

MANOMETERS

Pressure is a basic property of fluids from which quantity of flow, state of matter, liquid level in storage tanks, etc. is inferred. In applications like pneumatic transport of solids, actuation of control valves, control and automation of industrial machinery and processes, etc., the pressure of working fluids plays major roles. Moreover, pressure is responsible for circulation of blood in the human body and thus for sustenance of life. Hence the significance of accurate pressure measurement is well realized now.

Physically, pressure (P) is the force exerted by fluid normal to a unit area of confining surface. Mathematically, it is force F per unit area and expressed as

$$P = F/A \quad (1)$$

where A is the area.

The unit of pressure in the SI system is the newton per square meter, known as the pascal (Pa); larger related units are the kilopascal (kPa) and megapascal (MPa). In practice fluid pressure is the resultant of gravity (dominant in liquids at rest) and compression (dominant in gases) and exhibits the following characteristics, which form the basis of its measurement and other useful applications:

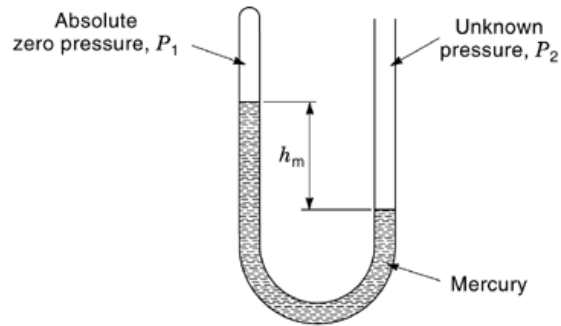
- (1) The pressure at any point is independent of direction, but its magnitude depends on the position in a fluid at rest—for example, on the depth from the free surface of a liquid.
- (2) The shape of the confining surface does not affect fluid pressure.
- (3) External pressure applied to a fluid at rest is equally transmitted to every point of the fluid. This principle, known as Pascal's law, is used in deadweight testers, hydraulic presses, etc.

Measurement involves the following three kinds of pressures.

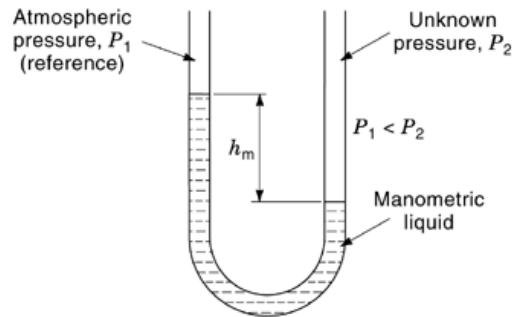
- (1) **Absolute Pressure** The difference between the unknown fluid pressure and absolute zero pressure or perfect vacuum, as indicated for example by barometers. It is independent of the local atmospheric pressure.
- (2) **Gauge pressure** The difference between the unknown pressure and local atmospheric pressure. It can be positive (above atmospheric) or negative (below atmospheric—vacuum). It is indicated for example by Bourdon pressure gauges.
- (3) **Differential pressure** The difference between two unknown pressures, as read for example by U-tube manometers and micromanometers.

Liquid manometers and electronic manometers with suitable arrangement or modifications for the two pressure connections may be adapted for measurement of any of the above three pressures, as shown in Fig. 1.

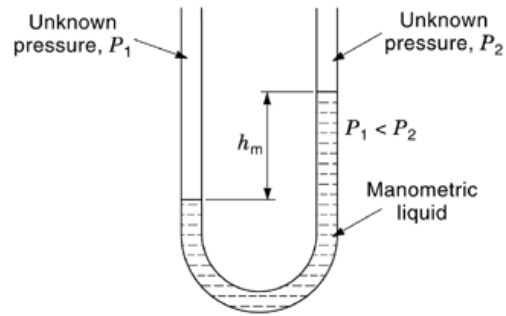
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(a) Absolute pressure



(b) Gauge pressure



(c) Differential pressure

Fig. 1. Schemes of pressure measurement.

Liquid Manometers

Measurement of pressures using liquid-filled manometers is one of the earliest techniques. They provide simple, accurate, and reliable means of measurement with no or few mechanical moving parts.

Simple U-Tube Manometer. The U-tube manometer is a direct-reading device whose operation depends on balancing of unknown differential pressure with the pressure due to a known column of liquid of known density exerted at a known depth. A vertical transparent U-shaped glass or clear acrylic tube partly

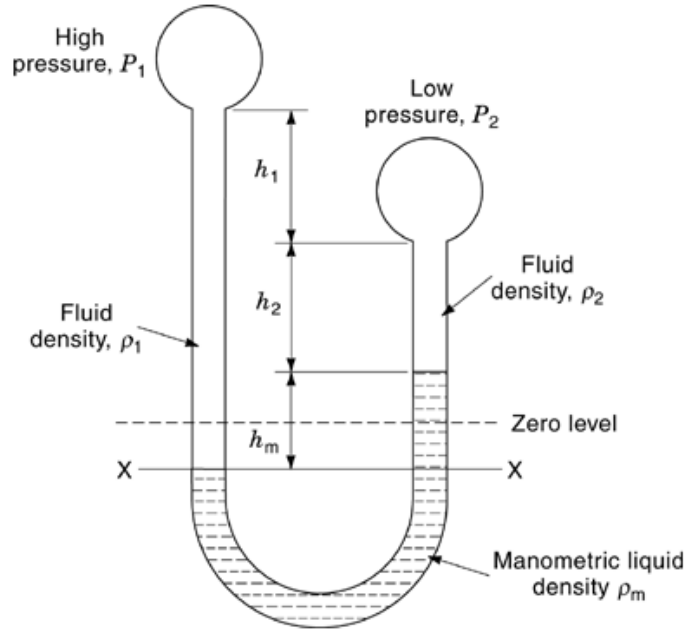


Fig. 2. Simple U-tube manometer.

filled with a liquid, called the *manometric liquid*, and equipped with a linear scale constitutes a simple U-tube manometer. Referring to Fig. 2, unknown pressures P_1 and P_2 applied to the tops of the limbs cause elevation of the manometric liquid of mass density ρ_m in the low-pressure side. The pressures at the same horizontal plane XX are equal for static equilibrium of the liquid:

$$P_1 + \rho_1 g_l (h_1 + h_2 + h_m) = P_2 + \rho_2 g_l h_2 + \rho_m g_l h_m \quad (2)$$

where g_l is the acceleration due to gravity at the given location. Rearranging the equation, we have

$$P_1 - P_2 = \rho_m g_l \left[1 + \frac{\rho_2 h_2}{\rho_m h_m} - \frac{\rho_1}{\rho_m} \left(\frac{h_1 + h_2}{h_m} + 1 \right) \right] h_m \quad (3)$$

For a given measurement the factor in brackets constitutes a correction to the measured vertical column height h_m . This is valid for the fluids whose densities ρ_1 and ρ_2 are comparable with that of the manometric liquid and also for fluid column heights h_1 and h_2 in the manometer and connecting tubes (1).

Equation (3) may be simplified in certain real situations. For example, when fluids 1 and 2 have nearly equal density, $\rho_1 = \rho_2 = \rho_f$, it becomes

$$P_1 - P_2 = \rho_m g_l \left[1 - \frac{\rho_f}{\rho_m} \left(\frac{h_1}{h_m} + 1 \right) \right] h_m \quad (4)$$

Further, when the pressure sources are connected to the manometer at the same level, then $h_1 = 0$, resulting in

$$P_1 - P_2 = \rho_m g_l \left(1 - \frac{\rho_f}{\rho_m} \right) h_m \quad (5)$$

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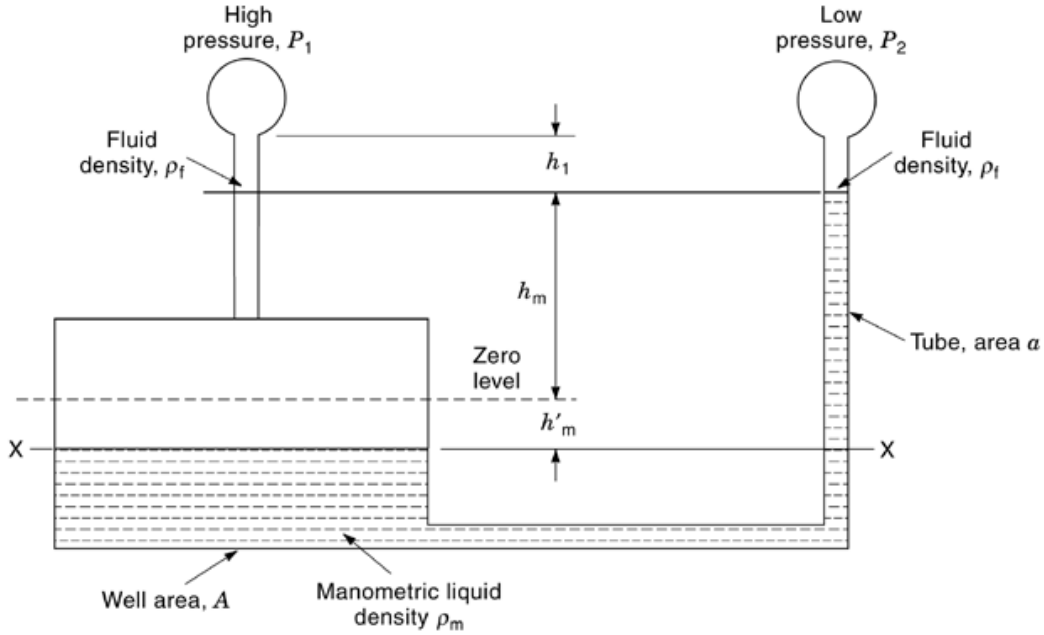


Fig. 3. Well manometer.

