COMPASSES

Modern life is unthinkable without extensive physical movement of humans and goods around the globe. Since the advent of Sputnik in 1957, extraterrestrial space has been added.

The general problem of developing instrumentation to aid in this transfer has been recognized for thousands of years. It has led from simple observation of geographic and astronomical features to use of magnetic compasses, global positioning systems with the aid of earth-bound satellites, and to inertial guidance systems. A qualitative step from the observation of external features like the stars to an internal device is the recognition by the Chinese and the Vikings in the eleventh century that pieces of magnetized iron point in the direction of the magnetic north when allowed to orient themselves freely.

As a result of these developments, the instruments became at the same time more precise and more accurate. For exam-

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ple, millimeter precision can now be achieved for differential changes of position on the earth's surface by using the global positioning system (GPS) with interferometric techniques, with accuracies in the meter range, and directions can be measured with large ring lasers with precisions reaching subarcseconds, with accuracies in the arcsecond range. Here, *accuracy* means the usual sense of difference between measured value and true value, whereas *precision* means scattering of measured values around the measured average.

One notes that the angular accuracy and precision achievable through astronomical observations at optical wavelengths with instruments mounted at sea level is limited through air turbulence to 0.1 arcsec. It can only be further reduced by special techniques.

POSITION AND DIRECTION ON EARTH, COMPASSES AND GYROS

Position and direction are in principle two very different categories, and the instrumentation required is correspondingly different.

Positioning of a Fixed Point

In positioning a fixed point, the position is to be recorded with respect to a coordinate system that should satisfy the following requirements, among others:

Realizable and easy interpretable coordinates
Good time stability
High resolution or precision
High reliabilty and repeatability, or accuracy

Instruments need to be developed that are ideally capable of a resolution and accuracy commensurate with the level of accuracy that the coordinate system is able to afford.

A variety of phenomena is at present used with positional accuracies in the meter range. On the earth's surface, starting with well-established stable surveying fix points (geodetic positions), triangulation instruments used in geodetic surveys can establish millimeter accuracy over short distances, up to kilometer distances. For larger distances, astronomical references are used. A large variety of astronomical bodies have historically served as references, starting with the sun for navigational purposes. The moon, with positional data versus time (ephemerides) of centimeter accuracy, is capable of yet higher accuracy. Among the stars, Polaris (North Star or Pole Star), the supergiant Ursae Minoris of stellar magnitude 2 in the constellation Ursa Minor, describes a circle around the celestial North Pole with a radius of only 49.5 arcmin and is thus well suited to locate latitude with arcsecond accuracy. It is a preferred reference for navigation, including extraterrestrial. One notes that 1 arcsec of latitude on the earth's surface corresponds to about 30 m.

Finally, for geodetic surveys requiring submillimeter accuracy over global distances, quasars are excellent fiducial radiation sources whose positions can be considered fixed for positioning purposes within our galaxy. On earth, their radio emission is monitored with a globally distributed set of radio antennas. By study of the correlation of the radio noise of the receiving antenna dishes, submillimeter accuracy is achieved, allowing the measurement of even continental drift, which is generally on the scale of millimeters per century. Such observations require, however, considerable expense of equipment. Quasars are suitable references mainly on account of their luminosity and their large distances from earth, up to several gigaparsecs (1 parsec = 3×10^{16} m), with corresponding very good positional stability, whose radio emission can be detected with radio dishes of sufficient size. In terms of angular resolution, coordinated radio dishes with dish diameters in the 10 m range achieve milliarcsecond accuracy as opposed to visible detection where air turbulence limits the resolution to angles larger than 100 marcsec.

Direction at a Point

The establishment of a direction in a given coordinate system is a fundamentally different problem. Mathematically, position is a scalar property at a point that can be established by measuring the three positional coordinates, while the direction is a vector with three vector components. It is, however, possible to evaluate a given direction, for example, the course (clockwise angle relative to the direction true north), by accurately measuring two successive positions of it and evaluating the vector connecting the two positions; this can be done with instruments of sufficient precision (not necessarily accuracy), for example, with GPS.

The time-honored process of dead reckoning is roughly the inverse of the process just described: knowing the starting position, the instantaneous direction of the vessel, its instantaneous speed, and the time of travel for each leg, the position of the vessel can be evaluated at any future time by a vector sum where each vector represents the direction and distance traveled with a given speed and direction at each leg.

There are a variety of physical principles available that are able to provide a reference direction independent of the position at which the measurement is performed. Such instruments may either evaluate a direction locally by differential analysis, or else supply an internally generated reference direction that is independent of time and motion of the carrier on which this instrument is mounted. Naturally, great care has to be exercised to render such an instrument insensitive to any rotation of the carrier vessel.

The discussion of such instruments is the main concern of this article.

A variety of physical effects have been employed in directional devices.

Magnetic Compasses. These instruments use the ability of a magnetized needle to align itself in the earth's magnetic field. Since in the present era the horizontal components of the magnetic induction of this field point with reasonable accuracy to a north geomagnetic pole that is close to the (geographic) North Pole, the direction of such a needle is, at least at latitudes far away from the polar regions, reasonably aligned with a meridian on the earth surface, with deviations (declinations) generally amounting to less than 10°.

Internally Defined Direction References. For instruments of this kind, any physical principle that establishes a direction in space that is constant is usable. A practically important class of instruments employ mechanical gyros. They work with the principle that the rotation axis of a rotating massive body maintains its direction under any translation if no torque is applied to it.

Electronic and Nuclear References. Besides electronic interferometers, a recent version of nuclear interferometers is the use of nuclei that have an angular momentum or spin associated with them. When techniques are employed that overcome the randomizing thermal effect such that sufficient nuclei can be aligned in a probe, a readout mechanism will then show a constant direction of such an assembly. Such instruments are called *nuclear magnetic resonance gyroscopes.*

Ring Lasers. These lasers operate with two or more laser beams, that is, photons, that circulate in opposite directions around a given area defined by three or more mirrors or, more commonly, by an optical fiber. The thus defined area can be represented by an area vector that provides a reference direction. The mechanism of excitation of the laser modes is in principle irrelevant. Besides the classic excitation via inversion through dc current with internal electrodes, high-frequency excitation is used with external electrodes, but modern approaches employ Brillouin stimulation and other methods as well (1).

Any rotation around the area vector or axis results in an output frequency that is proportional to the rate of rotation or the angular velocity. This principle can be used to define directions in various ways. A direct way is its use as a north finder, where a ring laser mounted on the surface of the rotating earth is tilted until its area vector is parallel to the earth rotation axis, which is then verified by measuring a maximum output rate of the ring laser. This vector then points to the geographic North Pole.

In a "strapped-down" version, the ring laser is clamped on the carrier. It records then any rotational component of it. Thus, given the original direction of the ring-laser axis, the integrated output is then used to evaluate the new direction. Since one ring laser can only measure angular velocity components around its own axis, one needs three such rings to completely evaluate the direction vector. Commercial devices with one to three rings in one instrument are on the market, including tiny gyro sensors for image stabilization in binoculars (2), inertial navigation systems in modern airliners, submarines, and missiles, and for establishing stable platforms (3).

