ACCELEROMETERS

Acceleration is an important parameter for general-purpose absolute motion measurements and vibration and shock sensing. Accelerometers are commercially available in a wide variety of ranges and types to meet diverse application requirements. They are manufactured to be small in size, light in weight, and rugged and robust to operate in harsh environments. They can be configured as active or passive sensors. An active accelerometer (e.g., piezoelectric) gives an output without the need for an external power supply, while a passive accelerometer only changes its electric properties (e.g., capacitance) and requires an external electrical power. In applications, the choice of active- or passive-type accelerometers is important, since active sensors cannot measure static or dc mode operations. For true static measurements, passive sensors must be selected.

Accelerometers can be classified in a number of ways, such as deflection or null-balance types, mechanical or electrical types, and dynamic or kinematic types. The majority of industrial accelerometers can be classified as either deflection type or null-balance type. Those used in vibration and shock measurements are usually the deflection types, whereas those used for measurements of motions of vehicles, aircraft, and so on for navigation purposes may be either type. In general, null-balance types are used when extreme accuracy is needed.

A large number of practical accelerometers are the deflection types; the general configuration is shown in Fig. 1. There are many different deflection-type accelerometers. Although their principles of operation are similar, they differ in minor details, such as the spring elements used, types of damping provided, and types of relative motion transducers employed. These types of accelerometers behave as second-order systems; the detailed mathematical analysis will be given in the following sections.

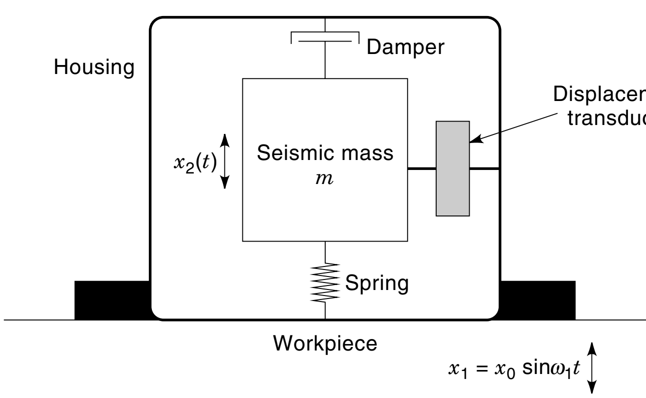

Figure 1. A typical deflection-type seismic accelerometer. In this basic accelerometer, the seismic mass is suspended by a spring or cantilever inside a rigid frame. The frame is connected to the vibrating structure; as vibrations take place the mass tends to remain fixed so that relative displacements can be picked up. They are manufactured in many different types and sizes with diverse characteristics.

Dynamic accelerometers have an operation that is based on measuring the force required to constrain a seismic mass to track the motion of the accelerated base, such as a spring-constrained-slug accelerometer. Although applicable to all, the mathematical treatment of the dynamic response of an accelerometer as a second-order system is given in detail in the section dedicated to seismic accelerometers. Another type is the kinematic accelerometer, which is based on timing the passage of an unconstrained proof mass from spaced points marked on the accelerated base, and is used for highly specific applications such as interspace spacecraft and gravimetry-type measurements.

For practical purposes, accelerometers can also be classified as mechanical or electrical types, depending on whether the restoring force or other measuring mechanism is based on mechanical properties (for example, the law of motion, distortion of a spring, or fluid dynamics) or on electrical or magnetic forces.

TYPES OF ACCELEROMETERS

Seismic Accelerometers

These accelerometers make use of a seismic mass that is suspended by a spring or a lever inside a rigid frame. The schematic diagram of a typical seismic accelerometer is shown in Fig. 1. The frame carrying the seismic mass is connected firmly to the vibrating source whose characteristics are to be measured. As the system vibrates, the mass tends to remain fixed in its position so that the motion can be registered as a relative displacement between the mass and the frame. This displacement is sensed by an appropriate transducer and the output signal is processed further. Nevertheless, the seismic mass does not remain absolutely steady, but for selected frequencies it can satisfactorily act as a reference position.

By proper selection of mass, spring, and damper combinations, the seismic instruments may be used for either acceleration or displacement measurements. In general, a large mass and soft spring are suitable for vibration and displacement measurements, while relatively small mass and a stiff spring are used in accelerometers. However, the term *seismic* is commonly applied to instruments that sense very low levels of vibration in the ground or in structures. They tend to have low natural frequencies.

The following equation may be written by using Newton's second law of motion to describe the response of seismic arrangements similar to shown in Fig. 1:

$$m\,d^2x_2/dt^2 + c\,dx_2/dt + kx_2 = c\,dx_1/dt + kx_1 + mg\cos(\theta) \quad (1)$$

where x_1 is the displacement of the vibration frame, x_2 is the displacement of the seismic mass, c is the velocity constant, θ is the angle between the sense axis and gravity, and k is the spring constant.

Taking $m\,d^2x_1/dt^2$ from both sides of the equation and rearranging gives

$$m\,d^2z/dt^2 + c\,dz/dt + kz = mg\cos(\theta) - m\,d^2x_1/dt^2 \qquad (2)$$

where $z = x_2 - x_1$ is the relative motion between the mass and the base.

In Eq. (1), it is assumed that the damping force on the seismic mass is proportional to velocity only. If a harmonic vibratory motion is impressed on the instrument such that

$$x_1 = x_0 \sin \omega_1 t \qquad (3)$$

where ω_1 is the frequency of vibration (rad/s). Writing

$$m\,d^2x_1/dt^2 = m\,x_0\omega_1^2 \sin \omega_1 t$$

modifies Eq. (2) as

$$-m\,d^2z/dt^2 + c\,dz/dt + kz = mg\cos(\theta) + ma_1 \sin \omega_1 t \qquad (4)$$

where $a_1 = m\ x_0\ \omega_1^2$.

Equation (4) will have transient and steady-state solutions. The steady-state solution of the differential equation (4) may be determined as

$$z = [mg\cos(\theta)/k] + [ma_1 \sin \omega_1 t/(k - m\omega_1^2 + jc\omega_1)] \qquad (5)$$

Rearranging Eq. (5) results in

$$z = [mg\cos(\theta)/\omega_n] + \{a_1 \sin(\omega_1 - \phi)/[\omega_n^2(1-r^2)^2 + (2\zeta r)^2]^{1/2}\} \qquad (6)$$

where $\omega_n = \sqrt{k/m}$ is the natural frequency of the seismic mass, $\zeta = (c/2)\sqrt{km}$ is the damping ratio, which also can be written in terms of critical damping ratio as $\zeta = c/c_c$, where $c_c = 2\sqrt{km}$, $\phi = \tan^{-1}[c\omega_1/(k - m\omega_1^2)]$ is the phase angle, and $r = \omega_1/\omega_n$ is the frequency ratio.

A plot of Eq. (6), $(x_1 - x_2)_0/x_0$ against frequency ratio ω_1/ω_n is illustrated in Fig. 2. This figure shows that the output amplitude is equal to the input amplitude when $c/c_c = 0.7$ and $\omega_1/\omega_n > 2$. The output becomes essentially a linear function of the input at high frequency ratios. For satisfactory system performance, the instrument constant c/c_c and ω_n should carefully be calculated or obtained from calibrations. In this way the anticipated accuracy of measurement may be predicted for frequencies of interest. A comprehensive treatment of the analysis is by McConnell (1); interested readers should refer to this text for further details.

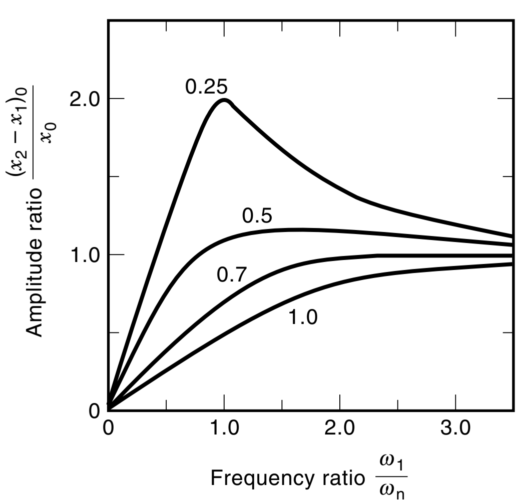

Figure 2. A typical displacement of a seismic instrument. The amplitude becomes large at low damping ratios. The instrument constants should be selected such that in measurements the frequency of vibration is to be much higher than the natural frequency, for example, greater than 2. Optimum results may be obtained when the value of the instrument constant c/c_c is about 0.7.

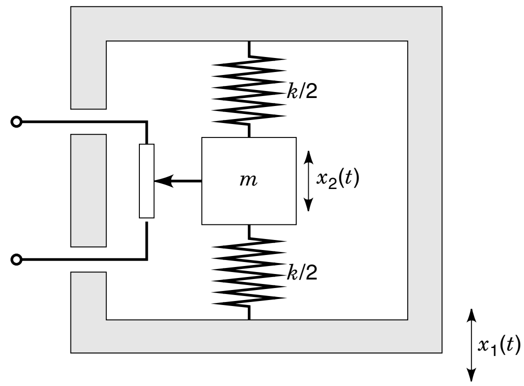

Figure 3. A potentiometer accelerometer. The relative displacement of the seismic mass is sensed by a potentiometer arrangement. The potentiometer adds extra weight, making these accelerometers relatively heavier. Suitable liquids filling the frame may be used as damping elements. These accelerometers are used in low-frequency applications.

If the seismic instrument has a low natural frequency and a displacement sensor is used to measure the relative motion z, then the output is proportional to the displacement of the transducer case. If the velocity sensor is used to measure the relative motion, the signal is proportional to the velocity of the transducer. This is valid for frequencies significantly above the natural frequency of the transducer. Velocity coil output produces a device commonly known as a geophone. It is not an accelerometer in the strict sense but it is similarly used. It excels at measuring low to medium frequency vibrations, as it offers exceptionally low self-generated noise output and very low output impedance. However, if the instrument has a high natural frequency and the displacement sensor is used, the measured output is proportional to the acceleration

$$kz = m\,d^2x_1/dt^2 \qquad (7)$$

This equation is true since displacement x_2 becomes negligible in comparison with x_1.

In these instruments the input acceleration a_0 can be calculated simply by measuring $(x_1 - x_2)_0$, the static deflection relative to the case. Generally, in acceleration measurements, unsatisfactory performance is observed at frequency ratios above 0.4. Thus, in such applications, the frequency of acceleration must be kept well below the natural frequency of the instrument. This can be done by constructing the instrument to have a low natural frequency by selecting soft springs and large masses.

Seismic instruments are constructed in a variety of ways. Figure 3 illustrates the use of a voltage divider potentiometer for sensing of the relative displacement between the frame and the seismic mass. In the majority of potentiometric instruments, the device is filled with a viscous liquid that interacts continuously with the frame and the seismic mass to provide damping. These accelerometers have a low frequency of operation (less than 100 Hz) and are mainly intended for slowly varying acceleration and low-frequency vibrations. A typical family of such instruments offers many different models, covering the range of ±1 g to ±50 g full scale. The natural frequency ranges from 12 Hz to 89 Hz, and the damping ratio ζ can be kept between 0.5 to 0.8 by using a temperature-compensated liquid-damping arrangement. Potentiometer resistance may be selected in the range of 1,000 Ω to 10,000 Ω, with a corresponding resolution of 0.45% to 0.25% of full scale. The cross-axis sensitivity is less than ±1%. The overall accuracy is ±1% of full scale or less at room temperatures. The size is about 50 mm^3 with a mass of about $\frac{1}{2}$ kg.

