HALL EFFECT TRANSDUCERS

Hall effect transducers belong to a family of semiconductors that produce an output voltage proportional to a magnetic field or magnetic induction. Initially, the Hall effect was a curiosity comparable to the Seebeck or Peltier effects, but soon it was realized that it could be used for the study of conduction mechanisms in semiconductors. From 1960 on, the first technical Hall effect applications were introduced—for example, the magnetic field measurement probe, magnetic multiplier, and current-to-voltage transducer. At present, a range of powerful Hall integrated circuits are available for magnetic field measurement purposes, the position control of objects, and angle position of wheels. The latest development was the discovery of the quantum Hall effect. This currently has no technical applications, but it has permitted electrical metrology to make considerable advances.

In 1879, E. H. Hall made the remarkable discovery (1) that a magnetic field perpendicular to a conducting foil deviates the charge carriers in a conductor in such a way that a voltage is generated transversally to the foil. The use and exploitation of this effect was, however, severely restricted to research purposes due to the shortage of good materials. Favorite materials during the exploration period were bismuth and even germanium, but the stability obtained with these elements was very disappointing due to their temperature characteristics. With the expansion of research on semiconductors after World War II, Hall measurements were refined with success for the determination of resistivity, carrier concentration, carrier type, and mobility in semiconductors. This research led to still better semiconductor materials; soon, the first industrial instruments for magnetic field measurements with sensitive calibrated Hall effect sensors were introduced on the market.

With the development of integrated circuits (IC) the applications domain of Hall devices was further extended because it became possible to integrate an amplifier with the Hall sensor in one unit. Such an IC can be used for several purposes: position detection, measurement of current, and electronic compass. In 1980, the German scientist Klaus von Klitzing (2) discovered a new effect that was strongly related to the classical Hall effect: the quantum Hall effect. By studying a two-dimensional electron gas at low temperatures in the presence of a very strong magnetic field (14.6 T), it was found that the ratio of the Hall voltage to the device current depends only on the Planck constant h (6.626×10^{-34} Js) and the elementary charge e of the electron (1.6021×10^{-19} C). The von Klitzing effect was very important to electrical metrology because it became possible to replace the material ohm standard in the same way as the volt standard was replaced by the Josephson junction.

HALL EFFECT

For the derivation of the classical Hall effect (3), a rectangularly conducting sample (Fig. 1) with width w, length L, and thickness t is considered. At the four side planes, the contacts 1, 2, . . . 6 have been added for the connection of a current source I (1, 4) and for potential measurements (2, 3, 5, 6). It is supposed that the width of the contacts is small in comparison with the distance l between (2, 3) and (5, 6). Also, the distances l_1 and l_2 are much larger than the width w. The sample is placed in a magnetic field with induction B perpendicular to its largest plane. Figure 1(b) shows that this plane is associated with a rectangular coordinate system with the x-axis in parallel with the length axis of the sample. As long as $B = 0$ in this system, the charge carriers in the sample move along the dotted straight lines between contacts 1 and 4.

The particular shape has the advantage that it can also be used for the determination of the classic resistivity of the

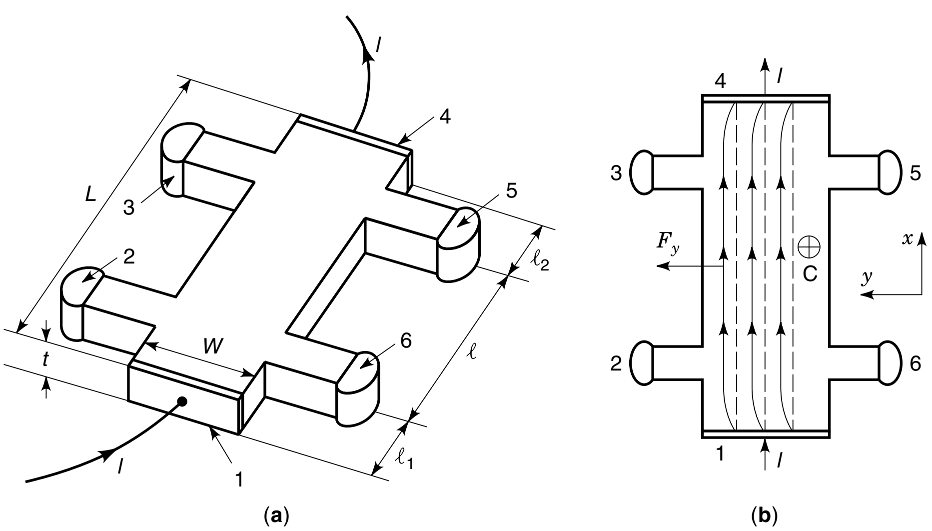

Figure 1. (a) Hall bridge-type sample with four voltage contacts (2, 3, 5, 6) and current contacts (1, 4); (b) deviation of current lines due to an applied B-field. The contacts 1 and 4 are supposed ideal, and ℓ_1, ℓ_2, and ℓ are much larger than the width W of the sample. The carrier density at the left and right side will differ due to the Lorentz force F_y.

sample material. The resistivity ρ_{xx} in the x-direction of the sample is determined with $B = 0$, by measuring the potential difference V_{23} (or V_{56}) between the contacts (2, 3) or (5, 6). It follows that

$$\rho_{xx} = \frac{V_{23}}{I}(wt/l)$$

If the contacts are symmetrically placed around the length axis of the sample, then we expect the voltage between contacts 2 and 6 (or 5 and 6) to be zero and in this case we also have $V_{23} = V_{56}$.

In the presence of a B-field, the moving charge carriers deviate from the straight lines due to the Lorentz force that the field exercises on the carriers. If the current flows in the x-direction, the carriers with an elementary charge q have a velocity v_x, and the corresponding force will be in the y-direction. The absolute value F_y of the Lorentz force is:

$$F_y = Bv_x q$$

As a result, the carrier concentration will be somewhat higher at the left-hand side than at the right-hand side of the sample [Fig. 1(b)]. It means that internally an electric field E_y counteracting the effect of the Lorentz force is generated. This field exercises a repelling Coulomb force qE_y on the carriers and in a steady-state situation both forces are equal and thus

$$E_y = Bv_x$$

The carrier concentration n (number of carriers per m^3) and the velocity v_x are related to the device current I, because the amount of charge displaced in 1 s is equal to the product $wtnqv_x$. Therefore we have

$$v_x = I/wtnq$$

Because the field E_y is constant for all points between contacts (2, 6) or (3, 5), it follows immediately that the voltage V_H between (3, 5) or (2, 6) is equal to $w \times E_y$

$$V_H = BI/tnq = R_H BI/t, \text{ with } R_H = 1/nq \tag{1}$$

The constant R_H is the Hall coefficient (independent of the width w), and it has the dimension m^3 C^{-1}. For the sensitivity of the device to be large, it is thus required that the sample is thin and the carrier concentration not too large. If these conditions are fulfilled, the mobility of the carriers (proportional to the carrier velocity) has to be large in order to give a reasonable conduction in the x-direction. This explains why in ordinary conductors the Hall effect is extremely small. From the foregoing derivation we can deduce some important facts.