If the density of manometric liquid is far greater than the fluid density, that is, $\rho_m \gg \rho_f$ (the usual case when measuring air pressures using water or mercury), then

$$P_1 - P_2 = \rho_m g h_m \quad (6)$$

Hence for a simple U-tube manometer, the unknown differential pressure is equal to the mass density at liquid temperature times the local value of acceleration due to gravity times the vertical height of displaced manometric liquid. As seen, the cross section of the U tube has no effect on the measurement, and the sensitivity depends only on the density of the liquid. The vertical column height is read from a built-in linear scale graduated in length units. Scales may also be directly graduated in desired pressure units. In a U-tube manometer with equal limbs, the rise of liquid on one limb accompanied by an equal drop in the other makes reading difficult. Sometimes this is overcome by doubling the scale readings from the reference zero.

Well Manometer. A practical modification of the simple U-tube manometer is the well or cistern manometer. One limb of U tube is made into a well of large cross-sectional area compared to the other limb. As differential pressure is applied (with higher pressure to the well), the liquid level in the narrow tube is displaced upwards. Because the well area is large, the accompanying drop of liquid in the well is relatively small and possibly negligible. In other words, the initial zero level in the well moves very little. Therefore only the height of the elevated liquid level in the tube need be measured, and that is straightforward. Referring to the well manometer in Fig. 3, the rise of the liquid column in the tube, h_m , does not equal the fall in the well, h'_m , but the loss of liquid in the well must equal gain of liquid in the tube:

$$\rho_m h'_m A = \rho_m h_m a \quad (7)$$

$$\frac{h'_m}{h_m} = \frac{a}{A} \quad (8)$$

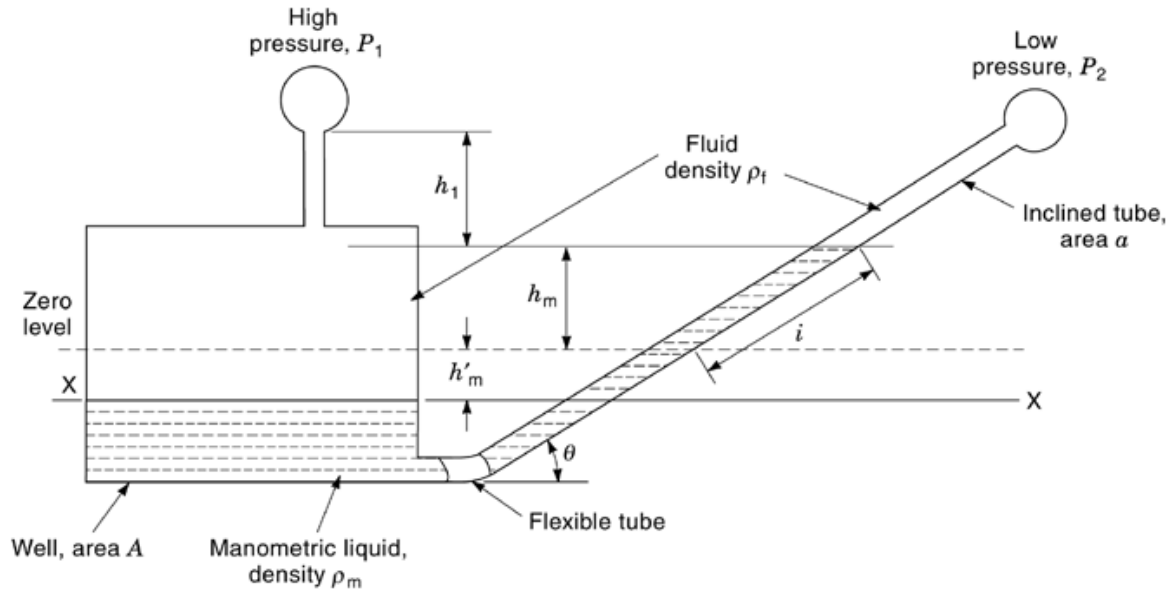


Fig. 4. Inclined manometer.

where a and A are the cross-sectional areas of the tube and the well respectively. For static equilibrium at section XX , the following expression may be written:

$$P_1 + \rho_f g l (h_1 + h_m + h'_m) = P_2 + \rho_f g l h_1 + \rho_m g l (h_m + h'_m) \quad (9)$$

Rearranging and combining with Eq. (8), the equation for well manometer is seen to be

$$P_1 - P_2 = (\rho_m - \rho_f) g l \left(1 + \frac{a}{A} \right) h_m \quad (10)$$

The area ratio of well to tube must be made as large as possible (500 times or more) to eliminate errors caused by the change in liquid level in the well, and if $a \ll A$ and $\rho_m \gg \rho_f$, as realized in air/water or air/mercury manometers, Eq. (10) reduces to Eq. (6). The scales are calibrated to correspond to equivalent pressure for most industrial applications. The small error due to the drop in the well may be corrected by suitably modifying the intervals of the scale graduation. Thus well manometers offer direct reading with a single indicating column and are widely used due to that convenience. However, their sensitivity to nonuniformity of the tube creates measurement uncertainty.

Inclined Manometer. For accurate and reliable measurement of low differential pressures (< 300 Pa) using liquid manometers, a large value of h_m is desirable. A modified form of the well manometer, popularly known as the inclined manometer or draft gauge, satisfies this requirement by expanding the scale at those pressures. The sensitivity and resolution of the scale at small pressures are increased by tilting the transparent tube of the well at an angle θ to the horizontal. Flexible tubes connect the well and the inclined tube. Because of the inclination, for a given differential pressure, the liquid column in the tube moves much more along its length. The level is read conveniently with an attached graduated scale. In Fig. 4, the vertical height h_m is

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related exactly to the distance the liquid moves along the inclined tube by

$$h_m = l \sin \theta \quad (11)$$

where l is the length of liquid column along the tube and θ is the inclination of the tube with respect to horizontal.

Combining Eq. (10) and Eq. (11),

$$P_1 - P_2 = (\rho_m - \rho_f)gl \left(1 + \frac{a}{A}\right) \sin \theta \quad (12)$$

For very small values of a/A and fluid densities much smaller than that of manometric liquid, the above equation reduces to

$$P_1 - P_2 = \rho_m gl \sin \theta \quad (13)$$

The calibrated scale may again be corrected for the level drop in the well. Though by reducing θ the length can be increased greatly, the extent of this improvement is limited by meniscus distortion due to surface tension, which depends on the cleanness of the tube. Each manometer must be individually calibrated. For good performance, the ratio l/h_m should not exceed 10 to 14, limiting θ to about 4° to 5° . In some designs, the tube can be set at more than one angle to cover various ranges. The zero of the inclined manometer can be adjusted by raising or lowering the well or moving a plunger in the well or by the expansion or contraction of a flexible bellows immersed in the well.

Inclined manometers are used for draft measurements and for continuous monitoring of differential pressure across filters in heating, ventilation, and air conditioning systems. Equipped with a photosensor for meniscus detection and alarm switches, it finds application in clean-room pressure monitoring in the health, food, drug, and nuclear industries.

Depending on the strength and sealing adopted, glass manometers are available up to about 2 MPa with mercury as the manometric liquid. The measuring range of the manometer depends on the physical length of the tube and also the density of the liquid; it is normally limited to 200 kPa. For equal pressure differentials, a water column height will be about 13.6 (the specific gravity of mercury) times greater than a mercury column height. Therefore, for a given manometer, the range can be extended by use of liquid with higher density and shrunk with lower density.

Accuracy depends on the use of a liquid with specific, clearly defined properties. Because of health hazards in using mercury, its use has declined in industry, but mercury manometers are used as primary standards for calibrating other instruments, due to their smaller uncertainty with sophisticated column measurement techniques. For fieldwork, in any case, the conventional liquid-filled manometers are inconvenient.

Micromanometers. In the manometers described so far, the accuracy depends greatly on reading the meniscus correctly. Manometers with sophisticated techniques for column height measurement, retaining the basic features of well and inclined-tube configurations, have been developed for measurement of small differential pressures with reduced uncertainty. Observation of the meniscus through a microscope, optical magnification of a scale floating in the liquid to facilitate reading, and the use of a micrometer for height measurements are some of the recent methods used in micromanometers. Micromanometers minimize errors due to human intervention in meniscus reading.