Linear variable differential transformers (LVDTs) offer another convenient means of measurement of the relative displacement between the seismic mass and the accelerometer housing. These devices have higher natural frequencies than potentiometer devices, up to 300 Hz. Since the LVDT has lower resistance to motion, it offers much better resolution. A typical family of liquid-damped differential-transformer accelerometers exhibits the following characteristics. The full scale ranges from ±2 g to ±700 g, the natural frequency from 35 Hz to 620 Hz, the nonlinearity 1% of full scale, the full-scale output is about 1 V with an LVDT excitation of 10 V at 2,000 Hz, the damping ratio ranges from 0.6 to 0.7, the residual voltage at the null position is less than 1%, and the hysteresis is less than 1% full scale. The size is 50 mm^3, with a mass of about 120 g.

Electrical resistance strain gauges are also used for displacement sensing of the seismic mass as shown in Fig. 4. In this case, the seismic mass is mounted on a cantilever beam rather than on springs. Resistance strain gauges are bonded on each side of the beam to sense the strain in the beam resulting from the vibrational displacement of the mass. Damping for the system is provided by a viscous liquid that entirely fills the housing. The output of the strain gauges is connected

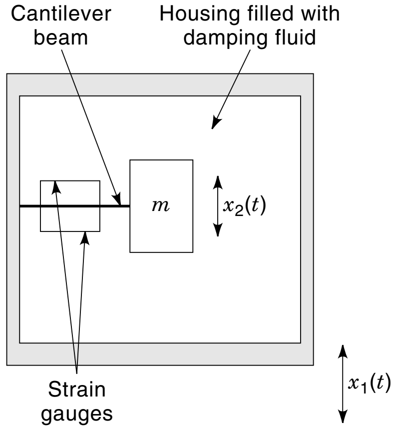

Figure 4. A strain-gauge seismic instrument. The displacement of the proof mass is sensed by piezoresistive strain gauges. The natural frequency of the system is low due to need of a long level beam to accommodate strain gauges. The signal is processed by bridge circuits.

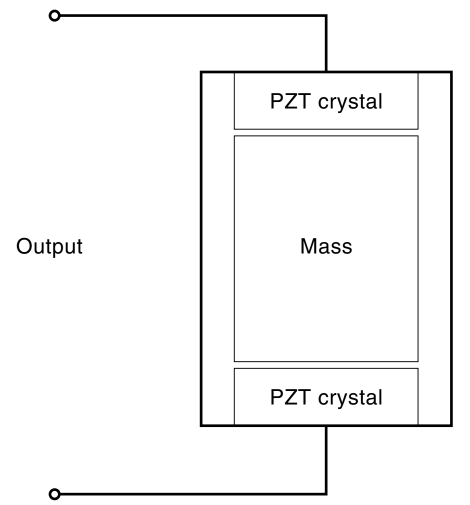

Figure 5. A compression-type piezoelectric accelerometer. The crystals are under compression at all times either by a mass or mass and spring arrangement. Acceleration causes a deformation of the crystal, thus producing a proportional electrical signal. They are small in size and widely used. They demonstrate poor performance at low frequencies.

to an appropriate bridge circuit. The natural frequency of such a system is about 300 Hz. The low natural frequency is due to the need for a sufficiently large cantilever beam to accommodate the mounting of the strain gauges. Other types of seismic instruments with piezoelectric transducers using seismic masses are discussed in detail in the section dedicated to piezoelectric-type accelerometers.

Seismic vibration instruments are affected seriously by temperature changes. Devices employing variable resistance displacement sensors will require correction factors to account for resistance change due to temperature. The damping of the instrument may also be also be affected by changes in the viscosity of the fluid due to temperature. For instance, the viscosity of silicone oil, often used in these instruments, is strongly dependent on temperature. One way of eliminating the temperature effect is to use an electrical resistance heater in the fluid to maintain the temperature at a constant value regardless of surrounding temperatures.

Piezoelectric Accelerometers

Piezoelectric accelerometers are used widely for general-purpose acceleration, shock, and vibration measurements. They are basically motion transducers with large output signals and comparatively small size. They are available with very high natural frequencies and are therefore suitable for high-frequency applications and shock measurements.

These devices utilize a mass in direct contact with the piezoelectric component or crystal as shown in Fig. 5. When a varying motion is applied to the accelerometer, the crystal experiences a varying force excitation ($F = ma$), causing a proportional electric charge q to be developed across it.

$$q = d_{ij}F = d_{ij}ma \tag{8}$$

where q is the charge developed and d_{ij} is the material's piezoelectric coefficient.

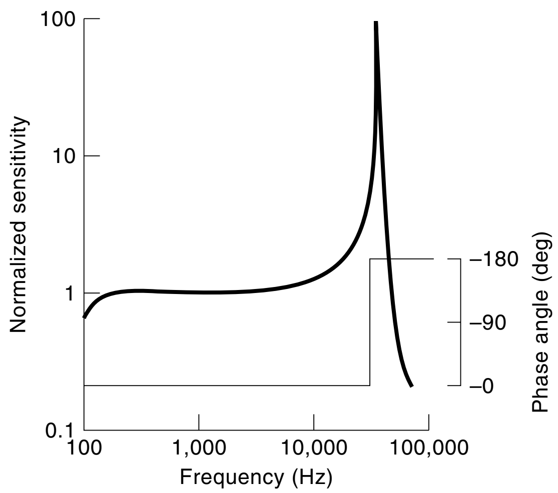

Figure 6. The frequency response of a typical piezoelectric accelerometer. Measurements are normally confined to the linear portion of the response curve. The upper frequency of the accelerometer is limited by the resonance of the PZT crystal. The phase angle is constant up to the resonance frequency.

As this equation shows, the output from the piezoelectric material is dependent on its mechanical properties, d_{ij}. Two commonly used piezoelectric crystals are lead-zirconate titanate ceramic (PZT) and crystalline quartz. They are both self-generating materials and produce a large electric charge for their size. The piezoelectric strain constant of PZT is about 150 times that of quartz. As a result, PZTs are much more sensitive and smaller in size than quartz counterparts. In the accelerometers, the mechanical spring constants for the piezoelectric components are high, and the inertial masses attached to them are small. Therefore, these accelerometers are useful for high-frequency applications. Figure 6 illustrates a typical frequency response of a PZT device. Typically, the roll-off starts near 100 Hz. These active devices have no dc response. Since piezoelectric accelerometers have comparatively low mechanical impedances, their effects on the motion of most structures is negligible. They are also manufactured to be rugged and have outputs that are stable with respect to time and environment.

Mathematically, their transfer function approximates to a third-order system as

$$e_o(s)/a(s) = (K_q/C\omega_n^2)\tau s/[(\tau s + 1)(s^2/\omega_n^2 + 2\zeta s/\omega_n + 1)] \quad (9)$$

where K_q is the piezoelectric constant related to charge (C · cm), τ is the time constant of the crystal, and s is the Laplace variable. It is worth noting that the crystal itself does not have a time constant τ, but the time constant is observed when the accelerometer is connected into an electric circuit, for example, and *RC* circuit.

The low-frequency response is limited by the piezoelectric characteristic $\tau s/(\tau s + 1)$, while the high-frequency response is related to mechanical response. The damping factor ζ is very small, usually less than 0.01 or near zero. Accurate low-frequency response requires large τ, which is usually achieved by use of high-impedance voltage amplifiers. At very low frequencies thermal effects can have severe influences on the operation characteristics.

In piezoelectric accelerometers, two basic design configurations are used: compression types and shear-stress types. In compression-type accelerometers, the crystal is held in compression by a preload element; therefore the vibration varies the stress in compressed mode. In a shear-stress accelerometer, vibration simply deforms the crystal in shear mode. The compression accelerometer has a relatively good mass-to-sensitivity ratio and hence exhibits better performance. But, since the housing acts as an integral part of the spring-mass system, it may produce spurious interfaces in the accelerometer output if excited around its proper natural frequency.

Microelectronic circuits have allowed the design of piezoelectric accelerometers with charge amplifiers and other signal-conditioning circuits built into the instrument housing. This arrangement allows greater sensitivity and high-frequency response and smaller size accelerometers, thus lowering the initial and implementation costs.

Piezoelectric accelerometers are available in a very wide range of specifications and are offered by a large number of manufacturers. For example, the specifications of a shock accelerometer may have 0.004 pC/g in sensitivity and a natural frequency of up to 250,000 Hz, while a unit designed for low-level seismic measurements might have 1,000 pC/g in sensitivity and only 7,000 Hz natural frequency. They are manufactured as small as 3 × 3 mm^2 in dimension with about 0.5 g in mass, including cables. They have excellent temperature ranges and some of them are designed to survive the intensive radiation environment of nuclear reactors. However, piezoelectric accelerometers tend to have larger cross-axis sensitivity than other types, about 2% to 4%. In some cases, large cross-axis sensitivity may be minimized during installations by the correct orientation of the device. These accelerometers may be mounted with threaded studs, with cement or wax adhesives, or with magnetic holders.

Electromechanical Force-Balance (Servo) Accelerometers

Electromechanical accelerometers, essentially servo or null-balance types, rely on the principle of feedback. In these instruments, an acceleration-sensitive mass is kept very close to a neutral position or zero displacement point by sensing the displacement and feeding back the effect of this displacement. A proportional magnetic force is generated to oppose the motion of the mass displaced from the neutral position, thus restoring this position just as a mechanical spring in a conventional accelerometer would do. The advantages of this approach are better linearity and elimination of hysteresis effects as compared with the mechanical springs. Also, in some cases, electrical damping can be provided, which is much less sensitive to temperature variations.

In high-vibration environments, force balance accelerometers benefit from two unique capabilities: velocity storage allows them to operate at saturation a small percentage of the time without actually losing information, and dynamic range change permits the useful range to be greater at high frequencies than near dc.

One very important feature of null-balance type instruments is the capability of testing the static and dynamic performances of the devices by introducing electrically excited test forces into the system. This remote self-checking feature can be quite convenient in complex and expensive tests in which it is extremely critical that the system operates cor-

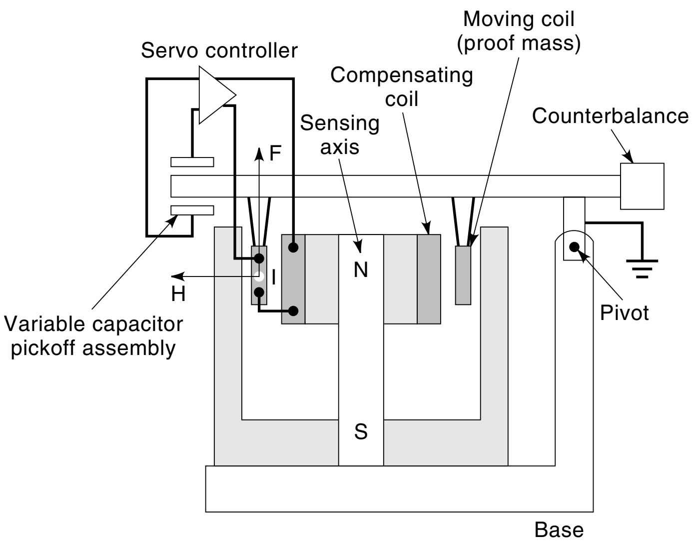

Figure 7. A basic coil and permanent magnet accelerometer. The coil is supported by an arm with minimum friction bearings to form a proof mass in a magnetic field. Displacement of the coil due to acceleration induces an electric potential in the coil to be sensed and processed. A servo system maintains the coil in a null position.

rectly before the test commences. These instruments are also useful in acceleration control systems, since the reference value of acceleration can be introduced by means of a proportional current from an external source. They are usually used for general-purpose motion measurements and monitoring low-frequency vibrations. They are specifically applied in measurements requiring better accuracy than that achieved by those accelerometers based on mechanical springs such as the force-to-displacement transducer.