1. Measurement of the Hall voltage permits determination of the average polarity of the charge carriers. It was indeed only supposed that the direction of the current was known, but this current could as well be established with holes or electrons depending on the type of semiconductor used. In the first case the left-hand side of the sample will have an excess positive charge and point 2 will become positive with respect to 6 (or 3 to 5). In the second case, 2 will be negative with respect to 6. For extrinsic p-type materials we find from R_H the carrier charge density (in C/m^3) $nq = 1/R_H$ and for the n-type this becomes $-1/R_H$.

2. From Eq. (1) of the Hall voltage we obtain the Hall resistance $R_{xy} = V_H/I$

$$R_{xy} = R_H B/t$$

This quantity increases linearly with the applied field B. The Hall effect device is also called a Hall transducer because it converts the value of the magnetic field into a voltage.

3. It can be seen that the current lines in the sample are slightly longer when the field is applied than when it is not. It can therefore be expected that the longitudinal resistivity ρ_{xx} increases when the field increases; this is the magnetoresistance effect. In the Hall sample, however, the magnetoresistance effect is of minor importance because the ratio $L:w$ is large.

Until now it has been supposed that the contact pairs are symmetrically placed with respect to the sample. If this is true, no Hall voltage can be generated between the contact pairs when the field is zero. In practice there always remains a slight asymmetry and, because of the finite resistivity in the x-direction, a small offset voltage V_{os} will add to the Hall voltage. This offset voltage can only be eliminated by reversing the direction of B or by rotating the Hall device over 180° around its x-axis and combining the two resulting Hall voltages. The two voltages corresponding to this procedure are, respectively, $V_1 = V_{os} + V_H$ and $V_2 = V_{os} - V_H$.The Hall voltage thus is found from $(V_1 - V_2)/2$. For ac measurements, the offset can be eliminated by blocking the dc component of the output voltage of the Hall device.

QUANTUM HALL EFFECT

In order to observe the quantization of the Hall resistance, several conditions have to be fulfilled: the charge carriers have to behave like a two-dimensional electron gas (2DEG), temperature has to be very low (e.g., below 4.2 K) and a very strong magnetic induction has to be applied (4,5,6).

A two-dimensional electron gas is the special condition that occurs in strong inversion or accumulation layers at planar semiconductor-semiconductor or semiconductor-insulator interfaces. In the high electric field at the interface, the electrons are only allowed to move freely in a very thin layer in parallel with the interface. When a magnetic field is applied perpendicular to the interface, the electrons will tend to move along circular paths due to the interaction of the field and the electron charge e. The frequency ω_c of this rotation is called the cyclotron frequency and it can in a classic way be derived from e, B and the cyclotron effective mass m_e by equating the centrifugal force and the centripetal force due to the B-field. From this it follows that

$$\omega_c = eB/m_e$$

Under normal conditions the electrons are scattered before a full revolution occurs. However, if the electron mobility is very high, a number of revolutions can exist, and the effect is that an angular momentum quantization occurs that influences the density states of the 2DEG. The net effect is that the resistivity ρ_{xx} shows an oscillating behavior as a function of the magnetic induction (Shubnikov–Haas effect). A necessary condition is that the product of the cyclotron frequency and the scattering time τ of the electrons is much larger than one: $\omega_c\tau \gg 1$. Because the scattering increases with temperature and ω_c is proportional to B, the experiments have to be performed at low temperatures (<40 K) and in the presence of a high field (e.g., 15 T) in order to observe quantization. Figure 2 shows the resulting Hall resistance R_{xy} and the magnetoresistivity ρ_{xx} as a function of B under these conditions. As derived in the preceding, the Hall resistance increases linearly with B as long as the temperature is high, for example, above 40 K, and the magnetoresistance also increases with B. If temperature is low enough (about 1 K), ρ_{xx} becomes very small for certain ranges of B and the Hall resistance R_H becomes quantized following the relation:

$$R_H(i) = h/ie^2, \quad i = 1, 2, 3, \ldots \tag{2}$$

In the special case $i = 1$, the Hall resistance becomes the von Klitzing constant R_K

$$R_K = R_H(1) \tag{3}$$

For practical applications, the 2D gas can be implemented in Si-metal-oxide semiconductor field-effect transistors (MOSFET) or in a heterostructure with layers of GaAs and AlGaAs.

Because the quantified Hall effect is used in metrology, scientists have tried to detect deviations from the theoretical behavior by experiments or by extending the models for the samples in order to see whether or not $R_H(i)$ is universally valid.

Dependence on the Semiconductor Material

As the 2D gas can be implemented in MOSFET structures as well as in AlGaAs/GaAs structures, both materials have been carefully investigated (7,8). Measurements have confirmed that the value of R_H is with certainty independent of the material. The precision obtained with these measurements was at a relative error level 3×10^{-10}. A small number of researchers have found anomalies, but it has been shown that these can always be attributed to bad samples (9). It was possible to develop a test for the samples, in order to see in advance if the correct conditions for observing the quantized Hall effect are fulfilled.

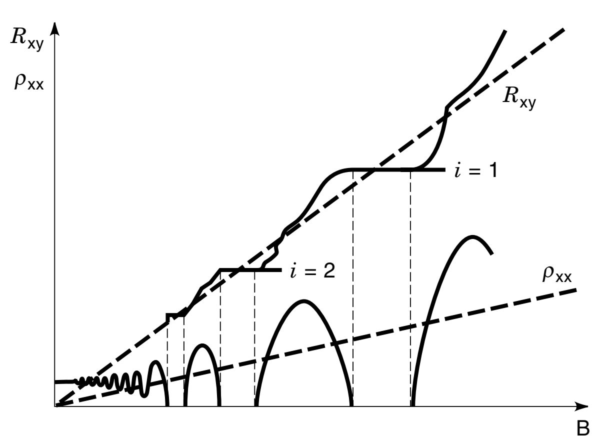

Figure 2. Hall resistance R_{xy} and longitudinal magnetoresistivity ρ_{xx} on an arbitrary scale and as a function of the induction B. The dotted lines correspond to temperatures above 44 K while the full lines are for less than 4.2 K. The Hall resistance shows quantized plateaus in this case.

Dependence on the Sample

Several heterostructure samples of the same material but with very different carrier mobilities have been studied (8) and it appears that the differences for R_H all fall within the uncertainty range of the measurement setup. On the other hand, the measurements also show that R_H varies as predicted with the integer i.

Width of the Sample

It is predicted by some theories (e.g., 10,11) that there could exist a relation between the width of the sample and R_H. However, studies performed on samples with widths of between 10 μm and 1 mm did not reveal deviations larger than the measurement precision (12).

Contact Resistance

For connection of the Hall sample to the outside world, metallic contacts are required. Good contact resistances are of the order of a few milliohms, but in some samples values in the kiloohm range have been observed. If the latter is the case, relative errors of the order 1×10^{-7} can be observed. The effect of bad contacts can not be predicted due to their microscopic nature. The experiments (13,14) show that this behavior is strongly influenced by the device current or temperature. It is estimated that the error caused by a good contact lies in the range 1×10^{-11}. It has to be noted that the errors encountered for samples with bad contacts are not due to the loading of the device by the voltmeter circuit. If the device is bad, there is always a measurable longitudinal voltage V_{xx} associated with it. Finally, the most important electrical attributes of a good quantized Hall resistor can be found in Ref. 15.