DIRECTION FINDING IN THE EARTH'S MAGNETIC FIELD, ERROR SOURCES, AND GYRO OPERATION

Direction Finding via the Earth's Magnetic Field

The Earth's Magnetic Field. The magnetic flux density ***B*** of the earth's field is a vector field whose flux lines define a direction anywhere within a few earth radii (the average earth

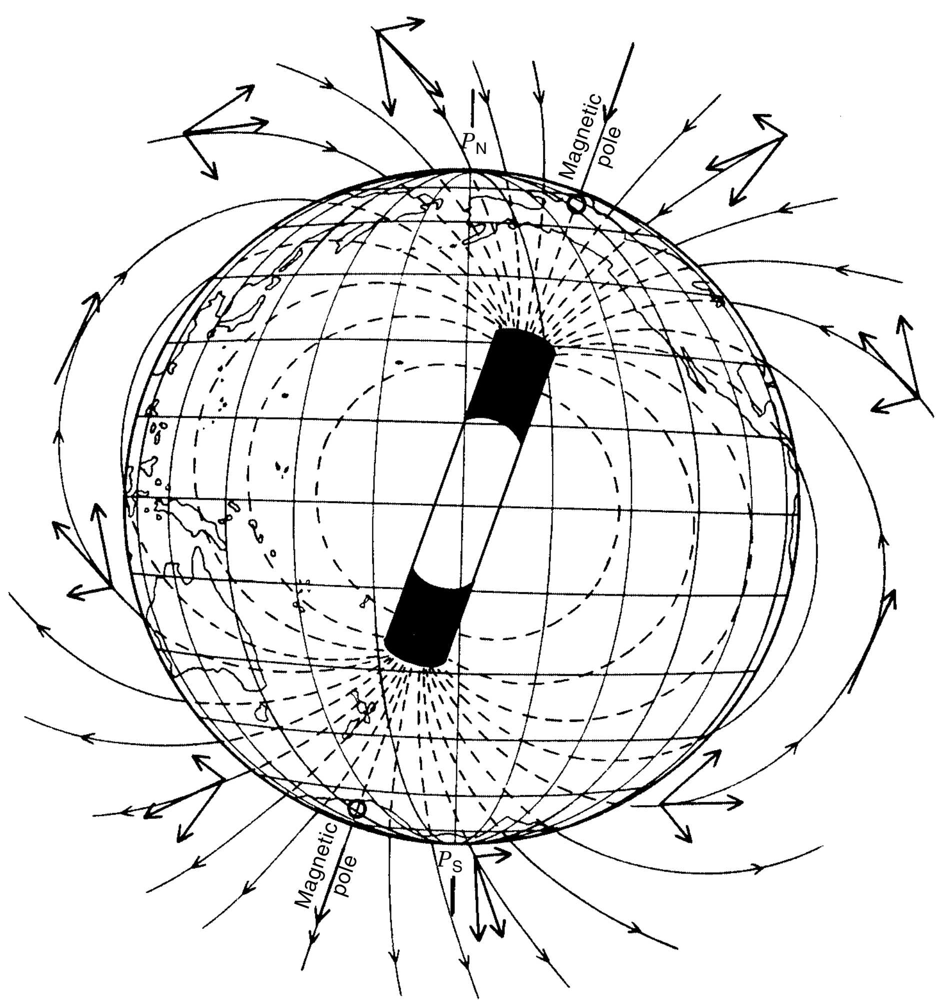

Figure 1. The earth's magnetic field. Dipole approximation. The vertical axis is the earth's kinematic axis, with poles N and S. [Source: Maloney, Ref. 7, p. 62.]

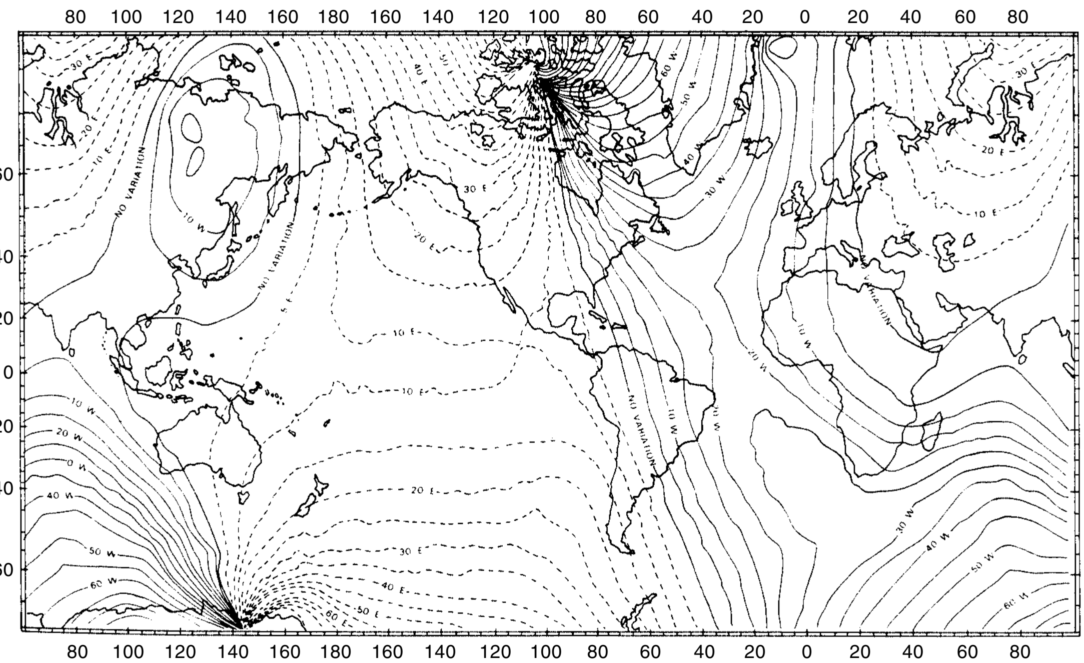

Figure 2. Deviation of horizontal compass directions from geographic South–North directions, or Variation. The poles in northern Canada and in Antarctica are clearly visible. [Source: Maloney, Ref. 7, p. 65.]

radius is 6371 km). Its direction can be sensed by placing a freely movable compass needle into it. The torque exerted by the earth's field on the compass dipole becomes zero when the dipole is aligned with the field at that location. Observations show that the earth's field itself can be described to a good approximation by a dipole (Fig. 1) with magnetic moment $m_E = 8.0052 \times 10^{22}\ \mathrm{A \cdot m^2}$ (4). This dipole is fixed at the earth's center and its axis is tilted by about 11° against the earth's axis (1996) (the number in parentheses is the year in which these coordinates were observed). An aligned needle whose movement is restricted such that it only shows the component parallel to the earth's surface will then point to the coordinates 79° North, 105° West (1996), which is called the north magnetic pole. The magnetic poles, by virtue of the fact that the magnetic flux lines at these points are normal to the earth's surface, constitute also perfect portholes for entry of charged particles from space (5). The following comments qualify the statements above:

1. There is no theory of earth magnetism that is able to predict the field's magnitude, direction, or time evolution sufficiently for navigational purposes.
2. The dipole approximation is good but a closer description of the actual earth field in terms of associated Legendre polynomials has several hundred terms in addition to the basic dipole term (4). The justification of using the dipole approximation stems from the fact that the dipole term in such an expansion is at least an order of magnitude larger than any subsequent term. An actual field distribution is shown in Fig. 2.
3. The south magnetic pole is not at the homologous point in the southern hemisphere; it is at 138° East, 65° South (1996).
4. The nature of the dipole-dipole interaction is such that opposite poles attract each other. In order to keep the notion that the magnetic pole closest to the geographic north pole is called north magnetic pole, the end of the compass needle that points north would have to be the needle's South Pole.
5. Local disturbances due to the presence of, man-made or other, magnetic depositions measurably distort the local field.
6. The field is slowly time dependent, both in magnitude and in direction. Daily changes exist. Figure 3 shows a measurement with a diurnal peak-to-peak excursion of 40 nT (6). These changes include a daily movement of

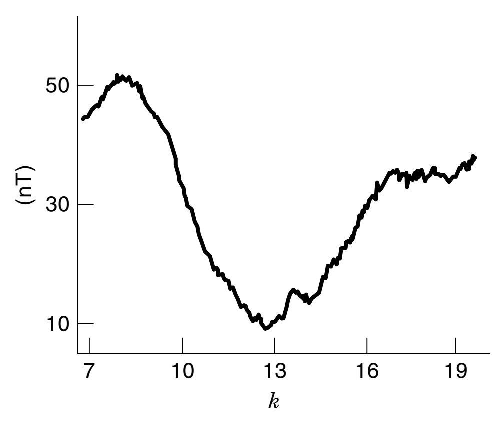

Figure 3. A measurement of a daily variation of the earth's magnetic field strength measured by nuclear magnetic resonance at Jussy, Switzerland, on 11 March 1959. A peak-to-peak change of 40 nT is apparent. [Source: Hochstrasser, Ref. 6, p. 232.]