There are a number of different electromechanical accelerometers: coil-and-magnetic types, induction types, etc.

Coil-and-Magnetic Accelerometers. These accelerometers are based on Ampere's law, that is, "a current-carrying conductor disposed within a magnetic field experiences a force proportional to the current, the length of the conductor within the field, the magnetic field density, and the sine of the angle between the conductor and the field."

Figure 7 illustrates one form of accelerometer making use of this principle. The coil is located within the cylindrical gap defined by a permanent magnet and a cylindrical soft iron flux return path. It is mounted by means of an arm situated on a minimum friction bearing or plexure so as to constitute an acceleration-sensitive seismic mass. A pickoff mechanism senses the displacement of the coil under acceleration and causes the coil to be supplied with a direct current via a suitable servo controller to restore or maintain a null condition.

Assuming a downward acceleration with the field being radial (90°), by using Ampere's law the force experienced by the coil may be written as

$$F = ma = ilB \tag{10}$$

or the current

$$i = ma/lB \tag{11}$$

where B is the effective flux density and l is the total effective length of the conductor in the magnetic field.

Current in the restoring circuit is linearly proportional to acceleration, provided (1) armature reaction effects are negligible and fully neutralized by a compensating coil in opposition to the moving coil, and (2) the gain of the servo system is large enough to prevent displacement of the coil from the region in which the magnetic field is constant.

In these accelerometers, the magnetic structure must be shielded adequately to make the system insensitive to external disturbances or the earth's magnetic field. Also, in the presence of acceleration there will be a temperature rise due to i^2R losses. The effect of these i^2R losses on the performance are determined by the thermal design and heat-transfer properties of the accelerometers. In many applications, special care must be exercised in choosing the appropriate accelerometer such that the temperature rises caused by unexpected accelerations cannot affect the scale factors or the bias conditions excessively. In others, digital signal conditioning can be used to produce a constant temperature rise after an initial transient at the turn on.

A simplified version of another servo accelerometer is given in Fig. 8. The acceleration a of the instrument case causes an inertial force f on the sensitive mass m, tending to make it pivot in its bearings or flexure mount. The rotation θ from the neutral position is sensed by an inductive pickup coil and amplified, demodulated, and filtered to produce a current i_a directly proportional to the motion from the null position. This current is passed through a precision stable resistor R to produce the output voltage signal and is applied to a coil suspended in a magnetic field. The current through the coil produces magnetic torque on the coil, which takes action to return the mass to the neutral position. The current required to produce magnetic torque that just balances the inertial torque due to acceleration is directly proportional to the acceleration a. Therefore the output voltage e_o becomes a measure of acceleration a. Since a nonzero displacement θ is necessary to produce the current i_a, the mass is not exactly returned the null position but becomes very close to zero because of the high-gain amplifier. Analysis of the block diagram reveals that

$$e_o/R = (mra - e_o K_c/R)(K_p K_a/K_s)/(s^2/\omega_{nl}^2 + 2\zeta_1 s/\omega_{n1} + 1) \tag{12}$$

where K_c, K_p, K_a, and K_s are constants. Rearranging this expression gives

$$mrR K_p K_a a/K_s = (s^2/\omega_{nl}^2 + 2\zeta_1 s/\omega_{nl} + 1 + K_c K_p K_a a/K_s)e_o \tag{13}$$

By designing the amplifier gain K_a to be large enough so that $K_c K_p K_a a/K_s \gg 1.0$, then

$$e_o/a(s) - K/(s^2/\omega_{nl}^2 + 2\zeta_1 s/\omega_{nl} + 1 + K_c K_p K_a a/K_s)e_o \tag{14}$$

where

$$K \cong mrR/K_c \quad \text{V/(m/s}^2\text{)} \tag{15}$$

$$\omega_n \cong \omega_{nl}\sqrt{K_c K_p K_a/K_s} \quad \text{rad/s} \tag{16}$$

$$\zeta \cong \zeta_1/\sqrt{K_c K_p K_a/K_s} \tag{17}$$

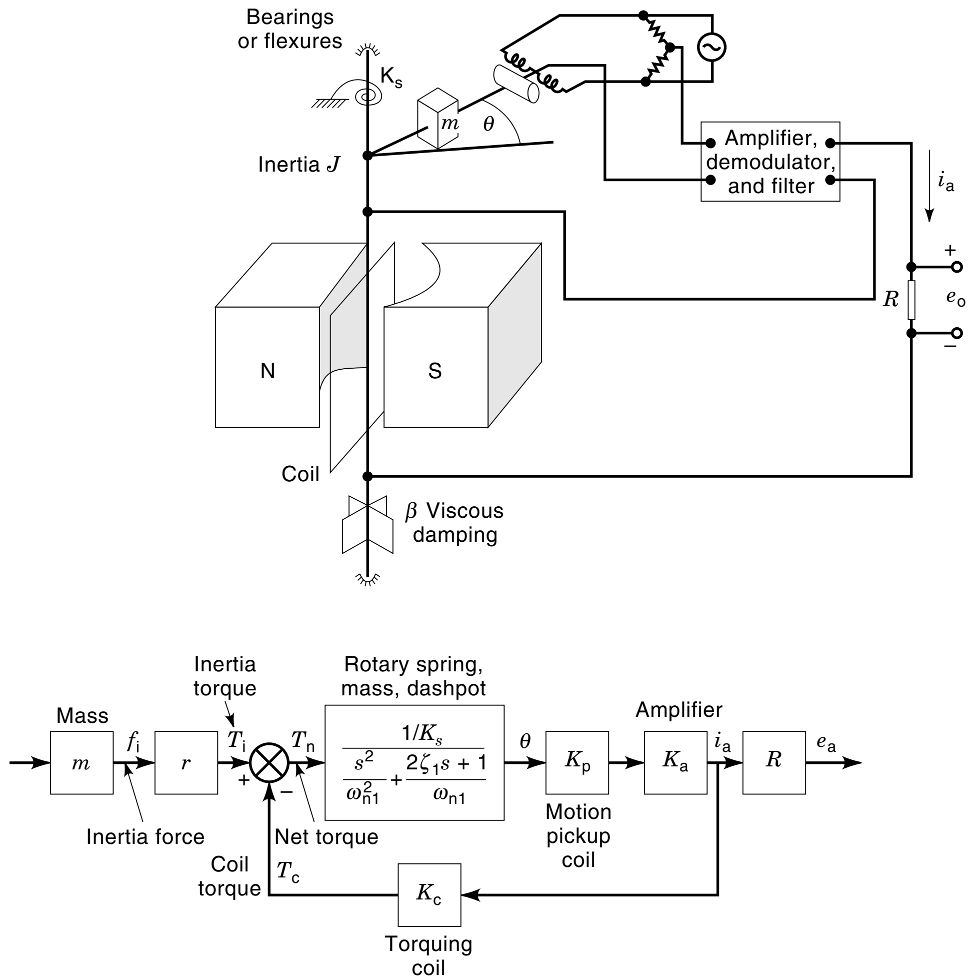

Figure 8. A simplified version of a rotational-type servo accelerometer. Acceleration of the instrument case causes an inertial force on the sensitive mass, tending to make it pivot in its bearings or flexure mount. The rotation from neutral is sensed by an inductive sensing apparatus and amplified and demodulated, and then filtered to produce a current directly proportional to the motion from the null position. The block diagram representation is useful in analysis.

Equation (15) shows that the sensitivity depends on the values of m, r, R, and K_c, all of which can be made constant. In this case, a high-gain feedback is useful in shifting the requirements for accuracy and stability from mechanical components to a selected few parameters for which the requirements can be met easily. As in all feedback systems, the gain cannot be made arbitrarily high because of dynamic instability; however, a sufficiently high gain can be achieved to obtain good performances. At very low frequencies, less than a few hertz, high gain can be used with no loss of stability, and modern integrated circuit (IC) amplifiers have static gains over one million. An excellent comprehensive treatment of this topic is given by Doebelin (2).

Induction Accelerometers. The cross-product relationship of current, magnetic field, and force gives the basis for induction-type electromagnetic accelerometers, which are essentially generators rather than motors. One type of instrument, the cup-and-magnet design, includes a pendulous element with a pickoff mechanism and a servo controller driving a tachometer coupling. A permanent magnet and a flux return ring, closely spaced with respect to an electrically conductive cylinder, are attached to the pendulus element. A rate-proportional drag force is obtained by electromagnetic induction effects betwen the magnet and conductor. The pickoff mechanism senses pendulum deflection under acceleration and causes the servo controller to turn the rotor in a sense to drag the pendulus element toward the null position. Under steady-state conditions motor speed is a measure of the acceleration acting on the instrument. Stable servo operation is achieved by employing a time-lead network to compensate the inertial time lag of the motor and magnet combination. The accuracy of the servo-type accelerometers is ultimately limited by consistency and stability of scale factors of coupling and cup-and-magnet devices as a function of time and temperature. Since the angular rate is proportional to acceleration, angular position represents a change in velocity. This is a useful feature, particularly in navigation applications.

Another accelerometer based on induction design uses the eddy-current induction torque generation. It was pointed out that the force-generating mechanism of an induction accelerometer consists of a stable magnetic field, usually supplied by a permanent magnet, which penetrates orthogonally through a uniform conduction sheet. The movement of the conducting sheet relative to the magnetic field in response to an acceleration results in a generated electromotive potential in each circuit in the conductor. This action is in accordance with the law of Faraday's principle. In induction-type accelerometers, the induced eddy currents are confined to the conductor sheet, making the system essentially a drag coupling.

A typical commercial instrument based on the servo-accelerometer principle might have a micromachined quartz flex-

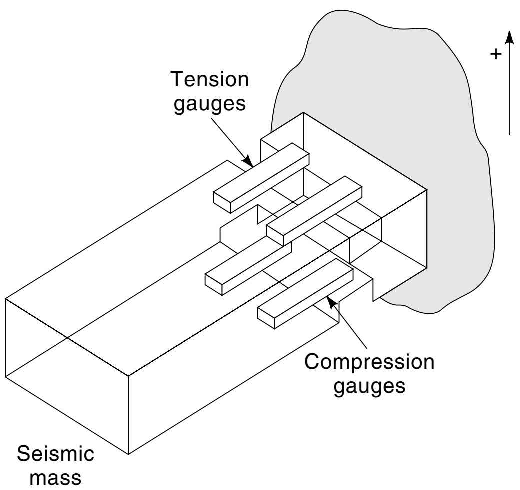

Figure 9. Bonding of piezoelectric and piezoresistive elements on to inertial system. As the inertial member vibrates, deformation of the tension and compression gauges causes resistance to change. The change in resistance is picked up and processed further. Accelerometers based on PZTs are particularly useful in medium- to high-frequency applications.

ure suspension, differential capacitance angle pickoff, air squeeze film plus servo lead compensation for system damping. Of the various available models, a 30 g range unit has a threshold and resolution of 1 μg, a frequency response that is flat to within 0.05% at 10 Hz and 2% at 100 Hz, a natural frequency of 1,500 Hz, a damping ratio from 0.3 to 0.8, and transverse or cross-axis sensitivity of 0.1%. If, for example, the output current is about 1.3 mA/g, a 250 Ω readout resistor would give about ±10 V full scale at 30 g. These accelerometers are good for precision work and used in many applications such as aircraft and missile control systems, measurement of tilt angles for borehole nagivation, and axle angular bending in aircraft weight and balance systems.