APPLICATIONS OF THE CLASSICAL HALL EFFECT

The Hall effect can be used in semiconductor research as well as in technical applications.

Research Applications

The Hall effect is used intensively in semiconductor research for the determination of carrier concentration (n) and mobility (μ) in combination with the determination of conductivity (σ) or resistivity (ρ). In the first instance, it is assumed that the layers under study are uniform. If the magnitude of the carrier charge is q, then Eq. (1) for R_H gives the concentrations for p-type and n-type semiconductors, respectively

$$p = 1/qR_H \text{ and } n = -1/qR_H \tag{4}$$

For the case where two carrier types are involved, the general relation between the conductivity mobilities μ_p, μ_n and the conductance σ is given by

$$\sigma = q(n\mu_n + p\mu_p) \tag{5}$$

and, if $p \gg n$, this reduces to

$$\sigma = pq\mu_p = \mu_p/R_H \tag{6a}$$

For electron carriers the corresponding equation is:

$$\sigma = nq\mu_n = \mu_n/R_H \tag{6b}$$

In principle, it is possible to determine the mobility μ_p from this equation by measuring σ and R_H. Nevertheless, some problems that can arise require corrections.

A first problem is the occurrence of carrier scattering mechanisms that require a correction factor to be introduced when μ_p or μ_n are calculated from Eq. (6). A scattering factor r has to be introduced in Eq. (6a) and its equivalent Eq. (6b) for the electrons as follows:

$$p = r/qR_H \text{ or } n = r/qR_H \tag{7}$$

Depending on the scattering mechanism, the scattering factor lies in the range 1 . . . 2. Therefore, the values of the conductivity mobilities derived from R_H have to be corrected with r

$$\mu_p = \mu_H/r \text{ or } \mu_n = \mu_H/r \tag{8}$$

The mobility μ_H is derived from the Hall voltage and σ is the Hall mobility and μ_p and μ_n are the ordinary conductivity mobilities. Although the latter are more important, the Hall mobility is easier to determine and can still give comparative information about the material.

Second, R_H depends also on the B-field and, when both types of carriers are present as well as a B-field, the Hall coefficient becomes rather complex. In Ref. 16 the following equation is given:

$$R_H = r\frac{(p - b^2n) + (\mu_n B)^2(p-n)}{q[(p+bn)^2 + (\mu_n B)^2(p-n)^2]} \tag{9}$$

The coefficient b is equal to the ratio of the mobilities μ_n and μ_p or $b = \mu_n/\mu_p$.

For $B \to 0$ and $B \to \infty$, the general expression [Eq. (9)] can be simplified to a low-field and a high-field expression, respectively. Depending on the mobilities it is possible to have semiconductors which obey either the low- or high-field expression, or the general equation [Eq. (9)]. The low-field expression will be valid when $B\mu_n \ll 1$ if $p \ll n$ or $B\mu_p \ll 1$ if $p \ll n$. Likewise, the high-field limit is valid when $B\mu_n \gg 1$ if $p \gg n$ or $B\mu_p \gg 1$ if $p \ll n$.

For some semiconductors the conduction mechanism changes with temperature. For example, in HgCdTe one finds that for T < 200 K . . . 300 K electrons are the main carriers. At the lower temperatures a mixed carrier conduction occurs and, as a result, R_H eventually can change its sign. Therefore, Hall measurements are always performed at different temperatures and induction in order to extract the necessary information from the measured data. An extra complication is the dependence of r on B.

In order to be able to determine the spatial resistivity and mobility profiles in nonuniform layers, it is necessary to remove small layers from the sample so that Hall coefficients can be measured as a function of thickness. This can be done by etching or, preferably, by anodic oxidation. In another method used for layers on an isolator, a pn-junction, a MOSFET or a moscapacitor is formed at its upper surface. If, for example, a metal gate layer is used, the penetration depth of the space-charge region can be controlled by means of the reverse bias of the junction and accurate measurements of the mobility and carrier concentration are the result.

In Ref. 3 one can find a review of the most common sample shapes used, and the precautions to be taken for the reduction of errors. Especially, if parameter estimation techniques are used the results can deliver much information about the material. Hall measurements on irregular sheets are also possible, provided some conditions are met. The theory for such samples was derived in 1958 by van der Pauw; see Ref. 17.

Technical Applications

Because it became possible to integrate the electronics and the Hall effect transducer on the same substrate, a number of important industrial applications have been demonstrated. The Hall transducer can be used for direct measurement of magnetic induction and power, switching applications, position control, and so on. A Hall device will not be destroyed by strong magnetic fields and when semiconductors of the III-V type (GaAs, InSb) are applied, it is possible to enhance the sensitivity and the allowable temperature range. With ion-implanted GaAs a temperature range of −40 to +250 °C is obtained. The advantage of using silicon lies in the fact that the integration of an amplifier with the Hall element is easy. However, sensitivity is lower due to the low mobility of the carriers in silicon.

In combination with magnets fixed on moving parts, the Hall transducer effectively becomes a position sensor. By monitoring the output, one can get an idea about the velocity or the acceleration of a moving object. Such a transducer will never wear out because there is no friction. Aging will also be very low because of the large mean time between failures (MTBF) of an integrated circuit. In contrast with optical devices, the operation of this transducer will not be disturbed by moisture, dust, or dirt except when the dust contains iron particles. The Hall device is also relatively fast: the rise and fall times are of the order 0.2 μs and magnetic ac fields with frequencies of up to 100 kHz can be detected. The substrate containing the transducer is either a ceramic material or a ferromagnetic substance. The latter is used if it is necessary to decrease as much as possible the permeability of the magnetic circuit in which the device is mounted.

Linear Hall Effect Transducers. The output of a linear Hall effect device is proportional to the product of the induction B and the device current I; in this way, it performs an analog multiplication. It is also obvious that such an element can be dedicated to the measurement of magnetic fields if I has a constant value.

Measurement of Magnetic Fields. The instrument using Hall effect sensors for the determination of magnetic induction (18–22) is called a gaussmeter although it should be termed a teslameter, as the unit of induction is tesla (T) in SI units.

