GRAVIMETERS

Gravity measurements reflect the earth's gravitational attraction, its centrifugal force, tidal accelerations due to the sun, moon, and planets, and other applied forces. They are invaluable to oil and mineral prospectors. They are used by physical scientists to determine the exact size and shape of the earth and they contribute to the gravity compensations applied to inertial navigation systems. If the earth was a

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sphere of uniform density, then its gravitational attraction on small bodies located on its surface would be constant everywhere. In fact, the earth's centrifugal forces of rotation have flattened it at the poles, making its polar radius approximately 21 km less than its equatorial radius. The outward centrifugal acceleration at the equator (which is nonexistent at the poles) decreases inward equatorial gravity accelerations by approximately 3400 mgal (1 mgal = 10^{-5} m/s^2 ≅ 1 μg or 1 part per 10^6). Equatorial gravity measurements also reflect a greater attraction to the whole earth (owing to the lower latitude bulge), and this results in an increase of approximately 4900 mgal. Polar gravity measurements reflect being closer to the center of mass which accounts for an increased measurement of approximately 6600 mgal. Taken collectively, polar gravity is roughly 5100 mgal stronger than equatorial gravity. Gravity measurements also decrease by about 1 mgal for every 3 m increase in height above mean sea level. Local and regional subterranean density variations also produce gravity changes as large as 200 to 300 mgal, although these tend to be smaller.

As explained by Einstein, the Equivalence Principle does not permit gravity to be measured at a point. What is measured is the *specific* force impressed on the instrument. The *gravitational* force is inferred from knowledge (and assumptions) of the acceleration of the instrument. Accelerometers and gravimeters are very similar devices because they are sensitive to the same types of inputs. They are, however, optimized for different measurement regimes in terms of dynamic range, frequency response, and operating environments.

The first gravity measurement device was the pendulum clock, invented by Huygens in 1656 (Newton's 14th year). The pendulum's period (T) and the acceleration of gravity (g) are inversely related by $T = 2\pi(I_m/mgh)^{1/2}$ where I_m is the pendulum's moment of inertia about its pivot axis, m is its mass, and h is the distance between its center of mass and its pivot point. French astronomers soon noticed such clocks lost time at the equator when compared to Paris-based observations. This was the first direct evidence that gravity lessens as latitude lessens. Pendulum measurements are time consuming and require elaborate executions and corrections. Moreover, neither I_m nor h can be measured with great precision. A pendulum's mechanical properties also change with time and transport. These mechanical changes create changes in the pendulum's period that are difficult to calibrate. Owing to these problems, pendulums have been completely replaced by two classes of high-precision, high-accuracy gravity measurement devices: (1) absolute gravity apparatuses (both portable and stationary) which use lasers and atomic clocks to measure and time freely falling body distances and (2) relative gravity meters (or gravimeters), which measure the force required to rebalance the gravity force acting on a leveled proof mass attached to a spring *against* the force exerted by the spring, as the meter is moved from one measurement point to the next.

ABSOLUTE GRAVITY MEASUREMENTS

Free-fall Acceleration Measurements

Neglecting air resistance, if a freely falling body is a distance x_0 from an overhead origin at time t_0 and moving with velocity v_0 m/s, then subsequent x_i distances occur t_i seconds after t_0 with time and distance related by (assuming g is constant from x_0 to x_i)

$$x_i = x_0 + v_0 t_i + \tfrac{1}{2} g t_i^2 \tag{1}$$

Designating t_1 as a starting time, t_2 an intermediate time, and t_3 a final time; and taking combinations of $x_i - x_j$ differences yields

$$g = 2\left[\frac{\Delta x_3 - (\Delta t_3/\Delta t_2)\Delta x_2}{\Delta t_3^2 - \Delta t_2 \Delta t_3}\right] \tag{2}$$

where $\Delta x_3 = x_3 - x_1$, $\Delta x_2 = x_2 - x_1$, $\Delta t_3 = t_3 - t_1$, $\Delta t_2 = t_2 - t_1$, and it's noted Eq. (2) is free of x_0 and v_0. Precise and accurate measurements of distance *and* time are required to accurately measure the absolute value of g in this manner. Today's most sensitive free-fall measuring instruments are portable and rely on stabilized lasers and atomic (rubidium) clocks to provide length and time standards. The drifts in these standards are low enough that they can be used for months without drift errors contributing at the parts per 10^9 level. The standards also are minimally affected by transit vibrations and environmental temperature changes (1). Since a mass dropped from rest falls approximately 5 m in 1 s, time and distance measurements precise to 1 part per 10^9 yield absolute gravity measurements with precisions of a few μgal (1 μgal = 10^{-8} m/s^2 ≅ 10^{-9} g).