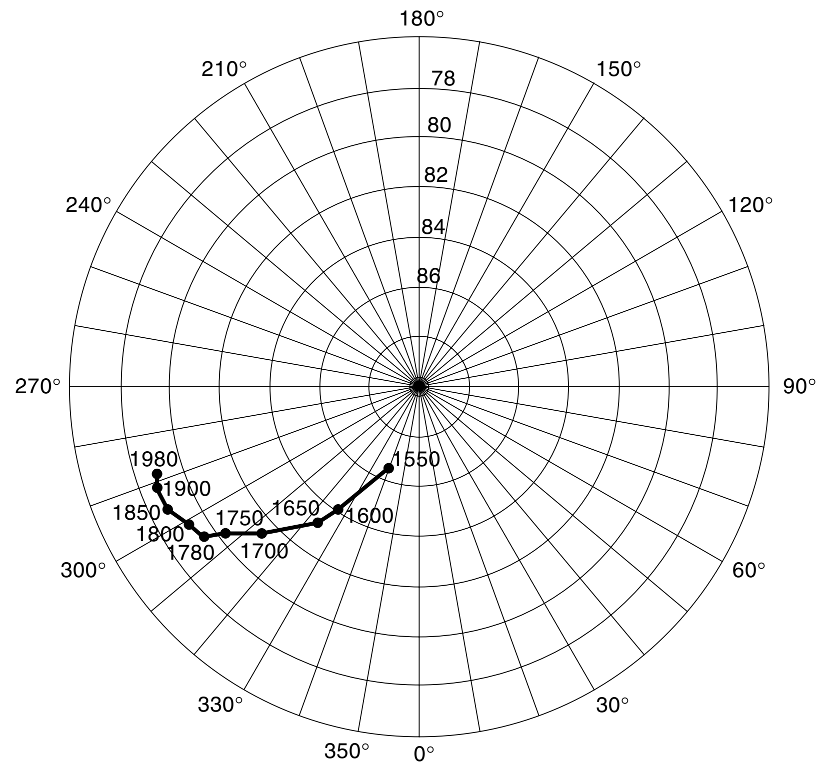

Figure 4. Movement of the geomagnetic North Pole between 1550 and 1980. The movement covers about 7 degrees of latitude and 50 degrees of longitude. Corresponding yearly variations depend on the location on earth. [Source: Reprinted from Ref. 4, p. 389, by permission of the publisher Academic Press Limited, London.]

the magnetic poles in elliptical paths with a major axis of about 50 miles (7).

The yearly change in magnitude is of the order of a few tens of nanoteslas depending on the location.

Paleomagnetic studies show that the field is known to have reversed itself several times in direction throughout geologic time. The movement of the north magnetic pole during the last 430 years is shown in Fig. 4 (4). The data are, however, not as smooth as the figure implies, as an estimate of the position in 1360 is 66° N, 291° E (69° W) (8). In recognition of this movement, modern atlases indicate only a “North magnetic pole area” (1998: near Bathurst Island, Canada).

7. Magnetic storms add additional time dependence. Their magnitudes may range up to 5 μT (6).

From detailed studies of the earth’s magnetic field it can be concluded that the accuracy of compasses is limited to about $\pm 0.5°$, due to the lack of stability of the earth’s field, local disturbances, and due to the difficulty of eliminating man-made fields in the vessel. This error is achievable with a compass whose vicinity has small residual stray fields that are properly compensated for, and whose declination is updated at regular intervals (at least annually).

It follows that a compass mounted in precision instruments, such as surveyor’s instruments, that are capable of ± 20 arcsec resolution can only give cursory initial guidance. Also, since 0.5° corresponds to about 55 km on the earth’s surface, it is impossible to direct, for example, a vessel with subkilometer accuracy on the globe using compasses alone. Meter accuracy of positioning can be achieved with GPS, and arcsecond accuracy of direction is achieved with inertial guidance systems, for example, with ring-laser gyros (RLG).

Analysis and Reduction of Disturbing Fields from Sources other than the Earth’s Field. Given that the earth’s field has at present an average magnitude of 53 μT and that saturation fields of permanent magnets reach magnitudes of 1 T, the necessity of elimination of or compensation for fields other than the earth’s field is an important consideration during installation and use of compasses.

Any extraneous ac fields can easily be removed by enclosure in a nonmagnetic metallic screen. This problem is, however, of lesser importance because the compass that is to sense the dc earth field is usually made with a large time constant of several seconds, thereby integrating to zero any ac fields with frequencies well above 1 Hz, for example, 50 Hz or 60 Hz fields generated by nearby power lines.

Time Constant. The large time constant for the needle is usually obtained by immersion in a liquid. While this method is very convenient, it is nevertheless not optimal from the point of view of minimizing the mechanical fluctuations or the mechanical noise. The latter is achieved by introducing feedback via a lossless mechanism, which, however, introduces an effective damping into the needle movement and thus lowers the noise temperature of the needle. Approaches of this kind are well known in electrical engineering but not in connection with magnetic compasses. The reason for this lack of development is probably the inherent, much larger limitation of precision due to the fluctuations of the earth’s field that make further development of the precision of magnetic compasses redundant for any but the shortest observation times. The compensation of extraneous dc fields can become a formidable problem.

Magnetic Stray Fields. There are two simple rules used to assess such fields’ magnitudes.

B *field produced by an electric dc current:* "One ampère produces 200 nT at one meter distance." This rule is derived from a current flowing in an infinite straight line. This field decreases with distance R as R^{-2}.

B *field produced by ferromagnetic material in the earth's field:* "One gram of iron produces 1 nT at 1 m distance." This rule is derived from the dipole field of a ferromagnet induced by the earth's field. The strength of a dipole field decreases with distance R as R^{-3}. The basis of this rule was first espoused in Ref. 9, p. 139.

The disturbances are such that the field due to the iron mass of a typical automobile or a ship needs to be carefully compensated for if a magnetic compass is to be used in such an environment.

The two rules are heeded to the extreme in high-resolution nuclear magnetic spectra in the earth's field. This method to obtain an accurate mapping of the magnetic field vector requires a housing of the apparatus that has to be set up at least 100 m away from power lines, and all ferromagnetic materials, including glass and bronze nails with traces of iron in them, have to be avoided in the construction of such sites (6).

Conversely, these rules require careful selection of the site of a compass on a vessel with a typical iron mass of the order of 10,000 tons. The second rule previously given suggests 10 μT at 10 m distance, of the same order as the horizontal field component (15 μT). Furthermore, the induced dipole moment of a ship depends not only on the ship's position on the globe but also on its alignment with the local field. Additionally, many types of ferromagnetic material may be present with vastly different nonlinearities, saturation, and remanences. This is dealt with approximately by distinguishing between a permanent part and a variable part of the disturbing field.