Piezoresistive Accelerometers

Piezoresistive accelerometers are essentially semiconductor strain gauges with large gauge factors. High gauge factors are obtained since the material resistivity is dependent primarily on the stress, not only on dimensions. This effect can be greatly enhanced by appropriate doping of semiconductors such as silicon. The increased sensitivity is critical for vibration measurements since it allows miniaturization of the accelerometer. Most piezoresistive accelerometers use two or four active gauges arranged in a Wheatstone bridge. Extra precision resistors are used, as part of the circuit, in series with the input to control the sensitivity, balancing and offsetting temperature effects. The mechanical construction of a piezoresistive accelerometer is shown in Fig. 9.

In some applications, overload stops are necessary to protect the gauges from high-amplitude inputs. These instruments are useful for acquiring vibration information at low frequencies, for example, below 1 Hz. In fact, the piezoresistive sensors are inherently true static acceleration measurement devices. Typical characteristics of piezoresistive accelerometers may be 100 mV/g as the sensitivity, 0 Hz to 750 Hz as the frequency range, 2,500 Hz as the resonance frequency, 25 g as the amplitude range, 2,000 g as the shock rating, and 0°C to 95°C as the temperature range, with a total mass of about 25 g.

Differential-Capacitance Accelerometers

Differential-capacitance accelerometers are based on the principle of the change of capacitance in proportion to applied acceleration. They come in different shapes and sizes. In one type, the seismic mass of the accelerometer is made as the movable element of an electrical oscillator as shown in Fig. 10. The seismic mass is supported by a resilient parallel-motion beam arrangement from the base. The system is set to have a certain defined nominal frequency when undisturbed. If the instrument is accelerated the frequency varies about and below the nominal value depending on the direction of acceleration.

The seismic mass carries an electrode located in opposition to a number of base-fixed electrodes that defined variable capacitors. The base-fixed electrodes are resistance coupled in the feedback path of a wideband, phase-inverting amplifier. The gain of the amplifier is predetermined to ensure maintance of oscillations over the range of variation of the capacitance determined by the applied acceleration. The value of the capacitance C for each of the variable capacitors is given by

$$C = \epsilon k S/h \tag{18}$$

where k is the dielectric constant, ϵ is the capacitivity of free space, S is the area of electrode, and h is the variable gap.

Denoting the magnitude of the gap for zero acceleration as h_0, the value of h in the presence of acceleration a may be written

$$h = h_0 + ma/K \tag{19}$$

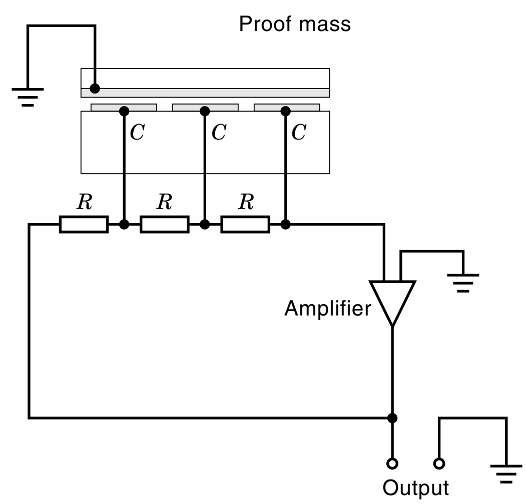

Figure 10. A typical differential-capacitance accelerometer. The proof mass is constrained in its null position by a spring. Under acceleration, variable frequencies are obtained in the electrical circuit. In a slightly different version the proof mass may be constrained by an electrostatic feedback force, thus resulting in a convenient mechanical simplicity.

where m is the value of the proof mass and K is the spring constant. Thus,

$$C = \epsilon kS/(h_0 + ma/K) \tag{20}$$

For example, the frequency of oscillation of the resistance-capacitance type circuit is given by the expression

$$f = \sqrt{6}/2\pi RC \tag{21}$$

Substituting this value of C in Eq. (20) gives

$$f = (h_0 + ma/K)\sqrt{6}/2\pi R\epsilon kS \tag{22}$$

Denoting the constant quantity $\sqrt{6}/2\pi R\epsilon kS$ as B and rewriting Eq. (22) gives

$$f = Bh_0 + Bma/K \tag{23}$$

The first term on the right-hand side expresses the fixed bias frequency f_0 and the second term denotes the change in frequency resulting from acceleration, so that the expression may be written as

$$f = f_0 + f_{\mathrm{a}} \tag{24}$$

If the output frequency is compared with an independent source of a constant frequency of f_0 whereby f_{a} may be determined.

A commonly used example of a capacitive-type accelerometer is based on a thin diaphragm with spiral flexures that provide the spring, proof mass, and moving plate necessary for the differential capacitor, as shown in Fig. 11. Plate motion between the electrodes pumps air parallel to the plate surface and through holes in the plate to provide squeeze film damping. Since air viscosity is less temperature sensitive than oil, the desired damping ratio of 0.7 hardly changes more than 15%. A family of such instruments are easily available with full-scale ranges from ±0.2 g (4 Hz flat response) to ±1,000 g (3,000 Hz), a cross-axis sensitivity less than 1%, and a full-scale output of ±1.5 V. The size of a typical device is about 25 mm^3 with a mass of 50 g.

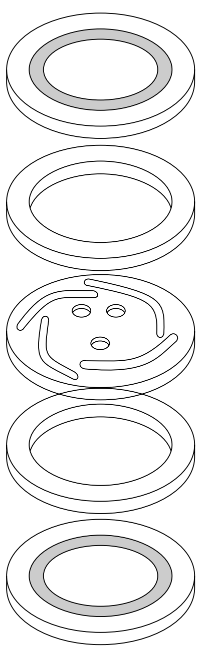

Figure 11. A diaphragm-type capacitive accelerometer. The seismic element is cushioned between the electrodes. Motion of the mass between the electrodes causes air movement passing through the holes, which provides a squeeze film damping. In some cases oil may be used as the damping element.

Strain-Gauge Accelerometers

Strain-gauge accelerometers are based on resistance properties of electrical conductors. If a conductor is stretched or compressed, its resistance alters because of dimensional changes and the changes in the fundamental property of material called piezoresistance. This indicates that the resistivity ρ of the conductor depends on the mechanical strain applied onto it. The dependence is expressed as the gauge factor

$$(dR/R)/(dL/L) = 1 + 2\nu + (d\rho/\rho)/(dL/L) \tag{25}$$

where 1 indicates the resistance change due to length, 2ν indicates resistance change due to area, and $(dp/p)/(dL/L)$ indicates the resistance change due to piezoresistivity.

In acceleration measurements, the resistance strain gauges can be selected from different types, such as unbonded metal-wire gauges, bonded metal-wire gauges, bonded metal-foil gauges, vacuum-deposited thin-metal-film gauges, bonded semiconductor gauges, and diffused semiconductor gauges. However, bonded and unbonded metal-wire gauges usually find wider applications in accelerometers. Occasionally, bonded semiconductor gauges, known as piezoresistive transducers, are used, but they suffer from high temperature sensitivities, nonlinearities, and some mounting difficulties. Nevertheless, in recent years, they have found new applications with the development of micromachine transducer technology, which is discussed in detail in the microaccelerometer section.

Unbonded-strain-gauge accelerometers use the strain wires as the spring element and as the motion transducer, using similar arrangements as in Fig. 9. They are useful for general-purpose motion and vibration measurements from low to medium frequencies. They are available in wide ranges and characteristics: typically ±5 g to ±200 g full scale, a natural frequency of 17 Hz to 800 Hz, 10 V excitation voltage ac or dc, full-scale output ±20 mV to ±50 mV, a resolution less than 0.1%, an inaccuracy less than 1% full scale, and a cross-axis sensitivity less than 2%. The damping ratio (using silicone oil damping) is 0.6 to 0.8 at room temperature. These instruments are small and light, usually with a mass less than 25 g.

Bonded-strain-gauge accelerometers generally use a mass supported by a thin flexure beam. The strain gauges are cemented onto the beam to achieve maximum sensitivity, temperature compensation, and sensitivity to both cross-axis and angular accelerations. Their characteristics are similar to unbonded-strain-gauge accelerometers but have greater sizes and weights. Often silicone oil is used for damping. Semiconductor strain gauges are widely used as strain sensors in cantilever-beam and mass types of accelerometers. They allow high outputs (0.2 V to 0.5 V full scale). Typically, a ±25 g acceleration unit has a flat response from 0 Hz to 750 Hz,

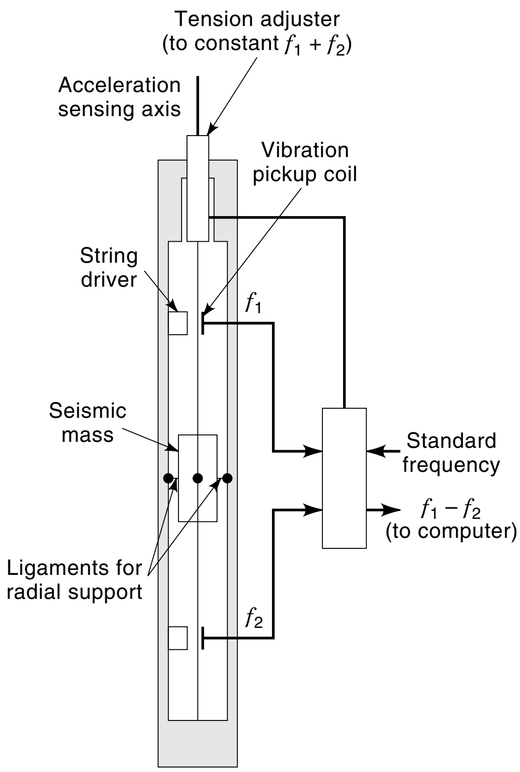

Figure 12. A vibrating-string accelerometer. A proof mass is attached to two strings of equal mass and length and it is supported radially by suitable bearings. The vibration frequencies of strings are dependent on the tension imposed by the acceleration of the system in the direction of the sensing axis.

a damping ratio of 0.7, a mass of 28 g, and an operational temperature of −18°C to ±93°C. A triaxial ±20,000 g model has a flat response from 0 kHz to 15 kHz, a damping ratio of 0.01, and a compensation temperature range of 0°C to 45°C and is 13 × 10 × 13 mm^3 in size and 10 g in mass.

Inertial Types: Cantilever and Suspended-Mass Accelerometers

There are a number of different inertial-type accelerometers, most of which are in development stages or used under very special circumstances, such as gyropendulum, reaction-rotor, vibrating-string, and centrifugal-force-balance designs. In many types, the force required to constrain the mass in the presence of the acceleration is supplied by an inertial system.

The vibrating-string instrument, Fig. 12, makes use of a proof mass supported longitudinally by a pair of tensioned, transversely vibrating strings with uniform cross section and equal lengths and masses. The frequency of vibration of the srings is set to several thousand cycles per second. The proof mass is supported radially in such a way that the acceleration normal to the strings does not affect the string tension. In the presence of acceleration along the sensing axis, a deferential tension exists on the two strings, thus altering the frequency of vibration. From the second law of motion the frequencies may be written as

$$f_1^2 = T_1/4m_s l, \quad f_2^2 = T_2/4m_s l \tag{26}$$

where T is the tension and m_s and l are the masses and lengths of strings, respectively.