From the preceding it follows that the Hall voltage is due to the component of the field normal to the main sheet plane of the device. If the angle between the field B and the normal is α [Fig. 3(a)], then the normal component is $B \cos \alpha$. For maximum sensitivity it is required that $\alpha = 0$ and if the field

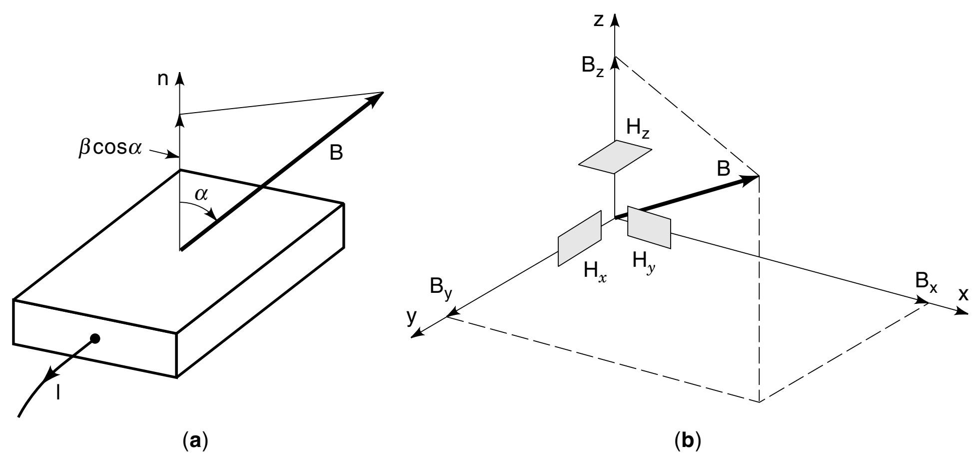

Figure 3. (a) Component normal to Hall device for an arbitrary direction of the field and (b) setup with three elements for the determination of magnitude and direction of field.

has an unknown direction it is necessary to search for maximum output by rotating the device in space. If three separate Hall devices are orthogonally arranged [Fig. 3(b)] so that their main planes coincide with the planes of a rectangular axis system, the outputs will be proportional to the spatial components B_x, B_y and B_z of B. The magnitude and direction of the vector $\boldsymbol{B}$ in space can then be derived from the three outputs. It is, however, required that the probes be carefully calibrated. This can be done by adjusting the individual device currents, or by measuring the three sensitivity factors and storing them in a memory for further use with the software calculations. If large spatial gradients exist in the field, then the device output represents a kind of average. This error can be reduced by using very small elements: mostly it is possible to adapt the magnetic circuit to minimize this effect.

As sensitivity depends on I it seems appropriate to take I as large as possible. However, this also increases both device dissipation and current consumption. Therefore the advantage gained by increasing I is rather marginal and it is better to amplify the Hall voltage electronically if a higher sensitivity is required. For spatial magnetic field measurements it is preferable that the Hall device has a ceramic substrate as this ensures that the field is not disturbed by the device. Due to the large field of applications of gaussmeters, several types of probes with a calibrated built-in Hall element are available on the market. For measurement of fields in air gaps, for example, in electrical machines or relays, thin probes are preferred but these probes are very sensitive to mechanical stress. The Hall device is mounted at the end of a long flat isolating stylus so that less accessible points can be probed as well. The transverse probe is sensitive for fields perpendicular to the stylus plane. The axial probe is cylindrical and the Hall device is mounted perpendicular to the axis of the cylinder; it measures fields parallel to the probe axis. Full-scale sensitivity can be as large as 15 T and rms readings of ac fields are also possible.

Measurement of Electrical Current. Although not strictly necessary, a magnetic circuit is commonly used with the Hall device for current measurement (19,23) purposes [Fig. 4(b)]. This has two advantages: the sensitivity of the measurement is increased, and the position of the current carrying conductor becomes unimportant. The magnetic field at a distance d of a straight conductor [Fig. 4(a)] carrying a current I, has a value $H = I/2\pi d$ and this is position sensitive. A Hall device at this position will give an output proportional to $\mu_0 H$. If the current encircles a magnetic circuit, the force lines [e.g., 1 in Fig. 4(b)] will concentrate in the magnetic material (mean length L, magnetic permeability μ) and in the air gap with width δ. The value B_g of the magnetic induction in the air gap becomes, with Ampère's law and because of the constant flux in the core and air gap section,

$$B_g = \mu\mu_0 I/(L + \mu\delta) \qquad (10)$$

For $\mu = 10^4$, $L = 0.1$ m, $\delta = 1$ mm (10^{-3} m) the value $\mu\delta = 10$ and L can thus be neglected. It means that in the range where $\mu\delta \gg L$, the gap induction B_g can be approximated by $\mu_0 I/\delta$ and the sensitivity becomes independent of μ and yet can be very high because δ is so small. If the wire encircles the magnetic circuit N times, then B_g also becomes N times larger. The material properties of the core material barely influence the precision of the measurement, but a very stable air-gap width δ is required. Also, when the current increases, the core induction increases and the material has a tendency to saturate. As a result, some of the magnetic field lines can

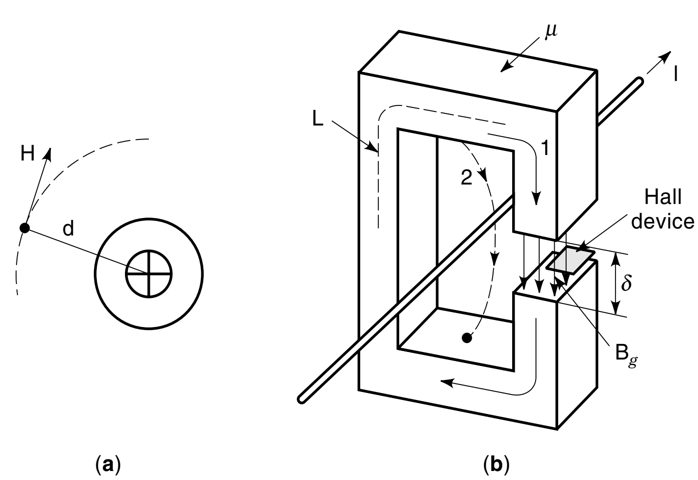

Figure 4. (a) The radial field around a current-carrying conductor depends on the position of the Hall element; (b) magnetic circuit with a Hall device in the air gap. At low saturation all field lines in the core follow the magnetic path (1). When saturation occurs, some of the field lines (2) are lost for the air gap.

leave the legs of the core and pass through the window area [e.g., line 2 in Fig. 4(b)]. Because a part of the flux is lost, the air-gap induction becomes too low and a nonlinearity is introduced in the transducer characteristic. The existence of an air gap is favorable in this respect because it reduces the core induction at the cost of a lower sensitivity. The design of such a circuit is therefore a trade-off between sensitivity and full-scale behavior. The nonlinearity can remain below $\pm 1\%$ with ordinary core materials such as ferrite. Besides the electrical offset of the Hall device, a magnetic offset is caused by the core material. This is due to the remanent magnetism that remains after a heavy magnetization of the core. The remanent field gives an additional electrical output offset. Reduction of the remanent magnetism is done by choosing a very soft magnetic material. It is not possible to correct for this remanent field using software because its value depends on the history of magnetization and, as precise models for the core behavior are lacking, a precise offset error correction is unpredictable. In Ref. 24, one can find how the problem of the intrinsic Hall plate offset voltage can be alleviated by combining the output of several elements. The simple circuit can still be very useful if the highest precision is not required because it requires only a Hall IC and a magnetic circuit, and power consumption is very low.

