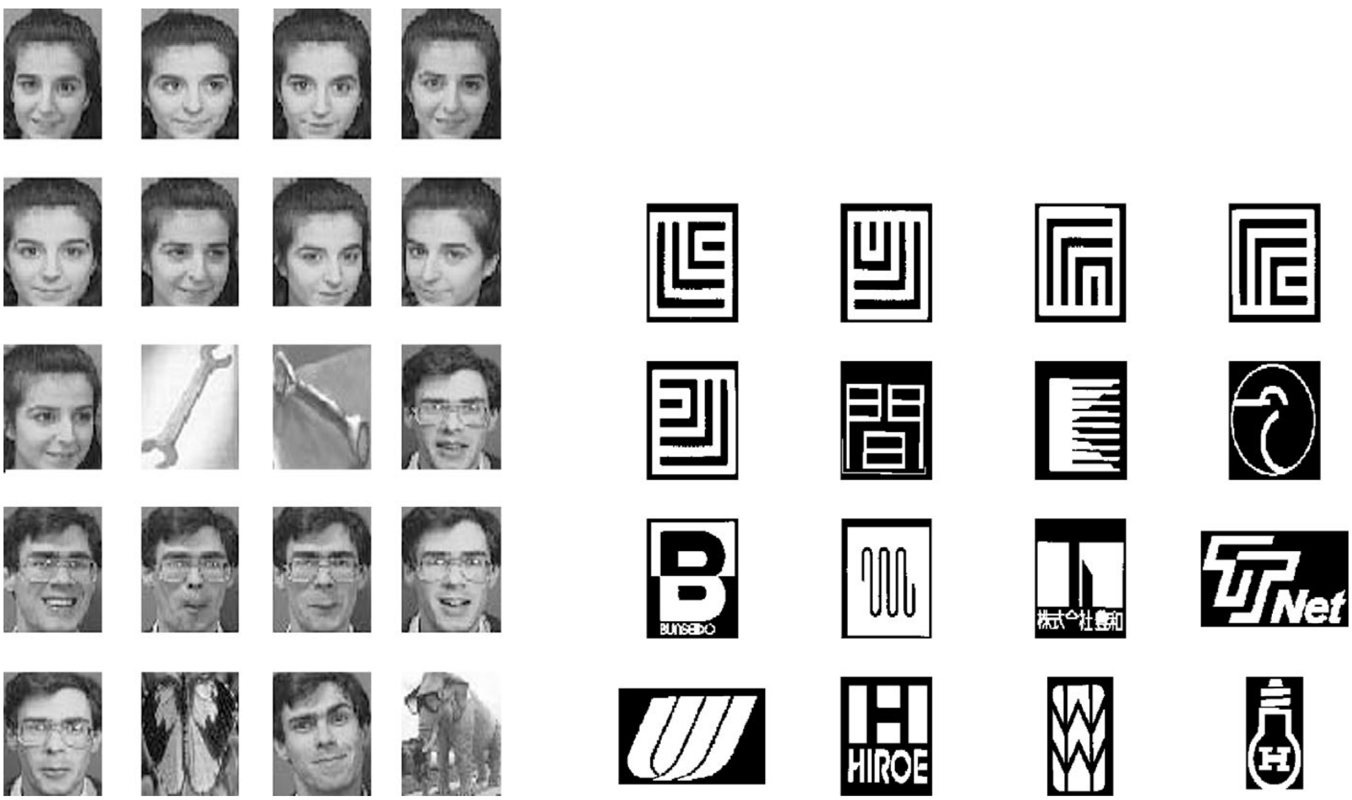

Figure 6. Retrieved result (from fuzzy edge map), (**a**) general purpose database (**b**) logo retrieval from (USPTO) database, with top left image as the query image.

The process is started with $x_i = x_p + 1$, and x_i is incremented one by one until $x_i < x_q$. In other words, we calculate the ambiguity by observing how the introduction of a gray level x_i of the fuzzy region affects the similarity measure among gray levels in each of the modified fuzzy subsets A' and B'. The ambiguity $\mathcal{A}$ is maximum for the gray level x_i in which the correlations of two modified fuzzy subsets are equal. The threshold level (T) for segmentation corresponds to gray value with maximum ambiguity $\mathcal{A}$. That is,

$$\mathcal{A}(T) = \max \arg\{A(x_i)\}; \quad \forall\ x_p < x_i < x_q \tag{38}$$

As an example, we explain the merits of the proposed method in Figs. (7) and (8). Figure (7) shows the original MR images and their gray-value histograms, whereas Fig. 8 represents the fuzzy second-order correlations $C_{A'}(x_{\min} : x_i)$ and $C_{B'}(x_i : x_{\max})$ of two modified fuzzy subsets A' and B' with respect to the gray level x_i of the fuzzy region and the ambiguity of each gray level x_i. The value of α is also given here. Figure (8) depicts the segmented image of the proposed method. The thresholds are determined according to the strength of ambiguity.