Methods of Compensation

Helmholtz Coils. A rigorous approach is to set up three orthogonal pairs of Helmholtz coils (10) whose currents are controlled by sensors of the three spatial components of the stray field. A pair of Helmholtz coils, with an axial separation equal to the coil radius, produces a rather homogeneous ***B*** field with a drop-off at the fringes varying with the fourth power of the distance and is therefore an excellent, precisely adjustable tool. As such it is the preferred tool where compensation down to picoteslas or less are required as in the production of magnetoencephalograms, where the fields to be measured themselves are at the nanotesla level. For regular compass applications in the field, this solution is too spacious and too expensive.

Compensating Permanent Magnets (11). The horizontal component of the disturbing field can be split into two orthogonal components. A straightforward method of compensation is therefore to place two small permanent magnets at 90° off the needle such that after any rotation of the vehicle with respect to the direction North, the needle shows in the same direction, that is, North plus the proper declination. The installation and adjustment of such compensating magnets require a proper empirical approach, since the direction and magnitude of the stray field are generally not known.

Flinders Bars (12). This is an ingenious method to compensate for the induced variable component of the magnetic dipole moment of a large vessel. One or more pieces of soft iron (with low remanence) are placed near the compass, such that their induced moment compensates the ship's magnetic moment. Since the field of the ship decreases with R^{-3}, relatively small pieces in close vicinity of the compass suffice; furthermore, any rotation of the ship or positioning of it at a different magnitude of the earth's field induces a proportional amount into ship and Flinders bar, and thus keeps the compensation valid.

Details for the use and adjustment of compasses are given in the excellent book by Maloney (7).

Degaussing Coils. These coils are used to neutralize the external field of the vessel, for example, to avoid detection by an adversary. Typically, however, the internal field increases as a consequence, and therefore the compensation has to be adjusted depending on whether the degaussing field is switched on or off.

Description of a Practical Magnetic Compass

A widely used instrument is the US Navy Standard No. 1 7.5 inch compass; see Fig. 5 (Ref. 7, p. 67). The housing is made from nonmagnetic material. Two bar permanent magnets of a nickel–cobalt–iron alloy are fixed and aligned with magnetic north on a compass card. A ring magnet may also be used. The card is in a bowl filled with alcohol or varsol to provide a passive dampening with a time constant of a few seconds. A mark, the lubber's line, at the compass rim indicates the ship's direction. The graduation on the compass card opposite to the lubber's line indicates the ship's course. Correction pieces of soft iron with negligible remanence and hard iron with a permanent dipole moment are placed in certain configurations outside the compass. The goal of initial correction is to compensate not only for the ship's horizontal dipole moment at various headings of the ship but also to an extent for the vertical magnetic moment of the ship and the earth's magnetic field, since generally only the horizontal component is used for navigation.

Astronomical Direction Finding

A very convenient fact is the position of Polaris close to the celestial north pole, which is equivalent to the direction of the kinematic earth axis. Although Polaris is by no means fixed (13), its motion is known to better than arcsecond accuracy, and for determination of positions or directions of such accuracy, the *American Ephemeris and Nautical Almanac* (14) can be used to correct the observation. The Almanac is updated every year.

An obvious disadvantage of this mode of direction finding is the restriction to nights and cloudless skies.

Gyros

A radical departure from the use of external aids is the use of instruments based on phenomena that provide a stable reference direction independent of external parameters. Gyros can be mechanical, based on nuclear spin, electron spin, or macroscopic quantum systems with an angular moment given by circulating photons (ring lasers) (15), electrons (16) including superconducting rings, atoms (atomic ring interferometers) (17), or in particular helium atoms (superfluid rings) (18). All gyros possess an internally defined direction, given either by the direction of the angular momentum or spin ***M*** of the spinning particle or quantum fluid, or by the area vec-

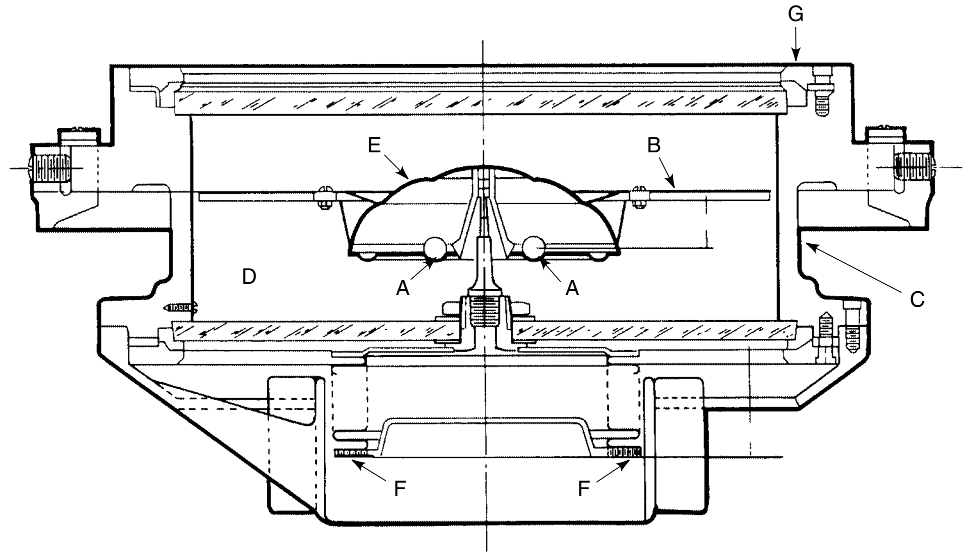

Figure 5. Schematic of a magnetic compass for a ship. The compass card is immersed in a bowl filled with liquid and an airbubble. The bowl is centered on a vertical axis, a pivot, which lets the card's magnets stay oriented toward North when the ship changes direction.

tor that is circumscribed by the moving particle. For spinning massive particles the time evolution in the absence of a torque $\boldsymbol{T}$ is described by

$$d\boldsymbol{M}/dt = \boldsymbol{T} = \mathbf{0} \tag{1}$$

that is, the vector $\boldsymbol{M}$ has then a constant direction. In particular, its direction stays constant under any movement of the vessel on which this system is placed if one ensures that the movement does not impart any torque to the system. For mechanical gyros, a Cardan suspension or similar gimbaling structures are designed to effect the decoupling of motions of the carrier from the gyro system; see Fig. 6.

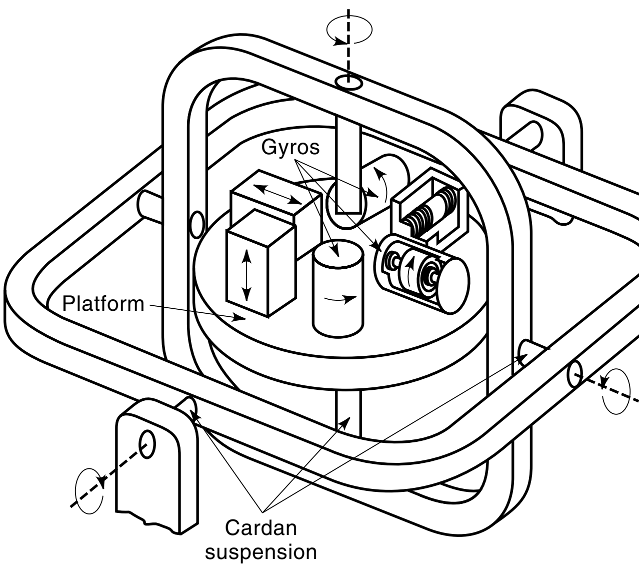

Figure 6. Schematic of an inertial navigation system with mechanical gyros and acceleration sensors on a navigational platform which is placed in a Cardan suspension. The platform keeps its attitude in space while the vessel on which it is mounted rotates freely in all directions. [Source: *Der Spiegel,* No. 10, 1984, p. 214.]