The quantity $T_1 - T_2$ is proportional to ma where a is the acceleration along the axis of the strings. An expression for the difference of the frequency-squared terms may be written as

$$f_1^2 - f_2^2 = (T_1 - T_2)/4m_s l = ma/4m_s l \tag{27}$$

hence

$$f_1 - f_2 = ma/[(f_1 + f_2)4m_s l] \tag{28}$$

The sum of frequencies $(f_1 + f_2)$ can be held constant by servoing the tension in the strings with reference to the frequency of a standard oscillator. Then, the difference between the frequencies becomes linearly proportional to acceleration. In some versions, the beamlike property of the vibratory elements is used by gripping them at nodal points corresponding to the fundamental mode of the vibration of the beam and at the respective centers of percussion of the common proof mass. The output frequency is proportional to acceleration, and the velocity is proportional to phase, thus offering an important advantage. The velocity change can be measured by something almost as simple as counting zero crossings. Improved versions of these devices lead to cantilever-type accelerometers, discussed next.

In this accelerometer, a small cantilever beam mounted on the block is placed against the vibrating surface, and an appropriate mechanism is provided for varying the beam length. The beam length is adjusted such that its natural frequency is equal to the frequency of the vibrating surface, and hence the resonance condition is obtained. Recently, slight variations of cantilever-beam arrangements are finding new applications in microaccelerometers.

In a different type of suspended-mass configuration, a pendulum is used that is pivoted to a shaft rotating about a vertical axis. Pickoff mechanisms are provided for the pendulum and the shaft speed. The system is servo controlled to maintain it at null position. Gravitational acceleration is balanced by the centrifugal acceleration. The shaft speed is proportional to the square root of the local value of the acceleration.

All inertial force accelerometers just described are absolute measure instruments. That is, their scale factors are can predetermined solely by establishing mass, length, and time quantities, as distinguished from voltage and spring stiffness.

Electrostatic-Force-Feedback Accelerometers

Electrostatic accelerometers are based on Coulomb's law between two charged electrodes. They measure the voltage in terms of force required to sustain a movable electrode of a known area, mass, and separation from an affixed electrode. The field between the electrodes is given by

$$E = Q/\epsilon k S \tag{29}$$

where E is the intensity or potential gradient (dV/dx); Q is the charge, S is the area of the conductor, and k is the dielectric constant of the space outside the conductor.

By using this expression it can be shown that the force per unit area of the charged conductor (in N/m^2) is given by

$$F/S = Q^2/2\epsilon kS^2 = \epsilon kE^2/2 \quad (30)$$

In an electrostatic-force-feedback accelerometer (similar in structure as in Fig. 9) an electrode of mass m and area S is mounted on a light pivoted arm for moving relative to the fixed electrodes. The nominal gap h between the pivoted and fixed electrodes is maintained by means of a force-balancing servo system capable of varying in the electrode potential in response to signals from a pickoff mechanism that senses relative changes in the gaps.

Considering one movable electrode and one stationary electrode and assuming that the movable electrode is maintained at a bias potential V_1 and the stationary one at a potential V_2. The electrical intensity E in the gap can be expressed as

$$E_1 = (V_1 - V_2)/h \quad (31)$$

so that the force of attraction may be found as

$$F_1 = \epsilon kE^2S/2h^2 = \epsilon k(V_1 - V_2)^2S/2h^2 \quad (32)$$

In the presence of acceleration, if V_2 is adjusted to restrain the movable electrode to the null position, the expression relating acceleration and electrical potential may be given by

$$a = F_1/m = \epsilon k(V_1 - V_2)^2S/2h^2m \quad (33)$$

The device so far described can measure acceleration in one direction only, and the output is quadratic, that is,

$$(V_1 - V_2) = D\sqrt{a} \quad (34)$$

where D is the constant of proportionality.

The output may linearized in a number of ways, one of them being the quarter-square method. If the servo controller applies a potential $-V_2$ to the other fixed electrode, the force of attraction between this electrode and the movable electrode becomes

$$a = F_1/m = \epsilon k(V_1 + V_2)^2S/2h^2m \quad (35)$$

and the force-balance equation of the movable electrode when the instrument experiences a downward acceleration a now is

$$ma = F_1 - F_2 = [(V_1 + V_2)^2 - (V_1 - V_2)^2]\epsilon kS/2h^2m$$

or (36)

$$ma = \epsilon kS(4V_1V_2)/2h^2m$$

Hence, if the bias potential V_1 is held constant and the gain of the control loop is high so that variations in the gap are negligible, the acceleration becomes a linear function of the controller output voltage V_2 as

$$a = V_2[(\epsilon kS2V_1)/h^2m] \quad (37)$$

The principal difficulty in mechanizing the electrostatic force accelerometer is the relatively high electric field intensity required to obtain an adequate force. Also, extremely good bearings are necessary. Damping can be provided electrically or by viscosity of the gaseous atmosphere in the interelectrode space if the gap h is sufficiently small. The scheme works best in micromachined instruments. Nonlinearity in the voltage breakdown phenomenon permits larger gradients in very small gaps.

The main advantages of electrostatic accelerometers are extreme mechanical simplicity, low power requirements, absence of inherent sources of hysteresis errors, zero temperature coefficients, and ease of shielding from stray fields.

Microaccelerometers

By the end of the 1970s it became apparent that the essentially planar integrated-circuit (IC) technology processing could be modified to fabricate three-dimensional electromechanical structures, called micromachining. Accelerometers and pressure sensors were among the first IC sensors. The first accelerometer was developed in 1979. Since then the technology has been progressing steadily, and now an extremely diverse range of accelerometers is readily available. Most sensors use bulk micromachining rather than surface micromachining techniques. In bulk micromachining the flexures, resonant beams, and all other critical components of the accelerometer are made from bulk silicon in order to exploit the full mechanical properties of a silicon single crystal. With proper design and film process, bulk micromachining yields an extremely stable and robust accelerometer.

The selective etching of multiple layers of deposited thin films, or surface micromachining, allows movable microstructures to be fabricated on silicon wafers. With surface micromachining, layers of structure material are disposed and patterned as shown in Fig. 13. These structures are formed by polysilicon and a sacrifical material such as silicon dioxide.

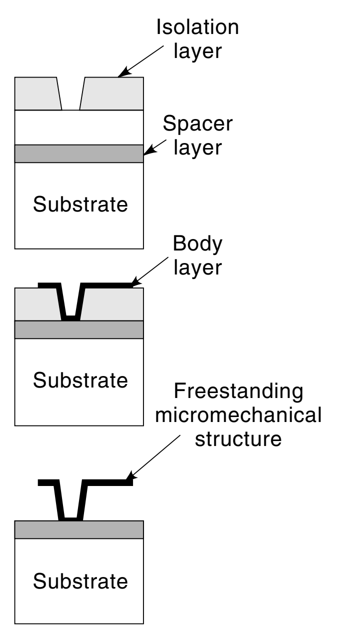

Figure 13. Steps of surface micromachining. The acceleration-sensitive three-dimenisonal structure is formed on a substrate and a sacrificial element. The sacrificial element is etched to leave a freestanding structure. The spacing between the structure and substrate is about 2 μm.

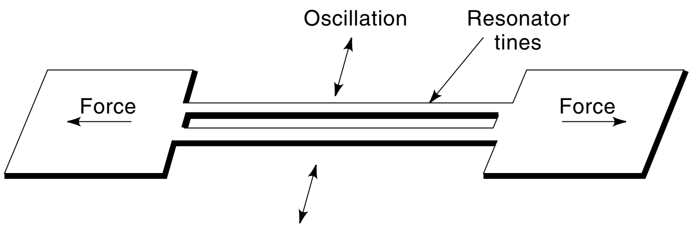

Figure 14. A double-ended tuning fork (DETF) acceleration transducer. Two beams are vibrated 180° out of phase to eliminate reaction forces at the beam ends. The resonant frequency of the beam is altered by acceleration. The signal-processing circuits are also integrated in the same chip.

The sacrificial material acts as an intermediate spacer layer and is etched away to produce a freestanding structure. Surface machining technology also allows smaller and more complex structures to be built in multiple layers on a single substrate.

The operational principles of microaccelerometers are very similar to capacitive force-balance or vibrating-beam accelerometers, discussed earlier. Manufacturing techniques may change from one manufacturer to another. However, in general, vibrating-beam accelerometers are preferred because of better air-gap properties and improved bias performance characteristics.

Vibrating-beam accelerometers, also termed resonant beam force transducers, are made in such a way that an acceleration along a positive input axis places the vibrating beam in tension. Thus the resonant frequency of the vibrating beam increases or decreases with the applied acceleration. A mechanically coupled beam structure also known as a double-ended tuning fork (DETF) is shown in Fig. 14.

In DETF, an electronic oscillator capacitively couples energy into two vibrating beams to keep them oscillating at their resonant frequency. The beams vibrate 180° out of phase to cancel reaction forces at the ends. The dynamic cancellation effect of the DETF design prevents energy from being lost through the ends of the beam. Hence, the dynamically balanced DETF resonator has a high Q factor, which leads to a stable oscillator circuit. The acceleration signal is output from the oscillator as a frequency-modulated square wave that can be used for a digital interface.

The vibrating beam accelerometer is similar in philosophy to the vibrating string accelerometer. Frequency output provides an easy interface with digital systems, and measurement of phase provides an easy integration to velocity. Static stiffness eliminates the tension and makes the device much smaller. A recent trend is to manufacture vibrating beam accelerometers as micromachined devices. With differential frequency arrangements, many common mode errors can be eliminated, including clock errors.

The frequency of resonance of the system must be much higher than any input acceleration, and this limits the measurable range. In one military micromachined accelerometer the following characteristics are given: a range of ±1200 g, a sensitivity of 1.11 Hz/g, a bandwidth of 2,500 Hz, and unloaded DETF frequency of 9,952 Hz, the frequency at +1,200 g is 11,221 Hz, the frequency at −1,200 g is 8,544 Hz, and the temperature sensitivity is 5 mg/°C. The accelerometer size is 6 mm diameter by 4.3 mm length, with a mass of about 9 g, and it has a turn-on time less then 60 s. The accelerometer is powered with +9 to +16 V dc and the nominal output is 9,000 Hz, square wave.

Surface micromachining has also been used to manufacture application-specific accelerometers, such as for air-bag applications in the automotive industry. In one type, a three-layer differential capacitor is created by alternate layers of polysilicon and phosphosilicate glass (PSG) on a 0.38 mm thick, 100 mm long wafer. A silicon wafer serves as the substrate for the mechanical structure. The trampoline-shaped middle layer is suspended by four supporting arms. This movable structure is the seismic mass for the accelerometer. The upper and lower polysilicon layers are fixed plates for the differential capacitors. The glass is sacrificially etched by hydrofluoric acid (HF).

CALIBRATIONS AND SENSITIVITY

Calibrations of accelerometers are necessary in acceleration, vibration, and shock sensing. The calibration methods can broadly be classified to be *static* or *dynamic* calibrations. The static calibration is conducted at one or several levels of constant accelerations. For example, if a tilting table calibration method is selected, the vertical component of the free fall is made use of without a choice of magnitude, but with the advantage of being readily available, and accurate to a few parts in 10^7. On the other hand, if a centrifuge is selected, it produces a constant acceleration as a function of speed of rotation, and the magnitudes can be chosen in a wide range from 0 g to well over 50,000 g, but with typical uncertainty of 1% in measuring the actual radius from the axis of rotation to the effective center of mass. The dynamic calibration is usually done by using an electrodynamic shaker.