Closed-Loop Hall Effect Current Transducer. The linearity and offset problems can be reduced considerably by applying feedback (25) to the simple circuit (Fig. 5). In this case, the Hall voltage amplifier drives a second winding N_2 coupled with the core and a resistor R. The voltage drop RI_2 over this resistor is taken as the new output voltage. By choosing the correct winding, the amplifier output current I_2 can counteract the effect of the unknown current I_1. The number of ampere turns generating the field is now equal to the difference of the ampere turns of the current and feedback windings. Equation (10) for the magnetic circuit becomes, with H_i and H_g, respectively, the magnetic field in the magnetic circuit and in the air gap

$$H_i L + H_g \delta = H_i(\delta + L/\mu) = N_1 I_1 - N_2 I_2 \tag{11}$$

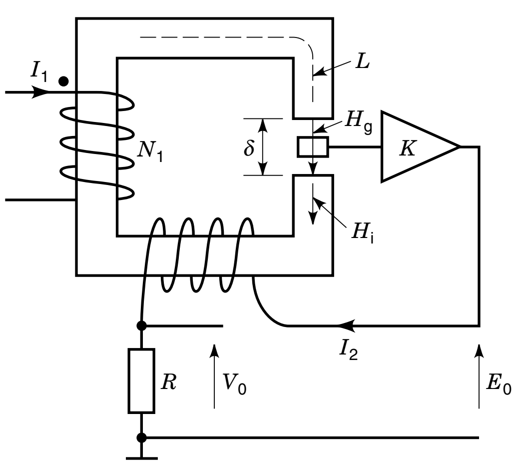

Figure 5. Closed-loop Hall effect device in magnetic circuit. If the loop-gain is very large the primary and secondary ampere turns will cancel and V_0 becomes proportional to I_1.

The amplifier output voltage E_0 is proportional to the air-gap induction and with an induction-to-voltage gain K, we obtain

$$E_0 = K\mu_0 H_g / R$$

As the corresponding current I_2 is E_0/R, the air-gap field becomes

$$H_g = N_1 I_1 / (\delta + L/\mu + N_2 K \mu_0 / R)$$

If the amplifier gain K is made very large, it is seen that $H_g \to 0$ and the field in the air gap disappears due to the feedback. It follows that $N_1 I_1 \to N_2 I_2$ and

$$V_0 \approx (N_1/N_2) I_1 \tag{12}$$

As is the case in every feedback system, the static error can be neglected if K is high enough. The output V_0 is a very accurate image of I_1 because the winding resistances, leakage inductances, and losses in the core, have no effect on accuracy. If the material is initially sufficiently demagnetized, the permeability will be very high and the magnetic offset error is reduced. In contrast with the open-loop system, the length of the gap barely influences the measurement precision; as a result, the temperature behavior is also enhanced. When the current I_1 exceeds the output current capability of the amplifier, for example, under short-circuit conditions, the core is likely to be saturated and, therefore, a means for demagnetizing the core is standard in such an instrument. Demagnetization is obtained by driving temporarily the winding N_2 with an exponentially damped oscillating (e.g., sinuoidal) current. If the initial value of this current drives the core into strong saturation, eventually the core will be demagnetized. For current probe applications the core is divided into two parts, which can be separated in such a way that the conductor can be enclosed by the core window without breaking the electric circuit. Especially, in this case it can be appreciated that the possible instability caused by imperfect core halves contact is eliminated by the feedback. The required current output capability of the amplifier depends entirely on the winding ratio N_2/N_1 and the full-scale current I_1

$$I_2 = I_1 N_1 / N_2 \tag{13}$$

For practical reasons, the winding ratio has to be limited. This means that the amplifier in general will have to deliver a much larger current than in the open-loop system and its power consumption will be large. Another problem is that additional circuitry is required for wideband measurements because the frequency response of the Hall device is limited to about 100 kHz. A solution is to split the circuit into two parts. First, the frequency response of the Hall amplifier will deliberately be limited to a few kilohertz by first-order lowpass filtering of the output V_0. The purpose is to reduce the effect of the poles in the open-loop transfer of the Hall device. Next, for the higher-frequency part of the spectrum, the two windings can be made to operate as a current transformer. The signal obtained from this current transformer is also first-order high-passed, with the same 3 dB cut-off frequency and transfer gain as for the Hall device. By combining the two filter outputs a flat frequency characteristic is then obtained. Typical probes show a flat frequency response from dc to 20 or 100 MHz and have full-scale ranges from 1—10 A with a full-scale output voltage of 50–200 mV.

Multiplier Applications of the Hall Effective Device. Although ac power can be measured by sampling the current and the voltage of a consumer, in general, the potential drop, and, hence, the dissipation in the current shunt, are relatively large. Further, it is difficult to protect the electronics unconditionally against a short-circuit at the consumer side. On the other hand, when the load current flows through the magnetizing coil of Fig. 4, a short will not harm the Hall device or the magnetizing coil because the coil is very rugged. As suggested by Eq. (1) the Hall device output is proportional to the product of B and I. In fact, one of the first applications of the device was as an analog multiplier (19,20). Although this application has been superseded completely by the use of the Gilbert cell analog multiplier ICs, the device still has attractive features for power applications. For example, for determination of the power an ac load consumes, the product of the momentary voltage $v(t)$ and current $i(t)$ has to be calculated. The former current-measuring circuit can easily be adapted for this purpose. It suffices to vary the Hall current I proportional to the voltage $v(t)$ so that the output voltage $v_0(t)$ of the device becomes proportional to the momentary power. By filtering this voltage, a measure for the average power is obtained. If $v_0(t)$ is fed to a voltage-to-frequency converter followed by a counter, the counter output will represent the energy consumed by the load. For household applications the preferable counter is a mechanical one because it gives a clear indication and holds the current value, even during a power loss. The advantage of using the Hall device lies in the fact that the current scale for the instrument can be adapted through a change of the winding N_1 in the same way as a classical Watt-hour meter with a rotating disk induction motor.

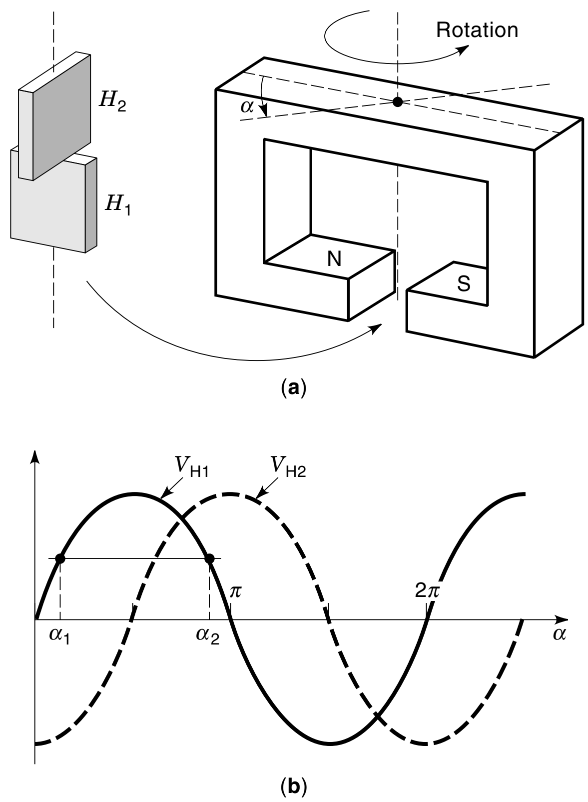

Figure 6. (a) Hall device used as an angle sensor: a magnet is fixed on a turning axis and the Hall device is mounted between the pole shoes of the magnet. The output of V_{H1} is proportional to sin α because for $\alpha = 0$ its plane coincides with the plane of the magnet. (b) A second device is necessary in order to remove all possible ambiguity from determination of α from the Hall voltages.