Prandtl Micromanometer. Figure 5 shows the Prandtl micromanometer, a modified version of the well-and-inclined-tube manometer. In this, the well and the transparent tube inclined at about 2° are connected by a flexible tube. The height of the liquid column is measured using a micrometer, which moves the well vertically.

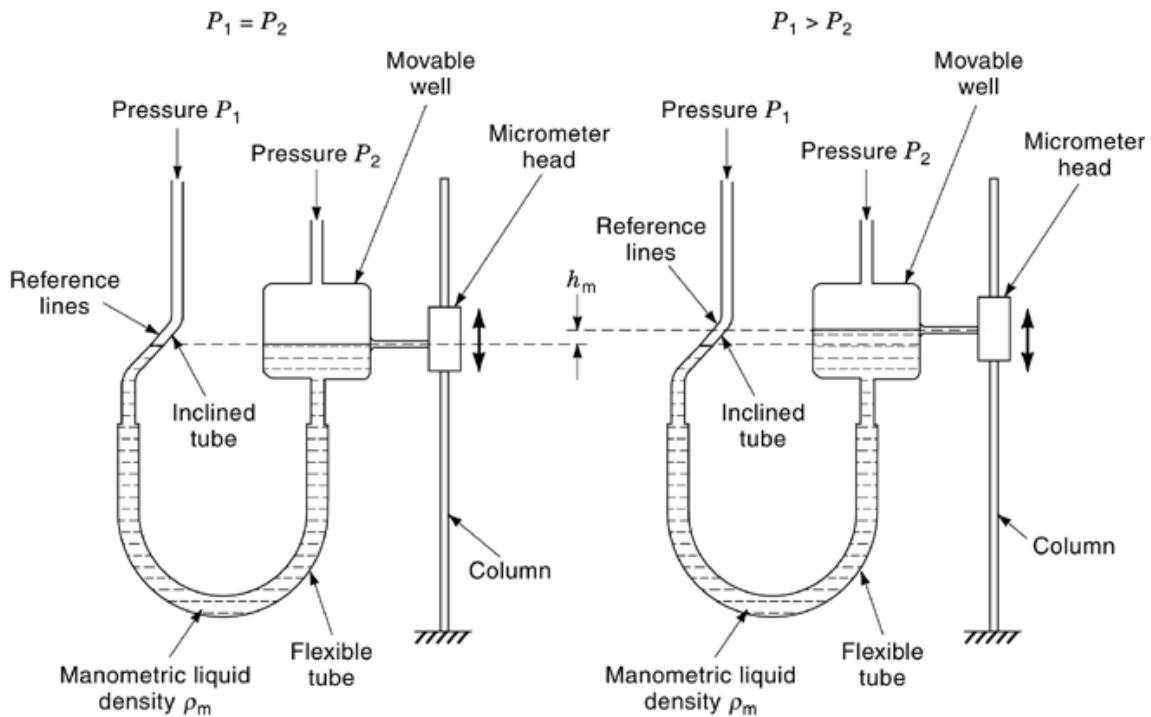


Fig. 5. Prandtl micromanometer.

With equal pressures applied to well and tube, the well is raised or lowered with respect to the tube so that the null or reference position is established. This corresponds to the position where the meniscus is within two close marks on the inclined portion of the transparent tube or coincident with a fixed hairline viewed through a microscope. Application of pressure moves the meniscus away from the reference position. It is then restored to the reference position by adjusting the well by means of micrometer. The difference in readings of the micrometer gives the liquid column height created by the differential pressure. The height is measured using the micrometer. With water, this micromanometer has a readability of $20 \mu\text{m}$, and with alcohol, $5 \mu\text{m}$, with rangeability from 0 to 10 kPa. For higher ranges, the height can be measured using digital techniques with height gauges and rotary encoders. As readings are taken with the meniscus in the same reference position, errors due to capillary effects are considerably reduced.

Micrometer Micromanometer. In this type, the liquid column displacements caused by the unknown differential pressures are measured using micrometers (2) as in Fig. 6. It consists of two large-diameter transparent tubes with their bottoms connected to form the basic U-tube manometer. Large-diameter tubes minimize capillary effects during measurement. Two micrometers with sharp pointers are fixed at or near the centers of the tubes. The contact of a movable pointer with the surface of the liquid may be sensed visually, and the height of the column is computed from the initial and final readings of micrometer. In another type, using electronic sensing and mercury as the liquid, two metal plates form capacitors with the mercury surfaces. At zero reference, the air gaps of the capacitors are equal, producing a null reading. With pressures applied, the air gaps become unequal, producing an error voltage, which causes a feedback servomechanism to restore the null point. The rotation of the shaft of the servo system, measured using a rotary encoder, gives the height of the liquid column and hence the applied differential pressure. The resolution of the digital readout system is of the order of $3 \mu\text{m}$, and the uncertainty is of the order of $\pm 0.004\%$ of the reading.

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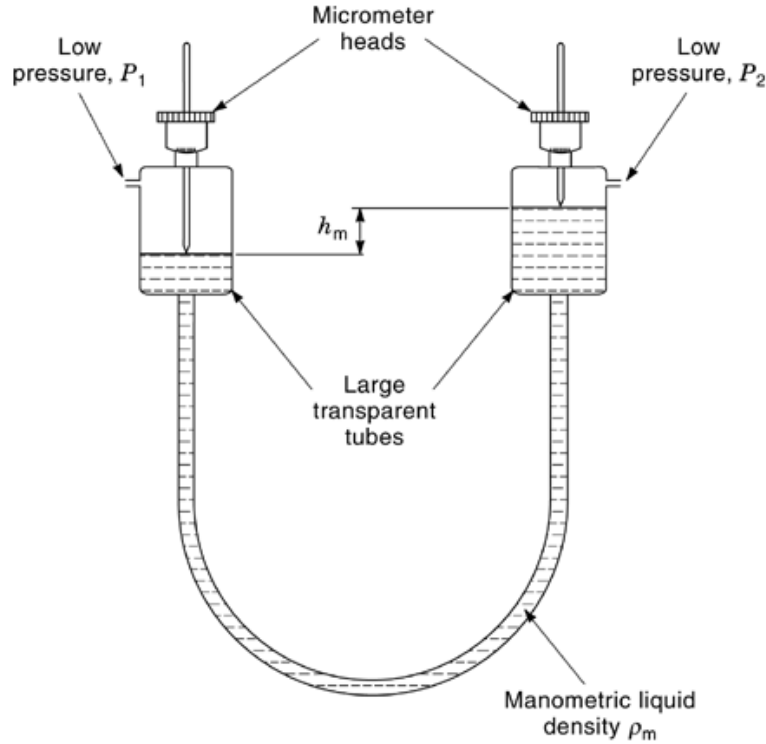


Fig. 6. Micrometer micromanometer.

Two-Liquid Micromanometer. The two-liquid micromanometer is a variation of the simple U-tube manometer with two identical wells of area A , connected to two identical limbs with area a . The U tube is filled with two immiscible and insoluble manometric liquids of nearly equal densities. With this arrangement, shown in Fig. 7, the sensitivity will be higher than with the conventional U tube, and it can be used in applications where the use of an inclined manometer is not allowed. The meniscus under application of differential pressure must always be inside one of the vertical tubes below the wells. With pressure applied, the liquid columns displace as in the figure. Under static equilibrium in the plane XX , the general governing equation is

$$\begin{aligned}
 P_1 + \rho_f g l (h_f + h'_m) + \rho_1 g l \left(h - h'_m + \frac{h_m}{2} \right) \\
 = P_2 + \rho_f g l (h_f - h'_m) + \rho_1 g l \left(h + h'_m - \frac{h_m}{2} \right) + \rho_2 g l h_m \quad (14)
 \end{aligned}$$

As the volume of liquid fall in the well will be equal to that of rise in the tube, then

$$h'_m = \frac{a}{A} \frac{h_m}{2} \quad (15)$$

$$P_1 - P_2 = \left[\rho_2 - \rho_1 \left(1 - \frac{a}{A} \right) - \rho_f \frac{a}{A} \right] g l h_m \quad (16)$$