The electrodynamic shaker is designed to oscillate in a sinusoidal motion with variable frequencies and amplitudes. They are stabilized at selected levels of calibration. This is an

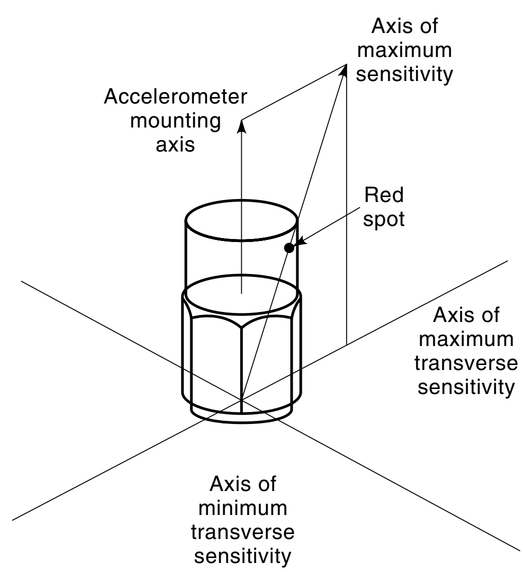

Figure 15. A vectorial illustration of cross-axis sensitivity. Accelerometers may sense vibrations not only in the direction of the main axis but also perpendicular to the main axis. These cross-axis responses are minimized in many accelerometers to a value less than 1%. Sometimes this sensitivity may be used to determine the correct orientation of the device.

absolute method that consists of measuring the displacement with laser interferometry and a precise frequency meter for accurate frequency measurements. The shaker must be driven by a power amplifier thus giving a sinusoidal output with minimal distortion. The National Institute of Science and Technology uses this method as a reference standard. Precision accelerometers, mostly of the piezoelectric types, are calibrated by the absolute method and then used as the working standards. A preferred method is the back-to-back calibration, where the test specimen is directly mounted on the working standard, which in turn is mounted on an electrodynamic shaker.

Sensitivity

A vibrational structure may have been subjected to different forms of vibrations, such as compressional, torsional, or transverse. A combination of all these vibrations may also take place simultaneously, which makes analysis and measurement difficult and complex. It was discussed earlier that the differential equations governing the vibrational motion of a structure depends on the number of degrees of freedom, which are described as a function of space coordinates $f(x, y, z, t)$. For example, the transverse vibrations of structures may be a fourth-order equation differential equation.

Fortunately, most common acceleration and vibration measurements are simple in nature, being either compressional or torsional. They can easily be expressed as second-order differential equations, as explained in the frequency response section. However, during measurements, most accelerometers are affected by transverse vibrations and their sensitivity can play a major role in the accuracy of the measurements.

The transverse, also known as cross-axis, sensitivity of an accelerometer is its response to acceleration in a plane perpendicular to the main accelerometer axis as shown in Fig. 15. The cross-axis sensitivity is normally expressed as a percentage of the main-axis sensitivity and should be as low as possible. There is no single value of cross-axis sensitivity, but it varies depending on the direction. The direction of minimum sensitivity is usually supplied by the manufacturers.

The measurement of the maximum cross-axis sensitivity is part of the individual calibration procedure and should always be less than 3% to 4%. If high levels of transverse vibration are present, this may result in erroneous overall results. In this case, separate arrangements should be made to establish the level and frequency contents of the cross-axis vibrations. The cross-axis sensitivity of typical accelerometers mentioned in the relevant sections were (2% to 3% for piezoelectric types and less than 1% for most others). In force-feedback accelerometers the transverse sensitivity is proportional to the input axis misalignment; therefore, it may be calibrated as a function of the temperature to within several microradians.

APPLICATIONS

This section is concerned with applications of accelerometers to measure physical properties such as acceleration, vibration and shock, and motions associated with inertial navigation. A full understanding of accelerometer dynamics is necessary in relation to characteristics of acceleration, vibration, and shock. The vibrations can be periodic, stationary random, nonstationary random, or transient.

Periodic Vibrations. In periodic vibrations, the motion of an object repeats itself in an oscillatory manner. This can be represented by a sinusoidal waveform

$$x(t) = X_{\text{peak}} \sin(\omega t) \tag{38}$$

where $x(t)$ is the time-dependent displacement, $\omega = 2\pi ft$ is the angular frequency, and X_{peak} is the maximum displacement from a reference point.

The velocity of the object is the time rate of change of displacement

$$u(t) = dx/dt = \omega X_{\text{peak}} \cos(\omega t) = U_{\text{peak}} \sin(\omega t + \pi/2) \tag{39}$$

where $u(t)$ is the time-dependent velocity, and $U_{\text{peak}} = \omega X_{\text{peak}}$ is the maximum velocity.

The acceleration of the of the object is the time rate change of velocity

$$a(t) = du/dt = d^2u/dt^2 = -\omega^2 X_{\text{peak}} \sin(\omega t) = A_{\text{peak}} \sin(\omega t + \pi) \tag{40}$$

where $a(t)$ is the time-dependent acceleration, and $A_{\text{peak}} = \omega^2 X_{\text{peak}} = \omega\, U_{\text{peak}}$ is the maximum acceleration.

From the preceding equations it can be seen that the basic form and the period of vibration remains the same in acceleration, velocity, and displacement. But velocity leads displacement by a phase angle of 90° and acceleration leads velocity by another 90°. The amplitudes of the three quantities are related as a function of frequency, shown in Fig. 16.

In nature, vibrations can be periodic but not necessarily sinusoidal. If they are periodic but nonsinusoidal, they can be expressed as a combination of a number of pure sinusoidal

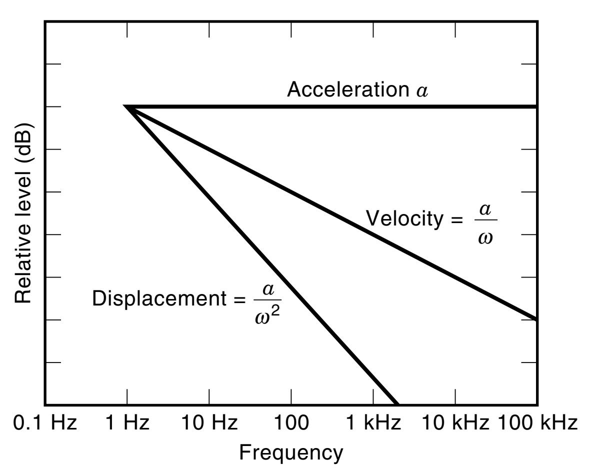

Figure 16. The logarithmic relationship between acceleration, velocity, and displacement. Velocity at a particular frequency can be obtained by dividing acceleration by a factor proportional to frequency. For displacement acceleration must be divided by a factor proportional to square of the frequency. Phase angles need to be determined separately, but they can be neglected in time-average measurements.

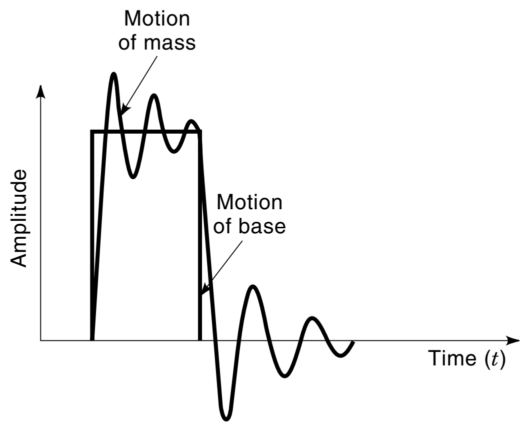

Figure 17. The time response of a shock excitation of a single-degree-of-freedom system. As the duration of the shock pulse increases sustained oscillations get shorter in time but larger in amplitude. The maximum system response may be as high as twice the magnitude of the shock pulse.

curves, describes by Fourier analysis as

$$x(t) = X_0 + X_1 \sin(\omega_1 t + \phi_1) + X_2 \sin(\omega_2 t + \phi_2) + \cdots X_n \sin(\omega_n t + \phi_n) \tag{41}$$

where ω_1, ω_2, . . ., ω_n are frequencies (rad/s), X_0, X_1, . . ., X_n are maximum amplitudes of respective frequencies, and ϕ_1, ϕ_2, . . ., ϕ_n are phase angles.

The number of terms may be infinite, and the higher the number of elements betters the approximation. These elements constitute the *frequency spectrum.* The vibrations can be represented in the time domain or frequency domain, both of which are extremely useful in the analysis. As an example, in Fig. 17, the time response of the seismic mass of an accelerometer is given against a rectangular pattern of excitation of the base.

Stationary Random Vibrations. Random vibrations occur often in nature, where they constitute irregular cycles of motion that never repeat themselves exactly. Theoretically, an infinitely long time record is necessary to obtain a complete description of these vibrations. However, statistical methods and probability theory can be used for the analysis by taking representative samples. Mathematical tools such as probability distributions, probability densities, frequency spectra, cross-correlations, auto correlations, digital Fourier transforms (DFTs), fast Fourier transforms (FFTs), auto-spectral-analysis, rms values, and digital filter analysis are some of the techniques that can be employed. Interested readers should refer to references for further information.

Transients and Shocks. Often, short duration and sudden occurrence vibrations need to be measured. Shock and transient vibrations may be described in terms of force, acceleration, velocity, or displacement. As in the case of random transients and shocks, statistical methods and Fourier transforms are used in the analysis.

Nonstationary Random Vibrations. In this case, the statistical properties of vibrations vary in time. Methods such as time averaging and other statistical techniques can be employed.

The majority of accelerometers described here can be viewed and analyzed as seismic instruments consisting of a mass, a spring, and a damper arrangement as shown in Fig. 1. Taking only the mass–spring system, if the system behaves linearly in a time invariant manner, the basic second-order differential equation for the motion of the mass alone under the influence of a force can be written as

$$f(t) = m\,d^2x/dt^2 + c\,dx/dt + kx \tag{42}$$

where $f(t)$ is the force, m the mass, c the velocity constant, and k the spring constant.

Nevertheless, in seismic accelerometers the base of the arrangement is in motion too. Therefore, Eq. (42) may be generalized by taking the effect motion of the base into account. Then this equation may be modified as

$$m\,d^2z/dt^2 + c\,dz/dt + kz = mg\cos(\theta) - m\,d^2x_1/dt^2 \tag{43}$$

where $z = x_2 - x_1$ is the relative motion between the mass and the base, x_1 the displacement of the base, x_2 the displacement of the mass, and θ the angle between the sense axis and gravity.

In order to lay a background for further analysis, taking the simple case, the complete solution to Eq. (42) can be obtained by applying the superposition principle. The superposition principle states that if there are simultaneously superimposed actions on a body, the total effect can be obtained by summing the effects of each individual action.

Equation (42) describes essentially a second-order system that can be expressed through Laplace transform as

$$X(s)/F(s) = 1/ms^2 + cs + k \tag{44}$$

or

$$X(s)/F(s) = K/(s^2/\omega_n^2 + 2\zeta s/\omega_n + 1) \tag{45}$$

where s is the Laplace operator, $K = l/k$ is the static sensitivity, $\omega_n = \sqrt{k/m}$ is the undamped critical frequency (rad/s, and $\zeta = (c/2)\sqrt{km}$ is the damping ratio.

As can be seen in the performance of accelerometers, the important parameters are the static sensitivity, the natural frequency, and the damping ratio, which are functions of mass, velocity, and spring constants, respectively. Accelerometers are designed to have different characteristics by suitable selection of these parameters.