Clinical Hall Effect Devices as Position Sensors. A number of industrial applications require knowledge of the position of a mechanical part; the Hall effect ICs have been developed especially for this task (22). For example, in brushless dc motors the angular position of the rotor has to be known in order to activate the drive electronics of the field coils. More recently, the Hall sensor has been applied in magnetic bearings (26) for magnetic levitation purposes.

If there is no magnetic field available, the Hall devices will be used in combination with permanent magnets (27). Suppose the rotation angle α of a permanent magnet has to be determined [Fig. 6(a)]. If a linear sensor H_1 is positioned in a zone between the pole shoes, where the field of a rotating magnet is homogeneous and has a value B the output V_{H1} will be proportional to the cosine of the angle of rotation α on the condition that the rotation axis coincides with the length-axis of the Hall sensor plane. From output V_{H1} it is, however, impossible to obtain α, as there are two angle solutions for every voltage [α_1 and α_2 in Fig. 6(b)]. This ambiguity can be resolved by placing a second transducer H_2 (output V_{H2}) perpendicular to H_1. From V_{H1} and V_{H2}, and taking into account the signs of both outputs, the position of the magnet can be determined in the whole α-range (0, 2π).

A digital output can be obtained by including a Schmitt-trigger circuit (Fig. 7), driven by the analog output. Mostly, an open-collector transistor forms the output as this permits adaptation of the IC to the logic level of the system connected to it. Sometimes a small magnet is mounted close to the sensor in order to provide for a magnetic bias. The output can be made to react in two ways as shown in Fig. 8. In Fig. 8(a), the purpose is to discriminate between the presence or absence of a magnetic pole. Without field the output is at the logical level "1" (line 1). When the magnet approaches the device its analog output increases. At the trip point of the Schmitt-trigger corresponding to a field B_1, the digital output falls to the logical level "0" (line 2). In a third phase, the magnet is removed from the device but output remains low (line 3) as long as B

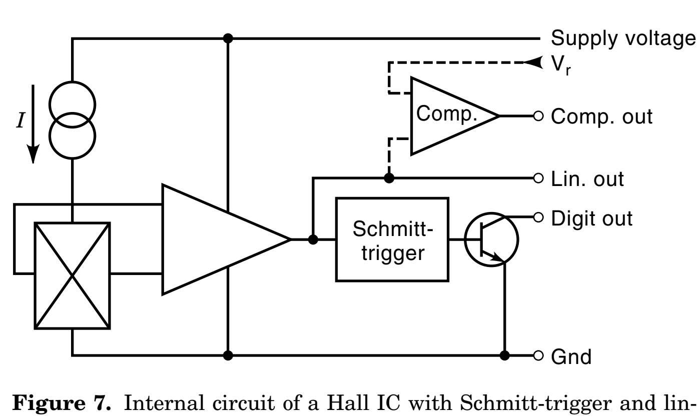

Figure 7. Internal circuit of a Hall IC with Schmitt-trigger and linear output. A comparator output is also useful for limit detection.

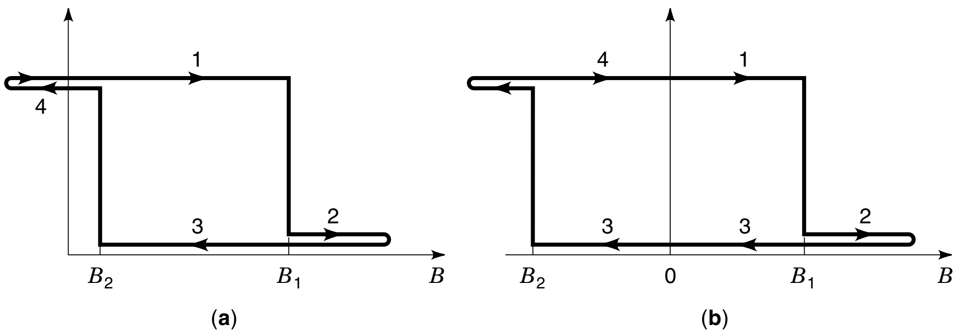

Figure 8. (a) Schmitt-trigger output for a unipolar sensitive Hall IC; (b) output for bipolar sensitive IC.

lies above the lower trip point B_2. In Fig. 8(b), the lower trip point corresponds to a negative field, that is, the other pole of the magnet. Operated in these two ways, the Hall device acts like a switch. The hysteresis of the Schmitt-trigger reduces the effect of bouncing and noise. There exist three common ways for detecting the position of a moving magnet: Fig. 9(a–c). It is assumed that the magnet moves along the indicated x-axis in all three cases. In Fig. 9(a), the magnet moves in a direction perpendicular to the Hall device plane. The analog output falls continuously with increasing displacement x, and a digital action is possible if the circuit is followed by a comparator. In Fig. 9(a), the comparator switches from low to high if the Hall voltage exceeds reference voltage V_d. The main drawback is the poor linearity of the analog output with respect to the movement of the magnet. In Fig. 9(b), the movement is along an axis in parallel with the device plane and the south pole is near the magnet. The output will be at maximum when the south pole is positioned at $x = 0$. The system can be used as a mechanical window detector: if the magnet is positioned within the range $(-x_1, +x_1)$, the comparator output is low and vice versa. The sensitivity is controlled by adapting the distance d between the magnet and the device. Linearity can be acceptable in a small part of the displacement curve around the point of the curve where the second derivative with respect to B vanishes. In the third case of Fig. 9(c), the magnet is turned over 90° and a linear device sensitive for negative and positive fields is used. Sensitivity can be controlled as in the former case and linearity can be extremely good but the sensitivity control will also influence the linearity. In the setup of Fig. 9(c), sometimes both magnet and Hall device are fixed but a high permeability screen moving between them can interrupt the field. Such a system can detect the teeth of a cogwheel or it can replace the mechanical breaker of the ignition in a car.

Quantum Hall Effect in Electrical Metrology. In the International System of Units (SI) the four base units are the meter (length), kilogram (mass), second (time) and the ampere (current). The ampere is the current that generates a force of 1 N per length between a pair of very thin and infinitely long conductors. This definition is artificial and of little practical value, but it ensures that the mechanical and electrical units of force are the same. Apart from this, some of the mechanical units derived are introduced for practical purposes, for exam-