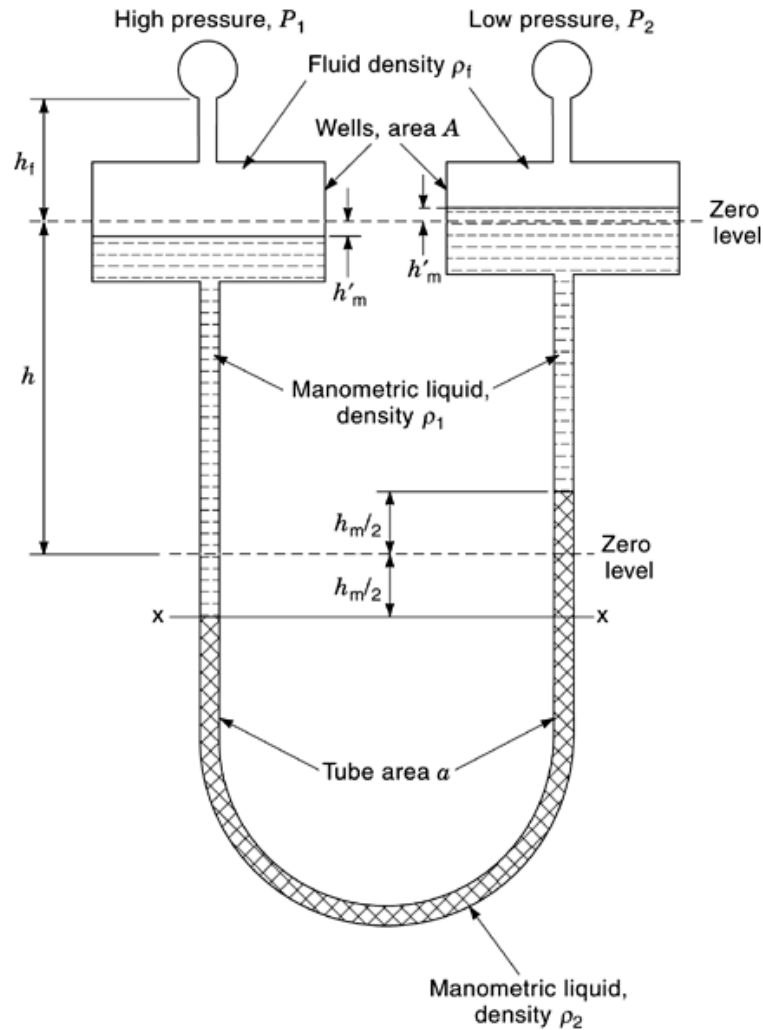


Fig. 7. Two-liquid micromanometer.

For $a \ll A$ and small values of ρ_f ,

$$P_1 - P_2 = (\rho_2 - \rho_1)g_i h_m \tag{17}$$

When ρ_2 and ρ_1 are close, a reasonably large value of h_m can be achieved. But the height magnification achieved by this micromanometer is limited by its slow response and unclear meniscus interface. With kerosene and alcohol of densities of 797 kg/m^3 and 777 kg/m^3 , a magnification of about 50 can be achieved (compared to water as the manometric liquid), with a sensitivity of $20 \mu\text{m}$ of water (3).

Sonar Micrometer. In a sonar micrometer, the mercury column height is sensed by an ultrasonic technique. Piezoelectric transceivers are located at the bottom of a specially constructed U tube. The manometer transmits pulses upward through the mercury column. Pulses reflected at the meniscus are received at the bottom, which starts and stops the digital counter at the shorter and larger columns respectively. The difference

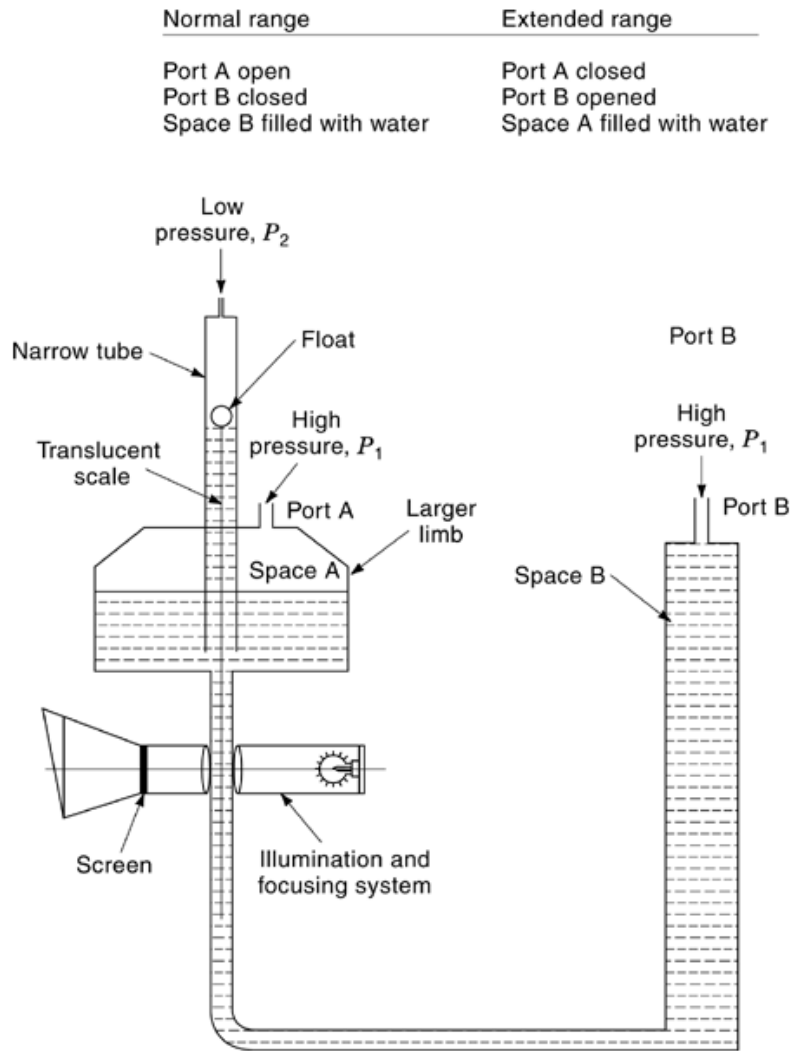


Fig. 8. High-range projection micromanometer.

in the travel times of pulses and sonic velocity in mercury yield the mercury column height due to the applied differential pressure. The accuracy is of the order of $\pm 0.003\%$ of the reading. To minimize the temperature effects on the density of mercury and the sonic velocity, the temperature of the system must be maintained constant with feedback control systems.

High-Range Projection Micromanometer. A high-range micromanometer due to Betz is illustrated in Fig. 8. It consists of a U tube, of which one limb has larger cross section, containing distilled water as manometric liquid. With gas pressures applied to the limbs, the liquid level raises in the narrow tube. A float, carrying a slim translucent graduated scale, rises along with liquid inside the narrow tube. The scale, sliding through a focusing system, produces a sharp illuminated and magnified image of the graduations of the scale on a ground glass screen. The liquid displacement is read directly from the scale markings, with fractional readings obtained from an auxiliary scale on the screen. Coarse zero adjustment of the manometer can be made by draining excess water, and fine zero adjustments by adjusting the position of the focusing system.

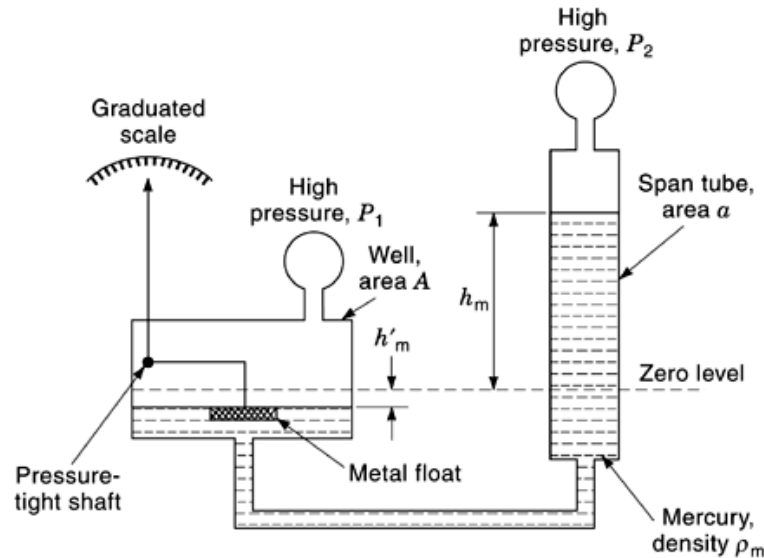


Fig. 9. Indicating or recording well manometer.

This is an accurate, reliable instrument used in research and industry, but its high internal volume makes it sluggish for dynamic applications. The range is about 4000 Pa, with sensitivity of about 1 Pa. Changing the tube cross section as indicated in the figure may double the range. For extended range, the scale readings are to be multiplied by 2. Differential pressure can be measured at operating pressures of up to about 2 MPa.

Indicating and Recording Manometers.

Well Manometer. A conventional well manometer can be modified to a recording or indicating type, as Fig. 9 depicts. A metal float in the well transmits the vertical movement of mercury to external devices through a pressure-tight shaft to a pointer moving on a graduated scale or a recording pen. The equation for the static balance of this recording manometer is

$$P_1 - P_2 = \rho_m g l \left(1 + \frac{A}{a} \right) h_m \quad (18)$$

The change h'_m in the liquid column height in the well is generally restricted to around 0.01 m. The range of this instrument may be extended by altering the cross-sectional area a of the small span tube. A measuring range as small as 0 to 2.5 kPa or as large as 0 to 150 kPa can be achieved with operating pressures up to 15 MPa. In a noncontact magnetic clutch method of measuring height, a magnetic follower follows the movement of the float in the well in a narrow nonmagnetic tube below the float and produces rotation of a magnetic arm, pen, or pointer moving over a scale. This does not require pressure-tight shafts or bearings.