Once the response is expressed in the form of Eqs. (44) and (45), analysis can be taken further either in the time domain or in the frequency domain. The time response of a typical second-order system for a unit-step input is given in Fig. 18. The Bode plot gain phase responses are depicted in Fig. 19. Detailed discussions about frequency response, damping, damping ratio, and linearity are made in relevant sections, and further information can be obtained in the references.

Systems in which a single structure moves in more than one direction are termed multi-degree-of-freedom systems. In this case, the accelerations become functions of dimensions as d^2x/dt^2, d^2y/dt^2, and d^2z/dt^2. Hence, in multichannel vibration tests multiple transducers must be used to create uniax-

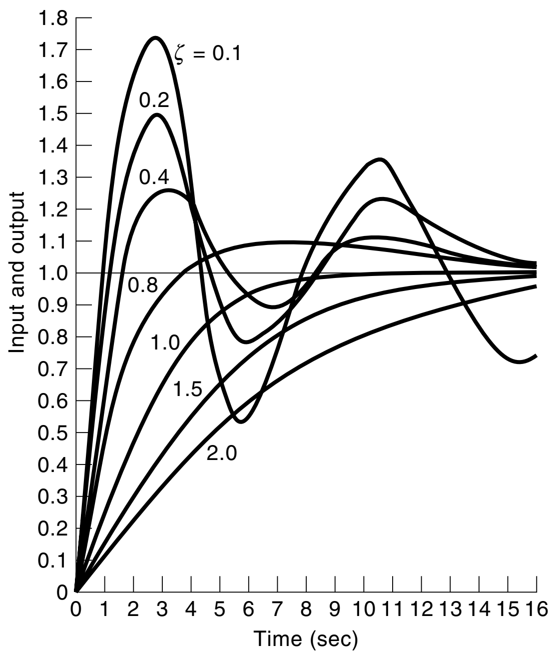

Figure 18. Unit-step-time responses of a second-order system with various damping ratios. The maximum overshoot, delay, rise, settling times, and frequency of oscillations depend on the damping ratio. A smaller damping ratio gives a faster response but larger overshoot. In many applications a damping ratio of 0.707 is preferred.

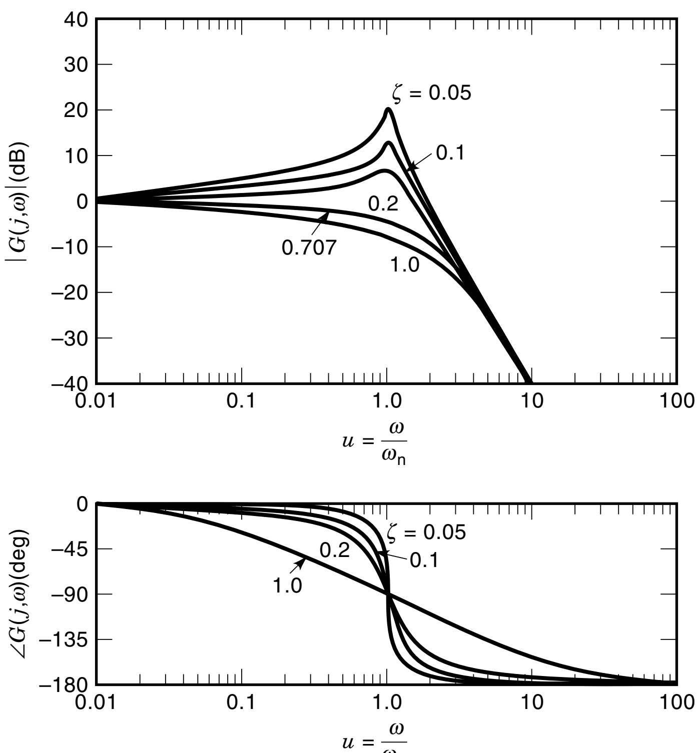

Figure 19. Bode plot of the gain and phase angle of the transfer function $G(j\omega)$ of a second-order system against frequency. Curves are functions of frequencies as well as damping ratios. These plots can be obtained theoretically or by practical tests conducted in frequency range.

ial, biaxial, or triaxial sensing points for measurements. Mathematically, a linear multi-degree-of-freedom system can be described by a set of coupled second-order linear differential equations, and when the frequency response is plotted it normally shows one resonance peak per degree of freedom.

Frequently, acceleration and vibration measurements of thin plates or small masses are required. Attaching an accelerometer with a comparable mass onto a thin plate or a small test piece can cause *mass loading*. Since acceleration is dependent on the mass, the vibration characteristics of the loaded test piece may be altered, thus yielding wrong measurements. In such cases, a correct interpretation of the results of the measuring instruments must be made. Some experimental techniques are also available for the correction of the test results in the form performing repetitive tests conducted by sequentially adding small known masses and by observing the differences.

In general, accelerometers are preferred over other displacement and velocity sensors due to the following reasons:

1. They have a wide frequency range from zero to very high values. Steady accelerations can easily be measured.
2. Acceleration is more frequently needed since destructive forces are often related to acceleration rather than to velocity or displacement.
3. Measurement of transients and shocks can readily be made, which is much easier than displacement of velocity sensing.
4. Displacement and velocity can be obtained by simple integration of acceleration by electronic circuitry. Integration is preferred over differentiation.

Selection, Full-Scale Range, and Overload Capability

Ultimate care must be exercised for the selection of correct accelerometer to meet the requirements of a particular application. At first glance there may seem to be a confusingly large selection of accelerometers available. But they can be classified in two main groups. The first group is the general-purpose accelerometers offered in various sensitivites, frequencies, and full scale and overload ranges, with different mechanical and electrical connection options. The second group of accelerometers have characteristics targeted toward a particular application. A list of manufacturers of accelerometers is supplied in Table 1.

In deciding about the application type, for example, general purpose or special, and the accelerometer to be employed, the following characteristics need to be considered: the transient response or cross-axis sensitivity; frequency range, sensitivity, mass and dynamic range; cross-axis response; and environmental conditions, temperature, cable noise, stability of bias, scale factor, and misalignment. Some useful hints about these characteristics will be given below.

The Frequency Range. Acceleration measurements are normally confined to using the linear portion of the response curve. The response is limited at low frequencies as well as at high frequencies by the natural resonances. As a rule of thumb the upper-frequency limit for the measurement can be set to one-third of the accelerometer's resonance frequency

Table 1. List of Manufacturers

Allied Signal, Inc.
101 Colombia Rd.
Dept. CAC
Morristown, NJ 07962
Tel.: 602-496-1000 or 800-707-4555
Fax: 602-496-1001

Bokam Engineering, Inc.
9552 Smoke Tree Avenue
Fountain Valley, CA 92708
Tel.: 714-962-3121
Fax: 714-962-5002

CEC Vibration
Division of Sensortronics
196 University Parkway
Pomona, CA 91768
Tel.: 909-468-1345 or 800-468-1345
Fax: 909-468-1346

Dytran Instrument, Inc.
Dynamic Transducers and Systems
21592 Marilla Street,
Chatsworth, CA 91311
Tel.: 800-899-7818
Fax: 800-899-7088

ENDEVCO
30700 Rancho Viejo Road
San Juan Capistrona, CA 92675
Tel.: 800-289-8204
Fax: 714-661-7231

Entran Devices, Inc.
10-T Washington Ave.
Fairfield, NJ 07004
Tel.: 800-635-0650

First Inertia Switch
G-10386 N. Holly Rd.
Dept. 10, P.O. Box 704
Grand Blanc, MI 48439
Tel.: 810-695-8333 or 800-543-0081
Fax: 810-695-0589

Instrumented Sensor Technologies
4701 A Moor Street
Okemos, MI 48864
Tel.: 517-349-8487
Fax: 517-349-8469

Jewel Electrical Instruments
124 Joliette Street
Manchester, NH 03102
Tel.: 603-669-6400 or 800-227-5955
Fax: 603-669-6962

Kistler Instrument Co.
75 John Glenn Dr.
Amherst, NY 14228-2171
Tel.: 800-755-5745

Lucas Control Products, Inc.
1000 Lucas Way
Hampton, VA 23666
Tel.: 800-745-8008
Fax: 800-745-8004

Metrix Instrument Co.
1711 Townhurst
Houston, TX 77043
Fax: 713-461-8223

Patriot
Sensors and Controls Corporation
650 Easy Street
Simi Valley, CA 93065
Tel.: 805-581-3985 or 800-581-0701
Fax: 805-583-1526

PCB Piezoelectronics, Inc.
3425 Walden Avenue
Depew, NY 14043
Tel.: 716-684-0001
Fax: 716-684-0987

PMC/BETA
9 Tek Circle
Natick, MA 01760
Tel.: 617-237-6020
Fax: 508-651-9762

Rutherford Controls
2697 International Pkwy
Building #3, Suite 122
Virginia Beach, VA 23452
Tel.: 800-899-5625
Fax: 804-427-9549

Sensotech, Inc.
1202 Chesapeak Ave.
Columbus, OH 43212
Tel.: 614-486-7723 or 800-867-3890
Fax: 614-486-0506

SETRA
45 Nagog Park
Acton, MA 01720
Tel.: 508-263-1400 or 800-257-3872
Fax: 508-264-0292

Silicon Microstructures, Inc.
46725 Fremond Blvd.
Fremond, CA 94358
Tel.: 510-490-5010
Fax: 510-490-1119

SKF Condition Monitoring
4141 Ruffin Road
San Diego, CA 92123
Tel.: 800-959-1366
Fax: 619-496-3531

Summit Instruments, Inc.
2236 N. Cleveland-Massillon Rd.
Akron, Ohio 44333-1255
Tel.: 800-291-3730
Fax: 216-659-3286

Wilcoxon Research
21-T Firstfield Road
Gaithersburg, MD 20878
Tel.: 800-842-7367
Fax: 301-330-8873

such that the vibrations measured will be less than 1 dB in linearity. It should be noted that an accelerometer's useful frequency range is significantly higher, that is, to one-half or two-thirds of its resonant frequency. The measurement frequencies may be set to higher values in applications in which lower linearity (say 3 dB) may be acceptable as in the case of monitoring of internal conditions of machines since the repeatability is more important than the linearity. The lower measuring frequency limit is determined by two factors. The first is the low-frequency cutoff of the associated preamplifiers. The second is the effect of ambient temperature fluctuations to which the accelerometer may be sensitive.

The Sensitivity, Mass, and Dynamic Range. Ideally, a higher transducer sensitivity is better, but compromises may have to be made for sensitivity versus frequency, range, overload capacity, size, and so on.

Accelerometer mass becomes important when using small and light test objects. The accelerometer should not load the structural member, since additional mass can significantly change the levels and frequency presence at measuring points and invalidate the results. As a general rule, the accelerometer mass should not be greater than one-tenth the effective mass of the part or the structure that it is mounted onto for measurements.

The dynamic range of the accelerometer should match the high or low acceleration levels of the measured objects. General-purpose accelerometers can be linear up to 5,000 g to 10,000 g, which is well into the range of most mechanical shocks. Special accelerometers can measure up to 100,000 g.

An important point in the practical application of accelerometers is that if mechanical damping is a problem, air damping is preferable to oil damping, since oil damping is extremely sensitive to viscosity changes. If the elements are stable against temperature, electronic damping may be sufficient.