Readout. There must be an engineering solution to the readout problem. For gyros the position of the rotation axis has to be read out with an accuracy that the particular gyro system is capable of. The interaction of the readout and gyro has to be minimized to avoid any backlash that affects the gyro axis. An equivalent problem exists in optical gyros where the required interference of the optical beams at the output may produce backscattering into the cavity, which in severe cases synchronizes the sensing beams inside and thus renders the device useless for interrogation of its rotation status or as a compass.

In modern gyros, a viable system has to have a resolution of 1 arcsec or better. To appreciate this one notes that 1 arcsec of latitude corresponds to about 30 m. A submarine, to get to its berth after a long time of inertial navigation without external references, needs arcsecond accuracy.

Of the many possible internal directional references, mechanical gyros with various types of suspension and readout capabilities as well as ring-laser gyros and fiber-optic gyros have been developed to engineering standards. These devices are all capable of arcsecond resolution, as far as the principle of operation and the resolution of the readout is concerned. In mechanical gyros, the rotor of a motor constitutes the spinning part; in this case, the coupling to the outside world is basically through the stator's rotating magnetic field, with no friction except for the bearing of the rotor itself. High accuracy gyros employ air bearings.

Superconducting Mechanical Gyro (19). An extreme case of development is the Gravity Probe B where a satellite houses a mechanical gyro (19) which is to measure a secular change of 44 marcsec per year with an error of ± 1 marcsec per year. The superconducting rotor is electrostatically suspended in a vacuum, and the readout is optical. The development of this gyro has so far lasted for about three decades, but the previous specifications are now secured (1997).

Superfluid Gyros. They are based on the rotation of superfluid ^3He or ^4He in a closed circuit. They are in the develop-

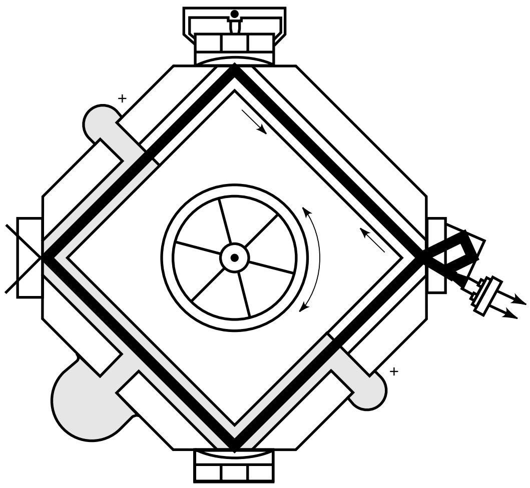

Figure 7. Schematic of a plane square ring-laser gyro with dc excitation. The interferometric readout is visible on top of the figure. The mirror on the left is moved in and out by a piezoelectric device, to compensate for pathlength changes. A dithering motor is mounted in the center to prevent the beams from locking at small rotation rates. [Source: G. T. Martin, *IEEE Spectrum,* Feb. 1986, p. 50. © 1986 IEEE.]

ment stage. Their basic resolution is theoretically far superior to ring lasers, and more so to classical mechanical gyros, but laboratory results (1997), while encouraging, show nevertheless that much work needs to be done to approach the anticipated performance limits.

Ring-Laser Gyros and Fiber-Optic Gyros. The counterrotating beams in an optical cavity form typically a triangle or a square (Fig. 7). The beams develop a phase difference as soon as the cavity is rotated around its area vector in absolute space. If the rotation in three dimensions needs to be known, a three-gyro system as in Fig. 8 is employed. In interferometer fiber-optic gyros (IFOGs, Fig. 9), this phase difference is measured by special readout techniques (20).

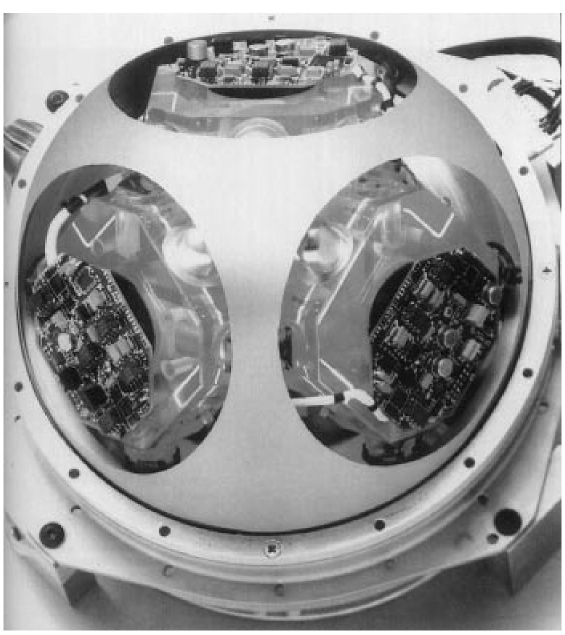

Figure 8. Three-gyro mount to sense three-dimensional rotation. [Source: G. T. Martin, *IEEE Spectrum,* Feb. 1986, p. 49. © 1986 IEEE.]

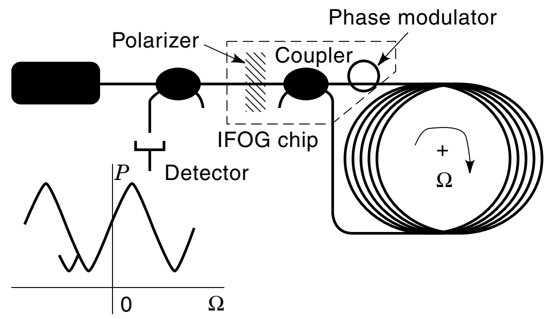

Figure 9. Principle of an interferometer fiber-optic gyro (IFOG). The fiber-optic loops on the right are excited by an external laser on the left whose beam is split into the two counterrotating beams by the coupler. A second coupler, immediately at the laser output, picks up the returning beams and feeds them into the detector where a phase difference is detected. [Source: Hotate, Ref. 3, p. 108.]

In self-oscillating loops, the requirement of phase closure forces a shift of the optical oscillation frequency, which is opposite for beams in opposite directions. The corresponding frequency difference Δf between the beams is then given by the Sagnac formula (15)

$$\Delta f = (1/2\pi)d\phi_o/dt = (4/\lambda L)\boldsymbol{A}\cdot\boldsymbol{\Omega} = (4/\lambda L)A(d\phi_r/dt)\cos\alpha \tag{2}$$

where $\boldsymbol{A} = (1/2)\oint \boldsymbol{r} \times d\boldsymbol{l}$ is the area enclosed by the beam path, represented by its normal vector, $\boldsymbol{\Omega}$ is the angular velocity vector to which the device is subjected, α is the angle between the two vectors, λ is the vacuum wavelength of the laser light, and L is the perimeter $L = \oint dl$, or the length of the cavity. This general definition of $\boldsymbol{A}$ and of L includes also nonplanar RLG configurations as in the quadrilateral Raytheon device, for example. After integration of Eq. (2), the optical phase ϕ_o of the RLG output becomes proportional to the mechanical angle of rotation, ϕ_r, if the direction of $\boldsymbol{\Omega}$ stays constant.

Certain gyros (21) work with four beams, with pairs of counterrotating beams. In those cases, the beat frequency doubles.