U-Tube Manometer. Figure 10 shows the simple U-tube mercury manometer modified to an indicating or recording type. It is mounted on a pivot or knife-edge and is free to tilt under the influence of applied differential pressure. The rotation of the meter is restricted by a calibrated spring. Flexible tubes are used for applying differential pressures to the manometer. Angular movement indicates the applied pressure difference. No pressure-tight constructions are necessary for this case.

Ring Balance Manometer. The ring balance manometer has a hollow partitioned ring or split ring made of metal or molded plastic as shown in Fig. 11. The ring is partly filled with the desired manometric liquid, forming two measuring chambers above the liquid level, to which the unknown pressures are admitted. The

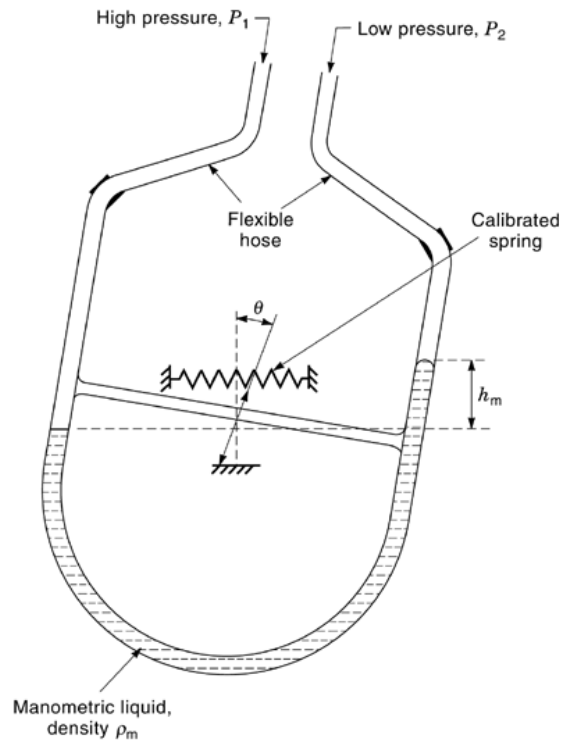


Fig. 10. Indicating or recording U-tube manometer.

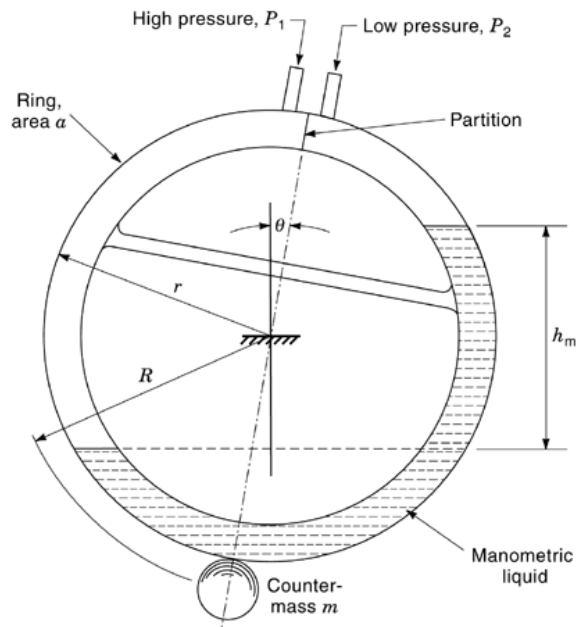


Fig. 11. Ring balance indicating or recording manometer.

liquid acts as a seal between the two pressure fluids. The ring assembly is supported at its center by means of a knife-edge resting on roller or ball bearings. A counter mass m , which restricts the rotation of the ring, is attached to the ring as shown. Unknown pressures are applied to chambers through flexible hoses, restricting their length and movement to a minimum. Pressure forces act at the two partitioned surfaces or closed ends of the ring, causing the ring to rotate. The moment of force due to differential pressure rotates the ring until the moment due to the counter mass restores the balance. For a ring balance manometer with ring radius r and with counter mass fixed at the radius R , when pressures P_1 and P_2 are applied to the two chambers, the ring rotates through an angle θ from the zero position. If a is the area of the cross section of the ring, for the static balance of moments about the center,

$$(P_1 - P_2)ar = mg_l R \sin \theta \quad (19)$$

$$P_1 - P_2 = \frac{R mg_l}{r a} \sin \theta \quad (20)$$

The angle θ is the measure of differential pressure. By altering the geometrical parameters of the ring and the value of the counter mass, different ranges can be covered. Using this manometer with oil and mercury as sealant, differential pressure ranges of about 40 Pa to 170 kPa are covered at operating pressures of up to 60 MPa. The uncertainty of a commercial ring balance manometer is about $\pm 1\%$ of range. The movement of the ring can be transmitted by means of linkages to either an indicating or a recording device. The choice of liquid and ring materials depends on their compatibility with the pressure fluids.

Bell Manometer. The movement of an inverted bell restrained by an opposing spring force in a liquid-sealed container under the action of applied pressure is related to the unknown pressures in these types of recording manometers. In the single-bell version, as in Fig. 12, the high pressure is applied inside the bell and the low pressure outside, above the liquid. The movement of the bell is transmitted to an indicating pointer or recording device through pressure-tight linkages. With unknown pressures P_1 and P_2 applied, under static equilibrium, neglecting the buoyancy forces, the pressure forces are balanced by restraining spring forces:

$$P_1 A_1 - P_2 A_2 = kh_m \quad (21)$$

For a small gap above the bell having negligible wall thickness, $A_1 = A_2 = A$, so

$$P_1 - P_2 = \frac{kh_m}{A} \quad (22)$$

The motion of the bell may be taken out mechanically or by electrical means. The instrument can also be directly calibrated to indicate pressure.

In yet another type, shown in Fig. 13, two identical bells are suspended from a beam balance moving into a sealing liquid. Under the action of pressure, the beam deflects from the equilibrium position. The rotation of the beam is restored to equilibrium by means of a counter mass m attached to the center of the beam at radius of r from the pivot. For static equilibrium of the beam, the moments of the forces must balance:

$$P_1 A l - P_2 A l = mg_l r \sin \theta \quad (23)$$

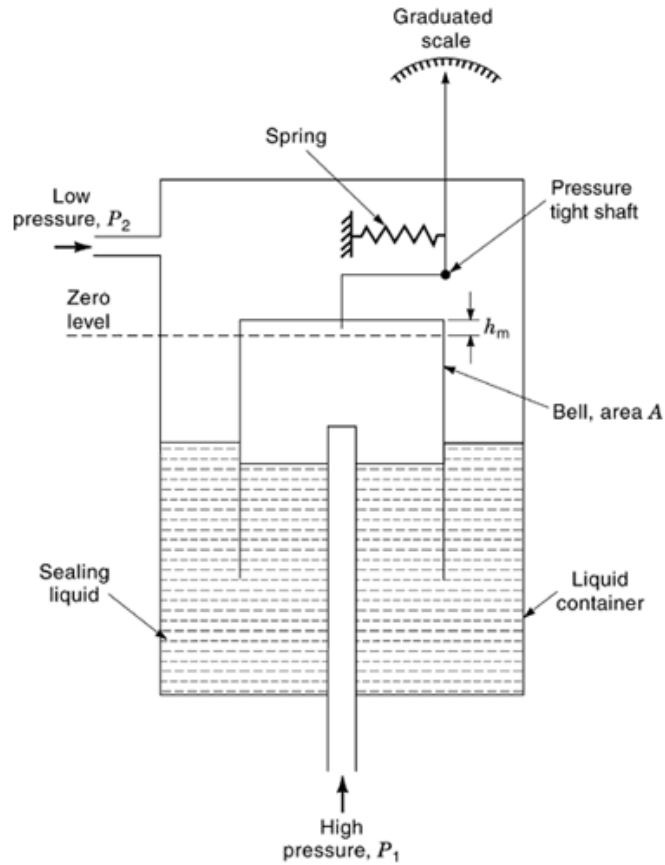


Fig. 12. Single-bell indicating or recording manometer.

where l is the pivotal distance of the bell to the point of suspension, marked in Fig. 13, and θ is the angle of deflection of beam. Thus

$$P_1 - P_2 = \frac{mgr}{Al} \sin \theta \quad (24)$$

$$= K \sin \theta \quad (25)$$

where k is a design constant for the meter. Overpressure and vacuum can be measured or recorded using these manometers. The sealing liquid is usually oil.

Bell manometers measure gas pressure differences in the range of 250 Pa to 4 kPa at pressures as high as 5 MPa.