The Transient Response. Shocks are characterized as sudden releases of energy in the form of short-duration pulses exhibiting various shapes and rise times. They have high magnitudes and wide frequency contents. In the applications where transient and shock measurements are involved, the overall linearity of the measuring system may be limited to high and low frequencies by phenomena known as zero shift and ringing, respectively. The zero shift is caused by both the phase nonlinearity in the preamplifiers and the accelerometer not returning to steady-state operation conditions after being subjected to high shocks. Ringing is caused by high-frequency components of the excitation near-reasonance frequency preventing the accelerometer to return back to its steady-state operation condition. To avoid measuring errors due to these effects the operational frequency of the measuring system should be limited to the linear range.

Full-Scale Range and Overload Capability. Most accelerometers are able to measure acceleration in both positive and negative directions. They are also designed to be able to accommodate overload capacity. Appropriate discussions are made on full scale range and overload capacity of accelerometers in the appropriate sections. Manufacturers also supply information on these two characteristics.

Environmental Conditions. In selection and implementation of accelerometers, environmental conditions such as temperature ranges, temperature transients, cable noise, magnetic field effects, humidity, and acoustic noise need to be considered. Manufacturers supply information on environmental conditions.

SIGNAL CONDITIONING

Common signal conditioners are appropriate for interfacing accelerometers to computers or other instruments for further signal processing. Caution needs to be exercised to provide the appropriate electric load to self-generating accelerometers. Generally, the generated raw signals are amplified and filtered suitably by the circuits within the accelerometer casing supplied by manufacturers. Nevertheless, piezoelectric and piezoresistive transducers require special signal conditioners with certain characteristics, which is discussed in the following section. Examples of signal conditioning circuits will also be given for microaccelerometers.

Signal Conditioning Piezoelectric Accelerometers. The piezoelectric accelerometer supplies a very small energy to the signal conditioner. It has a high capacitive source impedance. The equivalent circuit of a piezoelectric accelerometer can be regarded as an active capacitor that charges itself when loaded mechanically. The configuration of external signal conditioning elements are dependent on the equivalent circuit selected. The charge amplifier design of the conditioning circuit is the most common approach, since the system gain and low-frequency responses are well defined. The performance of the circuit is independent of cable length and capacitance of the accelerometer.

The charge amplifier consists of a charge converter output voltage, which occurs as a result of the charge input signal returning through the feedback capacitor to maintain the input voltage at the input level close to zero, as shown in Fig. 20. An important point about charge amplifiers is that their sensitivities can be standardized. They basically convert the input charge to voltage first and then amplify this voltage. With the help of basic operational-type feedback, the amplifier input is maintained at essentially 0 V; therefore it looks like a short circuit to the input. The charge converter output voltage that occurs as a result of a charge input signal is returned through the feedback capacitor to maintain the voltage at the input level near zero. Thus, the charge input is stored in the feedback capacitor, producing a voltage across it that is equal to the value of the charge input divided by the capacitance of the feedback capacitor. The complete transfer function of the circuit describing the relationship between the output voltage and the input acceleration magnitude may be determined by the following complex transform:

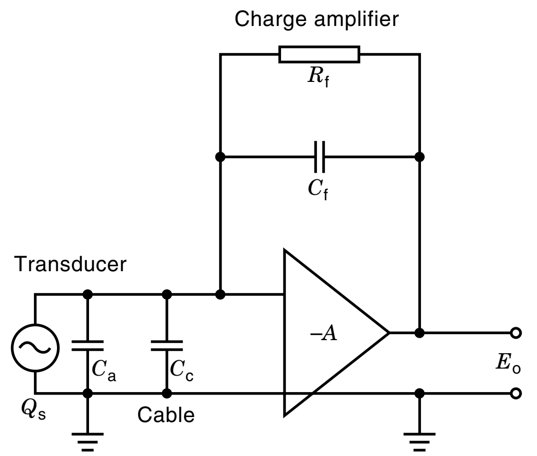

Figure 20. A typical charge amplifier. The transducer charge, which is proportional to acceleration, is first converted to voltage form to be amplified. The output voltage is a function of the input charge. The response of the amplifier can be approximated by a first-order system. In the PZT transducer the preamplifier is integrated within the same casing.

$$E_o/a_0 = S_a jR_fC_f\omega\{1 + jR_fC_f[1 + (C_a + C_c)/(1 + G) + C_f]\omega\} \quad (46)$$

where E_o is the charge converter output (V), a_0 the magnitude of acceleration (m/s^2), S_a the accelerometer sensitivity (mV/g), C_a the accelerometer capacitance (F), C_c the cable capacitance (F), C_f the feedback capacitance (F), R_f the feedback loop resistance, and G the amplifier open-loop gain.

In most applications, since C_f is selected to be large compared with $(C_a + C_c)/(1 + G)$, the system gain becomes independent of the cable length. In this case the denominator of the equation can be simplified to give a first-order system with roll-off at

$$f_{-3\,\mathrm{dB}} = \frac{1}{2\pi R_fC_f} \quad (47)$$

with a slope of 10 dB per decade. For practical purposes, the low-frequency response of this system is a function of well-defined electronic components and does not vary by cable length. This is an important feature when measuring low-frequency vibrations.

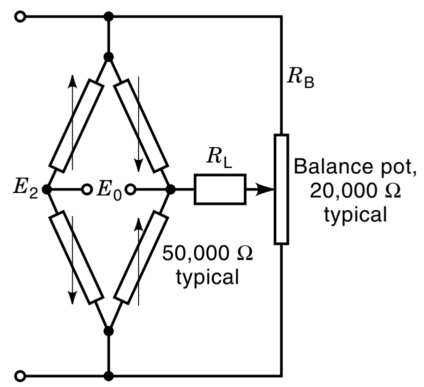

Figure 21. A bridge circuit for piezoresistive and strain gauge accelerometers. The strain gauges form the four arms of the bridge. The two extra resistors are used for balancing and for fine-adjustment purposes. This type of arrangement reduces temperature effects.

Many accelerometers are manufactured with preamplifiers and other signal-conditioning circuits integrated with the transducer enclosed in the same casing. Some accelerometer preamplifiers include integrators to convert the acceleration proportional outputs to either velocity or displacement proportional signals. To attenuate noise and vibration signals that lie outside the frequency range of interest most preamplifiers are equipped with a range of high-pass and low-pass filters. This avoids interference from electrical noise or signals inside the linear portion of the accelerometer frequency range. Nevertheless, it is worth mentioning that these devices usually have two time constants, external and internal. The mixture of these two time constants can lead to problems particularly at low frequencies. The internal time constant is usually fixed by the manufacturer through design and construction. Special care must be observed to account for the effect of external time constants in many applications by mainly observing impedance matching.

Signal Conditioning of Piezoresistive Transducers. Piezoresistive transducers generally have high-amplitude outputs, low-output impedances, and low intrinsic noise. Most of these transducers are designed for constant-voltage excitations. They are usually calibrated for constant-current excitations to make them independent of external influences. Many piezoresistive transducers are configured as full-bridge devices. Some have four active piezoresistive arms, together with two fixed precision resistors to permit shunt calibration in the signal conditioner as shown in Fig. 21.

Microaccelerometers. In microaccelerometers signal-conditioning circuitry is integrated within the same chip with the sensor as shown in Fig. 22. A typical example of the signal-conditioning circuitry is given in Fig. 23 in block diagram form. In this type of accelerometer, the electronic system is essentially a crystal-controlled oscillator circuit and the output signal of the oscillator is a frequency-modulated acceleration signal. Some circuits provide a buffered square-wave output that can directly be interfaced digitally. In this case the need for analog-to-digital conversion is eliminated, thus removing one of the major sources of errors. In other types of accelerometers signal conditioning circuits such as analog-to-digital converters are retained within the chip.

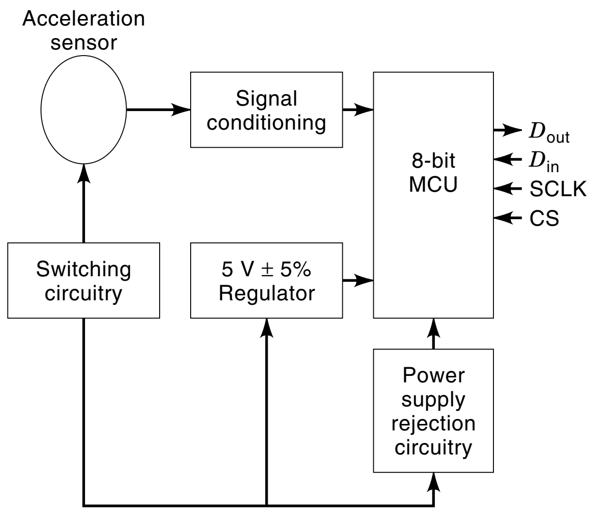

Figure 22. A block diagram of an accelerometer combined with microcontroller unit (MCU). The signal-conditioning, switching, and power-supply circuits are integrated to form a microaccelerometer. The device can directly be interfaced with a digital signal processor or a computer by using the data out D_{out}, data in D_{in}, chip select CS, and clock synchronization SCLK pins. In some cases analog-to-digital converters and memory are also integrated.

Force Feedback Accelerometers. These often must be digitized for use in digital systems. If they are used in inertial navigation systems there may be specific problems introduced by the accuracy requirements. The dynamic range may exceed 24 bits, and the system must operate in real time. Accurate integration may be required to get velocity changes as an output. A common solution is to use voltage-to-frequency or current-to-frequency converters to convert the analog signals to a train of velocity-weighted pulses. These converters cost as much and add as much to the error budget as the accelerometer.

Here, it is worth mentioning that global positioning systems (GPSs) are becoming add-ons to many position-sensing

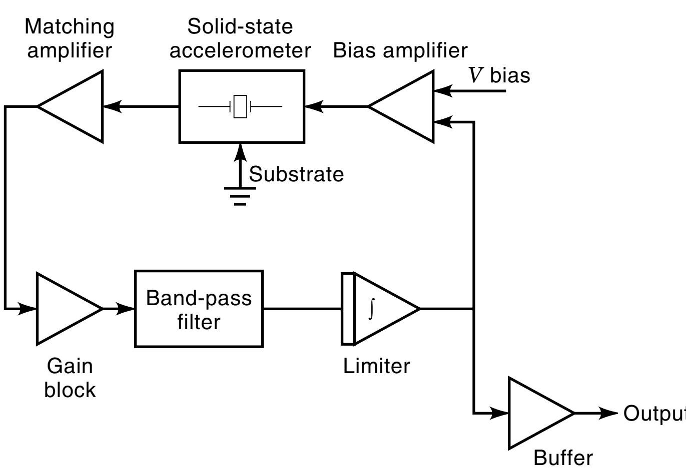

Figure 23. A block diagram of a signal-conditioning circuit of a microaccelerometer. The output signal of the oscillator is a frequency-modulated acceleration signal. The circuit provides a buffered square-wave frequency output that can be read directly into a digital device.

mechanisms. Because of antenna dynamics, shadowing, multipath effects, and the need for redundancy in critical systems such as aircraft, many of these systems will require inertial aiding tied in with accelerometers and gyros. With the development of micromachining, small and cost-effective GPS-assisted inertial systems will be available in the near future. These developments will require extensive signal processing with a high degree of accuracy. Dynamic ranges of the order of one million to one (e.g., 30 to 32 bits) need to be dealt with. A challenge awaits the signal-processing practitioner in achieving these accuracy requirements.

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ACCESS CONTROL. See DATA SECURITY.
ACCESS, MULTIPLE. See MULTIPLE ACCESS SCHEMES.
ACCOUNTANCY. See ACCOUNTING.
ACCOUNTANTS. See ACCOUNTING.