Scale Factor. This denotes the sensitivity of the transduction from the mechanical rotation rate to the optical frequency, or conversely the ratio of equivalent electronic phase change of the ring-laser output frequency to the mechanical rotation angle of the device. It is

$$S = 4A/(\lambda L)[\mathrm{Hz}/(\mathrm{rad/s})] \tag{3}$$

In normal usage, the scale factor is, however, given as S' with the units counts per arcsecond or

$$S' = [4A/(\lambda L)](\pi/180)(1/3600) \quad (\mathrm{counts/arcsec}) \tag{4}$$

A typical RLG operating at $\lambda = 633$ nm, with an area $A = 1\ \mathrm{dm}^2$ and a length $L = 40$ cm then has a scale factor $S' = 0.76$ counts/arcsec.

Sensitivity to Attitude. The scalar product $\boldsymbol{A} \cdot \boldsymbol{\Omega} = A\Omega \cos\alpha$ in the Sagnac Eq. (2) gives rise to a sensitivity to the tilt angle α,

$$|d\Delta f/d\alpha| = S\frac{d\phi_m}{dt}\sin\alpha \tag{5}$$

Equation (5) is important for North-finding ring-laser gyros, where the device is oriented on the rotating earth until the output is maximized, or with equivalent strategies. The uncertainty $d\alpha$ is a measure of the achievable error of the RLG as an indicator of the direction North, that is, the direction of the kinetic axis of earth rotation. Since $d\alpha \propto 1/\sin\alpha$, better sensitivity is basically obtained by finding the direction(s) orthogonal to North.

Limits of Accuracy of Ring-Laser Gyros. Contrary to mechanical gyros, ring-laser gyros possess no rotating mass. Errors due to torque, Eq. (1), that give rise to a variety of effects in mechanical gyros are therefore absent. Absent also are basic time constants when the ring laser changes its rotation rate or its attitude. Equally, no spin-up time is required to initialize the sensor. In practice, the electronics surrounding the gyro cause warm-up times that are typically in the millisecond range.

Basic quantum limits have been reached, which allow, for example, one to measure a rotation as slow as the earth's with very high precision.

Quantum Noise and Optical 1/f Noise. The photon statistics in a laser beam produce a frequency fluctuation whose white one-sided power spectral density $S_{\Delta f}$ is given by

$$S_{\Delta f} = hf_0^3/(PQ^2) \tag{6}$$

where h is Planck's constant, $= 6.6 \times 10^{-34}$ W·s^2, f_0 is the average optical oscillation frequency (Hz), P is the optical power in the output beam (W), and Q is the passive quality factor of the cavity in which the beam is created.

For a standard RLG operating with two independent counterrotating light beams, the beat frequency Δf has twice this noise power density, provided that the beams are uncorrelated and each has the same optical power P. In a four-frequency gyro, the noise power quadruples, but there is a net gain in the signal-to-noise ratio of the latter, because the doubled output frequency [see Eq. (2)] is accompanied by a root-mean-square frequency fluctuation that is only $\sqrt{2}$ times larger.

In ring-laser systems a $1/f$ noise component has also been observed empirically. Its power spectral density is

$$S_{\Delta f, 1/\nu} \cong 4(f_0^2/Q^4)(1/\nu) \tag{7}$$

where ν is the Fourier frequency of the oscillator. The inverse quartic power dependence of Q suggests, however, only a small contribution to optical cavities with large Q's as in RLGs.

Time Averaging. When averaging the beat frequency between two beams over a time duration T, the average has an rms error (24)

$$\overline{\Delta} f_{\mathrm{rms}} = [hf_0^3/(PQ^2)]^{1/2}(1/T)^{1/2} \tag{8}$$

The error of the average frequency is thus reduced proportional to $1/\sqrt{T}$. The stability of the system with time sets a useful upper limit of measuring time and with it the ultimate limit of accuracy.

An RLG as mentioned before, with a quality factor $Q = 1 \times 10^9$, a power output of 100 μW, and a measurement time of 1 min has an rms frequency fluctuation of 3.4 mHz from Eq. (8). The equivalent mechanical phase excursion due to rotation follows from S' above as 4.5 marcsec in 1 min of observation time.

As a North finder [Eq. (5)] the compass error of the same device, again with 1 min of observation, is about 6 arcsec on the earth, if measured with $\alpha = 90°$.

RLGs are so far superior to IFOGs on account of their larger quality factors achievable.

Optical Frequency. The beat frequency versus rotation rate, or the scale factor, can be increased by increasing the optical frequency, that is, lowering the wavelength. Gas-ring lasers are using a helium-neon mixture, approximately 7 : 1, whereby the helium may be actually the isotope ^{3}He, which achieves a better conversion of input power to optical power than the naturally occurring ^{4}He, and an approximately equal mixture of the two naturally occurring neon isotopes ^{20}Ne and ^{22}Ne to minimize the cross coupling between the two counterrotating beams. Other lasing gases with larger optical frequencies have been found to be inadequate for RLGs.

The optical cavity needs to be excited in one and only one mode clockwise (CW) as well as counterclockwise (CCW). This limits the power, whose upper limit is given by the onset of multimode excitation. Fiber-optic rings are operated at wavelengths in the infrared. The preferred wavelength is 1.55 μm with a fortuitous coincidence of minimum absorption in the fused silica fiber used and europium-doped fibers that can be used as in-line amplifiers.

Area and Path Length. For a given geometry of the ring the area is proportional to L^2, and it follows from Eq. (2) that larger-area rings are more sensitive. For usual navigational applications decimeter-sized rings operating with helium and neon are quite sufficient, mainly due to the large quality factor [Eq. (2)] that can be achieved with high-quality mirrors. The latter have reflection losses approaching 1 ppm (1997).

Fiber-optic rings have quality factors that are several orders of magnitude smaller. The effective area is therefore increased by making spools of many turns. With n complete turns the effective area is increased n-fold. The output frequency (or the equivalent phase shift) is then also increased n-fold.

Optimum RLG Geometry. Of all possible planar and nonplanar polygons, plane regular polygons have the largest area to perimeter ratio and therefore the largest scale factor [see Eq. (3)]. Furthermore, given a fixed circumscribed circle for them and employing mirrors with a fixed finesse, the optimum geometry of a gas-ring laser is a square. With this geometry, the gain in the area to perimeter ratio overcompensates the additional loss when going from an equilateral triangle to a square. Square rings also have additional advantages (15). However, in practice triangular-shaped as well as square-shaped RLGs are used.

These geometric considerations do not apply to fiber-optic rings. There the loss mechanisms are distributed in the fiber, and the total loss and therefore the noise power are roughly proportional to the total length L of the fiber. The signal is

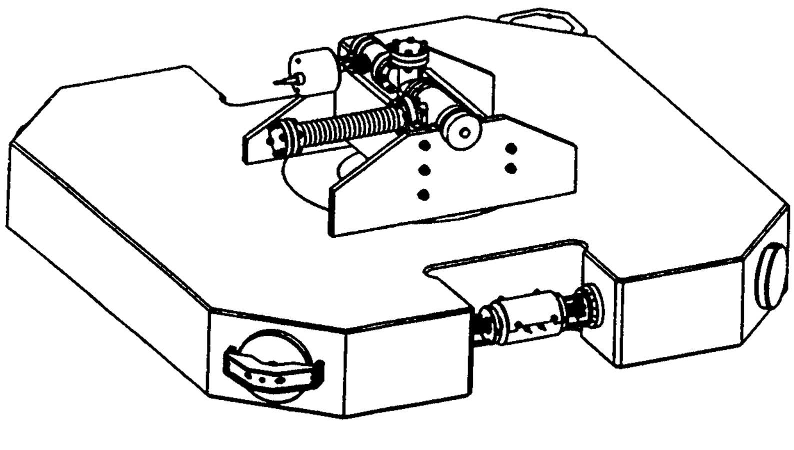

Figure 10. Schematic of a high-resolution research ring laser for detection of fluctuations of the earth rotation. The main differences to RLGs are the large size with a 1 m^2 area mounted in a stiff 600 kg Zerodur block, a very-high-Q cavity, and RF excitation.

also proportional to the length; thus the beat frequency to rms frequency fluctuation ratio generally increases in fiber-optic rings with $\sqrt{L}$.