Barometers

Manometers basically measure differential pressures between the two limbs. If one limb is evacuated to zero absolute pressure or absolute vacuum, then the height of liquid column will correspond to the local atmospheric

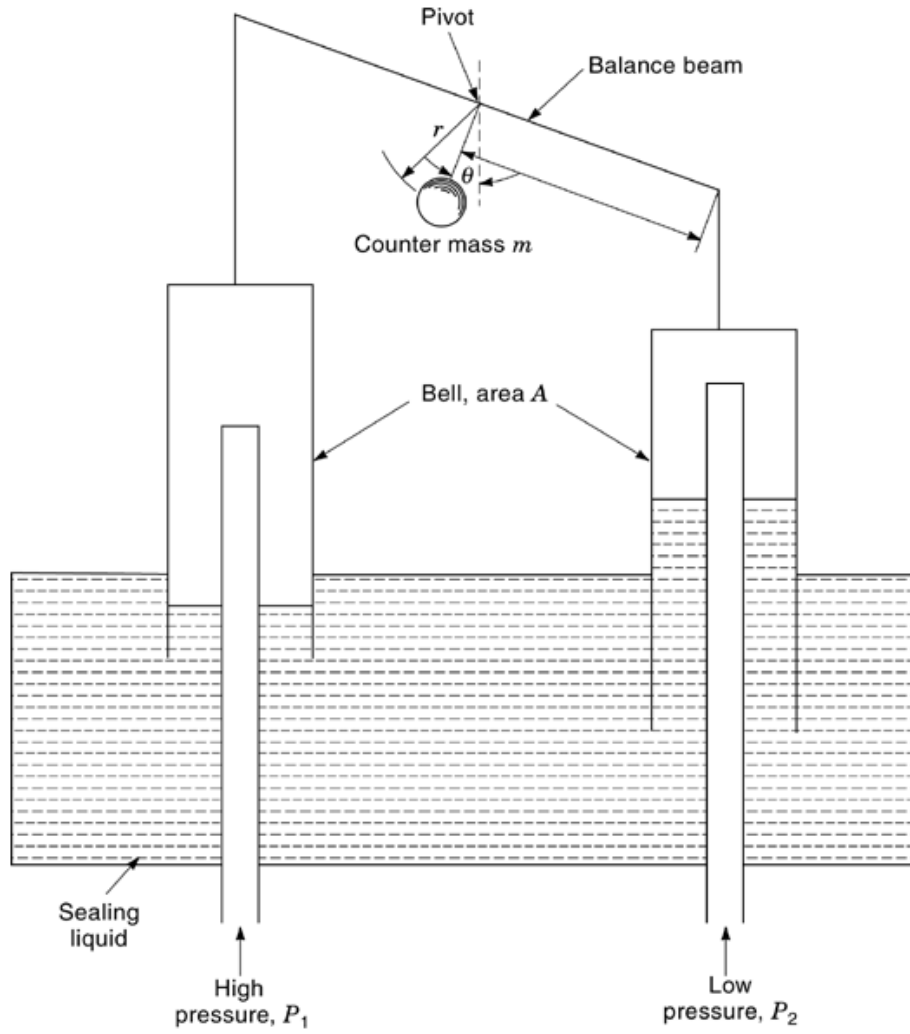


Fig. 13. Twin-bell indicating or recording manometer.

pressure. The barometer, invented by Evangelista Torricelli in 1643, is a modification of the well manometer. It has a glass tube with one end closed and with its open end immersed in a well partly filled with mercury, which is vented to the atmosphere. The tube, filled with pure mercury free of air and other foreign matter, is inverted in the well. Mercury will drop in the tube, and above the mercury column (except for vapor pressure) absolute zero reference pressure or perfect vacuum exists. The pressure exerted by the atmosphere on the mercury surface in the well is balanced by the pressure due to gravity on the mass of mercury column per unit area in the evacuated tube.

Referring to Fig. 14, the barometric pressure P_b is given by the expression

$$P_b = \rho_m g h_m \quad (26)$$

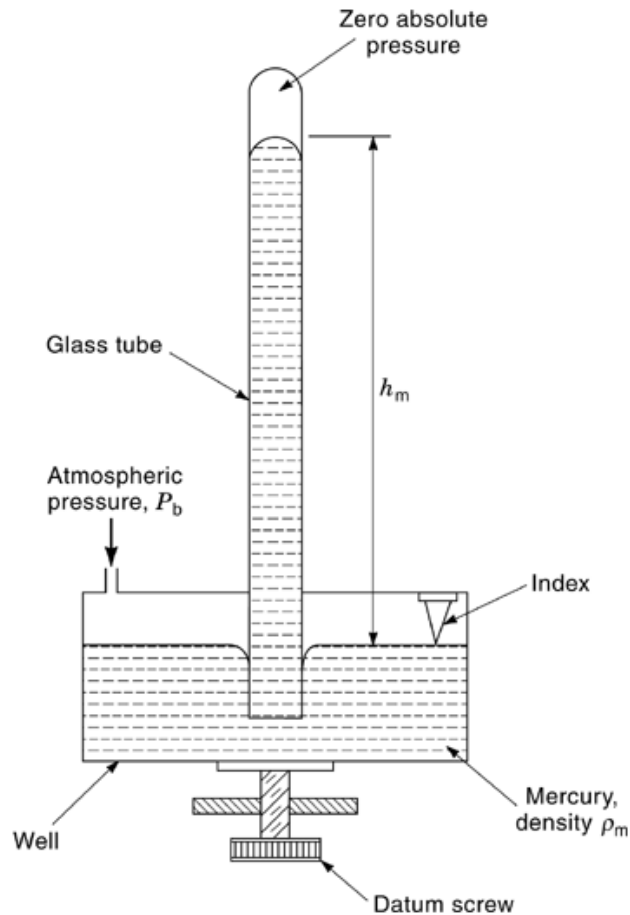


Fig. 14. Barometer.

The density of mercury, ρ_m , must correspond to the site temperature, and g_l must be the local value. The reference pressure above the mercury column will actually be the vapor pressure of mercury at the local temperature (about 0.7 Pa at 21° C). The correction due to nonzero reference is small compared to atmospheric pressure and is usually neglected. The accuracy of the barometer depends on the zero setting of the scale at the mercury level in the well. The datum adjustment screw and the index provide fixed zero reference for the scale during use.

When the scale calibration and the density of mercury are at a temperature other than local temperature, a correction must be applied to the indicated column height to obtain the correct height to be used, and corrections must also be made for variation of density with temperature and gravity. The temperature correction is negligible if the calibration temperature of the scale, the site temperature, and the reference temperature are equal. If the barometer is used at another elevation, a correction for altitude must be taken into account. The other factors that influence the uncertainty are alignment of the zero level, the method of reading the meniscus, verticality of the scale, capillary effects of mercury, and elevation. The calibration uncertainty of the barometer for measurement of steady pressure varies between $\pm 0.001\%$ and $\pm 0.03\%$ of the reading.

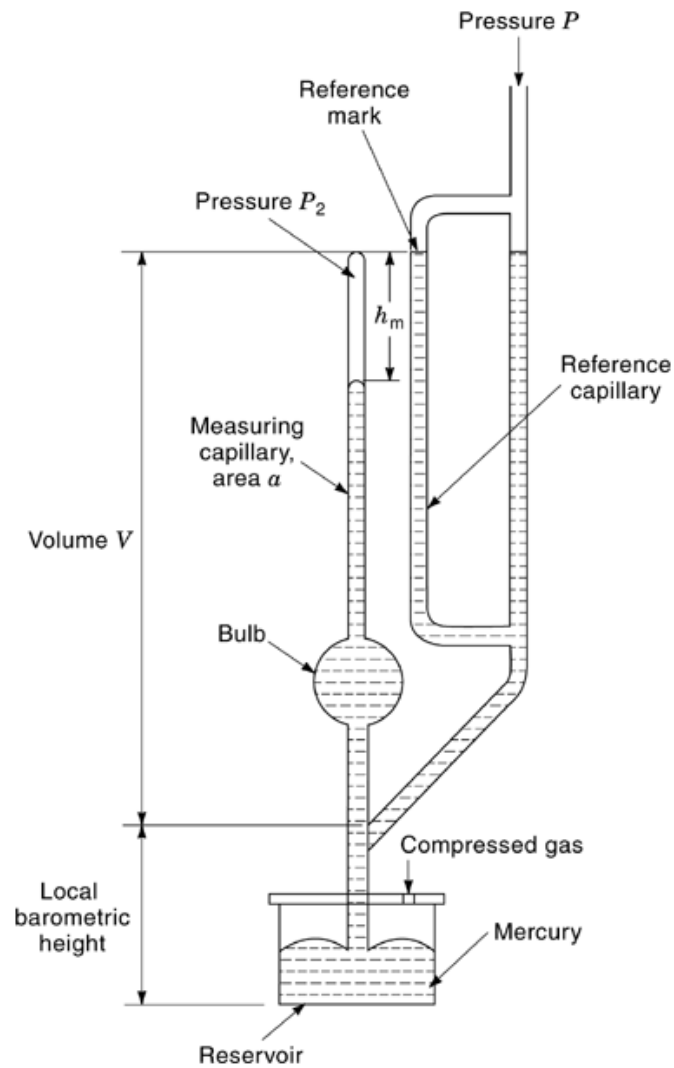


Fig. 15. McLeod gauge.