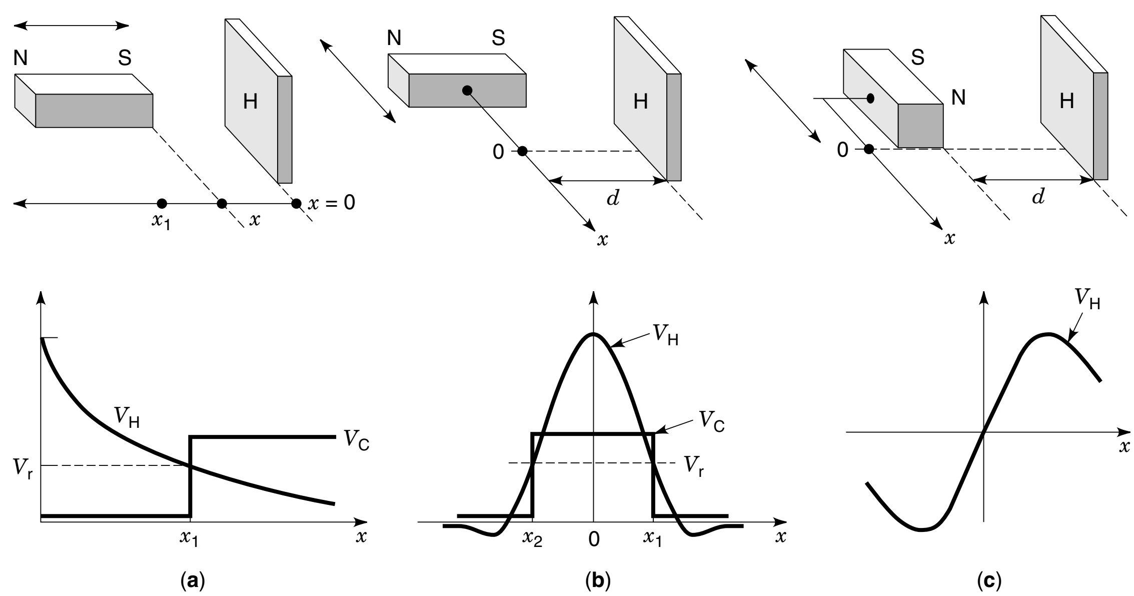

Figure 9. Three ways to detect a magnet movement: (a) magnet axis and movement normal to Hall plane and resulting linear (V_H) and comparator (V_C) output voltages; (b) movement axis parallel with Hall plane gives a window function for V_C; (c) output is linear over a large displacement range.

ple, the Henry, coulomb. These secondary quantities can always be expressed in the base units. The magnitude of magnetic permeability in the SI system is fixed to $4\pi \times 10^{-7}$ H/m. The second is defined by means of the period of a well-defined transition in Cesium 133. The velocity of light (c) has a fixed value and the meter is the distance traveled in a vacuum by light during 1/299.792.458 s. This makes the use of the material standard meter obsolete. All electrical units can be defined in this system if two of them, for example, the volt and the ampere, are related to the mechanical units meter, second, kilogram. This comparison can be implemented via a kelvin current or volt balance. In practice the ampere has been replaced by the ohm as the latter is related to the ampere by Ohm's law.

Because the realization of physical standards requires complex instruments, national metrology laboratories still possess material standards such as Weston cells and 1 Ω resistors for their electrical measurements. These standards are considered to be a kind of flywheel in which an image of the physical unit is conserved for a limited time. By regularly comparing the physical standards with the material standards, the correction for each of these is determined. The national volt (V_{LAB}) and ohm (Ω_{LAB}) of a nation is then defined from the average values of these material standards. Before the invention of the Josephson junction and the quantified Hall effect, it was imperative to perform regularly cumbersome international comparisons in order to determine the corrections between the national standards V_{LAB} and Ω_{LAB} of the different countries.

In 1956, Thomson and Lampard (28,29) derived a striking theorem concerning a special cylindrical four-electrode capacitor structure. It was proved that for the structure of Fig. 10(a), independently of the shape of the cross section of the electrodes A, B, C, and D, the capacities C_1 and C_2 per unit length (1 m) of opposite electrodes obey the expression

$$\exp(-\pi C_1 \epsilon_0) + \exp(-\pi C_2 \epsilon_0) = 1 \tag{14}$$

with ϵ_0 the permittivity of the absolute vacuum. The value of ϵ_0 is known because, in vacuum, the velocity of light is $c = (\mu_0 \epsilon_0)^{-0.5}$ and c as well as μ_0 are fixed. For a symmetrical capacitor Fig. 10(b), $C_1 = C_2 = C$ and from Eq. (14) we obtain $C = 1.953549043$ pF/m. In practical applications, a capacitor with finite length is used and this length is varied by inserting a guard tube S in the space between the four electrodes (30–32). By careful construction it is possible to reduce the effect of the fringing fields at both ends of the capacitor. In this way, the physical length of the capacitor is changed by displacing the guard tube from l_1 to l_2 and the corresponding variation of capacity is 1.953549043 $(l_2 - l_1)$ pF. The capacity variation in this system is, therefore, only determined by one dimension; the displacement $l_2 - l_1$ of the guard tube can be measured very accurately with a laser interferometer. The very complicated setup for this system has been called the calculable capacitor because of its capacitance formula. By employing a chain of ac comparator bridges the material ohm standard can be related to the impedance of the calculable capacitor, and thus with the mechanical units.

The quantum Hall effect has opened the door to a new material ohm standard, which is solely dependent on the universal physical constants e and h. On the other hand, it is generally accepted (33) that the von Klitzing constant R_K, that is, the value of R_H for $i = 1$, can also be calculated from μ_0, c and the dimensionless fine structure constant α. This constant can be derived via complex quantum electrodynamic calculations from measurements of the anomalous magnetic moment of the electron. The relation between μ_0, c, α, h and e is given by

$$R_K = R_H(i = 1) = h/e^2 = (\mu_0 c/2)\alpha^{-1} \tag{15}$$

with a value $\alpha = 1/137.0359898$. Since μ_0 and c have well-defined values, the accuracy of R_K and α is the same in Eq. (15).

As it became clear that the quantum Hall effect could be used for definition of the unit of resistance, metrologists succeeded in refining the measurement procedures so that R_K could be expressed in Ω_{LAB}. It also became possible to compare the existing material ohm standards with a relative error of 10^{-8}. At that moment the reproducibility of the quantum Hall measurements was better than the implementation of the material ohm in the International System of Units (SI).

In 1990, the Comité Consultatif d'Electricité (CCE) decided, based on an analysis of all the known international comparisons of the material resistance standards by the Bureau International des Poids et Mesures (BIPM), to attach a conventional value to the von Klitzing constant

$$R_{K-90} = 25812.807\ \Omega$$

The relative uncertainty for this value is 2×10^{-7} and this definition makes it possible to replace the maintained standards with the quantum Hall effect device. At the same time, for the second quantum effect used in metrology, that is, the Josephson effect, a conventional value $K_{J\text{-}90}$ has also been as-

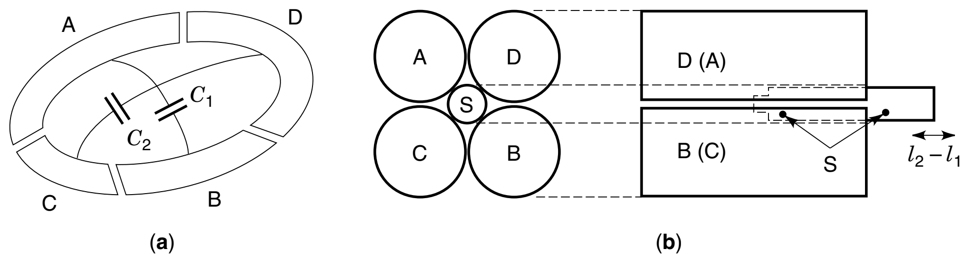

Figure 10. (a) Calculable capacitor structure with four electrodes; (b) simplified system with four cylindrical electrodes A, B, C and D. The effective length is varied by moving a screen S in the opening between the cylinders. Not shown are a similar but fixed screen at the left side, the screen surrounding the tubes, interferometer, and trimming screws. The smaller part of the screen is the "spike." It can be shown that this spike reduces the effects of irregularities in the cross section of the capacitor (Ref. 32).