Practical RLG. See Fig. 8. The beam path is encased in a material with a very low coefficient of thermal expansion; at this time (1997) the two materials ultralow expansion (ULE) quartz and Zerodur, with thermal expansion coefficients approaching 0.01 ppm/K, are used. Compared to these artificial materials, the best-known naturally occurring low-expansion material is fused silica with 0.55 ppm/K, which is, however, inadequate, since the amount of electronics required to keep a single optical cavity mode tuned to the laser gain medium at even modest ambient temperature changes becomes prohibitive.

The maturity of these devices is underlined by the fact that as early as in the mid-1970s a "laser navigation system" was planned for the Sojourner that landed on Mars on 4 July 1997 (25).

Figure 10 shows a high-resolution ring laser specifically constructed for geodetic research (26). The beams circumscribe a plane square area of 1 m^2. It is designed to resolve the earth's rotation rate to about one part in 10^8, which comes close to the known fluctuations of the earth's rotation vector $\mathbf{\Omega}$.

MORE DETAILED ANALYSIS OF THE EARTH'S MAGNETIC FIELD, COORDINATE SYSTEMS, AND NEW DEVELOPMENTS

Some Details about the Earth Magnetic Field

Units. The SI system is used here as it is throughout this encyclopedia. In order to facilitate the reading of older literature, three hard-to-eradicate non-SI units are translated.

$$\text{One gauss} = 1\,\text{G} = 0.0001\,\text{T} = 100\,\mu\text{T}$$

$$\text{One gamma} = 1\,\gamma = 1\,\text{nT}$$

$$\text{One oersted} = 1\,\text{Oe} = 1000/4\pi\ \text{A/m} = 79.6\,\text{A/m}$$

The Magnetic Dipole Approximation. The magnetic flux density $\boldsymbol{B}$ of a dipole at far field is given in coordinate-free form by (10)

$$\boldsymbol{B} = [\mu_0/(4\pi R^3)]\left(3\frac{\boldsymbol{R}\cdot\boldsymbol{m}}{R^2}\boldsymbol{R} - \boldsymbol{m}\right) \qquad (9)$$

whereby the dipole with magnetic dipole moment $\boldsymbol{m}$ is placed at R = 0. The far field of $\boldsymbol{B}$ is then given at the vector distance $\boldsymbol{R}$.

With the equivalent magnetic moment at the earth's center pointing in the direction of magnetic north, the far field at the earth surface is given by

$$\boldsymbol{B} = [\mu_0 m/(4\pi R_\text{E}^3)](\boldsymbol{a}_\text{R} 2\sin\phi_m - \boldsymbol{a}_{\phi_m}\cos\phi_m) \qquad (10)$$

and its magnitude by

$$B = [\mu_0 m/(4\pi R_\text{E}^3)]\sqrt{1 + 3\sin^2\phi_m} \qquad (11)$$

Here, $R_\text{E} = 6.37 \times 10^6$ m is the average earth radius, $m \cong 8 \times 10^{22}\ \text{A}\cdot\text{m}^2$ just given is the magnitude of the equivalent dipole approximating the earth's field, ϕ_m is the magnetic latitude (positive for northern latitudes, negative for southern latitudes), $\boldsymbol{a}_R$ is a unit vector at the field point normal to and pointing away from the earth surface, and $\boldsymbol{a}_\theta$ is a unit vector pointing to magnetic north. The magnitude near the north magnetic pole ($\phi_m = 90°$) is 62 μT, whereas at the magnetic equator ($\phi_m = 0°$) it is 31 μT. The average field at the earth surface is 53 μT but the average horizontal component of it is only 15 μT.

The curl of any $\boldsymbol{B}$ field is always 0.

Coordinate Systems. In the literature, generally three coordinate systems are in use.

Right-Handed Spherical Coordinate System r_s, θ_s, ϕ_s. Here the radius r_s is measured from the earth's center, the polar angle is measured starting at the (geographic) North Pole with $\theta_s = 0°$ and ending at the (geographic) South Pole with $\theta_s = 180°$, and the azimuth is measured from a reference meridian (Greenwich) onwards east for a full circle, or 360°.

Geographic-Navigational Coordinate System for the Earth Surface ϕ, λ. Here the latitude ϕ is counted from the equator on, positive towards North, negative towards South. The longitude (meridian) λ is counted from Greenwich on positive towards West and negative towards East, so that the meridian opposite the Greenwich meridian has both assignments, ±180°. For most purposes of navigation, the earth's surface can be considered as a perfect sphere of radius 6370.7 km except for local features like mountains and valleys. This coordinate system is not right-handed.

Coordinate Transformations. The piecewise linear transformations between the geographic-navigational and spherical coordinate systems can be written with the aid of the signum function sgn(x) [sgn(x = 0) = 0, sgn(x > 0) = 1, sgn(x < 0) = −1] as

$$\begin{aligned} \phi_s &= -\lambda + \mathrm{sgn}(\lambda)[\mathrm{sgn}(\lambda) + 1]180^\circ \\ \lambda &= -\phi_s + \mathrm{sgn}(\phi_s - 180^\circ)[\mathrm{sgn}(\phi_s - 180^\circ) + 1]180^\circ \\ \theta_s &= 90^\circ - \phi \\ \phi &= 90^\circ - \theta_s \end{aligned} \tag{12}$$

Magnetic Coordinate System ϕ_m, λ_m. This system is similar to the geographic-navigational system except that the magnetic latitudes $\phi_m = \pm 90^\circ$ are defined by the geomagnetic poles.

The local angle between a magnetic meridian and the corresponding local geographic meridian is the declination D (degrees).

Because of local and temporal changes of the direction of the magnetic field that may amount up to about 0.5° even for observations over one day and/or over short distances (6), the magnetic coordinates, including the most important declination, are not reliable when better accuracies are required. The compass mounted on a level or theodolite for surveying purposes therefore is exclusively used for cursory orientation in the field, not for measurements, considering that these instruments are generally capable of performing at the subarcminute level for civil engineering tasks.

Calculation of Declination. Given the spherical coordinates of the north geomagnetic pole θ_{s,N_m} and ϕ_{s,N_m}, a location X with coordinates $\theta_{s,X}$ and $\phi_{s,X}$, and the dipole approximation of the earth's field, the declination D at X becomes, through application of spherical-trigonometric equations,

$$\begin{aligned} D &= \cos^{-1}\left[\frac{\cos\theta_{s,X} - \cos\theta_{s,N_m}\cos\phi_{m,X}}{\sin\theta_{s,N_m}\sin\phi_{m,X}}\right] \\ \phi_{m,X} &= \cos^{-1}[\cos\theta_{s,N_m}\cos\theta_{s,X} \\ &\quad + \sin\theta_{s,N_m}\sin\theta_{s,X}\cos(\phi_{s,X} - \phi_{s,N_m})] \end{aligned} \tag{13}$$

where $\phi_{m,X}$ is the great circle distance from the north geomagnetic pole to the location X expressed as an arc, in degrees, as seen from earth's center. Care has to be exercised in applications of these formulas, as the relation of the spherical coordinates to the navigational coordinates is only piecewise linear.