McLeod Gauge

Manometers can measure absolute pressures of about 13 Pa. However, McLeod gauges using mercury measure low absolute pressures of ideal gases in the range of 0.001 Pa to 130 Pa, with measurement uncertainty of $\pm 1\%$ or more, depending on the measured pressure. Using a pressure-dividing technique, the lower range is extended by allowing gas through a series of orifices (4). The low outlet pressure is related to the high inlet pressure measured by the gauge. The operation depends on Boyle's law, which states that the product of volume and pressure is constant at constant temperature for a given mass of gas. In principle, a known volume V of sample gas under unknown pressure P is compressed isothermally to a higher pressure, the resulting volume and pressure are measured using a simple mercury manometer, and unknown pressure is calculated. In Fig. 15, with mercury in the reservoir, the applied unknown pressure fills the gauge completely.

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Subsequently the mercury level in the reservoir is raised so that a known volume of sample gas is isolated and sealed. Further movement of mercury in the bulb and measuring capillary compresses the trapped gas at constant temperature. It is raised till it reaches the reference mark in the reference capillary tube. Under these conditions, the remaining volume is read from the measuring capillary. The difference in the mercury column height h_m in the measuring and the reference capillary is a measure of the unknown applied pressure.

Applying Boyle's law to an isolated sample of gas at initial and final conditions, we have

$$P_1 V_1 = P_2 V_2 \quad (27)$$

$$\begin{aligned} P_1 &= P, & V_1 &= V \\ P_2 &= P + \rho_m g l h_m \end{aligned} \quad (28)$$

$$V_2 = ah_m \quad (29)$$

where a is the cross-sectional area of both measuring and reference capillaries. Thus

$$P = \frac{ah_m^2}{V - ah_m} \rho_m g l \quad (30)$$

If the final compressed volume ah_m is much smaller than the initial volume V , Eq. (30) simplifies to

$$P = \frac{ah_m^2}{V} \rho_m g l \quad (31)$$

Thus the pressure is related to physical dimensions of the gauge, which can be determined with reasonable accuracy. The measuring capillary may be directly calibrated in terms of pressure. The amplification of the applied pressure depends on the ratio of the initial to the final volume of gas. By an increase in initial volume, significant improvement in accuracy may be achieved. The scale of the gauge is nonlinear, and its sensitivity depends on the applied pressure. It may be made linear by always trapping a constant volume of gas in the measuring capillary instead of referring to a zero mark. For very small constant volume V_c , Eq. (31) reduces to

$$P = \frac{V_c \rho_m g l}{V} h_m \quad (32)$$

where h_m is the difference in mercury level between the reference mark on the measuring capillary and the level of mercury in the reference capillary in a linear gauge. The reference mark in the capillary can be fixed by raising the mercury up to the closed end of the measuring capillary with absolute zero vacuum prevailing in the gauge. The movement of mercury in the gauge can be effected by any of following means:

- (1) Raising or lowering the mercury reservoir
- (2) Moving a mechanical plunger in the mercury reservoir
- (3) Admitting dry compressed gas into the reservoir
- (4) Swiveling the gauge

In swivel gauges (5), with the measuring capillary horizontal, the gauge is filled at unknown pressure to make up the isolated gas. A 90° rotation of the gauge forces the mercury to rise in the tube by the action of gravity. Identical measuring and reference capillaries avoid capillary effects.

As the measurement using this gauge is not continuous, steady conditions must exist in the vacuum system, and it is unsuitable for mixtures of gases containing condensable components. For gases deviating from ideal gas laws it requires special calibration, taking account of the compressibility factor. Moisture traps must be installed to avoid the entry of condensable vapor into the gauge (6). The cross-sectional area of the capillary tube and the initial volume can be determined from gravimetric methods by weighing distilled water that fills the volume.

The Ishii effect, due to evaporation of mercury from the reservoir, can cause serious underestimation of the pressure, depending on the gases measured, as the gas molecules encountered by mercury vapor are removed from the tubes. This error may be reduced by artificially cooling the mercury at the reservoir.

Since the measured pressure is related to dimensions of the McLeod gauge, it serves as a primary vacuum standard for calibration of vacuum gauges.

Factors Influencing Liquid Manometers

For measurements with less uncertainty, correct values of variables at the measurement conditions and locations must be used for calculating the differential pressures. The following major factors influence the performance of manometers, and the readings must be corrected for them in accurate work.

Temperature. The density of the manometric liquid depends on the local temperature, and the column height on length variations of scale with temperature. The density ρ_m is at the mean temperature of the liquid, T_m , and is described by the general equation.

$$\rho_m = \frac{\rho_0}{1 + \beta(T_m - T_0)} \quad (33)$$

where ρ_0 is the density of the liquid at temperature T_0 and β is the coefficient of volumetric expansion for temperatures, which may be obtained from standard handbooks on physical properties of substances. Also, when the height h_m of the manometric liquid is measured, the scale's expansion due to temperature must be considered.

Acceleration Due to Gravity. The local value of the acceleration due to gravity, g_l , at the location of the manometer must either be experimentally determined (for less uncertainty) or be evaluated from the knowledge of geographical location using empirical expressions. But the calculation gives only an approximate estimate, ignoring the variations of composition of the earth with location. The measured column height may be corrected to any base conditions of gravity by multiplying by the ratio of actual specific gravity to standard specific gravity.

Surface Tension. The surface tension of the liquids and their wetting of the wall of the manometer affect the column height and its measurement. The shape of the interface between the pressure fluids and manometric liquid depends on their densities and on the cohesive and adhesive forces between the fluids and the wall of the tubes. The meniscus is concave upward for water–air–glass manometers dominated by adhesive forces, and water rises in a tube by capillary action. In mercury–air–glass manometers the meniscus is concave downward because of cohesive forces, and the mercury level is depressed. Significant errors can be caused by the displacement of liquid by capillary action due to surface tension in manometers having tubes of unequal cross sections.

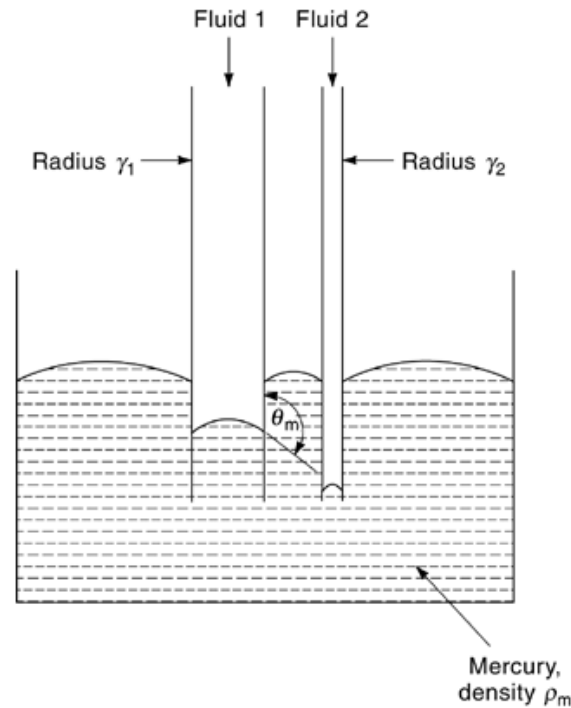


Fig. 16. Capillary effect in mercury manometer.

The correction in the column height h_m due to capillary effects for a manometer is given by the expression

$$\pm \frac{2 \cos \theta_m}{\rho_m g l} \left(\frac{\sigma_1}{r_1} - \frac{\sigma_2}{r_2} \right) \quad (34)$$

where θ_m is the angle of contact between the manometric liquid and the glass, σ_1 and σ_2 are the surface tensions of the manometric liquid with respect to pressure fluids 1 and 2 above it, and r_1 and r_2 are the radii of the tubes (Fig. 16). This correction is positive for a mercury manometer and negative for a water manometer, and the calculated correction must be added to the measured liquid column height. For the same fluid in identical limbs of a U tube, the capillary effect is negligible. Tapping the tubes slightly before reading the column height and reading it at the center of the meniscus will reduce errors. Use of larger-diameter (>0.01 m) tubes will also be effective in reduction of capillary effects.

Evaporation. Evaporation loss of manometric liquid will shift the initial zero, and this must be corrected for by checking the zero intermittently or adding liquid to the manometer to nullify the zero-shift error.