Figure 1 is a simplified diagram of the free-fall, absolute gravity meter at the Joint Institute for Laboratory Astrophysics (JILA) (2). A stabilized laser illuminates a Michelson interferometer formed between one light beam, reflected from a falling corner cube (a mirror that reflects the laser directly

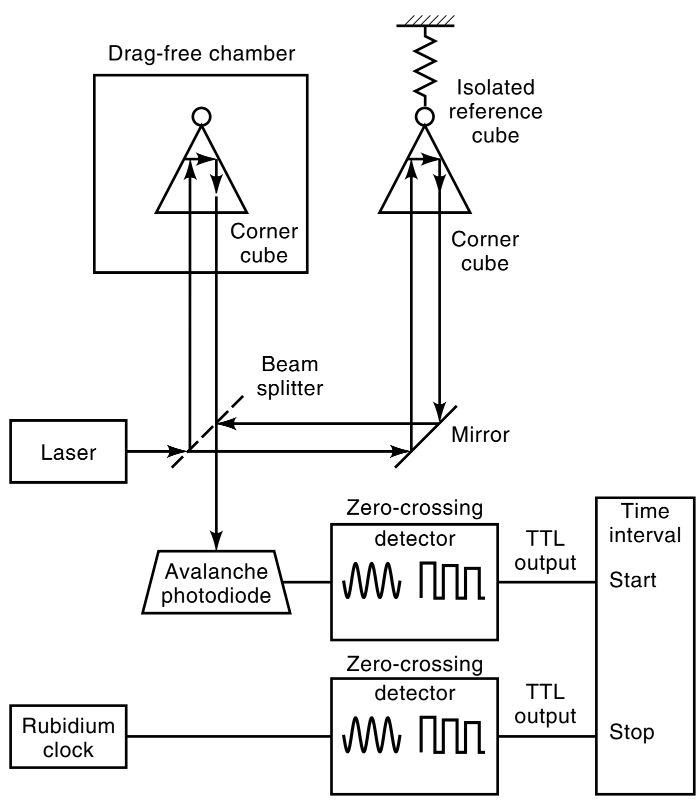

Figure 1. Block diagram of the Joint Institute for Laboratory Astrophysics free-fall method, patterned after Niebauer et al. (2).

back, regardless of the cube's orientation) and a second light beam reflected from a stationary reference corner cube. The free-falling cube resides in a drag-free vacuum chamber to eliminate air resistance. The interference of the two reflected beams makes moving fringes (light or dark bands) sensed by an avalanche photodiode, which activates the timing devices. An electronic scaler, linked to the atomic clock, determines the time between whole numbers of fringes to within a nanosecond (10^{-9} s). Since the wavelength of the laser is also accurately known, the distance traveled by the free-falling cube is accurately determined by counting the number of fringes passing during the time intervals Δt_3 and Δt_2 in Eq. (2). Because each fringe covers a distance of one-half the laser's wavelength (λ), the actual determination of g is from a modified version of Eq. (2), namely

$$g = \lambda \left[\frac{f_3 - (\Delta t_3/\Delta t_2) f_2}{\Delta t_3^2 - \Delta t_2 \Delta t_3} \right] \tag{3}$$

where f_3 and f_2 are the counted fringes during Δt_3 and Δt_2. The most accurate and precise free-fall absolute gravity measurements taken to date are discussed in Niebauer et al. (1). Accuracies of 2 μgal and repeatabilities of 1 μgal are claimed using a portable device weighing approximately 300 kg and possessing a 3 m^2 footprint.

Engineering challenges related to free-fall devices include making the vacuum chamber as free of electrostatic and magnetic effects as possible and ensuring that the laser beam reflected to the falling cube is accurately vertical. The latter is addressed by placing a dish of mercury under the point where gravity is measured. The optics base is then positioned and adjusted to make a vertical beam using the mercury as a reference. Another challenge is to optimally reduce microseismic motions. This is addressed by suspending the reference cube from the mass of a long-period vertical seismometer. Making the transmitted laser frequencies as stable as possible and improving the accuracies of the reference atomic clocks are ongoing challenges. Design of the release mechanism such that no impulse is applied is also a major challenge.