Distortion of the Earth's Magnetic Field by Ferromagnetic Materials. The magnetic dipole moment $\boldsymbol{m}$ of a piece of material with relative permeability μ_r and volume V that is placed in a field H is estimated by

$$\boldsymbol{m} = (\mu_r - 1)V\boldsymbol{H} \tag{14}$$

One kilogram of iron with a mass density of 8580 kg/m^3 placed in a magnetic field H = 50 μT/(μ_0) = 40 A/m with a relative permeability of about 2000 satisfies the rule for distortion by magnetic dipoles given previously.

Size of Torque. Given the magnetic dipole moment $\boldsymbol{m}$, the torque exerted on this moment in a field $\boldsymbol{B}$ is

$$\boldsymbol{T} = \boldsymbol{m} \times \boldsymbol{B} \tag{15}$$

The torque on a compass needle is small. With the magnetic moment of a needle of 2 A · m^2, the torque corresponds to that of a 10 mW electric motor running at 1000 rpm. Here the relation $P = T\omega$ is used, where P is the power generated (W), and ω is the angular frequency (rad/s).

Some Details about Ring Lasers and Other Gyros. A ring laser is an example of an ordered spin system with correlation lengths that may extend to megameters depending on the quality factor of the optical cavity in which the light beam is generated and on the power output. It is one of a very few macroscopic quantum systems. This one uses photons; others are superfluid systems using liquid ^{3}He or ^{4}He and electronic superconductors using electrons (Cooper pairs), and magnetic systems (quantum Hall effect). Gyros with low-noise electronic beams are also under active investigation. Of these novel principles, only the RLG and fiber-optic gyros are at this time engineered into mature systems. A recent addition to these tools are atomic interferometers in ring arrangements.

All these systems have in common a closed path enclosing a finite area around which the quantum fluid is circulating. In ring lasers this circulation is internally generated. In other systems the quantum fluid is injected by an external source whereby with specific devices (beam splitters or generally flux splitters), two different paths are created. At the point of rejoining there is a detection system that detects the phase difference, which is then a measure of the rotation rate.

Frequency Difference and Phase Difference in Optical Gyros; Sense of Rotation. The general problem of obtaining an output signal that is linearly related to the rotation of the device is solved as follows. Given a frequency split of the counterrotating beams $f_{1,2} = f_0 \pm \Delta f/2$ during a rotation, with a corresponding split in the wave numbers $k_{1,2} = k_0 \mp \Delta k/2$, and phase splits after rotation of $\phi_{o1,2} = \phi_{o0} \pm \Delta\phi_o/2$. The split may have a slow time dependence. The two optical beams are overlaid to each other, say in the x direction. The rapidly oscillating electric fields of the beams, with frequencies on the order of 10^{14} Hz to 10^{15} Hz and splits ranging up to 10^6 Hz, having equal amplitudes E_0, are then

$$E_{1,2} = E_0 \exp[j(2\pi f_{1,2}t - k_{1,2}x)] \tag{16}$$

The observed output power of the combined beams is then

$$P_{out} \propto |E_1 + E_2|^2 = 2E_0^2[\cos(2\pi\Delta f t - 2\pi\Delta k x + \Delta\phi_o) + 1] \tag{17}$$

Observation at $x = 0$ results in a cosinusoidal pattern with frequency Δf (ring lasers) and phase $\Delta\phi$ (nonoscillating Sagnac interferometers) superimposed on a background of constant power. The time dependence is then detected by a low-frequency optical ac detector, typically by a silicon junction device. If the direction of rotation is desired, the beams are slightly misaligned to produce a moving interferometric fringe pattern. Two detectors are then placed a quarter-fringe apart. Their phase differences versus time determine the direction of rotation uniquely.

It also follows from the extreme smallness of the Sagnac frequency relative to the optical frequency that the beam path must be extraordinarily symmetric in clockwise and anticlockwise direction. Any nonreciprocity will lead to an additional non-Sagnac effect, either an additional phase or a frequency bias depending on the nature of the nonreciprocity.

Modifications of Optical Beams. Two new types of optical beams are investigated: One investigation centers on the possibility of correlated emission of the amplifying plasma with the goal of creating beams whose quantum noise is correlated, so that the noise of the interfering beams can be greatly reduced (27). Another approach is to use squeezed light (28), where the quantum noise of the amplitude fluctuations is greatly increased in order to substantially reduce the frequency fluctuation, which keeps the Heisenberg uncertainty relation intact, but affords RLGs with much lower (frequency) noise.

Polarization-Preserving Fiber. When the beams are guided by optical fibers, the original polarization may not be maintained if irregularities are present along the fiber. Special fibers have been manufactured with a slightly elliptical cross section that enforces the same polarization throughout. This is necessary for several reasons, one of the most important is the necessity to have the polarizations of the two beams aligned when they are brought to interference.

Quality Factor Q. This factor enters with its inverse square in the noise and is therefore an important factor in the sensitivity of a gyro, whether as an interferometer or as a self-oscillating device.

In ring lasers with an amplifying gas at low pressure of the order of 1 torr, the overall quality factor Q of the optical cavity is mainly determined by the losses of the mirrors (29). Given a square cavity with four mirrors, each with a relative power loss $1 - R$, where R is the power reflectivity of a mirror, $Q = (\pi L/2\lambda)/(1 - R)$. In a square research ring laser (Fig. 10) with good mirrors approaching $1 - R = 1$ ppm each, $L = 4$ m, and $\lambda = 633$ nm, a quality factor of 9.9×10^{12} can be achieved (26). Q increases linearly with the perimeter of the cavity, as the losses are localized.

In fiber-optic gyros, the losses are distributed along the fiber. With a total loss of α dB/m (1000α dB/km), the quality factor becomes independent of the path length and is $Q = (2\pi/\lambda)(10/\ln 10)(1/\alpha)$. A good fiber with $\alpha = 0.001$ dB/m (1 dB/km) at $\lambda = 1.55$ μm gives rise to $Q = 1.8 \times 10^{10}$, almost three orders of magnitude less and independent of the length of the cavity.

GLOSSARY

Agonic line. A line on the earth's surface that connects points with zero magnetic variation.

Cardinal directions. North, East, South, and West.

Compass error. Algebraic sum of variation and deviation.

Declination. Variation.

Deviation. Errors caused in the compass direction by local effects.

Dip. Inclination of a perfectly balanced compass needle (inclinometer or dip circle) with respect to the local horizontal.

Isogonic line. A line on the earth's surface that connects points with equal magnetic variation.

Point. Subdivision of the compass circle into 32 points. 1 point = 11.25°.

Variation. Angle, in degrees, between the magnetic meridian and the corresponding geographic meridian. It is the direction of the horizontal component of the magnetic field with respect to the geographic coordinate system.

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COMPLEMENTARITY. See Duality, mathematics.

COMPLEX PROGRAMMABLE LOGIC ARRAYS. See Integrated software; Programmable logic arrays.

COMPLEXITY AND COMPUTABILITY. See Computational complexity theory.

COMPLEXITY THEORY. See Algorithm theory.

COMPONENTS. See Smart cards.

COMPONENTS, SLOT LINE. See Slotline components.

COMPOSITE INSULATORS. See Outdoor insulation.

COMPOUND SEMICONDUCTORS. See III-VI semiconductors; Photorefractive properties of gallium arsenide and superlattices.

COMPOUND-SEMICONDUCTOR TRANSISTORS. See Heterojunction bipolar transistor.

COMPRESSION OF DATA. See Data compression codes, lossy.

COMPRESSION, VIDEO. See Video compression methods; Video compression standards.

COMPRESSOR, EXPANDER. See Compandors.