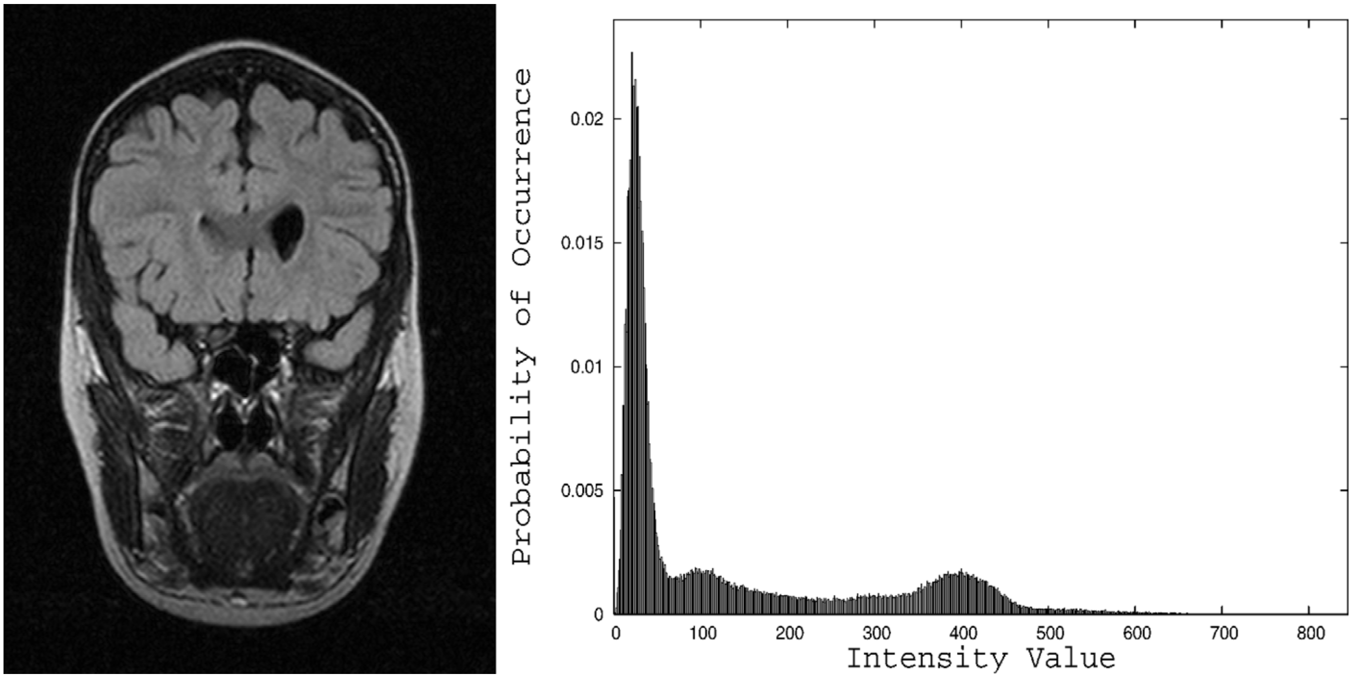

Figure 7. Original image and corresponding histogram.