Symmetric Rise and Fall Absolute Gravity Measurements

Sakuma (3) has developed an up-and-down corner cube absolute gravity measuring system. The reflector is initially catapulted upward, and measurements are made of both the upward deceleration and the downward acceleration. Key advantages of the up-and-down approach are the cancellations of air resistance effects and systematic timing errors. Thus the cube need not reside in a vacuum chamber. Key disadvantages are the mechanical vibrations caused by the upward launching of the mass, its nonportability, and its overall mechanical complexity. The device is permanently mounted on a seismically stabilized platform in Paris.

If the distances x_1 and x_2 (from overhead origin) are passed by the catapulted cube at times t_1 and t_2, and the free-falling cube later passes x_2 at t_3 and x_1 at t_4, then the *mean* time values of the x_1 and x_2 passages are equal, that is, $(t_4 + t_1)/2 = (t_3 + t_2)/2$. Using this fact, letting $x_1 - x_2 = \Delta x$, and applying Eq. (1) gives the up-and-down calculation of g as

$$g = \frac{8\Delta x}{(t_4 - t_1)^2 - (t_3 - t_2)^2} \tag{4}$$

Sakuma's nonportable instrument also claims precision and accuracy levels of a few μgals.

RELATIVE GRAVITY MEASUREMENTS

Absolute gravity measurements give acceleration values in terms of the basic units of length and time. Such measurements are important to physicists, astronomers, and geodesists determining the dimensions of the earth. For most other purposes, such as oil and mineral prospecting, understanding the earth's deep structure, and inertial navigation gravity compensations; it is the relatively small variation in gravity from point to nearby point that is important. Such variations can be quickly measured with an easy-to-carry instrument called a gravimeter.

The majority of gravimeters in use today balance the gravity force acting on a so-called proof mass suspended from a metallic spring or quartz fiber *against* the force exerted by the spring. Such a gravimeter is illustrated in Fig. 2. At a starting point where the absolute value of gravity is often known, the gravimeter is leveled and the spring tension is adjusted until it is balanced against the gravitational force acting on the proof mass. The gravimeter is then moved to the next point, leveled, and balanced again. The change in spring tension required to bring the gravimeter back into balance is the actual measurement. This is equivalent to the change in the acceleration of gravity between the two points. Repeated rebalancing (nulling) of the instrument between closely spaced points yields two desirable by-products: (1) lower spring hysteresis (inability to return to its original ten-

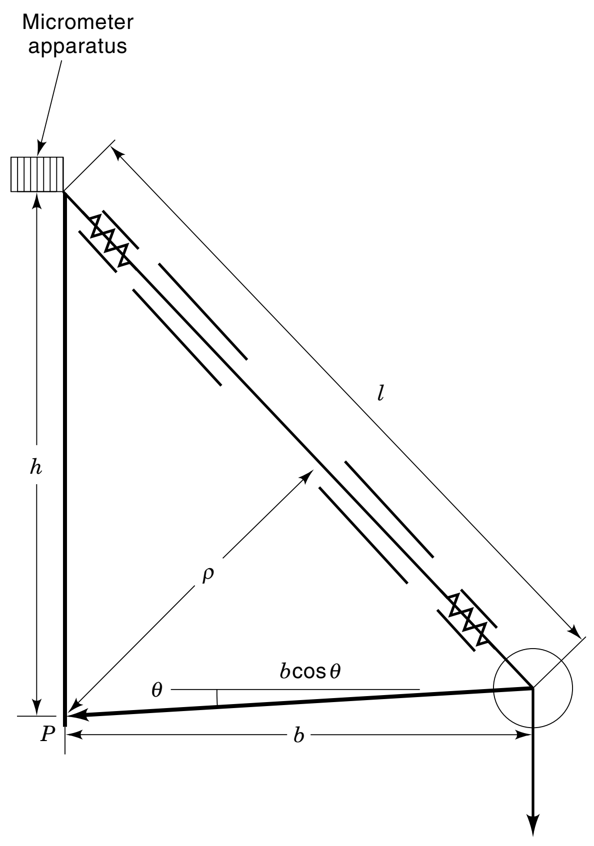

Figure 2. Principles of force balance gravimeter.

sion standard after compensating for a tension change caused by an external gravity change) and (2) the gravimeter's beam (b in Fig. 2) is kept close to a horizontal position that reduces sensitivity to leveling errors.