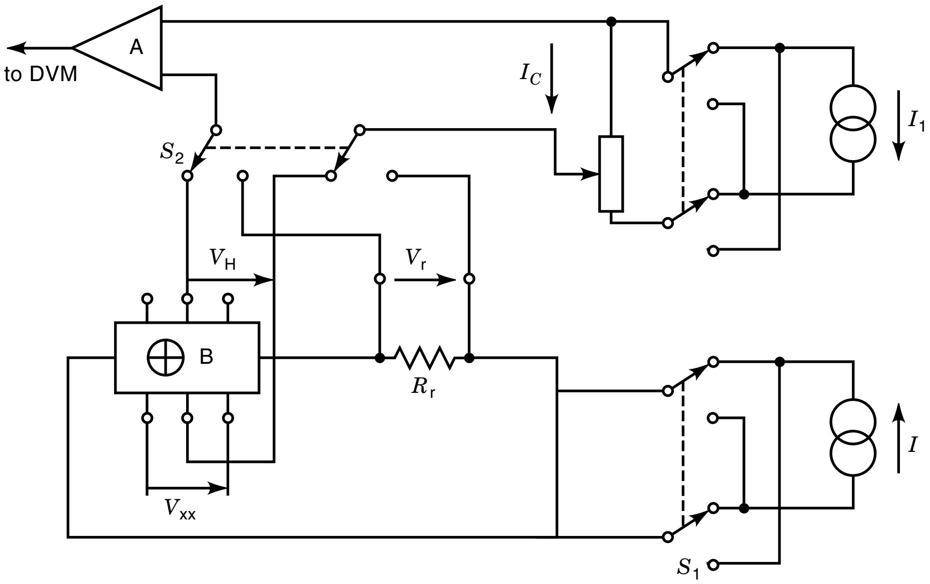

Figure 11. Potentiometric setup for comparing the quantified Hall resistance with the laboratory standard resistor.

signed to the Josephson constant. The material standards for the volt and ohm are now replaced by the Josephson junction and the quantum Hall device; the outputs of these devices are independent of the site on earth and the shape or materials used for their construction. Intercomparisons are therefore no longer as critical.

Measurement Setup. Comparisons of the Hall resistance with standard resistors can be performed in different ways: potentiometric or with a cryogenic current comparator bridge (CCC). Figures 11 and 12 show generic and simplified diagrams representative for various measurement setups. In the potentiometric method (Fig. 11) a reference resistor R_r of the same nominal value of $R_H(i)$ is used. As the plateau $i = 1$ is difficult to obtain with reasonable fields, often the integers

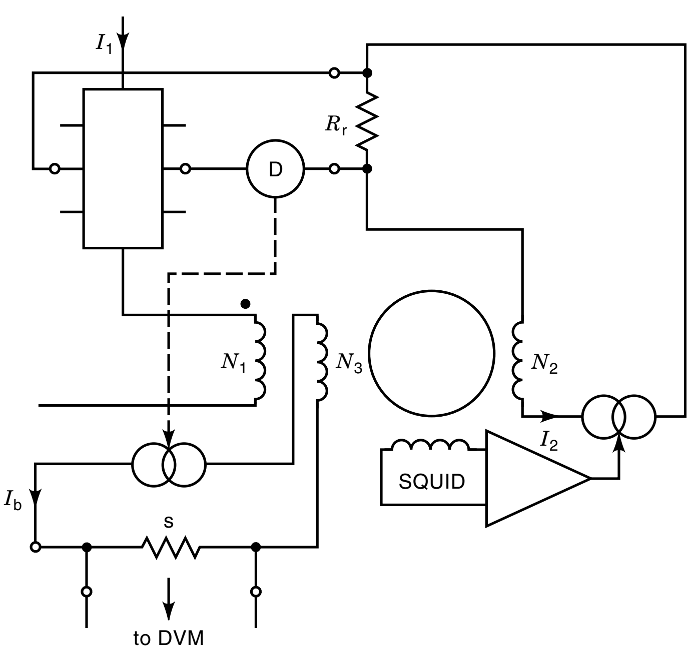

Figure 12. Cryogenic current comparator setup for comparing the quantified Hall resistance with the laboratory standard resistor.

$i = 2$ or $i = 4$ are preferred. Superconducting magnets that generate an induction of 15 T between its poles are commonly used. The value I of the dc current source is in the 50 μA range and this alleviates the work of the temperature controller because of the low power dissipation in R_r. Measurement of the voltage V_{xx} permits inspection if the correct conditions for the occurrence of the quantum Hall effect are met. The Hall plateaus will appear when V_{xx} is very small. A zero detector is lacking and, instead, a high-gain amplifier followed by a digital voltmeter (DVM) is used. A compensating source V_C is derived from a potentiometer fed with a second current source I_1. By changing the position of S_2, two voltage differences are measured: $\Delta_1 = V_H - V_C$ and $\Delta_2 = V_r - V_C$. From the Δ-values measured it is possible to calculate the current I and V_C and thus V_H. To eliminate the drifts of the current sources and the thermal emfs it is necessary to perform measurements over a longer period, for example, an hour, and calculate averages. Because all current sources can change direction (by turning over S_1) the effect of thermal voltages can be eliminated. If the CCC is used as shown in the simplified circuit of Fig. 12, it is possible to compare R_H with smaller resistors, for example, 100 Ω because the turns ratio N_2/N_1 can be varied. In Fig. 11, there are two balance conditions to be met: $V_H = R_r I_2$ for the zero detector balance, and $N_1 I_1 = N_2 I_2$ for the zero flux condition in the core. The magnetic flux is measured by the very sensitive superconducting quantum interference device (SQUID), and the output of this device controls the current source I_2 in such a way that the core flux becomes almost zero. If the balance of D is obtained by varying N_1, then we find with $V_H = R_H I_1$, the quantum Hall resistance $R_H = R_1 N_1/N_2$. In order to increase the precision to the level 10^{-9} or better, a fixed ratio N_1/N_2 is more appropriate. In this case a third current winding N_3 is added to the comparator and a balance current I_b derived from the detector output performs the automatic balancing. This balance current is measured by means of a shunt resistor s and a DVM. The balance current is much smaller than the main current and, therefore, the limited precision of the digital voltmeter does not impair the measurement accuracy.

If the two methods are compared (Ref. 5), it can be calculated for a specific case that the potentiometric method generates more system noise than the CCC. In a typical case the CCC method is therefore almost 45 times faster than the potentiometric method. As a result, the measurement time is reduced to a few minutes and the random uncertainty still is at the level 2×10^{-9}.

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DANIËL H. J. BAERT
University of Ghent

HALL RESISTANCE. See HALL EFFECT TRANSDUCERS.
HALOGEN LAMPS. See FILAMENT LAMPS.