Manometric Liquids

Any liquid that is immiscible with the pressure fluids and possesses well-defined physical characteristics may be used as a manometric liquid. Selection of the liquid for a specific application depends on the magnitude of the measured differential pressure, required resolution, and physical properties of the liquid such as density,

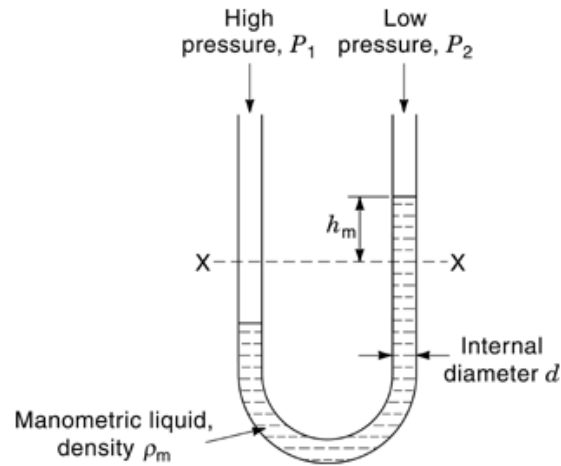


Fig. 17. Dynamics of a manometer.

surface tension, volatility, wetting property, and corrosiveness. Distilled water and mercury are the most commonly preferred liquids for manometers, as the variation of their properties with temperature is well established. Coloring agents are often used, as well as additives to reduce surface tension. Other typical liquids used are ethyl alcohol, kerosene, carbon tetrachloride, acetylene tetrabromide, and silicone oils. The densities of water and mercury are about 1000 kg/m^3 and $13,560 \text{ kg/m}^3$ at typical ambient temperatures. Alcohol, with density of about 800 kg/m^3 , has less surface tension, but its density is highly dependent on temperature, and it absorbs water, so it is useful mainly for constant-temperature dry-gas measurements. Silicone liquid, with density of about 820 kg/m^3 and possessing surface tension about $\frac{1}{4}$ that of distilled water, is suitable for inclined manometers with 2 mm to 3 mm diameter tubes (7).

Electronic Manometers. Electronic manometers with thin flat diaphragm elements have higher sensitivity. Their operation depends on variation of resistance, capacitance, inductance, or piezoelectric effects caused by the movement of the diaphragms due to applied pressure. The electrical signal output is further processed for indication of actual pressures. The advent of electronic techniques to measure the deflection of the diaphragms has improved the sensitivity, resolution, and means of compensation of nonlinearity effects.

Another type of electronic manometer has an optical sensor on the exterior of the measurement tube of a well-type manometer, which senses the meniscus. This sensor is made to slide outside the manometer, using a servomotor. When the liquid level changes due to pressure, the built-in servo feedback mechanism moves the optical meniscus sensor so that it is in line with the meniscus. The number of rotations of the servomotor gives a measure of the pressure. This type of manometer avoids errors due to human reading.

Manometric Measurements on Oscillating Fluids

Though liquid manometers can be used straightforwardly for steady differential pressure measurements, their dynamic characteristics assume significance in turbulent flows, characterized by instantaneous fluctuation of pressure. Referring to Fig. 17, for an isolated mass of liquid in a U tube, oscillating about the equilibrium position XX under the influence of applied pressure, the predominant forces governing the motion of the liquid mass are gravitational force due to mass of the liquid, drag force due to shearing stress along the tube wall, and inertial force due to liquid motion (8).

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The mass of displaced liquid, under the action of differential pressure $P_1 - P_2$, is proportional to the column height h_m . At any instant of time, the gravitational force tends to restore it to equilibrium at $h_m = 0$. Hence the liquid mass becomes analogous to a spring with displacement h_m . The resultant of the above forces causes the inertial motion of the liquid mass. According to Newton's second law of motion for the oscillating liquid column, it can be expressed that pressure force – gravitational force – drag force = Inertial force

$$\frac{\pi d^2}{4}(P_1 - P_2) - 2\frac{\pi d^2}{4}\rho_m g_l h_m - \pi d l \tau = k_0 \frac{\pi d^2}{4} l \rho_m \frac{d^2 h_m}{dt^2} \quad (35)$$

where d is the diameter of the manometer tube, ρ_m is the density of the manometric liquid, l is the total length of liquid column in the manometer, τ is the shear stress, g_l is the acceleration due to gravity, and k_0 is the mass correction factor due to velocity distribution in the tube. The value of k_0 is 1.33 for laminar flows. The second derivative of the displacement h_m yields the average acceleration of the liquid mass. The shear stress and the mass correction factor depend on the type of flow in the tube. Laminar flow occurs for tube Reynolds number (ratio of inertial forces to viscous forces) less than 2000, and turbulent for greater than 4000.

For steady laminar flow, the shear stress along the wall for liquid of dynamic viscosity μ_m is expressed as

$$\tau = \frac{8\mu_m}{d} \frac{dh_m}{dt} \quad (36)$$

Substituting Eq. (36) into Eq. (35) and rearranging gives the linear second-order dynamic equation for the manometer:

$$0.67 \frac{l}{g_l} \frac{dh_m^2}{dt^2} + \frac{16\mu_m l}{d^2 \rho_m g_l} \frac{dh_m}{dt} + h_m = \frac{P_1 - P_2}{2\rho_m g_l} \quad (37)$$

Expressing this in standard operator form, where d/dt is indicated as D , we have

$$\left(0.67 \frac{D^2}{g_l/l} + \frac{16\mu_m l}{d^2 \rho_m g_l} D + 1\right) h_m = \frac{1}{2\rho_m g_l} (P_1 - P_2) \quad (38)$$

The transfer function relating output h_m to the input $P_1 - P_2$ of Eq. (38) is

$$\frac{h_m}{P_1 - P_2}(D) = \frac{1}{\frac{2\rho_m g_l}{g_l/0.67l} + \frac{16\mu_m l}{d^2 \rho_m g_l} D + 1} \quad (39)$$

This is of the form

$$\frac{h_m}{P_1 - P_2}(D) = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1}$$

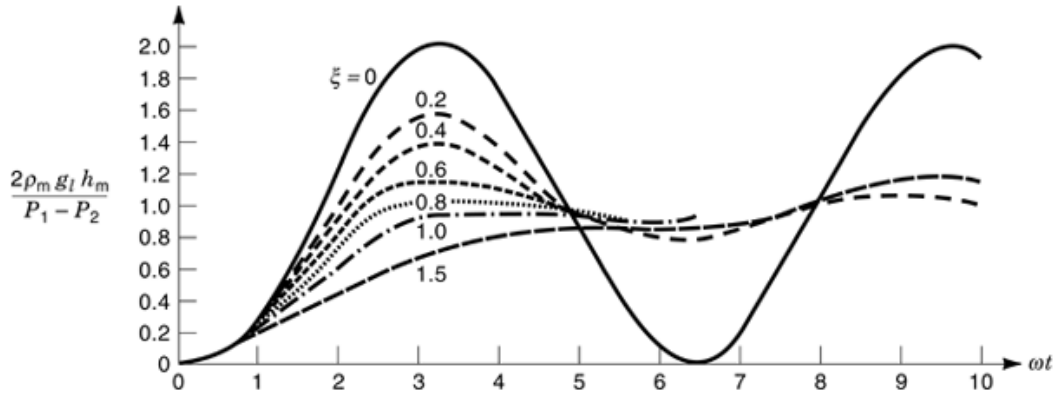


Fig. 18. Typical response of liquid manometer for step input pressure.

Thus, the sensitivity

$$K = \frac{1}{2\rho_m g l}$$

the natural frequency

$$\omega = 1.22 \sqrt{\frac{g l}{l}}$$

and the damping ratio

$$\xi = 9.8 \frac{\mu_m}{d^2 \rho_m} \sqrt{\frac{l}{g l}}$$

On application of differential pressure, the manometer reads a steady value only after a period of time.

In practice the damping ratio ξ for a liquid manometer, with either mercury or water, will be less than 1; that is, the manometer is an underdamped second-order system. For this case, the solution of Eq. (39) for a step pressure differential input is given by

$$\frac{2\rho_m g l h_m}{P_1 - P_2} = 1 - \frac{e^{-\xi \omega t}}{\sqrt{1 - \xi^2}} \sin \left(\sqrt{1 - \xi^2} \omega t + \sin^{-1} \sqrt{1 - \xi^2} \right) \quad (40)$$

It gives the dynamic response of a liquid manometer for laminar flow. Figure 18 shows the response characteristics of the manometer. The natural frequency indicates the speed of response for step input. Increasing ξ reduces oscillations and slows down the response.

For steady turbulent flows in smooth pipes, the shear stress is given by

$$\tau = 0.03955 \frac{\mu_m^{0.25} \rho_m^{0.75} dh_m^{1.75}}{d^{0.25} dt} \quad (41)$$

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In contrast with Eq. (36), the shearing stress is nonlinear, being proportional to the 1.75 power of the velocity. This results in a nonlinear second-order differential equation of motion, making analytical solution difficult. The differences in response of linear and nonlinear systems for sinusoidal excitation are usually small.

The damping ratio ξ depends on the frequency and amplitude of the oscillations. When the velocity distribution is nearly uniform, the effective mass may be taken as the actual mass, but ω is different than the laminar case. The value of k_0 is 1. Though the damping will be higher than in the laminar case, the magnitude will be small. Hence the damping due to nonlinear dependence may be ignored.

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