Principles of Zero-length Spring, Rebalancing Force, Unstable Gravimeters

A spring is said to be zero-length if its tension is proportional to its actual length. Thus, if all external forces were removed, the spring would collapse to zero length. The key advantage of such a spring is that if its tension supports the beam b and mass m in Fig. 2 in the horizontal beam position, it will support them in any position. Referring to Fig. 2 and the above definition, the spring's tension T is given by

$$T = k(l - l_0) \tag{5}$$

where k is the spring constant, l is the spring's length, and l_0 is the very small length (see discussion below) at which the tension is zero. In a state of true equilibrium or balancing of the two forces, the moments of the weight of the mass (mg) and of the spring's tension (T) about the pivot point P are equal. That is

$$\rho T = mgb\cos\theta \tag{6}$$

where the perpendicular distances ρ and b cos θ are shown on Fig. 2, θ being the small angle the beam b makes with the horizontal. Applying trigonometric laws to Fig. 2 and inserting Eq. (5) into Eq. (6) yields

$$l = \sqrt{h^2 + b^2 - 2hb\sin\theta} = \frac{khl_0}{kh - mg} \tag{7}$$

where h is the distance between the beam pivot point P and the spring end attached to some micrometer screw apparatus (see Fig. 2). Since θ is nearly zero, it follows from Eq. (7) that

$$g\frac{d\theta}{dg} = \left(\frac{l - l_0}{l_0}\right)\left(\frac{h}{b} + \frac{b}{h}\right) \tag{8}$$

From Eq. (8) one sees the gravimeter's sensitivity $d\theta/dg$, that is the change of the beam's angle for a given change in gravity, is greater the smaller l_0 can be made. In practice l_0 is made very small (hence the zero-length name) by winding the coils of a helical spring such that the wire is twisted about an axis in its own length as it is wound (4). Such a gravimeter is classified *unstable* (or *astatic*) because it is a moving system that approaches a point of instability where very small changes in gravity produce relatively larger proof mass displacements.

In zero-length gravimeters the spring is attached to one end of the beam near the proof mass. The spring's other end is attached to the micrometer apparatus. By adjusting the micrometer, the force on the main beam is altered such that when a change in gravity occurs, the beam is returned to the same angle with the horizontal. The change in gravity is shown as an arbitrary scale division on the micrometer's dial, which is easily converted to gravity units. Current zero-length spring gravimeters typically detect changes in gravity at the 1 part/10^8 level (0.01 mgal or 10 μgal). This level of sensitivity requires the spring constant k remain fixed at 1 part/10^8 and l_0 be held constant to an even higher degree. Unfortunately both k and l_0 vary with temperature, mainly through the change of the spring's elastic modulus. Therefore gravimeters require a constant temperature environment. This is achieved by housing them in sealed vacuum flasks or in electrically controlled thermostats. Current gravimeter designs also minimize barometric pressure and magnetic effects.

Gravimeter Range, Accuracy, Precision, Calibration, Drift, and Tidal Effects

A single spring constant k value [see Eq. (5)] cannot give high accuracy measurements over the large range between equatorial and polar gravity. Moreover, if the spring is subjected to large differences in g between force rebalances, it suffers from increased hysteresis. Gravimeter calibration determines k for the specific gravity range to be surveyed. Readings at two or more stations where g is already known gives an average value of k over the range of these stations. The station range must be at least the range of the subsequent survey but not larger than the instrument's range. The latter can be between 5 mgal (geophysical prospecting gravimeters) to over 7000 mgal (global geodetic gravimeters). If the range to be surveyed is large, the calibration stations should be widely separated in latitude. Gravimeters can also be calibrated by tilting them to measure variable components of the g vector. This approach is much more time consuming than field calibrations (5).