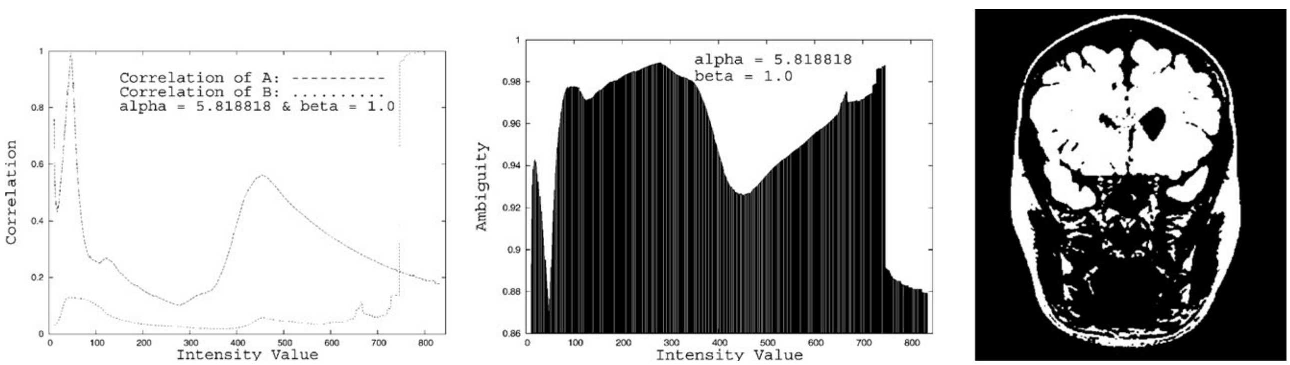

Figure 8. Correlations of two fuzzy subsets, measure of ambiguity and segmented image (proposed).

Other Advances

Over the years, applications of fuzzy theory in image processing and recognition has developed extensively in many other domains. Fuzzy morphology is a tool that has received considerable attention among researchers in the field of image processing (116, 117). Bloch (118, 119) used fuzzy theory to define spatial positioning of objects in images. A fuzzy error diffusion method has been proposed in Reference (120) to perform dithering to hide quantization errors in images. Zahlmann et al. (121) applied a hybrid fuzzy image, processing system to assess the damage to the blood vessels in the retina because of diabetis. A fractal coding scheme using a fuzzy image metric has been proposed in Reference (122). Adaptive schemes of digital watermarking in images and videos using fuzzy-adaptive resonance theory (fuzzy-ART) classifier has been given in Reference 123. In Reference 124, fuzzy theory has been used to represent the uncertain location of a normal Eucledian point, and its application in doppler image sequence processing has been demonstrated. Fuzzy theory have also been used in intelligent Web image retrieval purposes (125–127). In Reference 126, an image search engine named (STRICT) has been designed using fuzzy similarity measures. The authors in Reference 127 combine fuzzy text and image retrieval techniques to present a comprehensive image search engine.

For an image, the histogram, which gives the frequency (probability) of occurrence of each gray value, and the co-occurrence matrix, which gives the frequency (joint-probability) of occurrence of two gray vajues seperated by a specific distance, are the first- and second-order statistics. In Reference 128, the authors used fuzzy theory to explain the inherent imprecision in the gray values of an image and defined the first- and second-order fuzzy statistics of digital images, namely, fuzzy histogram and fuzzy co-occurrence matrix, respectively. Fuzzy theory has also been used in various other applications such as automatic taxget detection and tracking, stereovision matching, urban structure detection in synthetic aperture radar (SAR) images, and image reconstrution (129–132).

CONCLUSIONS AND DISCUSSION

The problem of image processing and recognition under fuzziness and uncertainty has been considered. The role of fuzzy logic in representing and managing the uncertainties in these tasks was explained. Various fuzzy set theoretic tools for measuring information on grayness ambiguity and spatial ambiguity in an image were listed along with their characteristics. Some examples of image processing operations (e.g., segmentation, skeleton extraction, and edge detection), whose outputs are responsible for the overall performance of a recognition (vision) system, were considered to demonstrate the effectiveness of these tools in providing both soft and hard decisions. The significance of retaining the gray information in the form of class membership for a soft decision is evident. Uncertainty in determining a membership function in this regard and the tools for its management were also stated. Finally, a few real-life applications of these methodologies are described.

In conclusion, *gray information is expensive and informative. Once it is thrown away, there is no way to get it back. Therefore, one should try to retain this information as long as possible throughout the decision-making tasks for its full use. When it is required to make a crisp decision at the highest level, one can always throw away or ignore this informaion*.

Most of the algorithms and tools described here were developed by the author with his colleagues. Processing of color images has not been considered here. Some significant results on color image information and processing in the notion of fuzzy logic are available in References 133–137.

Note that fuzzy set theory has led to the development of the concept of soft computing as a foundation for the conception and design of a high Machine IQ (MIQ) system. The merits of fuzzy set theory have also been integrated with those of other soft computing tools (e.g., artificial neural networks, genetic algorithms, and rough sets) with a hope of building more efficient processing and recognition systems.

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