There is an inevitable slow and regular change in the length of any gravimeter's spring. The rate of change or *drift* can be determined by returning to a local base as often as the desired accuracy requires (typically every 3 to 4 h). Repeated readings at the same base station over several days produces an oscillatory-shaped drift curve due to tidal effects. The latter result from changes in the gravitational attraction of the sun and moon as their positions change with respect to the earth. Depending on the solar and lunar positions, tidal effects can produce changes in gravity as large as 0.3 mgal over a period as short as 6 h (4). Tidal corrections can be calculated from knowledge of the positions of the sun and moon. However, these effects vary smoothly and slowly and they usually make up part of the gravimeter drift correction itself (unless the required accuracy dictates they be removed). The (tide-free) drift-rate of a fixed-site, specially modified, zero-length, LaCoste-Romberg gravimeter has been reduced to approximately 0.05 mgal/month. This instrument has measured relative gravity at precision and accuracy levels of 1 μgal (6). Portable, mass produced geodetic gravimeters weigh approximately 2 kg and come with a heating battery to maintain constant temperature. These devices typically measure gravity differences at precision (repeatability) levels of 5 μgal to 10 μgal.

RESEARCH AND DEVELOPMENT AREAS

Superconducting Gravimeters

Ultrasensitive cryogenic gravimeters (accelerometers) are being developed wherein the mechanical spring is replaced by a magnetic field produced by current flow in superconducting coils. In the Goodkind (7) device, the field supports a small sphere whose position, determined by the balance between the gravity field and the magnetic field, is monitored electronically. The signal-to-noise ratios of these measurements remain high over a very broad frequency range, covering both the tidal spectrum and very-low-frequency seismic bands (as low as 1 cycle/annum). This instrument has determined major tide components to accuracies better than 0.01 μgal. The nontidal signal along very low frequencies is mainly due to atmospheric pressure variations (which can be independently recorded and removed) and the so-called Chandler component of the centrifugal force, a roughly 4 μgal signal having a period of around 435 days. Superconducting gravimeters remain stable to a few μgals over such periods and can lead to a better understanding of the Chandler phenomena. Paik et al. (8,9) are developing a superconducting six-axis accelerometer that senses the three translational and three rotational rigid body motions of a single levitated niobium proof mass. This device constitutes a complete inertial measurement unit that is free of drift-prone gyroscopes. Such an instrument could become the core of a passive, nonjammable, virtually drift-free inertial navigation system of the future.

Accurate Shipborne and Airborne Gravity Measurements

When a gravimeter is placed on a moving ship it measures (along its sensitivity axis) the resultant accelerations due to gravity as well as the ship's roll, pitch, and yaw motions. Assuming a velocity of 10 knots, a shipborne gravimeter is typically subjected to high frequency ($>$ 1 Hz) horizontal and vertical ship vibration accelerations as large as 0.01 g or 10,000 mgal, 0.01 Hz to 1 Hz ship heaves causing vertical accelerations as high as 0.1 g or 100,000 mgal, and 0.001 Hz to 0.01 Hz ship fishtail or swaying motions which mainly produce horizontal accelerations (4). Therefore only long-period mean gravity values (typically corresponding to frequencies $<$ 0.001 Hz) can be precisely and accurately extracted from shipborne gravimeter surveys (using low-pass filtering and averaging techniques over the applicable times and distances). All the motional accelerations cited above can be minimized by placing the gravimeter near the ship's metacenter and mounting it on a shock absorbent, gyroscopically stabilized platform, that is more or less kept level or perpendicular to the average vertical. The stabilized platform also controls cross couplings between the horizontal forces and the highly sensitive vertical response.

In addition to averaging out the ship motion accelerations, one must apply the so-called Eotvos correction to account for the east–west component of the ship's motion. The outward directed centrifugal force component at an earth surface point is given by $R_E\omega^2\cos^2\phi$, where R_E is the earth's radius, ω is its angular velocity, and ϕ is the geodetic latitude. At the equator this acceleration is approximately 3400 mgal. If the gravimeter itself has a velocity then the centrifugal force acting on it will be different than if it is stationary. An eastward component of gravimeter velocity numerically adds to the earth rotation effect. This increases the outward centrifugal force and decreases the gravity reading (dominated by the inward attraction of the earth's matter). A westward component of gravimeter velocity has the opposite effect. For an arbitrary gravimeter velocity vector of magnitude V, making a heading angle α with astronomic (true) north, the Eotvos correction is given by

$$E = 2\omega V\cos\phi\sin\alpha + \frac{V^2}{R_E} \tag{9}$$

An east–west velocity of 20 km/h along the equator produces an Eotvos correction of approximately 80 mgal. From Eq. (9) one sees that dE errors in the computed Eotvos corrections are related to dV velocity and $d\alpha$ heading errors by

$$dE = k_1V\cos\phi\cos\alpha d\alpha + (k_2\cos\phi\sin\alpha + k_3V)\,dV \tag{10}$$

where the k_i are constants. Equation (10) implies an east-west course is most sensitive to velocity errors and a north-south course is most sensitive to heading errors. Therefore, in addition to minimizing the gravimeter's errors caused by the ship's motions and ocean waves, accurate estimates of the ship's positions, velocities, and orientations are required. A shipborne, multiple-antenna (attitude determining) global positioning system (GPS) receiver affordably provides such navigation data. For higher accuracy navigation, GPS data can be integrated with the gyroscopic and accelerometer outputs of an onboard inertial navigation system (INS). The potential of extracting vehicle-borne gravity vectors from integrated GPS/INS data has been examined (10). Therein one pursues the INS's insensitivity to gravity and the GPS's sensitivity to it.

Extraneous accelerations related to high-velocity airborne gravimeter surveys are much more problematic. In particular, large and rapid changes in altitude, linear acceleration, roll, and heading often occur. Even at a low flight speed of 100 km/h, the Eotvos correction in Eq. (9) is around 400 mgal. Thus a 1% error in the estimated speed or a 1 degree error in estimated azimuth introduce gravity errors of approximately 5 mgal. Since gravity decreases by about 1 mgal for every 3 m increase in altitude, and changes in elevation with time produce vertical accelerations, accurate vertical positioning is critical. A GPS or GPS/INS system can potentially meet these navigation requirements. Significant error sources coming from the aircraft's vertical and horizontal accelerations can be further minimized by using gyroscopically stabilized or gimbal suspended gravimeters, flying during periods of low atmospheric turbulence, and increasing the averaging time over which measurements are made (which reduces the resolution of the final mean gravity values). Brozena et al. (11) claimed 2 mgal to 3 mgal accuracies averaged over 20 km.

Gravitational Field Sensors

Jekeli (12) gives a semantically engaging explanation on how a single gravimeter *reacts* to the earth's centrifugal force and *applied* forces such as the one opposite and equal in magnitude to the gravitational attraction of the earth's matter; tidal accelerations caused by the sun, moon, and planets; atmospheric pressures; and any host vehicle accelerations. A gravimeter is insensitive however to a gravitational field (be it the earth's, sun's, moon's, etc.). A gravity gradiometer is a gravitational field sensor. The simplest gradiometer is made up of two closely spaced gravimeters (baseline lengths typically 15 cm to 30 cm) with sensitivity axes pointing in the same (or opposite) directions. By *differencing* (or adding) the two gravimeter outputs, the external forces cancel (at least to a first degree) and one is left *sensing* a gradient of the total gravitation of the solar system along the baseline. The earth's gravitational field clearly dominates those of other bodies and the measured gradient includes a centrifugal force effect. Short-baseline gravity gradient signals are very weak and reflect the higher spatial frequency components of the earth's gravitational spectrum. Engineering challenges abound in the design, fabrication, calibration and mobile operation of ultrasensitive, short-baseline gravity gradiometers. For an overview of these challenges see Jekeli (13). An omnidirectional gradiometer (or full-tensor gradiometer since it measures the full second-order tensor of the gravitational potential scalar) is an elaborate array of 12 or more gravimeters. Gravity gradient measurements interest oil and mineral prospectors and their line integrals can become real-time gravity vector compensations applied to an accompanying inertial navigation system.

Another type of gravitational field sensor is a satellite-to-satellite electromagnetic ranging device. Like gravity gradients, intersatellite range-rate changes (or line-of-sight accelerations) are functionally related to the difference of the gravity vectors at the two satellite positions. Since satellites are in free-fall and are usually separated by significant distances, onboard gravimeters (accelerometers) can measure the nongravitational forces present at each satellite position (such as atmospheric drag and solar radiation pressure). These external effects can then be removed from the Doppler based satellite-to-satellite range-rate measurement.

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GRAVITATIONAL SPECTRA. See GRAVIMETERS.
GREENHOUSE GASES. See AIR POLLUTION CONTROL.